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2-hop Scheme for Data Collection in Wireless Sensor Networks

By An Hong Vuong

A thesis submitted to
School of Information Science,
Japan Advanced Institute of Science and Technology,
in partial fulfillment of the requirements
for the degree of
Master of Information Science
Graduate Program in Information Science

Written under the direction of
Associate Professor Azman Osman Lim

September, 2012

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Professor Yasuo Tan
Professor Yoichi Shinoda

August, 2012 (Submitted)

Abstract

Sensors in wireless sensor networks (WSNs) usually form a tree topology and the sensed data are transmitted to a sink using multihop communication. However, multihop communication causes the phenomenon of unbalanced energy consumption, in which sensors close to the sink are overused due to transmitting not only their own sensed data but also data from other sensors, and they will die out early, resulting in network collapse although there may be still significant amount of energy in other sensors. Therefore, network lifetime could be prolonged if the energy consumption of all sensors in the network is balanced. In our research, the network lifetime of a WSN is defined as the time elapsed since the network started operating until one sensor run out of battery.

For any data gathering applications, especially in large-scale networks, a small delay in data collection is desired. In this research, data collection delay is defined as the time for all packets from all sensors in network to be received by the sink. However, also with multihop communication, a packet is relayed by many nodes before arriving at the sink, which causes a high delay in data collection.

There are two main objectives of our research, first, to increase the network lifetime of WSN by balancing energy consumption of each sensor throughout the network, and second, to reduce the delay in data collection. To obtain this, we propose a scheme called "2-hop scheme". Our proposed scheme exploits energy tradeoff between hop-by-hop transmission and 2-hop transmission. Hop-by-hop transmission is a basic communication pattern in WSN, where a packet is forwarded to the next hop until it reaches the sink. This basic communication consumes less energy at each hop; however, it causes high load of packets relay at nodes near the sink, resulting unbalanced energy consumption, as mentioned above. 2-hop transmission is another pattern of communication in WSN (in our research, we assume that each sensor can increase the transmitting power so that the transmission range is increased), where a packet is forwarded not to the next hop, but instead to the 2-hop-away node. For illustration, let us consider three sensors S_3, S_2, and S_1. The next hop of S_3 is S_2 and the next hop of S_2 is S_1. In hop-by-hop transmission, packets from S_3 are forwarded to S_2. In 2-hop transmission, packets from S_3 are forwarded not to S_2, but to S_1. 2-hop transmission consumes more energy than hop-by-hop transmission; however, it can help reduce the load (the total number of packets to be transmitted) at each node. By elegantly combining these two patterns of transmission, energy consumption of sensors in the network could be balanced and the network lifetime could then be prolonged.

More specifically, a packet is transmitted in hop-by-hop transmission with a probability p, and is transmitted in 2-hop transmission with probability $1 - p$. By choosing optimal transmission probability p for each node, the expected energy consumption of all nodes could be balanced and then, the network lifetime is proved to be maximized.

We analyze our proposed scheme for chain topology networks, binary tree topology networks and then for general tree topology networks. For chain and binary tree topology networks, we give a method to find optimal transmission probability for each node, so that energy consumption is balanced and network lifetime is maximized. For general tree topology network, it is difficult to find optimal transmission probability for each node; we then assign the same transmission probability for all nodes in the network, and use a simulator to estimate how much our proposed scheme could help increase the network lifetime compared to hop-by-hop scheme (hop-by-hop scheme is the conventional scheme where data is sent only in hop-by-hop transmission). Simulation results show that our proposed scheme outperforms the hop-by-hop scheme not only in term of network lifetime but also in term of data collection delay. With chain and binary tree networks, we also analyze the cases when initial battery levels in sensors are different to each other and give a method to find optimal transmission probabilities for the sensors.

Keywords: Wireless sensor networks (WSNs); 2-hop scheme; balanced energy consumption; network lifetime; data collection delay.

