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Author(s)	河井, 達治
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Description	Supervisor: Hajime Ishihara, 情報科学研究科, 修士

On Basic Structures of General Topology in Constructive Mathematics

Tatsuji Kawai (1010018)

School of Information Science,
Japan Advanced Institute of Science and Technology

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This thesis describes two notions of general topology in constructive mathematics, basic pair and concrete space, in the framework of Constructive Zermelo-Frankel Set Theory (CZF). As well as summarizing some of the existing results on these relatively new notions, we make contributions toward clarifying the structure of the categories of basic pairs (**BP**) and concrete spaces (**CSpa**).

Our primary contribution is in showing that the categories **BP** and **CSpa** are both complete and cocomplete as in the case of the classical category of topological spaces. Working in the extension of CZF, we showed that both categories have arbitrary (co)products and (co)equalisers for any parallel pair of arrows and hence are complete and cocomplete. The distinguishing feature of our construction is a uniform application of the notion of a generalized geometric theory to deal with predicativity problems which typically arise in predicative foundations such as CZF. Completeness and cocompleteness of both categories suggests that these categories have rich structures, allowing us to carry out the construction of product and quotient spaces which are familiar in the setting of classical general topology.

Our secondary contribution is in showing that **CSpa** is a coreflective subcategory of **BP**. The result partly clarified the relation between two categories which has been unknown.