Turbo Equalization: Fundamentals, Information Theoretic Considerations, and Extensions

A Tutorial on IEEE VTC-Spring 2012

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Part II
Chained Turbo Equalization (CHATUE) for Block Transmission without Guard Interval - Application to Uplink SC-FDMA -

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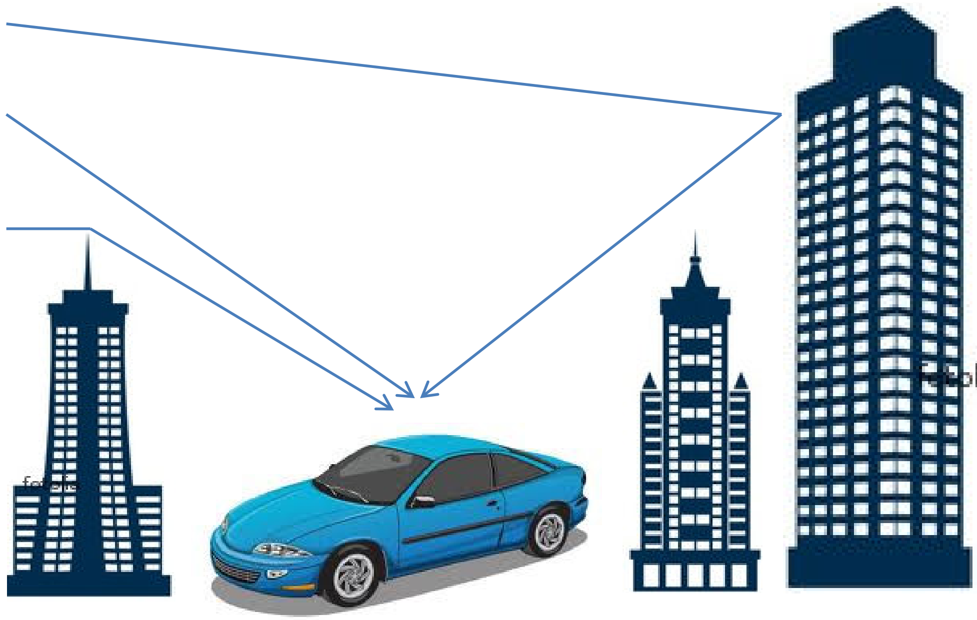
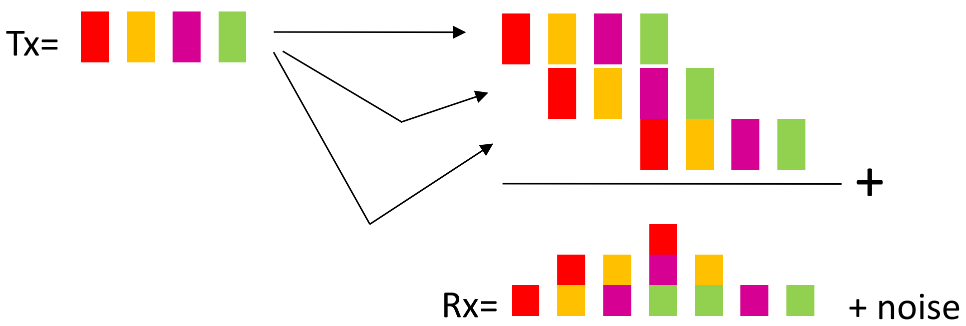
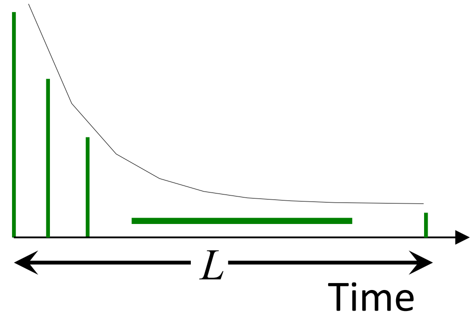
Outline of Presentation

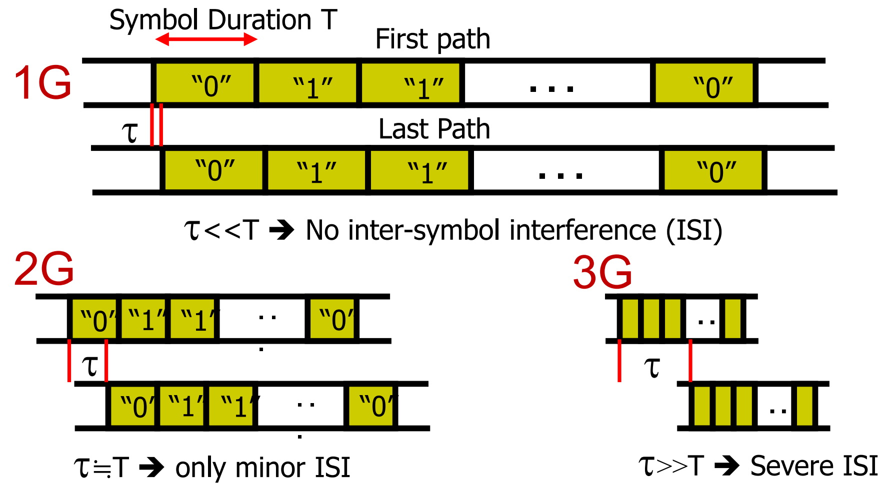
1. Motivations
2. Basic Principle
   1. System Model
   2. The Concept of CHATUE Algorithm
   3. Performance Evaluation
3. Applications
   1. Uplink SC-FDMA
   2. Mathematical Formulation
   3. Performance Evaluation
4. Conclusions

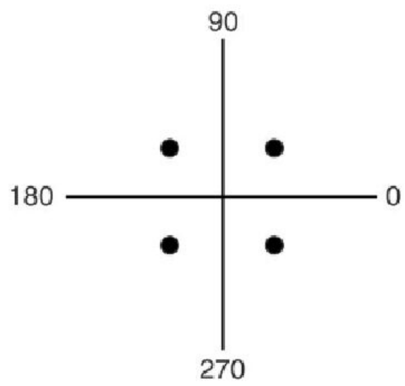
General Problem of Wireless Communications