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List of Abbreviations and Symbols

DGC	Data Gathering Cycle
$\boldsymbol{N}$	Number of sensors in the network
$\boldsymbol{L}$	Number of levels in a binary tree network
$\boldsymbol{S_i}$	The i^{th} sensor in the network
$\boldsymbol{p_i}$	Transmission probability of sensor S_i
$\boldsymbol{B_i}$	Initial battery level of sensor S_i
$\boldsymbol{\alpha}$	Path loss exponent. In this research, $\alpha = 2, 2.5, 3, 3.5$
$\boldsymbol{\epsilon_{elec}}$	Energy spent by the electronic circuit when transmitting or receiving one bit data. In this research, $\epsilon_{elec} = 50\ nJ/bit$
$\boldsymbol{\epsilon_{amp}}$	Transmission amplifier. In this research, $\epsilon_{amp} = 100\ pJ/bit/m^{\alpha}$
$\boldsymbol{d_1}$	Maximum hop-by-hop transmission range. In this research, $d_1 = 20\ m$
$\boldsymbol{d_2}$	Maximum 2-hop transmission range. In this research, $d_2 = 2d_1 = 40\ m$
$\boldsymbol{m}$	Packet size. In this research, $m = 1024\ bits$
$\boldsymbol{\epsilon_t(d_1)}$	Energy for transmitting one $m-bit$ packet over distance d_1 in hop-by-hop transmission
$\boldsymbol{\epsilon_t(d_2)}$	Energy for transmitting one $m-bit$ packet over distance d_2 in 2-hop transmission
$\boldsymbol{\epsilon_r}$	Energy for receiving one $m-bit$ packet

$\boldsymbol{\varepsilon_i}$	Energy consumption of sensor S_i in *one DGC*
$\boldsymbol{\xi_i}$	Energy consumption of sensor S_i in the *whole network lifetime*
$\boldsymbol{T_{h2h}}$	Network lifetime with hop-by-hop scheme
$\boldsymbol{T_{2hop}}$	Network lifetime with 2-hop scheme
$\boldsymbol{D_{h2h}}$	Data collection delay with hop-by-hop scheme
$\boldsymbol{D_{2hop}}$	Data collection delay with 2-hop scheme
$\boldsymbol{E}[\boldsymbol{\lambda}]$	Expected value of a random variable λ

Chapter 1

Introduction

1.1 Research Background and Motivation

Wireless sensor networking is an emerging technology that has a wide range of potential applications including environment monitoring, smart spaces, medical systems and robotic exploration. Such wireless sensor network (WSN) consists of many sensors distributed spatially to monitor physical or environmental conditions (see **Fig. 1.1**). Sensors are equipped with a radio transceiver enabling them to transmit their sensed data, such as temperature, humidity, sound, pollutants, pressure or motion wirelessly through the network to a main location, called the sink node.

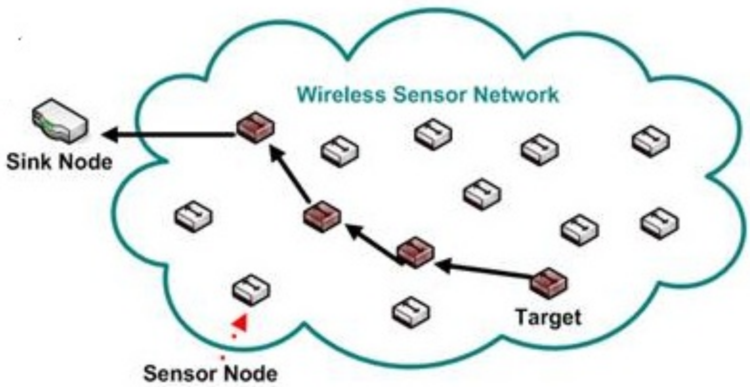

Figure 1.1: Wireless sensor network

Sensors in the network are expected to operate for a long time, maybe months or even years. However, they usually operate on small, inexpensive batteries, which have a constraint in energy supply. Moreover, in many scenarios, it is impractical or even impossible to replace or recharge those batteries after the sensors have been deployed. Therefore, prolonging the operational time of the network is an important consideration while designing WSNs.

Motivated by this, our research aims to prolong the post-deployment network lifetime. The network lifetime is regarded as the time elapsed from when the network started operating until at least one sensor in the network runs out of battery.