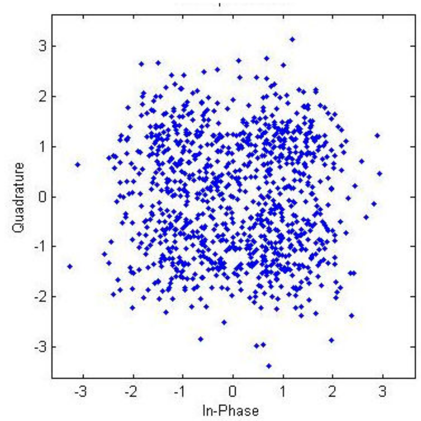




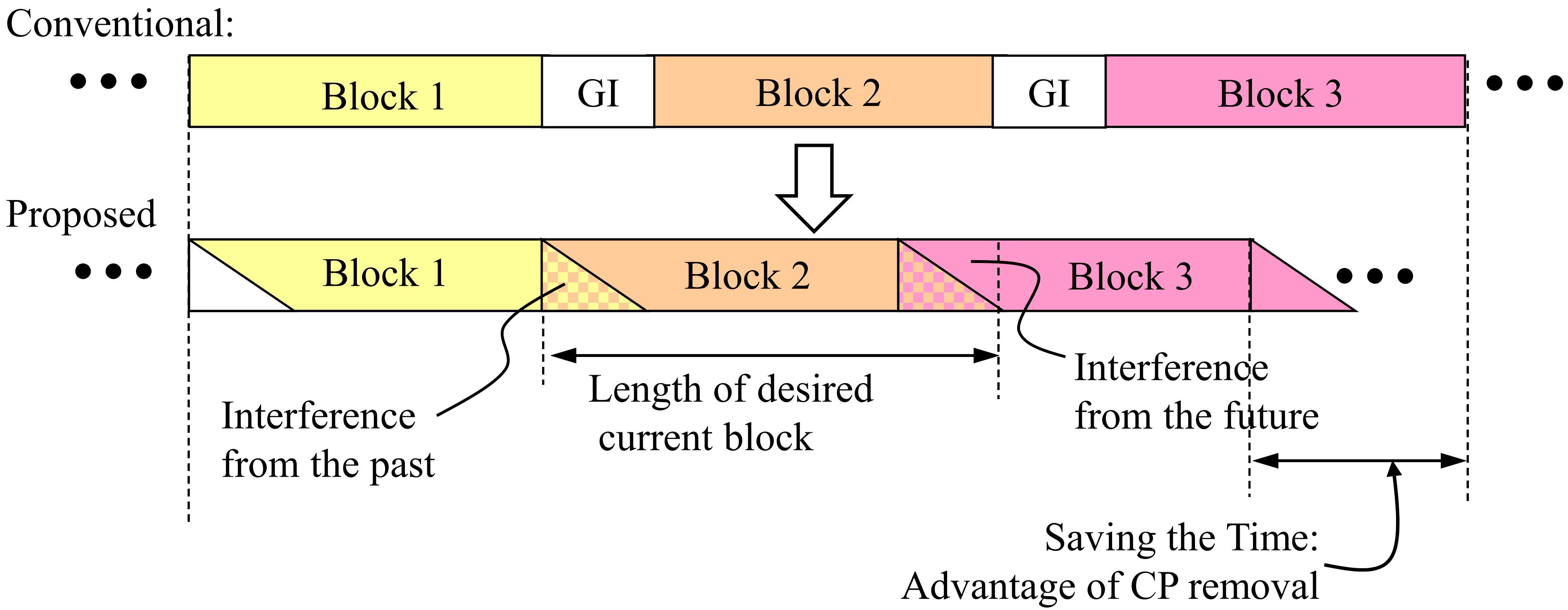






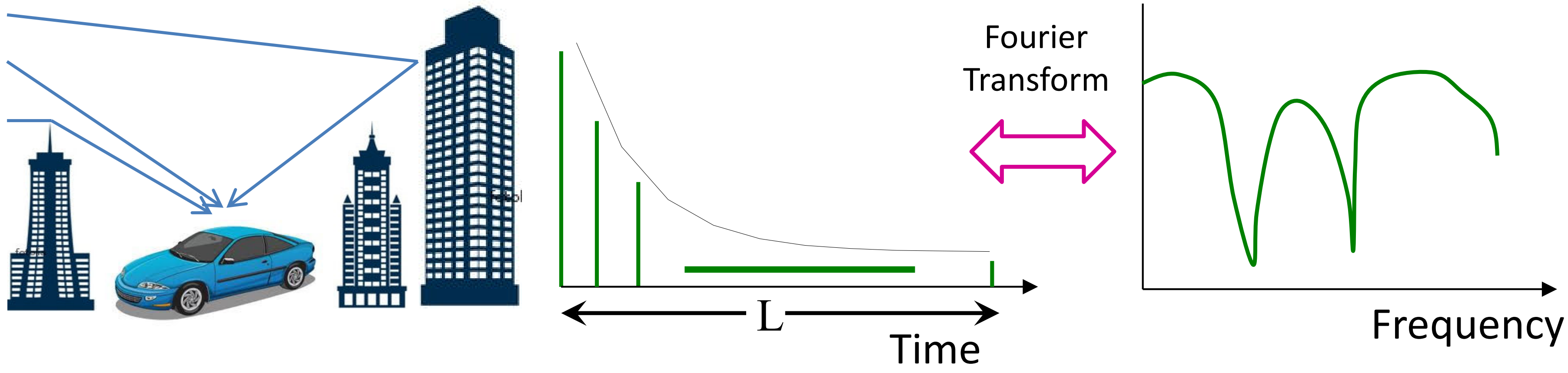


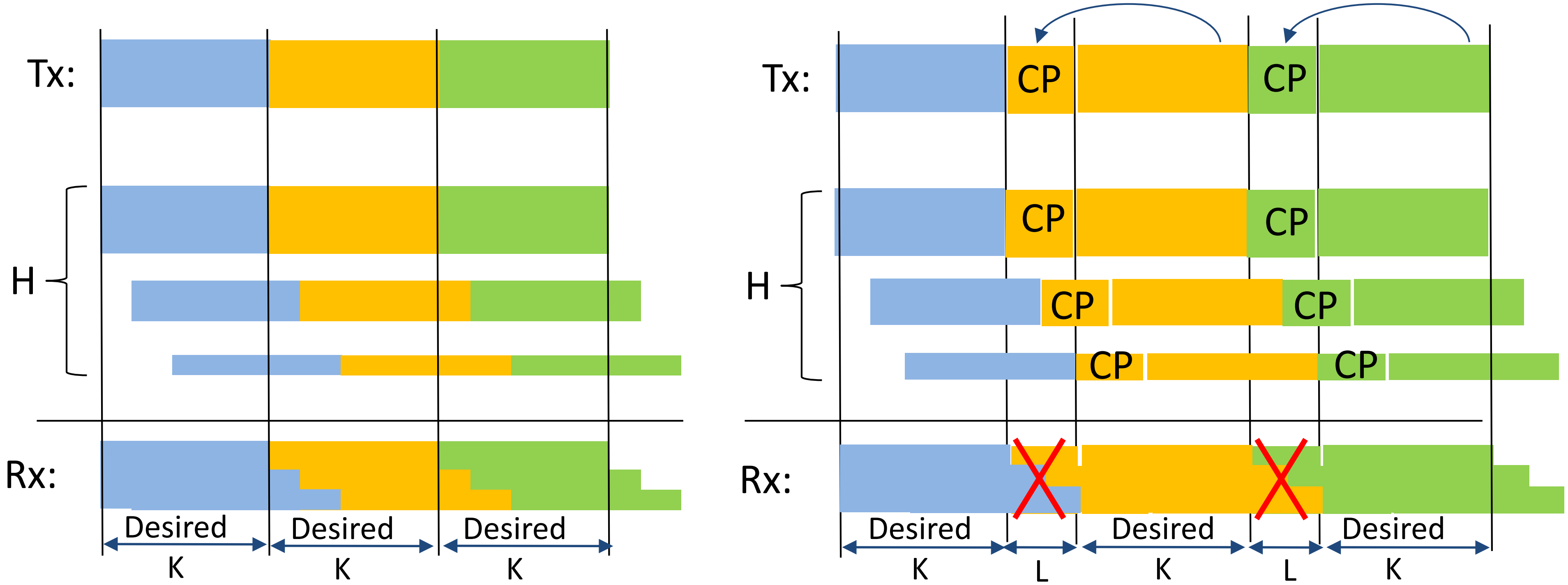
Motivation



- Normal Guard Interval (GI) (cyclic prefix): 4.69 μs (Cover 1.4km)
- LTE-Advanced SC-FDMA symbol length= 66.7 μs
- Data rate loss 4.69/66.7=7.03%
- GSM: 3.69 μs
- W-CDMA : 0.69 μs

The Standard Technique

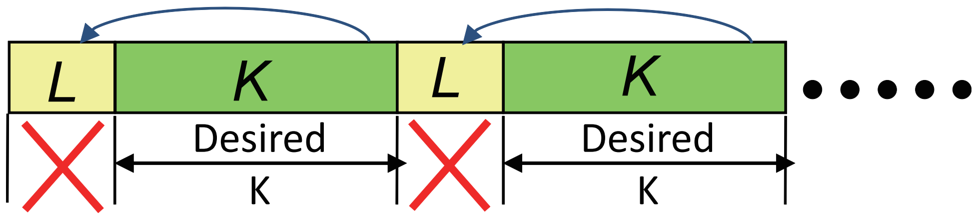




CP: Cyclic Prefix

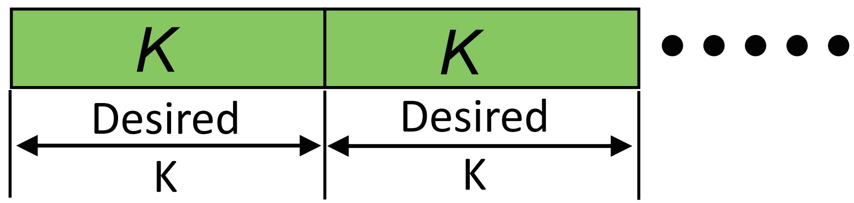
The Benefit of Guard Interval Removal

With Guard Interval:



- Rate Loss: YES
  $$\frac{S}{N} = \frac{E_b}{N_0} \cdot R \cdot \frac{K}{(K+L)} \cdot M$$
- Power Loss: YES
  $$R = \frac{S}{N} \cdot \frac{N_0}{E_b} \cdot \frac{(K+L)}{K} \cdot \frac{1}{M}$$

Without Guard Interval:



- Rate Loss: NO
  $$\frac{S}{N} = \frac{E_b}{N_0} \cdot R \cdot \frac{K}{(K+0)} \cdot M$$
- Power Loss: NO
  $$R = \frac{S}{N} \cdot \frac{N_0}{E_b} \cdot \frac{(K+0)}{K} \cdot \frac{1}{M}$$

Notation:
$\frac{S}{N}$: Signal-to-Noise Power Ratio, R: Coding rate, $\frac{E_b}{N_0}$: Energy bit per noise, M: Number of bits per symbol, K: Block length, L: GI length

CHATUE Algorithm: The Basic Principle

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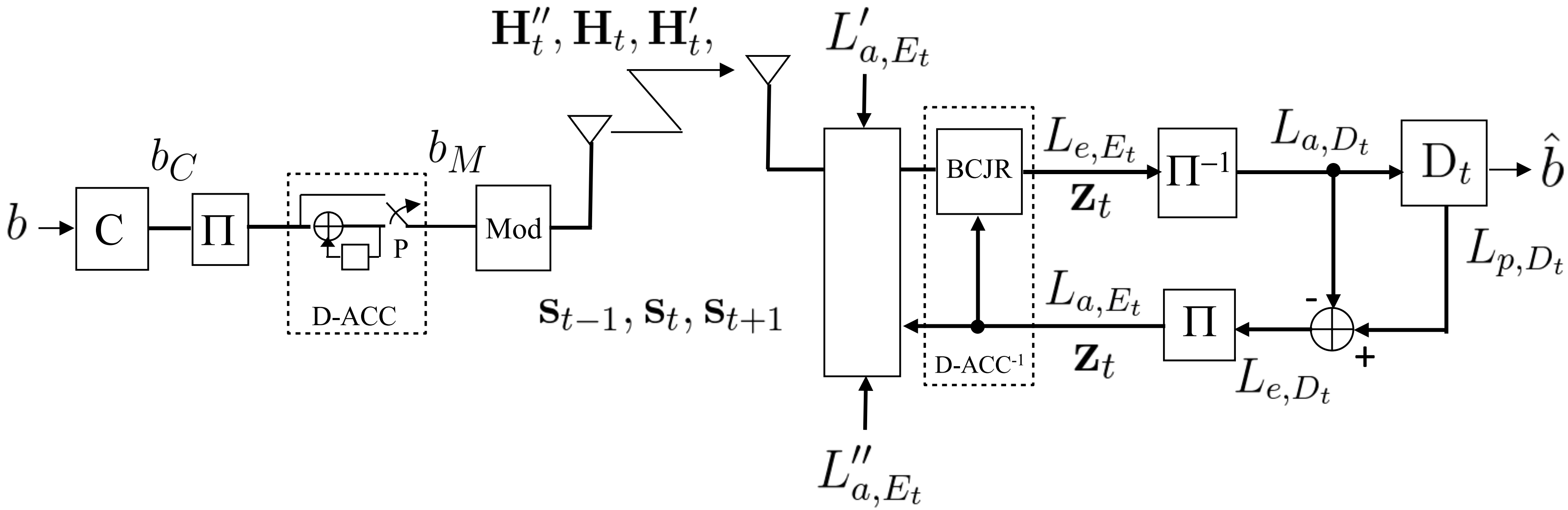
References (Suggested for Further Reading):

1 K. Anwar, H. Zhou, and T. Matsumoto, "Chained Turbo Equalization for Block Transmission without Guard Interval", 2010 IEEE 71st Vehicular Technology Conference (VTC 2010-Spring), pp.1-5, May 2010, Taiwan.

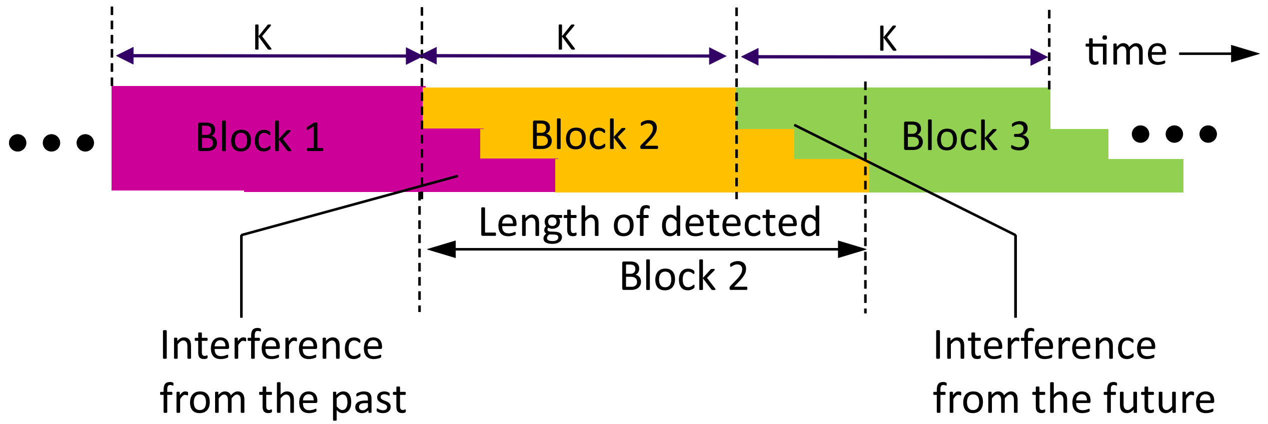
2 K. Anwar and T. Matsumoto, "Low Complexity Time-Concatenated Turbo Equalization for Block Transmission without Guard Interval: Part 1– The Concept", *Wireless Pers. Commun.*, Springer, DOI: 10.1007/s11277-012-0563-0 (Online: 24 March 2012).

3 K. Kansanen and T. Matsumoto, "An Analytical Method for MMSE MIMO Turbo Equalizer EXIT Chart Computation", *IEEE Transaction on Wireless Communications,* Vol. 6, No. 1, pp. 59–63.



$$\mathbf{s}_t = [s_t^{[0]}, s_t^{[1]}, \cdots, s_t^{[k]} \cdots s_t^{[K-1]}]^T \in \mathbb{C}^{K\times 1}. \quad (1)$$

$$\mathbf{y}_t = \mathbf{H}_t\mathbf{s}_t + \mathbf{H}'_{t-1}\mathbf{s}'_{t-1} + \mathbf{H}''_{t+1}\mathbf{s}''_{t+1} + \mathbf{n} \in \mathbb{C}^{(K+L-1)\times 1}, \quad (2)$$

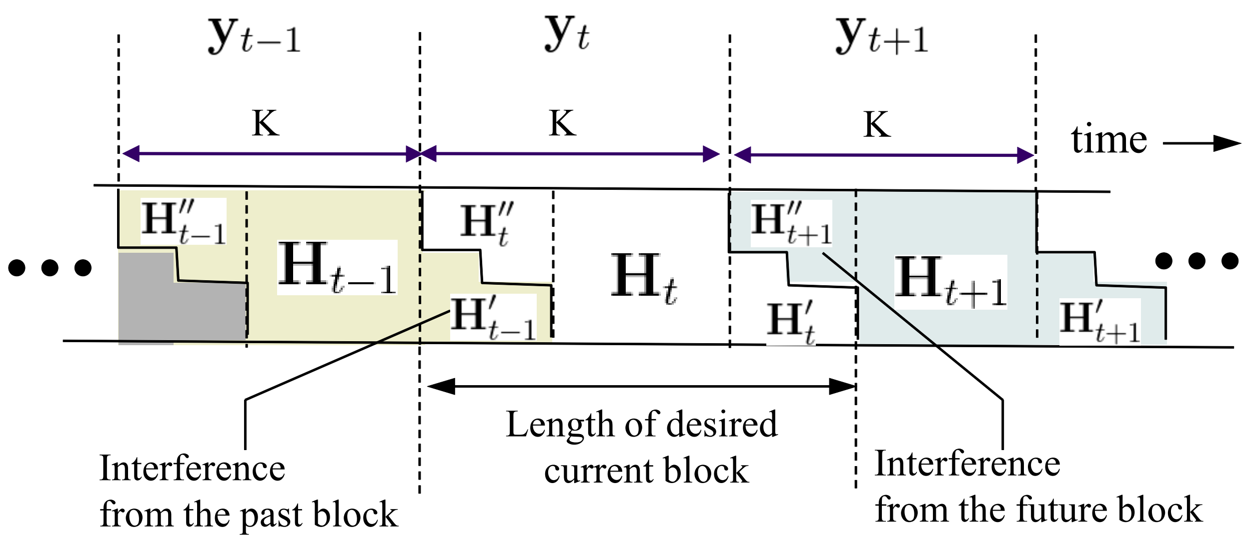


Channel Model

$$\mathbf{H}_t = \begin{bmatrix} h_0^{[0]} & & & 0 \\ \vdots & h_0^{[1]} & & \\ h_{L-1}^{[0]} & \vdots & \ddots & \\ & h_{L-1}^{[1]} & \vdots & h_0^{[K-1]} \\ & & \ddots & \vdots \\ 0 & & & h_{L-1}^{[K-1]} \end{bmatrix} \in \mathbb{C}^{(K+L-1)\times K}, \quad (3)$$

$$\mathbf{H}'_{t-1} = \begin{bmatrix} & h_{L-1}^{[K-L+1]} & \cdots & h_1^{[K-1]} \\ & & \ddots & \vdots \\ & & & h_{L-1}^{[K-1]} \\ & & & \\ 0 & & & \end{bmatrix}, \; \mathbf{H}''_{t+1} = \begin{bmatrix} & & & 0 \\ & & & \\ h_0^{[0]} & & & \\ \vdots & \ddots & & \\ h_{L-2}^{[0]} & \cdots & h_0^{[L-2]} & \end{bmatrix}$$