1.2 Research Problem: Unbalanced Energy Consumption

There are two basic communication patterns in wireless network that a node can utilize to transmit data to the sink: direct transmission and multi-hop transmission. Direct transmission, where data is directly transmitted to the sink without any relay, is very energy-expensive and may quickly drain out the sensors' batteries, especially for those located far away from the sink. In practical, to avoid sending large data over long distance, multi-hop communication is required. This type of transmission drains less energy at each node; however, the load at nodes near the sink becomes big, for they have to handle more data from the others. As a result, they quickly run out of battery, causing a decreased network lifetime. This phenomenon is called the unbalanced energy consumption problem due to multi-hop communication. **Fig. 1.2** illustrates an unbalanced energy system, where the load at node 1 (the number of packets it has to transmit) is five times bigger than the load at node 5.

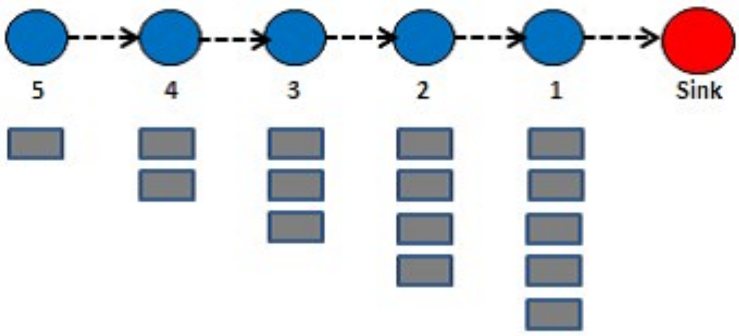

Figure 1.2: Unbalanced energy consumption caused by multi-hop transmission

1.3 Related Works

The problem of prolonging the lifetime of a sensor network has received significant attention in the last few years. Li et al. [1] proposed power-aware routing protocols to reduce energy consumption by selecting minimum-energy routing paths for transmitting packets, while Ma et al. [2] contributed a novel interference-free TDMA sleep scheduling problem called contiguous link scheduling, which assigns sensors with consecutive time slots to reduce the frequency of state transitions. However, strategies like energy aware routing or periodical sleeping focus on minimizing the total energy consumption of end-by-end packet delivery and do not explicitly solve the unbalanced energy consumption problem which can result in short network lifetime.

Cluster-head rotation schemes such as LEACH [3] and HEED [4] can achieve fairly even energy consumption among nodes within the clusters by periodically performing cluster-head rotation among all nodes in the cluster However, to achieve desirable balance of energy consumption, cluster-head the cluster-head selection algorithm must be performed frequently, which may add excessive processing and communication overheads to the network, resulting in much energy wastage.

Data aggregation, which the key idea is to combine data from different sensors to eliminate redundant transmissions (for example, in a sensor network for getting the maximum temperature of a region, after node A has received data from nodes B and C, it is not necessary for A to forwards all the data from B and C to the next hop, but instead A just needs to perform a MAX aggregation function on the temperatures and sends the maximum one to the next hop), can be used as a strategy to balance energy consumption and hence increase the network lifetime, as discussed in [5]. Li and Mohapatra [6] studied the problem of mitigating energy holes by traffic compression and aggregation. However, because data aggregation cannot be used in some cases, such as in a network deployed to get the exact temperature of all points in an area; those studies do not explore the possibility of avoiding energy holes in those kinds of data-gathering sensor networks.

Communication control protocol is another approach for the energy consumption balancing problem. Howitt and Wang [7] proposed Energy Balanced Chain (EBC) to balance energy consumption by optimizing hop distances. The communication topology is predetermined based on the anticipated traffic in the network. Power-adjusted transmission is another attractive scheme for balancing energy consumption in wireless sensor networks. First proposed in [8] by Guo et al. is mixed-routing scheme, where each node alternatively sends data in direct and multi-hop transmission. Efthymiou et al. [9] proposed a slice model and designed a probabilistic data propagation algorithm for balancing energy consumption in sensor networks where the nodes are uniformly deployed in a fan-shaped or circular region, and all nodes are assumed to have the same packet generation rate. Zhang et al. [10] also exploited the energy tradeoff between direct transmission and hop-by-hop transmission to balance energy consumption for all nodes in chain networks; then they derive the energy balanced solution for general topology networks by dividing the network into sections and approximately mapping it on chain models. Those studies imply that direct transmission can be performed by the sensors; however, if the sensors are too far from the sink, or the energy level at a node is not high enough, direct transmission may not work. In this research, we assume that the sensors can adjust the transmitting power so that data can be received by the 2-hop-away node. This pattern of communication is called "2-hop transmission". 2-hop transmission consumes less energy than direct transmission and therefore, it can also work in large-scale network, where direct transmission is impossible due to the long distance between some nodes and the sink. We address the problem of balancing energy consumption by combining multi-hop and 2-hop communication.

1.4 Research Objective

There are two main objectives of our research, first, to increase the network lifetime of WSN by balancing energy consumption of each sensor throughout the network, and second, to reduce the delay in data collection. To obtain this, we propose a scheme called "2-hop scheme". In this scheme, a sensor can perform two types of transmission: hop-by-hop and 2-hop transmission. 2-hop transmission requires more energy than hop-by-hop transmission, so if sensors far from the sink (where the load is not high) transmits more data in 2-hop transmissions than those near the sink (where the load is high), the overall energy consumption can be balanced throughout the network.

1.5 Structure of this Thesis

- Chapter 1 is about the research background and motivation, research problem, research objective and some related works.
- Chapter 2 is about the system models like data gathering models, energy model, communication model and problem statement.
- Chapter 3 is a theoretical analysis on
  - Network lifetime increase with 2-hop scheme in chain and binary tree topology networks.
  - Network lifetime increase comparison with direct scheme and 2-hop scheme.
  - Data collection delay comparison between hop-by-hop scheme, 2-hop scheme and direct scheme.
- Chapter 4 is about simulation studies on 2-hop scheme for
  - Network lifetime in general tree topology networks.
  - Data collection delay in general tree topology networks.
- Chapter 5 is the concluding remarks, contributions and future works.

Chapter 2

System Models, Definitions and Problem Statement

2.1 Data Gathering Model

Our analysis is for data gathering sensor networks where each sensor *periodically* transmits its sensed data to the sink. In most data gathering applications, usually the time between two adjacent data transmission cycles (duty cycles) is long, may be several minutes, hours or even days. Therefore, to avoid idle listening, sensors usually turn off their radio circuits when there is no data to transmit. In our model, we assume that a synchronized sleep/wake up scheme like T-MAC [11], S-MAC [12] or contiguous link scheduling [2] is exploited. The process in which all sensors wake up, generate the sensed data and transmit the data to the sink is defined as one Data Gathering Cycle (DGC) In one DGC, we assume that each sensor generates only one packet and sends that packet, together with all packets received from other sensors, to the sink. Between two adjacent DGCs, all sensors turn off their radios to save energy. **Fig. 2.1** illustrates the definition of DGC. In this study, all sensors in the network have the same data generation rate, and the amount of data generated by every sensor in each DGC is m bits.

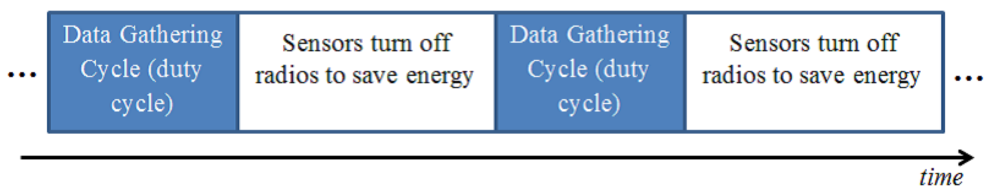

Figure 2.1: Data gathering model in WSN

2.2 Energy Model

2.2.1 First-order Radio Model

A typical sensor consists of three components: sensing component, data processing component and communication component. Although the first two components also dissipate sensor's energy, the energy spent by them is much smaller compared to energy for transmitting and receiving data. Thus, energy consumption of a sensor can be considered as the energy dissipated by the communication component, which is a subsystem consisting of transmitter/receiver electronics, a transmitting amplifier and an antenna. Heinzelman et al. [13] proposed a model, called the first-order radio model (see **Fig. 2.2**) for calculating transmitting and receiving energy of a sensor node.