Avoiding the Confusion on the Channel Models



$\mathbf{H}_{t-1}$: A Past channel matrix

$\mathbf{H}'_{t-1}$: A Past channel matrix with the past form

$\mathbf{H}''_{t-1}$: A Past channel matrix with the future form

$\mathbf{H}_t$: A Current channel matrix

$\mathbf{H}'_t$: A Current channel matrix with the past form

$\mathbf{H}''_t$: A Current channel matrix with the future form

Channel Model: Examples

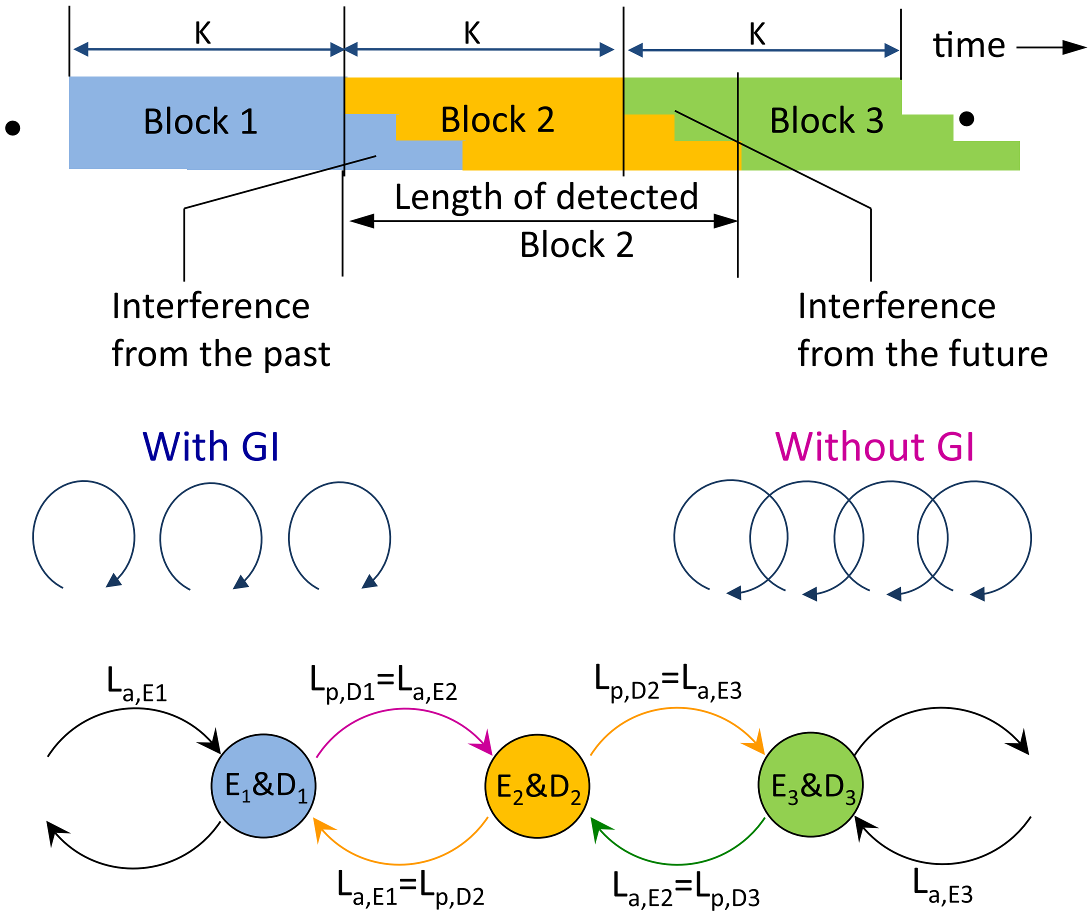
Given the channel responses of the past, the current, and the future blocks, respectively, as

$$\mathbf{h}_{t-1} = [1, 0.7], \mathbf{h}_t = [1, 0.5], \mathbf{h}_{t+1} = [1, 0.3],$$

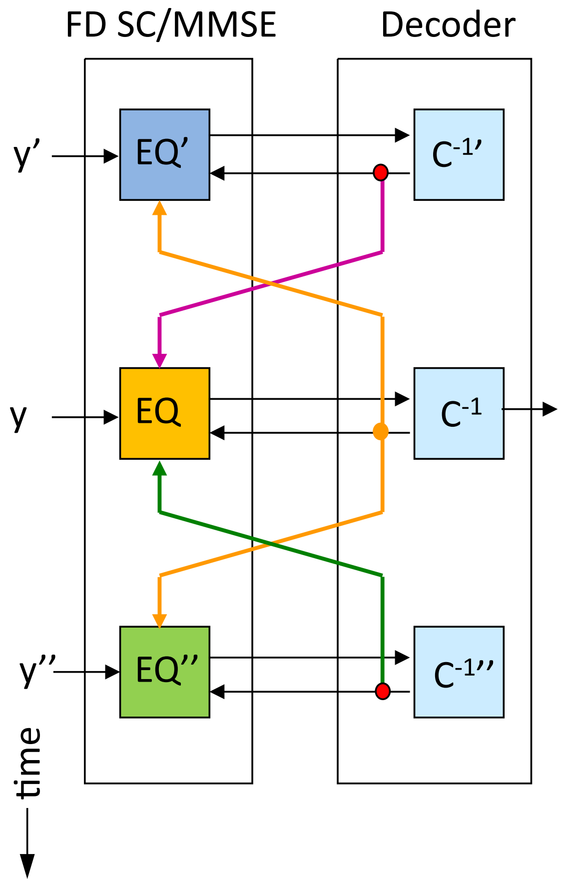
and block length $K = 4$, write the channels of:

1. $\mathbf{H}'_{t-1}$
2. $\mathbf{H}''_{t}$
3. $\mathbf{H}''_{t+1}$
4. $\mathbf{H}'_{t}$

Solution by the CHATUE Algorithm



Chained Equalization



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[1] [1] K. Anwar, Z. Hui, and T. Matsumoto, "Chained Turbo Equalization for Block Transmission without Guard Interval", in *IEEE VTC-Spring 2010*, Taiwan, May 2010.

[2] [2] K. Anwar and T. Matsumoto, "Low-complexity Time-concatenated Turbo Equalization for Block Transmission: Part 1 – The Concept", *Wireless Personal Comm., Springer*, March 2012 (DOI 10.1007/s11277-012-0563-0).

Two Key Principles

1. Retrieval of Circularity: Matrix $\mathbf{J}$

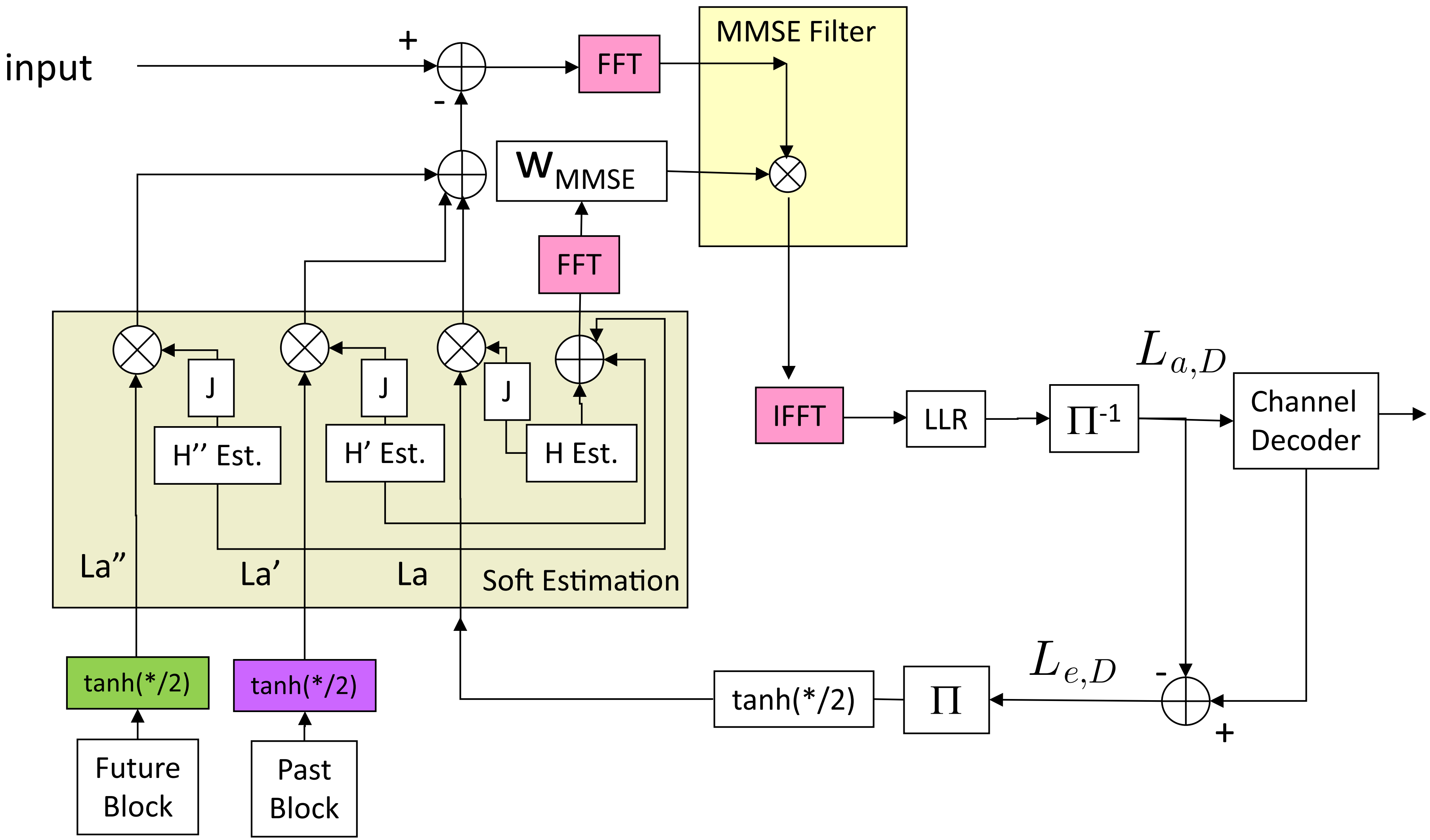
$$\mathbf{J} = \left[\begin{array}{c|c} 0_{(K-L+1)\times(L-1)} & \mathbf{I}_{K\times K} \\ \mathbf{I}_{(L-1)\times(L-1)} & \end{array}\right] \in \mathbb{C}^{K\times(K+L-1)} \tag{4}$$

2. ISI and IBI Removal: Modified FD/SC-MMSE

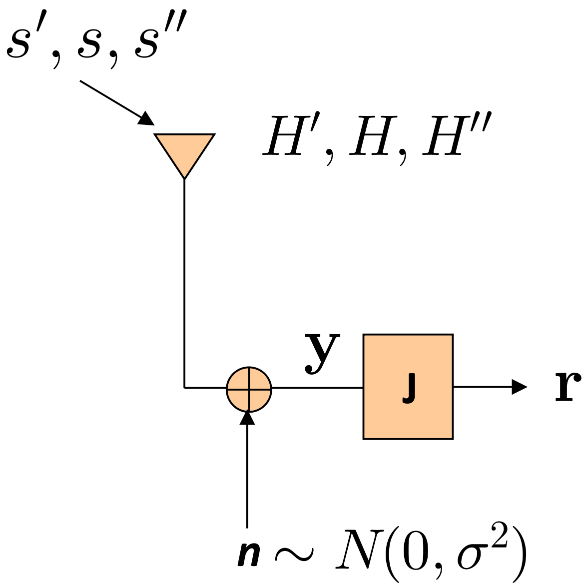
Example of Matrix $\mathbf{J}$ with $L = 3, K = 3$ and $h_t = [h_0, h_1, h_2]$

$$\mathbf{J} = \left[\begin{array}{cc|ccc} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right], \quad \mathbf{JH}_t = \begin{bmatrix} h_2 & h_1 & h_0 \\ h_0 & h_2 & h_1 \\ h_1 & h_0 & h_2 \end{bmatrix} \tag{5}$$

Modified SC-MMSE for CHATUE: ISI and IBI Removal



Soft Canceller MMSE for Chained Equalization



- Receive signal

$$\mathbf{y}_t = \mathbf{H}_t \mathbf{s}_t + \mathbf{H}'_{t-1} \mathbf{s}'_{t-1} + \mathbf{H}''_{t+1} \mathbf{s}''_{t+1} + \mathbf{n} \quad (6)$$

- Restore the circularity of the channel

$$\begin{aligned} \mathbf{r}_t &= \mathbf{J}\mathbf{y}, \\ &= \mathbf{J}\mathbf{H}_t \mathbf{s}_t + \mathbf{J}\mathbf{H}'_{t-1} \mathbf{s}'_{t-1} + \mathbf{J}\mathbf{H}''_{t+1} \mathbf{s}''_{t+1} + \mathbf{J}\mathbf{n} \quad (7) \end{aligned}$$

- Soft Replica

$$\hat{\mathbf{r}}_t = \mathbf{J}\mathbf{H}_t \hat{\mathbf{s}}_t + \mathbf{J}\mathbf{H}'_{t-1} \hat{\mathbf{s}}'_{t-1} + \mathbf{J}\mathbf{H}''_{t+1} \hat{\mathbf{s}}''_{t+1} \quad (8)$$

- Cancellation $\tilde{\mathbf{r}}_t = \mathbf{r}_t - \hat{\mathbf{r}}_t$. To simplify the expression, subscribe notation $_t$ may be removed.)
- Minimize the error:

$$w(k) = \arg \min_{w(k)^H} |w(k)^H [\tilde{r} + \mathbf{h}(k)\hat{s}(k)] - s(k)|^2 \quad (9)$$

with $\mathbf{h}(k)$ is the k-th column of matrix $\mathbf{JH}$.

Modified SC-MMSE for CHATUE: Time Domain

- The Wiener-Hopft Solution

$$\mathbb{E}\left[\frac{\partial|w(k)^H[\tilde{r}+\mathbf{h}(k)\hat{s}(k)]-s(k)|^2}{\partial w(k)^H}\right]=0 \qquad (10)$$

- MMSE Weight

$$\begin{aligned} w(k) &= \left(\mathbb{E}[\tilde{\mathbf{r}}\tilde{\mathbf{r}}^H+\mathbf{h}(k)|\hat{\mathbf{s}}(k)|^2\mathbf{h}(k)^H\right)^{-1}\mathbf{h}(k), \\ &= \left(\mathbf{J}\mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^H\mathbf{J}^H+\mathbf{J}\mathbf{H}'\boldsymbol{\Lambda}'\mathbf{H}'^H\mathbf{J}^H+\mathbf{J}\mathbf{H}''\boldsymbol{\Lambda}''\mathbf{H}''^H\mathbf{J}^H\right. \\ &\quad \left.\sigma^2\mathbf{J}\mathbf{J}^H+\mathbf{h}(k)|\hat{\mathbf{s}}(k)|^2\mathbf{h}(k)^H\right)^{-1}\mathbf{h}(k), \\ &= \left(\boldsymbol{\Sigma}+\mathbf{h}(k)|\hat{\mathbf{s}}(k)|^2\mathbf{h}(k)^H\right)^{-1}\mathbf{h}(k) \qquad (11) \end{aligned}$$

where

$$\boldsymbol{\Lambda} = \mathrm{diag}\left(1-|\hat{\mathbf{s}}(0)|^2, 1-|\hat{\mathbf{s}}(1)|^2, \cdots, 1-|\hat{\mathbf{s}}(K-1)|^2\right), \qquad (12)$$

$$\boldsymbol{\Lambda}' = \mathrm{diag}\left(0,\cdots,0, 1-|\hat{\mathbf{s}}(L-1)|^2, \cdots, 1-|\hat{\mathbf{s}}(-1)|^2\right), \qquad (13)$$

$$\boldsymbol{\Lambda}'' = \mathrm{diag}\left(1-|\hat{\mathbf{s}}(K)|^2, \cdots, 1-|\hat{\mathbf{s}}(K+L-1)|^2, 0,\cdots,0\right) \qquad (14)$$

Output of CHATUE SC/MMSE

$$\mathbf{w}(k)^H = \mathbf{h}(k)^H \left[\boldsymbol{\Sigma} + \mathbf{h}(k)|\hat{s}(k)|^2\mathbf{h}(k)^H\right]^{-1},$$
$$\stackrel{(a)}{=} \left(1+\gamma(k)|\hat{s}(k)|^2\right)^{-1}\mathbf{h}(k)^H\boldsymbol{\Sigma}^{-1} \quad (15)$$

Therefore, time domain final output:

$$\mathbf{z}(k) = \left(1+\gamma(k)|\hat{s}(k)|^2\right)^{-1}\mathbf{h}(k)^H\boldsymbol{\Sigma}^{-1}\left(\tilde{r}(k)+\mathbf{h}(k)\hat{s}(k)\right) \quad (16)$$

By performing block-wise processing, symbol wise inversion is not required:

$$\mathbf{z} = \left(\mathbf{I}_K + \boldsymbol{\Gamma}\mathbf{S}\right)^{-1}\left(\boldsymbol{\Gamma}\hat{\mathbf{s}} + \mathbf{H}^H\mathbf{J}^H\boldsymbol{\Sigma}\tilde{\mathbf{r}}\right), \quad (17)$$
$$\mathbf{S} = \mathrm{diag}\left[|\hat{\mathbf{s}}|^2\right], \quad (18)$$
$$\boldsymbol{\Gamma} = \mathrm{diag}\Big[\mathbf{H}^H\mathbf{J}^H\Big(\mathbf{J}\mathbf{H}\boldsymbol{\Lambda}(\mathbf{J}\mathbf{H})^H + \mathbf{J}\mathbf{H}'\boldsymbol{\Lambda}'(\mathbf{J}\mathbf{H}')^H + \mathbf{J}\mathbf{H}''\boldsymbol{\Lambda}''(\mathbf{J}\mathbf{H}'')^H + \sigma^2\mathbf{J}\mathbf{J}^H\Big)^{-1}\mathbf{J}\mathbf{H}\Big] \quad (19)$$

Note: (a). $\gamma(k) = \mathbf{h}(k)^H\boldsymbol{\Sigma}^{-1}\mathbf{h}(k)$

Output of Block-Wise Processing CHATUE

$$\mathbf{\Sigma} = \mathbf{J}\mathbf{H}\mathbf{\Lambda}\mathbf{H}^H\mathbf{J}^H + \mathbf{J}\mathbf{H}'\mathbf{\Lambda}'\mathbf{H}'^H\mathbf{J}^H + \mathbf{J}\mathbf{H}''\mathbf{\Lambda}''\mathbf{H}''^H\mathbf{J}^H, \quad (20)$$

$$\mathbf{\Gamma} = \text{diag}\left[\mathbf{H}^H\mathbf{J}^H\mathbf{\Sigma}^{-1}\mathbf{J}\mathbf{H}\right], \quad (21)$$

$$\mathbf{S} = \text{diag}(|\hat{\mathbf{s}}|^2) \quad (22)$$

Lemma: $\mathbf{J}\mathbf{H} = \mathbf{F}^H\mathbf{\Phi}\mathbf{F} \rightarrow \mathbf{H}^H\mathbf{J}^H = \mathbf{F}^H\mathbf{\Phi}\mathbf{F}$

$$\mathbf{\Sigma} = \mathbf{F}^H\mathbf{\Phi}\mathbf{F}\mathbf{\Lambda}\mathbf{F}^H\mathbf{\Phi}^H\mathbf{F} + \mathbf{J}\mathbf{H}'\mathbf{\Lambda}'\mathbf{H}'^H\mathbf{J}^H + \mathbf{J}\mathbf{H}''\mathbf{\Lambda}''\mathbf{H}''^H\mathbf{J}^H, \quad (23)$$

$$\mathbf{\Gamma} = \text{diag}\left[\mathbf{F}^H\mathbf{\Phi}^H\mathbf{F}\mathbf{\Sigma}^{-1}\mathbf{F}^H\mathbf{\Phi}\mathbf{F}\right] \overset{(a)}{=} \text{diag}\left[\mathbf{F}^H\mathbf{\Phi}^H\mathbf{X}^{-1}\mathbf{\Phi}\mathbf{F}\right] \quad (24)$$

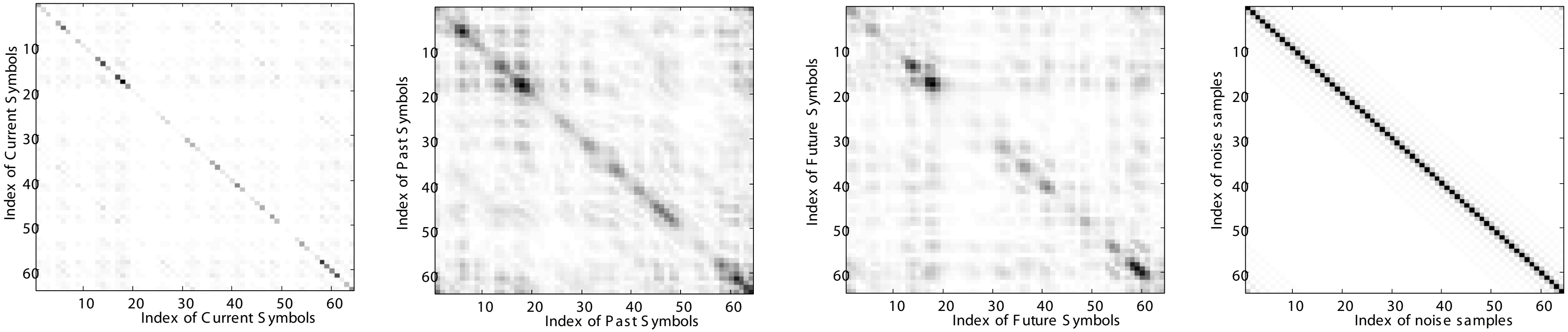
Finally, output of $\mathbf{z}$:

$$\mathbf{z} = \left[\mathbf{I}_K + \mathbf{\Gamma}\mathbf{s}\right]^{-1}\left[\mathbf{\Gamma}\hat{\mathbf{s}} + \mathbf{H}^H\mathbf{J}^H\mathbf{\Sigma}^{-1}\tilde{\mathbf{r}}\right],$$
$$\overset{(b)}{=} \left[\mathbf{I}_K + \mathbf{\Gamma}\mathbf{s}\right]^{-1}\left[\mathbf{\Gamma}\hat{\mathbf{s}} + \mathbf{F}^H\mathbf{\Phi}^H\mathbf{X}^{-1}\mathbf{F}\tilde{\mathbf{r}}\right] \quad (25)$$

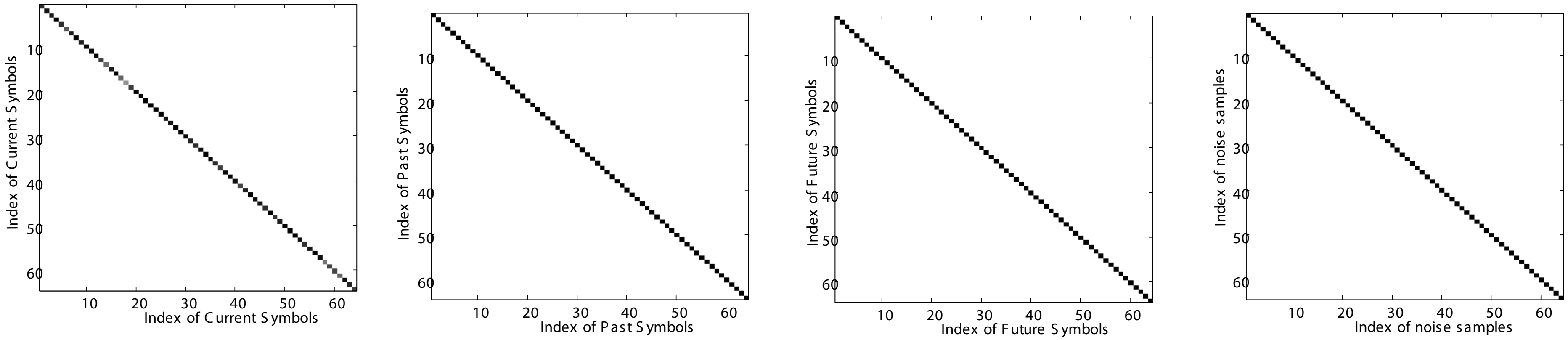
Note: (a). $\mathbf{X} = \mathbf{F}\mathbf{\Sigma}\mathbf{F}^H$, (b). $\mathbf{X}^{-1}$ still require simplification.