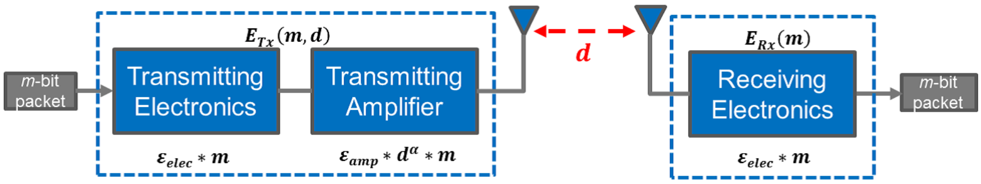

Figure 2.2: First-order radio model

According to this model, the energy $E_{Tx}(m, d)$ to transmit an m-bit packet over a distance d is:

$$E_{Tx}(m, d) = (\epsilon_{elec} + \epsilon_{amp} * d^{\alpha}) * m \tag{2.1}$$

where ϵ_{elec} is the energy dissipated per bit in the transmitting electronics, $\epsilon_{amp} * d^{\alpha}$ is the energy dissipated in the amplifier to transmit one bit over distance d, and α is the path loss exponent, usually $2 \leq \alpha \leq 4$ (where 2 is for propagation in free space, 4 is for relatively lossy environments and for the case of full specular reflection from the earth surface, the so-called flat-earth model). In some environments, such as buildings, stadiums and other indoor environments, the path loss exponent can reach values in the range of 4 to 6. Path loss exponent represents the reduction in power density (attenuation) of an electromagnetic wave as it propagates through space [14]. The energy $E_{Rx}(m)$ to receive an m-bit packet is:

$$E_{Rx}(m) = \epsilon_{elec} * m \tag{2.2}$$

here ϵ_{elec} is the energy dissipated in the receiving electronics for successful reception of a single bit. The energy dissipated in the transmitting electronics on data sending is the same as energy dissipated in receiving electronics on data receiving. The first-order radio model has been widely used in many studies for measuring the energy consumption in wireless communications, such as [1], [10], [15], [16].

2.2.2 Energy Consumption Calculation

Because of the fact that, compared with data communication, other kinds of energy such as energy for data processing, energy for sensing, etc. are much smaller, in this research, we do not take those types of energy into account. Thereby, the total energy consumption by a sensor is considered as the energy spent for communication. In one DGC, if a sensor transmits a total of f_{Tx} m-bit packets over distance d and receives a total of f_{Rx} m-bit packets, then the total energy consumption ε of that sensor in one DGC is:

$$\varepsilon = f_{Tx} * E_{Tx}(m, d) + f_{Rx} * E_{Rx}(m) \tag{2.3}$$

2.3 Communication Model

In our work, we assume that all the transmissions in the network are reliable. Taking into account the effects of data loss is considered as future work. The communication adopted here is similar to the one proposed in [9], [10]; but instead of using direct transmission, we use 2-hop transmission. More specifically, (see **Fig. 2.3**), a sensor (e.g., S_5) forwards a packet to the next hop (S_3) using hop-by-hop transmission with probability p and transmits the packet to the 2-hop neighbor (S_1) using 2-hop transmission with probability $1-p$. Our scheme does not consume plenty of energy as the aforementioned direct scheme; thereby, it still works well in large-scale networks. p_i is called the transmission probability and this communication model is called the "2-hop scheme".