Approximation to Diagonal

$$X = \Phi F \Lambda F^H \Phi^H + FJH'\Lambda'H'^H J^H F^H + FJH''\Lambda''H''^H J^H F^H + F\sigma^2 JJ^H F^H$$



$$X \approx \Phi \Lambda \Phi^H + \frac{1}{K}\mathrm{tr}(JH'\Lambda'H'^H J^H)I + \frac{1}{K}\mathrm{tr}(JH''\Lambda''H''^H J^H)I + \frac{1}{K}\mathrm{tr}(\sigma^2 JJ^H)I$$



Note: The mutual information (MI) of the past, the current, and the future blocks are $I'_{a,E_t} = I_{a,E_t} = I''_{a,E_t} = 0.5$.

Extrinsic LLR Formulation

$\mathbf{z}_t$ can be expressed as being equivalent to a Gaussian channel output as,

$$\mathbf{z}_t = \mu_t \mathbf{s}_t + \mathbf{v}_t \in \mathbb{C}^{K\times 1} \quad (26)$$

$$\mu_t = \mathrm{E}[\mathbf{z}_t \cdot \mathbf{s}_t^*] = \frac{1}{K}\mathrm{tr}\{\mathbf{\Gamma}(\mathbf{I}_K + \mathbf{\Gamma}\mathbf{S}_t)^{-1}\}, \quad (27)$$

where $\mathbf{v}_t$ is the equivalent noise vector with variance being $\sigma_t^2 = \mu_t(1-\mu_t)$. In (27), we used the approximation,

$$\mathbf{S}_t = \mathrm{diag}\{|\hat{\mathbf{s}}_t|^2\} \approx \frac{1}{K}\sum_{k=1}^{K} |\hat{s}_t^{[k]}|^2 \cdot \mathbf{I}_K \in \mathbb{C}^{K\times K}. \quad (28)$$

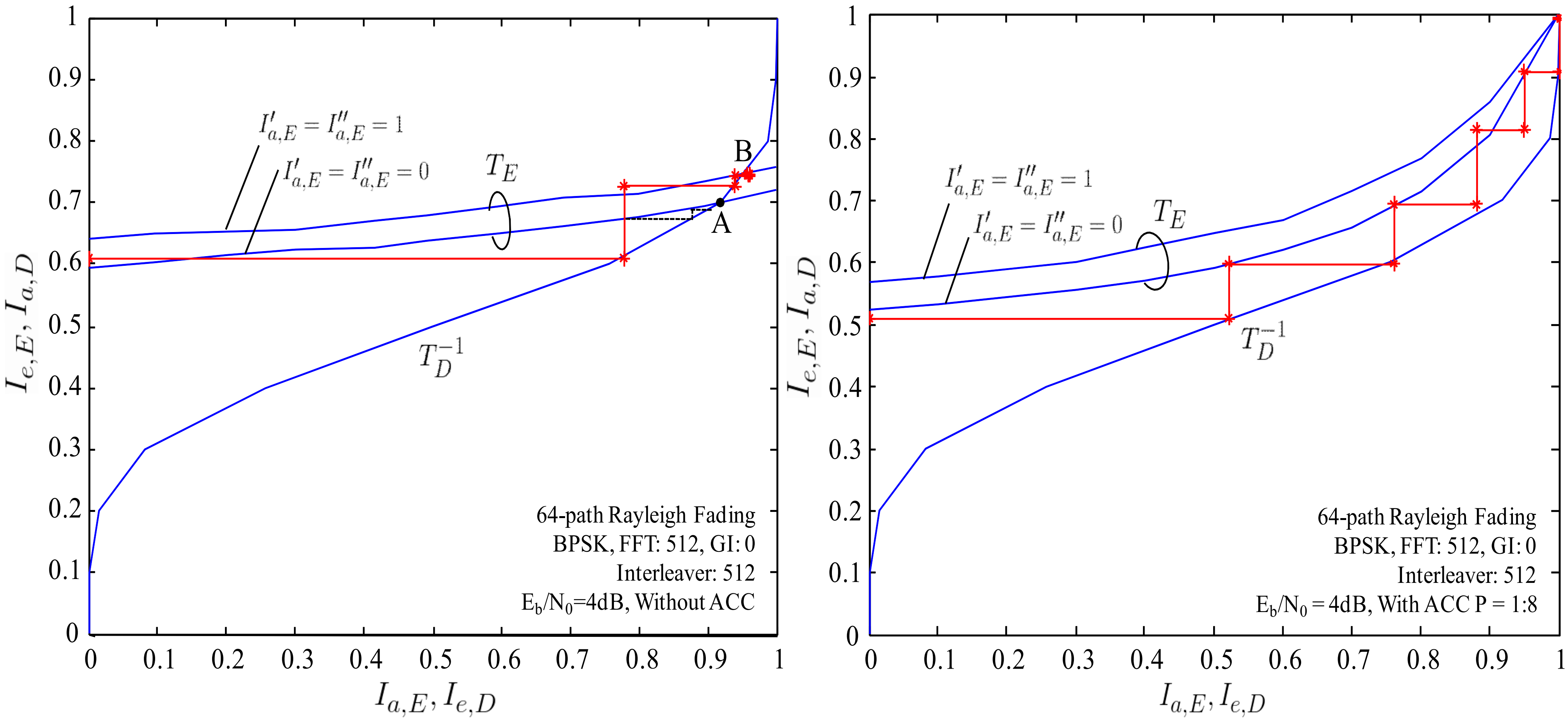
Finally, the extrinsic LLR of the transmitted binary symbol is

$$L_{e,E_t}[s_t^{[k]}] = \ln \frac{\Pr(z_t^{[k]}|s_t^{[k]} = +1)}{\Pr(z_t^{[k]}|s_t^{[k]} = -1)} = \frac{4\Re(z_t^{[k]})}{1-\mu_t}, \quad (29)$$

with $z_t^{[k]}$ being the k-th component of $\mathbf{z}_t$ and $\Re(z_t^{[k]})$ denoting the real part of the complex $z_t^{[k]}$.

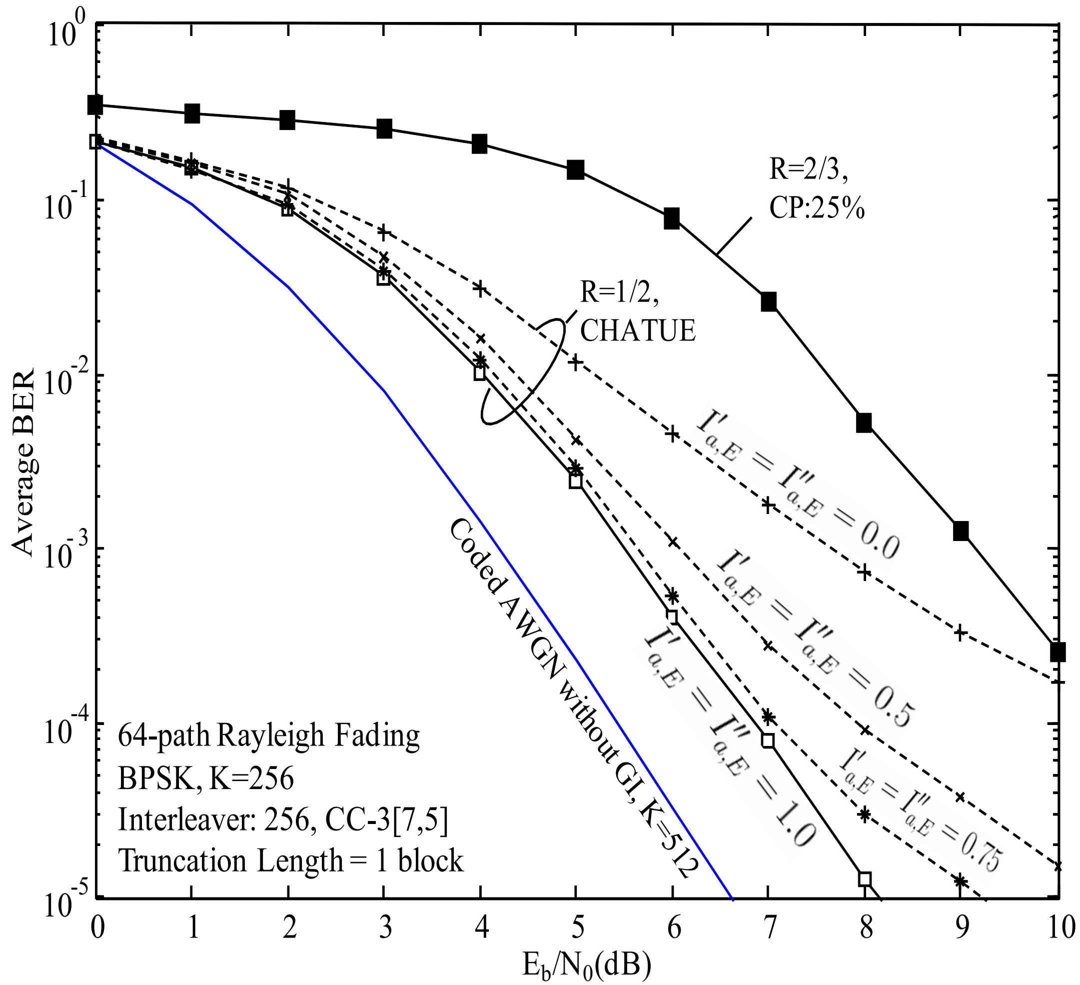
EXIT Chart Analysis

$E_b/N_0 = 4$ dB



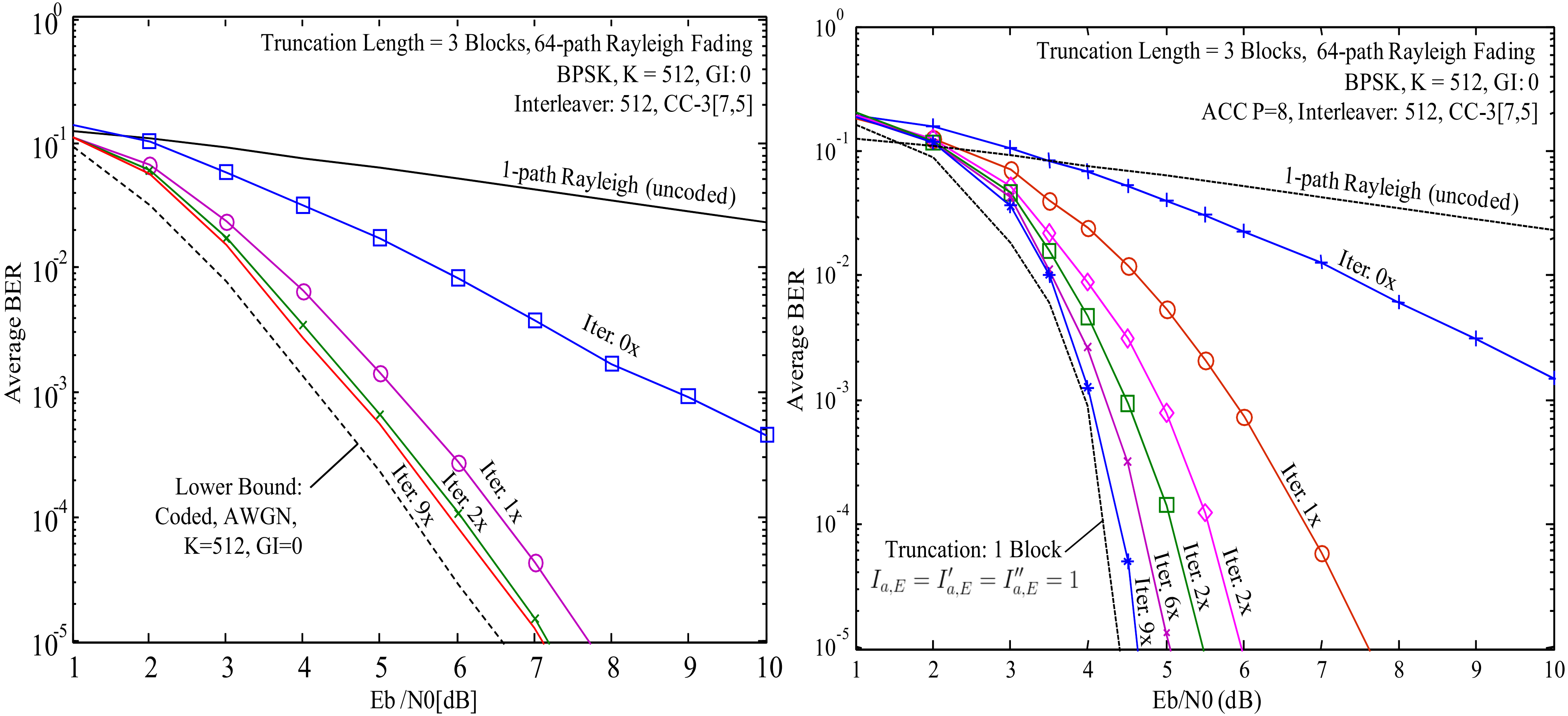
Note: EXIT analysis of the CHATUE Algorithm without and with doped accumulator (D-ACC) with $P = 8$.