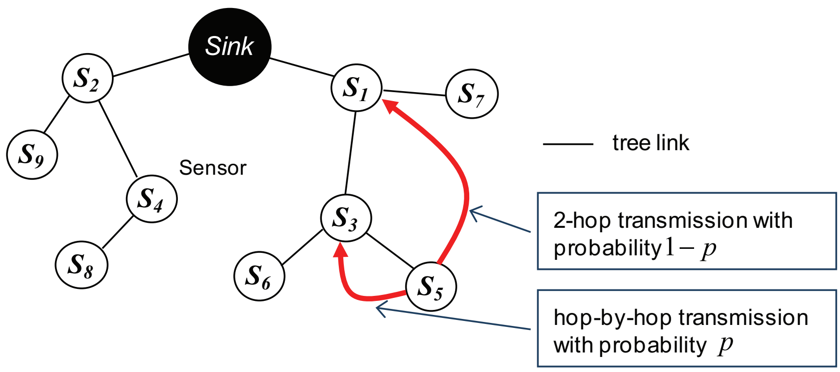

Figure 2.3: Proposed 2-hop scheme

2.4 Problem Statement

Let us consider a WSN consisting of N sensors $S_1, S_2, S_3, ..., S_N$. At initial time (deployment time), the battery levels in all sensors are assumed to be the same. The energy consumption of S_i in **one DGC** is denoted by $\boldsymbol{\varepsilon_i}$. The energy consumed by S_i in the **whole network lifetime** is denoted by $\boldsymbol{\xi_i}$. For a random variable λ, we denote by $E[\lambda]$

its expectation value. That means, $E[\varepsilon_i]$ is the expected energy consumption of S_i in one DGC and $E[\xi_i]$ is the expected energy consumption of S_i in the whole network lifetime.

A network is said to be energy balanced if each sensor has the same expected energy consumption in the whole network lifetime, i.e.,

$$E[\xi_i] = E[\xi_j] \quad i, j = 1, 2, ..., N \tag{2.4}$$

In our scheme, the transmission probability p_i is preassigned for sensor S_i and remains unchanged once the network starts to work. If p_i is large, S_i tends to send a packet in hop-by-hop transmission, so $E[\xi_i]$ is small. On the other hand, if p_i is small, a packet is more likely to be transmitted in 2-hop transmission, and $E[\xi_i]$ is large. Therefore, by assigning small probabilities for sensors far from the sink, and large probabilities for those close to the sink, energy unbalance can be reduced and if we assign optimal probabilities for sensors in the network, balanced energy consumption can be achieved. Thus, the problem of balancing energy consumption can be transformed to the *optimal transmission probability allocation* problem and our objective is now transformed to computing the optimal transmission probability for each sensor so that balanced energy consumption can be achieved.

Theorem 1 $E[\xi_i] = E[\xi_j]$ *if and only if* $E[\varepsilon_i] = E[\varepsilon_j] \quad \forall i, j = 1, 2, ..., N$

Proof See *Appendix A*.

From *Theorem 1*, the problem of the research can be stated as calculating optimal transmission probabilities so that $E[\varepsilon_i] = E[\varepsilon_j] \quad \forall i, i = 1, 2, ..., N$

Chapter 3

Theoretical Analysis on Network Lifetime and Data Collection Delay with 2-hop Scheme

3.1 Network Lifetime Increase with 2-hop Scheme in Chain Topology Networks

3.1.1 Chain Topology Networks

The chain topology network we consider in this chapter is a general chain network, where the sensors are deployed irregularly in an area. Each sensor select one of its near-sink neighbors for next hop packet relay. The sink is assumed to be at one end of the chain without loss of generality. The sensors are marked with 1 to N from the sink to the farthermost sensor (see **Fig. 3.1**). Each sensor S_i is assigned a transmission probability p_i. Because S_1 does not transmit any packet in 2-hop transmission, $p_1 = 1$.

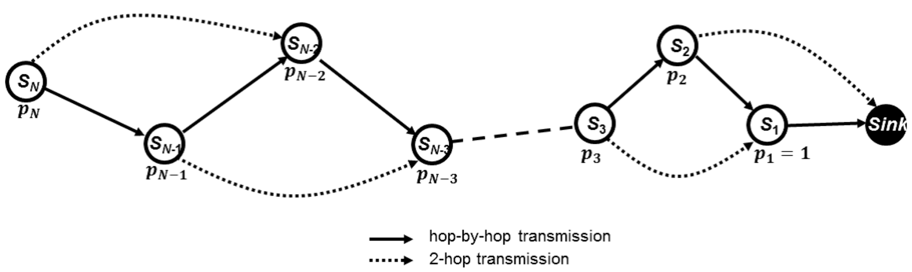

Figure 3.1: A chain topology network of N sensors

In one DGC, we denote by $f_{1,i}$ and $f_{2,i}$ the number of packets S_i forwards to sensor S_{i-1} using hop-by-hop transmission and to S_{i-2} using 2-hop transmission, respectively. The expectation value of a random variable λ is denoted by $E[\lambda]$.