The Advantage of GI/CP Removal



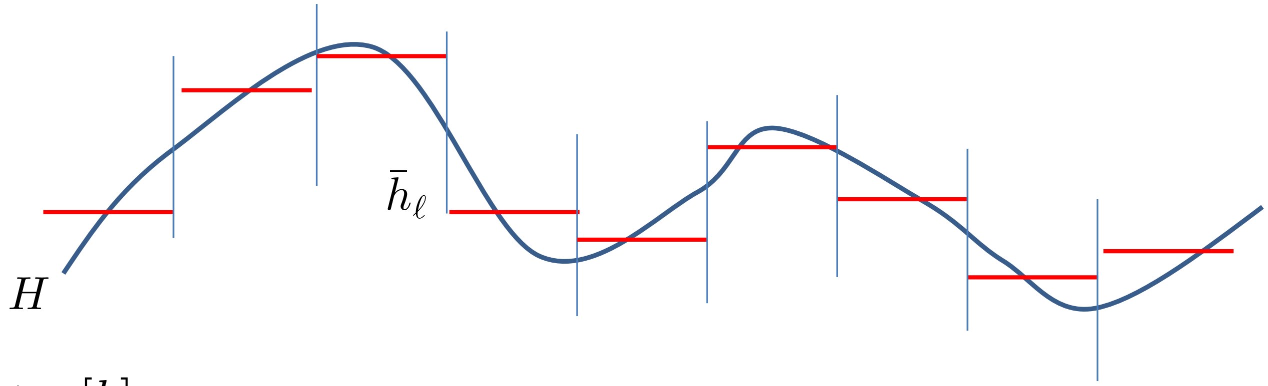
Note: The total power (over all path) is $\sum_{\ell=0}^{L-1} |h_\ell|^2 = 1$. The block length is kept by $K_{\mathrm{CHATUE}} = K_{\mathrm{SCCP}} + L$ with advantage of $\frac{N}{\mathcal{R}_{\mathrm{CHATUE}}} = \frac{N}{\mathcal{R}_{\mathrm{SCCP}}} + 1$.

BER Performances



Note: BER performances of the CHATUE Algorithm without and with doped accumulator (D-ACC) with $P = 8$.

How to Solve the Channel Variation?

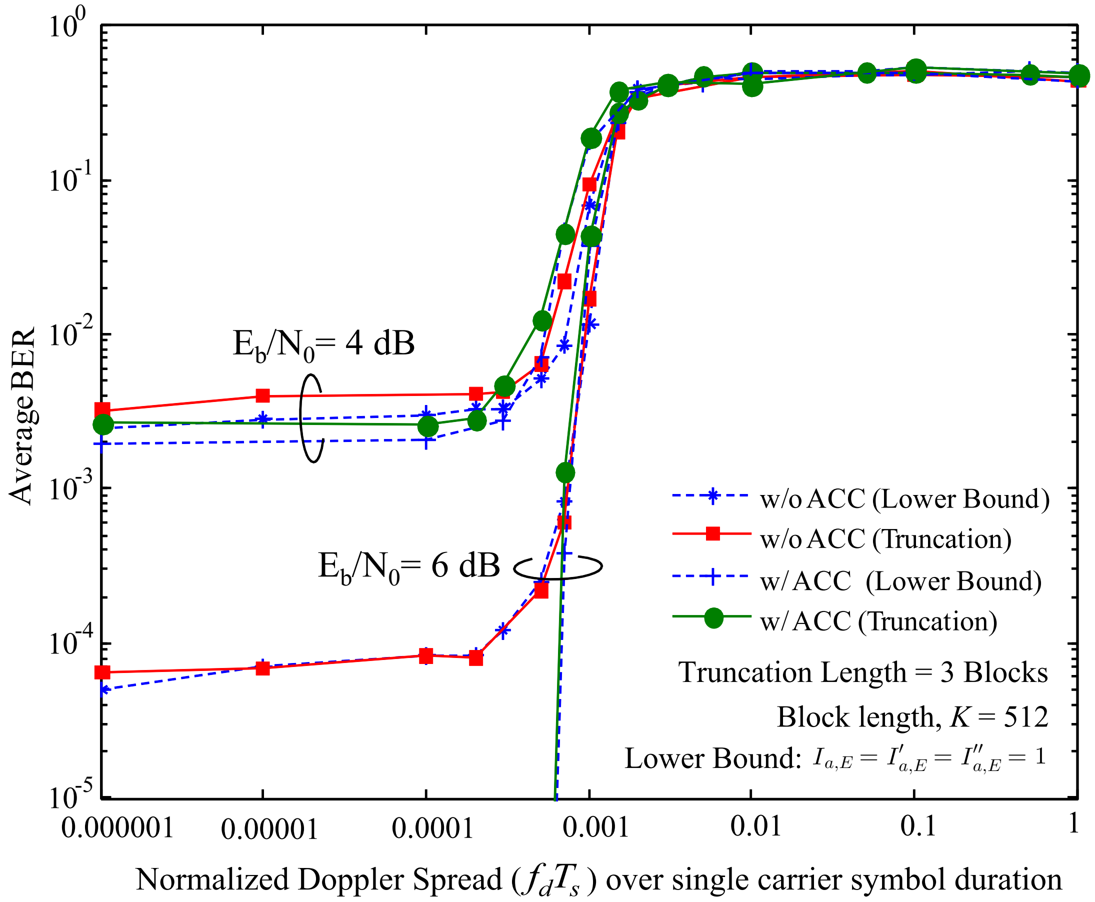


$$\bar{h}_\ell = \frac{1}{K}\sum_{k=0}^{K-1} h_\ell^{[k]},$$

$$\mathbf{H}_t = \begin{bmatrix} h_0^{[0]} & & & 0 \\ \vdots & h_0^{[1]} & & \\ h_{L-1}^{[0]} & \vdots & \ddots & \\ & h_{L-1}^{[1]} & \vdots & h_0^{[K-1]} \\ & & \ddots & \vdots \\ 0 & & & h_{L-1}^{[K-1]} \end{bmatrix}_t \approx \begin{bmatrix} \bar{h}_0 & & & 0 \\ \vdots & \bar{h}_0 & & \\ \bar{h}_{L-1} & \vdots & \ddots & \\ & \bar{h}_{L-1} & \vdots & \bar{h}_0 \\ & & \ddots & \vdots \\ 0 & & & \bar{h}_{L-1} \end{bmatrix}_t$$

$$= \bar{\mathbf{H}}_t \in \mathbb{C}^{(K+L-1)\times K}$$

Performances Evaluation



- In general, broadband communication (e.g. 4G) is sensitive to Doppler shift.
- Highway speed: 100 km/h at 3.5 GHz (WiMAX) $\rightarrow f_d T_s = 0.00162/512 = 0.0000031$

CHATUE Algorithm: Application to SC-FDMA Systems

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References (Suggested for Further Reading):

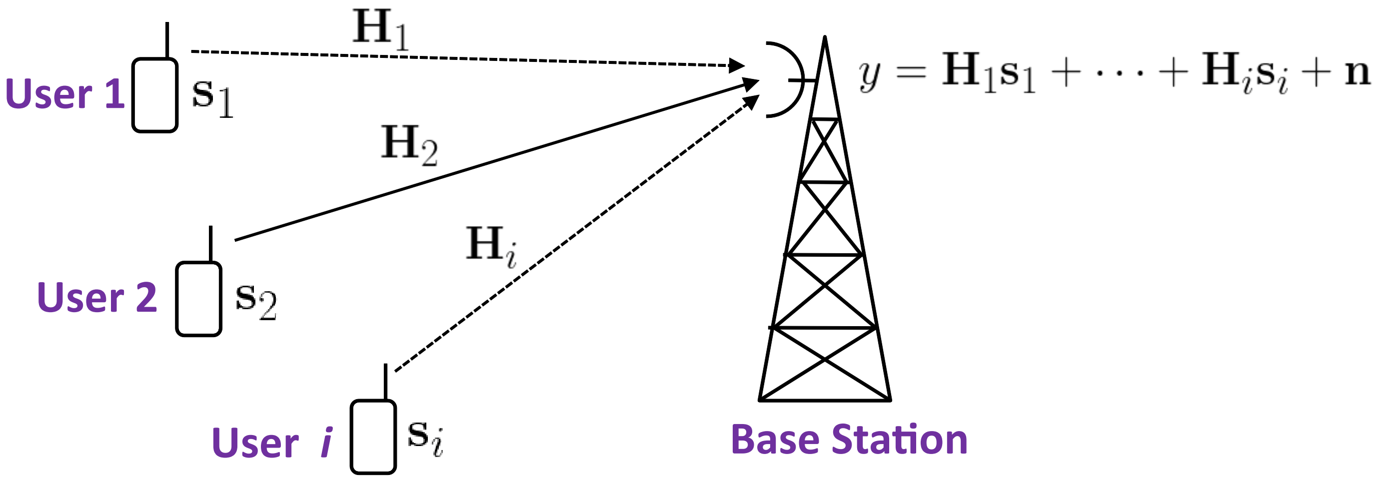
1 H. Zhou, K. Anwar, and T. Matsumoto, "Chained Turbo Equalization for SC-FDMA Systems without Cyclic Prefix", *IEEE Globecom 2010 Workshop on Broadband Single Carrier and Frequency Domain Communications*, pp.1318-1322, Dec. 2010, USA.

2 H. Zhou, K. Anwar, and T. Matsumoto, "Low Complexity Time-Concatenated Turbo Equalization for Block Transmission without Guard Interval: Part 2 – Application to SC-FDMA," *Wireless Personal Communications*, Springer, Sept. 2011 (DOI: 10.1007/s11277-011-0409-1).

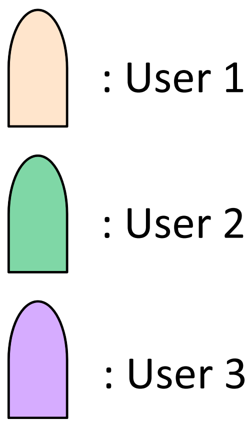
Application to (4G) Uplink SC-FDMA without GI

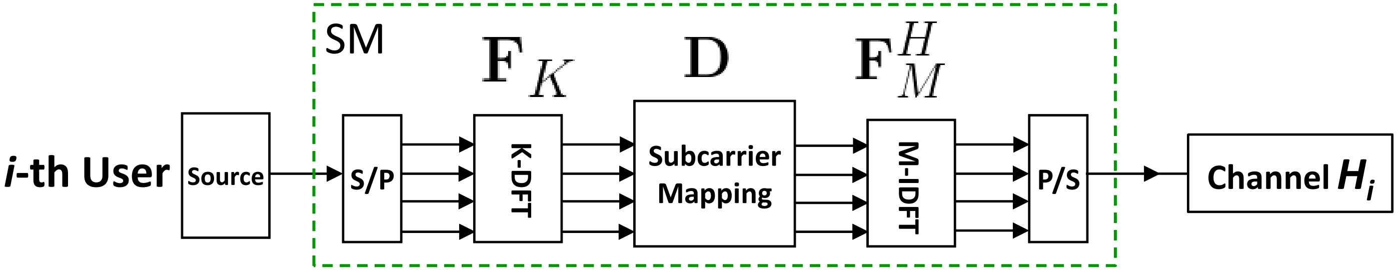
Ref.: H. Zhou, K. Anwar and T. Matsumoto, "Low Complexity Time-Concatenated Turbo Equalization for Block Transmission without Guard Interval: Part 2. Application to SC-FDMA", *Wireless Personal Commun.*, Springer, DOI 10.1007/s11277-011-0409-1, Pub. online: 25 Sept. 2011.

SC-FDMA: A Review

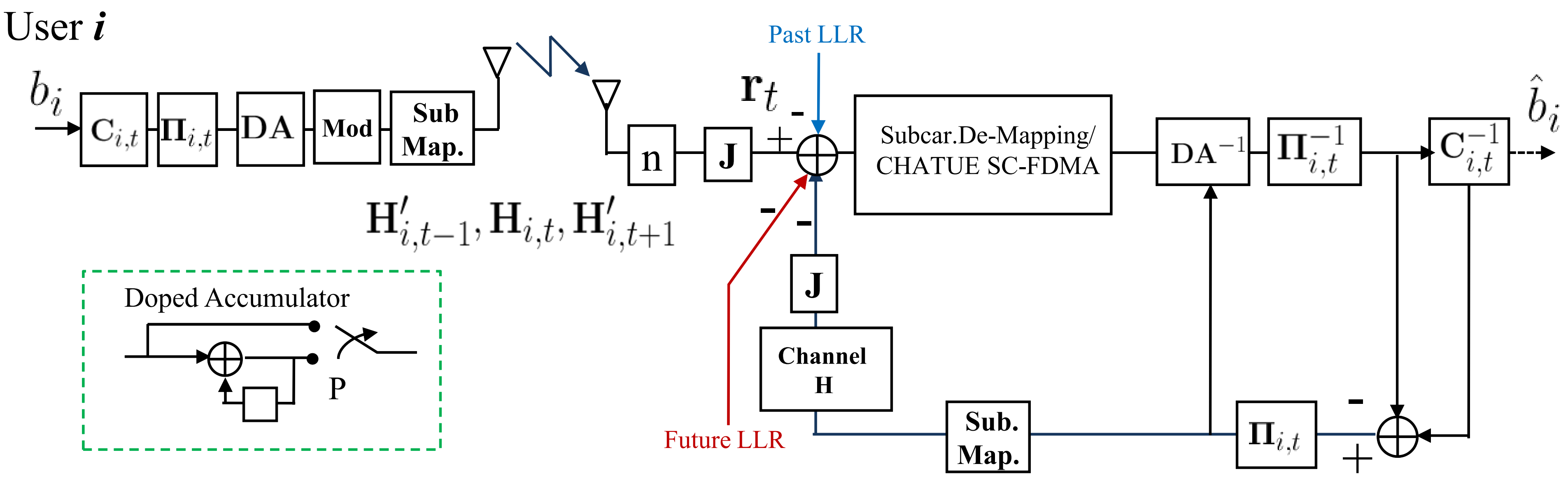


Subcarrier Allocation:





SC-FDMA: System Model



The received composite signal is

$$\mathbf{r}_t = \sum_{i=1}^{I} \mathbf{r}_{i,t} + \mathbf{J}\mathbf{n}, \qquad (30)$$

where

$$\mathbf{r}_{i,t} = \mathbf{J}\mathbf{H}_{i,t}\mathbf{F}_M^H\mathbf{D}_i\mathbf{F}_K\mathbf{s}_{i,t} + \mathbf{J}\mathbf{H}'_{i,t-1}\mathbf{F}_M^H\mathbf{D}_i\mathbf{F}_K\mathbf{s}'_{i,t-1} + \mathbf{J}\mathbf{H}''_{i,t+1}\mathbf{F}_M^H\mathbf{D}_i\mathbf{F}_K\mathbf{s}''_{i,t+1}$$

Matrix $\mathbf{D}$ for Subcarrier Allocations: Example

$\mathbf{D}_i$ is a $M \times K$ matrix for the i-th user, by which, the κ-th sub-carrier component of the K-point Discrete Fourier Transform (DFT) is mapped to the m-th sub-carrier of the M-point DFT, where $0 \leq \kappa \leq K-1$, and $0 \leq m \leq M-1$.
For localized sub-carrier mapping,

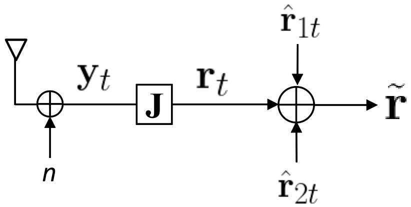
$$\mathbf{D}_i = \begin{cases} 1, & m = R_u \cdot K + \kappa \\ 0, & \text{otherwise} \end{cases} \tag{31}$$

and for distributed sub-carrier mapping,

$$\mathbf{D}_i = \begin{cases} 1, & m = R_u + \frac{M}{K} \cdot \kappa \\ 0, & \text{otherwise} \end{cases} \tag{32}$$

with R_u indicating the resource unit allocation, which is subjected to $0 \leq R_u \leq \frac{M}{K} - 1$.

Soft Cancellation in CHATUE-SC-FDMA



Soft Cancellation $\tilde{\mathbf{r}}_t = \mathbf{r}_t - \hat{\mathbf{r}}_t$

- Soft Replica

$$\hat{\mathbf{r}}_t = \sum_{i=1}^{I} \mathbf{J}\mathbf{H}_{i,t}\mathbf{F}_M^H\mathbf{D}_i\mathbf{F}_K\hat{\mathbf{s}}_{i,t} + \sum_{i=1}^{I} \mathbf{J}\mathbf{H}'_{i,t-1}\mathbf{F}_M^H\mathbf{D}_i\mathbf{F}_K\hat{\mathbf{s}}'_{i,t-1} + \sum_{i=1}^{I} \mathbf{J}\mathbf{H}''_{i,t+1}\mathbf{F}_M^H\mathbf{D}_i\mathbf{F}_K\hat{\mathbf{s}}''_{i,t+1} \quad (33)$$

- Soft Symbol Estimates

$$\hat{s}_{i,t}(k) = \mathrm{E}[s_{i,t}(k)|L_{e,C_i^{-1}}] = \tanh\{L_{e,C_i^{-1}}[s_{i,t}(k)]/2\}, \quad (34)$$

$$\hat{s}'_{i,t-1}(k) = \mathrm{E}[s'_{i,t-1}(k)|L'_{p,C_{i,t-1}^{-1}}] = \tanh\{L'_{p,C_{i,t-1}^{-1}}[s'_{i,t-1}(k)]/2\}, \quad (35)$$

$$\hat{s}''_{i,t+1}(k) = \mathrm{E}[s''_{i,t+1}(k)|L''_{p,C_{i,t+1}^{-1}}] = \tanh\{L''_{p,C_{i,t+1}^{-1}}[s''_{i,t+1}(k)]/2\}. \quad (36)$$

CHATUE-SC-FDMA Output

$$\begin{aligned}\mathbf{z}_{i,t} &= (\mathbf{I}_k + \mathbf{\Gamma}_{i,t}\mathbf{S}_{i,t})^{-1}[\mathbf{\Gamma}_{i,t}\hat{\mathbf{s}}_{i,t} + \mathbf{F}_K^H\mathbf{\Phi}_{i,t}^H\mathbf{F}_K\mathbf{\Sigma}_{i,t}^{-1}\tilde{\mathbf{r}}_{i,t}] \\ &= (\mathbf{I}_k + \mathbf{\Gamma}_{i,t}\mathbf{S}_{i,t})^{-1}[\mathbf{\Gamma}_{i,t}\hat{\mathbf{s}}_{i,t} + \mathbf{F}_K^H\mathbf{\Phi}_{i,t}^H\mathbf{X}^{-1}\mathbf{F}_K\tilde{\mathbf{r}}_{\mathbf{i},\mathbf{t}}] \in \mathbb{C}^{K\times 1}, \quad (37)\end{aligned}$$

where the $\mathbf{\Gamma}_{i,t}$ can be expressed as

$$\begin{aligned}\mathbf{\Gamma}_{i,t} &= \mathrm{diag}[\bar{\mathbf{H}}_{i,t}^H\mathbf{\Sigma}_{\mathbf{i},\mathbf{t}}{}^{-1}\bar{\mathbf{H}}_{i,t}] \\ &= \mathrm{diag}[\mathbf{F}_K^H\mathbf{\Phi}_{i,t}^H\mathbf{F}_K\mathbf{\Sigma}_{\mathbf{i},\mathbf{t}}{}^{-1}\mathbf{F}_K^H\mathbf{\Phi}_{i,t}\mathbf{F}_K] \\ &= \mathrm{diag}[\mathbf{F}_K^H\mathbf{\Phi}_{i,t}^H\mathbf{X}^{-1}\mathbf{\Phi}_{i,t}\mathbf{F}_K] \in \mathbb{C}^{K\times K} \quad (38)\end{aligned}$$

with $\mathbf{X}$ being the frequency domain covariance matrices given by

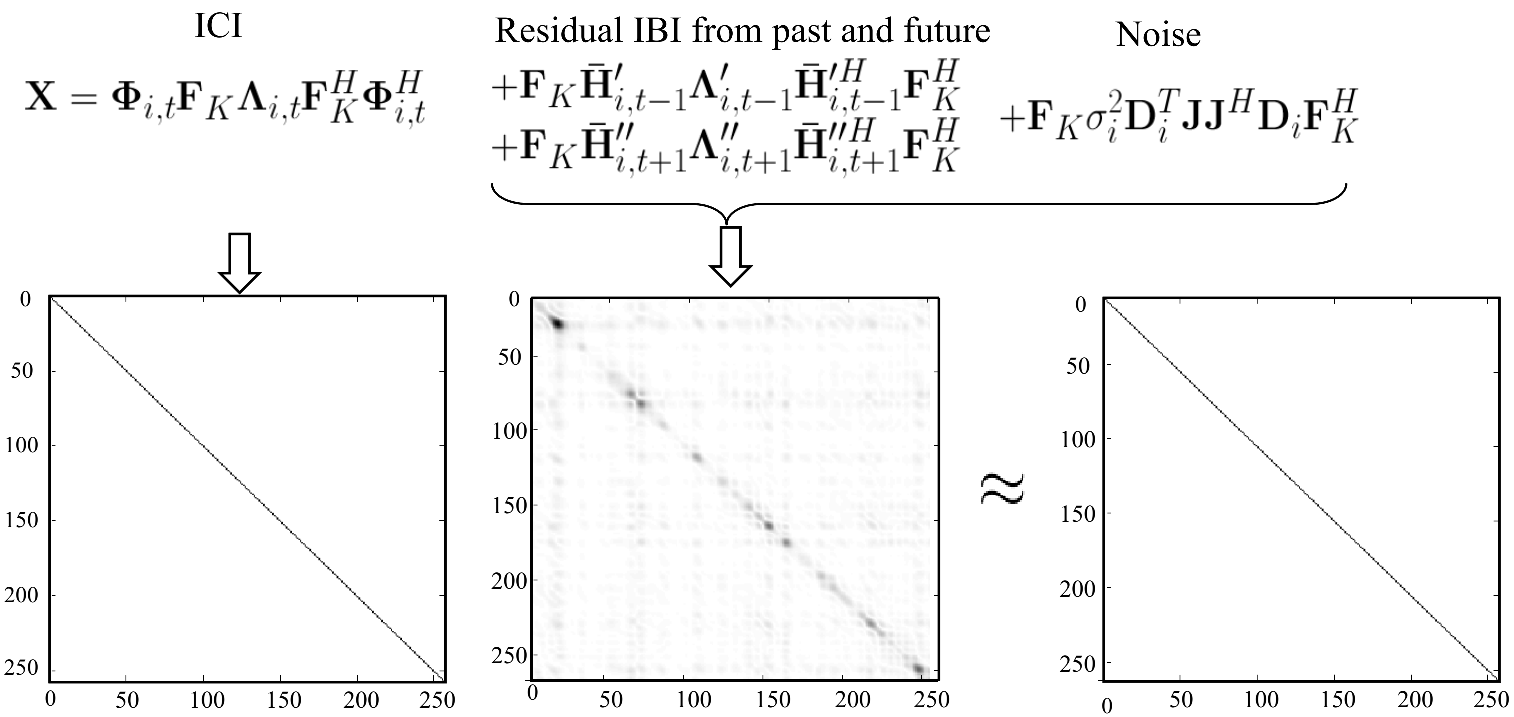
$$\begin{aligned}\mathbf{X} &= \mathbf{F_K}\mathbf{\Sigma}_{\mathbf{i},\mathbf{t}}\mathbf{F_K}{}^H = \mathbf{\Phi}_{i,t}\mathbf{F}_K\mathbf{\Lambda}_{i,t}\mathbf{F}_K^H\mathbf{\Phi}_{i,t}^H + \mathbf{F}_K\sigma_i^2\mathbf{D}_i^T\mathbf{J}\mathbf{J}^H\mathbf{D}_i\mathbf{F}_K^H \\ &+ \mathbf{F}_K\bar{\mathbf{H}}'_{i,t-1}\mathbf{\Lambda}'_{i,t-1}\bar{\mathbf{H}}'^H_{i,t-1}\mathbf{F}_K^H \\ &+ \mathbf{F}_K\bar{\mathbf{H}}''_{i,t+1}\mathbf{\Lambda}''_{i,t+1}\bar{\mathbf{H}}''^H_{i,t+1}\mathbf{F}_K^H \in \mathbb{C}^{K\times K} \quad (39)\end{aligned}$$

and $\sigma_i^2 = \frac{K}{M}\sigma_n^2$ for the i-th user.

Approximation for Computational Complexity Reduction

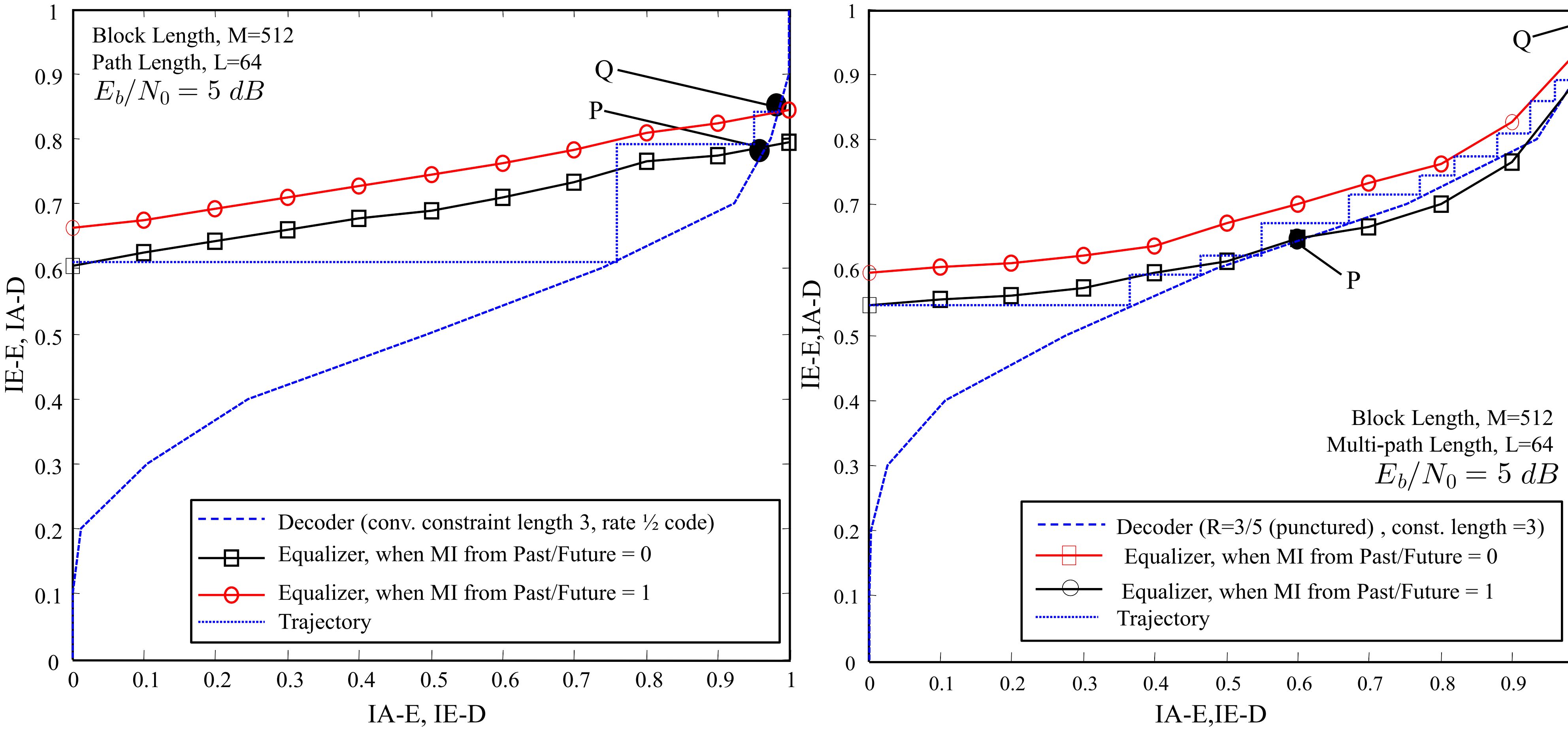
Based on $\mathbf{F\Lambda F}^H \approx \frac{1}{K}\mathrm{tr}[\mathbf{\Lambda}]\mathbf{I}_K$, we can perform:

$$\begin{aligned}\mathbf{X} &\approx \mathbf{\Phi}_{i,t}\mathbf{\Lambda}_{i,t}\mathbf{\Phi}_{i,t}^H + \mathrm{diag}(\mathbf{F}_K\sigma_i^2\mathbf{D}_i^T\mathbf{J}\mathbf{J}^H\mathbf{D}_i\mathbf{F}_K^H + \mathbf{F}_K\bar{\mathbf{H}}'_{i,t-1}\mathbf{\Lambda}'_{i,t-1}\bar{\mathbf{H}}'^H_{i,t-1}\mathbf{F}_K^H \\ &\qquad + \mathbf{F}_K\bar{\mathbf{H}}''_{i,t+1}\mathbf{\Lambda}''_{i,t+1}\bar{\mathbf{H}}''^H_{i,t+1}\mathbf{F}_K^H) \\ &\approx \mathbf{\Phi}_{i,t}\mathbf{\Lambda}_{i,t}\mathbf{\Phi}_{i,t}^H + \frac{1}{K}\mathrm{tr}\left[\sigma_i^2\mathbf{D}_i^T\mathbf{J}\mathbf{J}^H\mathbf{D}_i + \bar{\mathbf{H}}'_{i,t-1}\mathbf{\Lambda}'_{i,t-1}\bar{\mathbf{H}}'^H_{i,t-1} + \bar{\mathbf{H}}''_{i,t+1}\mathbf{\Lambda}''_{i,t+1}\bar{\mathbf{H}}''^H_{i,t+1}\right]\mathbf{I}_K\end{aligned} \tag{40}$$



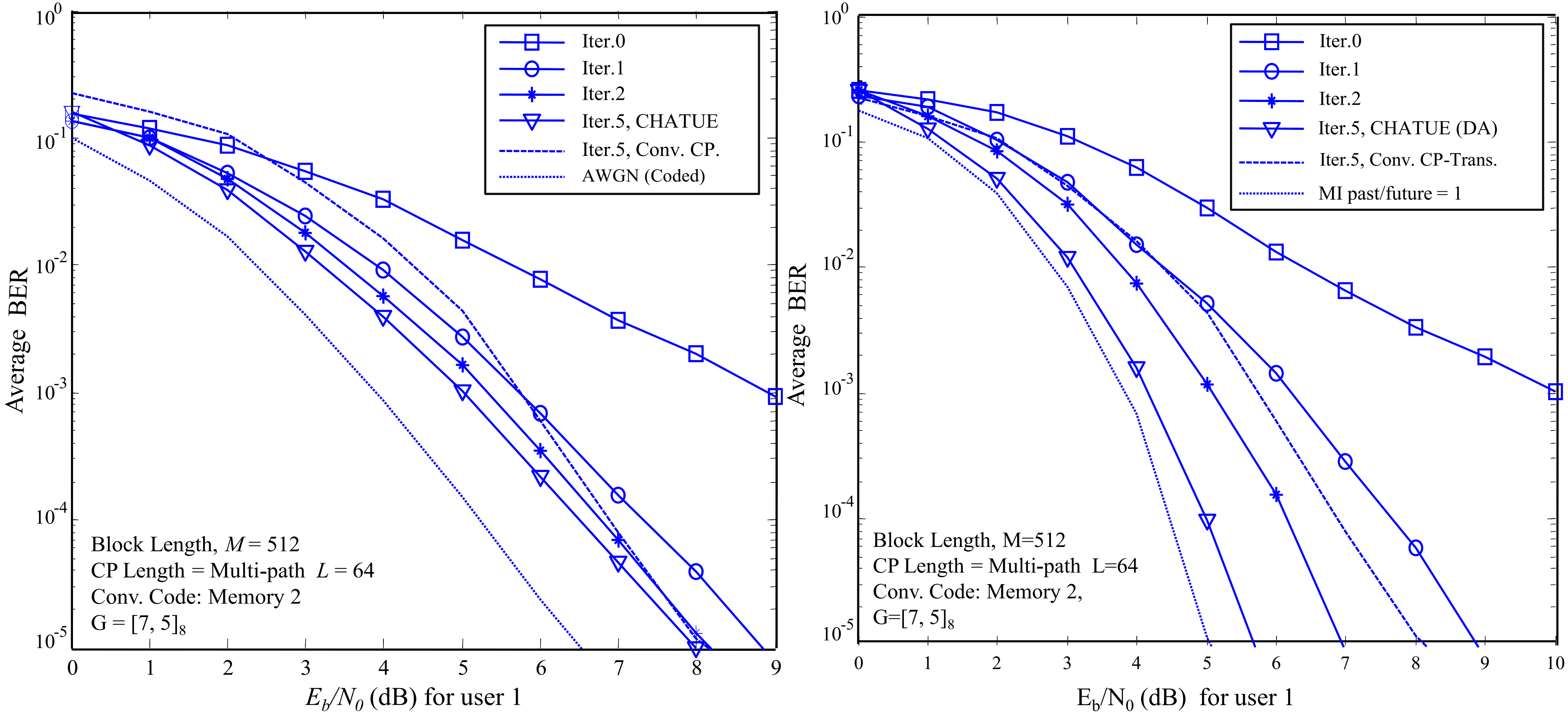
EXIT Chart Analysis for CHATUE-SC-FDMA

$E_b/N_0 = 5$ dB



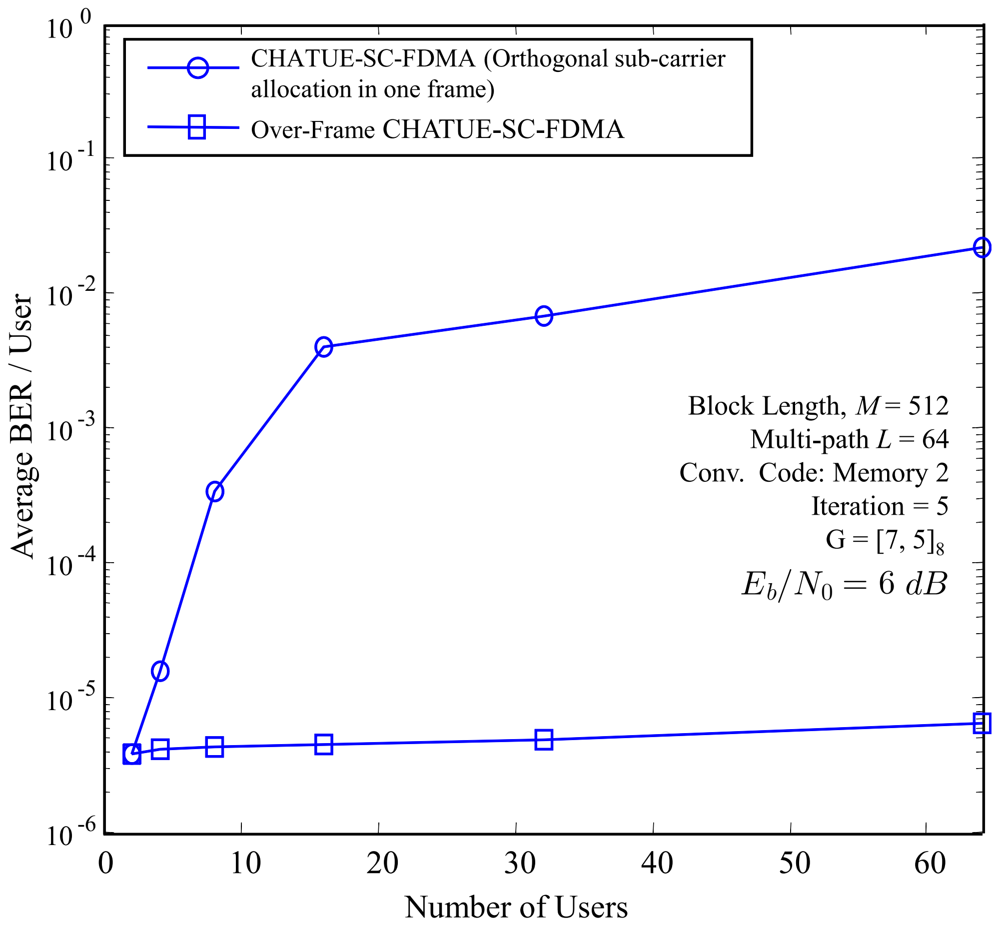
Note: EXIT analysis of the CHATUE Algorithm without and with doped accumulator (D-ACC) of foping rate $P = 8$.

BER Performances of CHATUE-SC-FDMA: User 1



Note: BER performances of the CHATUE Algorithm without and with doped accumulator (D-ACC) with $P = 8$.

CHATUE-SC-FDMA: Performance of Multiple Users



It is found that the BER performance is not significantly affected, even when the 512 sub-carriers are shared by 64 users.

Conclusions

- GI causes power loss or total rate-loss with a factor of $K/(K+L)$.
- CHATUE Algorithm can excellently improve the performance of transmission without GI with low computational complexity (it is also applicable for system with insufficient GI).
- Better performance (ICI, ISI, IBI Removal) can be achieved as demonstrated by: BER performance & Trajectory Analysis of the EXIT chart
- Further Advantages: (1) Lower Code Rate, (2) Turbo Cliff, (3) Multi-User Systems, (4) Uplink 4G SC-FDMA
- Using the CHATUE Algorithm for SC-FDMA, the next generation 4G systems is possible without cyclic prefix/guard interval.
- A comparison of CHATUE SC-FDMA with the conventional SC-FDMA with GI/CP-Transmission verifies that the performance is almost similar, but CHATUE SC-FDMA has a better spectral efficiency by eliminating the necessity of the GI/CP transmission.