Lemma 1 $p_i = \frac{E[f_{1,i}]}{E[f_{1,i}]+E[f_{2,i}]}, \quad \forall i = 1, 2, ..., N$

Proof See *Appendix B*.
It can be inferred from *Lemma 1*, and equation (B.3) that

$$E[f_{1,i}] = p_i E[n_i] \tag{3.1}$$

and

$$E[f_{2,i}] = (1 - p_i) E[n_i] \tag{3.2}$$

In practical, most current sensor motes cannot transmit a packet with power as small as possible and usually there is a minimum transmission power. In this research, we assume that all sensors in the network are identical, they use the same power P_1 for hop-by-hop transmission and the same power P_2 for 2-hop transmission. Let d_1 and d_2 be the maximum distance that a packet can be transmitted with power P_1 and P_2, respectively. The expectation value of energy consumption of S_i in one DGC is

$$E[\varepsilon_i] = E[f_{1,i}]\epsilon_t(d_1) + E[f_{2,i}]\epsilon_t(d_2) + (E[f_{1,i}] + E[f_{2,i}] - 1)\epsilon_r \tag{3.3}$$

Substituting (3.1) and (3.2) into (3.3), we have

$$E[\varepsilon_i] = p_i E[n_i]\epsilon_t(d_1) + (1 - p_i)E[n_i]\epsilon_t(d_2) + (E[n_i] - 1)\epsilon_r \tag{3.4}$$

where $\epsilon_t(d_1)$ and $\epsilon_t(d_2)$ are the energy to transmit one packet over distance d_1 and d_2, respectively. ϵ_r is the energy to receive one packet. They are calculated based on the *First-order radio model* mentioned above.

3.1.2 Optimal Transmission Probabilities

From *Problem Statement* section above, we have known that the objective of this research can be transformed to calculating optimal transmission probabilities to balance energy consumption throughout the network. However, we now may come up with a question "is it true that balancing energy consumption also maximizes the network lifetime?". After all, it is the network lifetime that needs to be maximized. Theorem 2 below answers "yes" to that question.

Lemma 2 *Assuming all sensors in the network have the same amount of initial battery; then the network lifetime is maximized if and only if* $\max_{1 \le i \le N}\{E[\varepsilon_i]\}$ *is minimized.*

Proof See *Appendix B*.

Theorem 2 *For a set of transmission probabilities* $(p_N, p_{N-1}, ..., p_3, p_2)$*, if* $E[\varepsilon_N] = E[\varepsilon_{N-1}] = ... = E[\varepsilon_2] = E[\varepsilon_1]$*; then with* $(p_N, p_{N-1}, ..., p_3, p_2)$*, the network lifetime is also maximized.*

Proof See *Appendix A*.
Now, we will find $(p_N, p_{N-1}, ..., p_2)$ to balance energy consumption of all sensors in the

network. From *Theorem 1*, we conclude that, the energy consumption is balanced if and only if $E[\varepsilon_i] = E[\varepsilon_{i-1}], \forall i = 1, 2, ..., N$. From (3.3), $E[\varepsilon_i] = E[\varepsilon_{i-1}] \Leftrightarrow$

$$\begin{aligned} & E[f_{1,i}]\epsilon_t(d_1) + E[f_{2,i}]\epsilon_t(d_2) + (E[f_{1,i}] + E[f_{2,i}] - 1)\epsilon_r = \\ & E[f_{1,i-1}]\epsilon_t(d_1) + E[f_{2,i-1}]\epsilon_t(d_2) + (E[f_{1,i-1}] + E[f_{2,i-1}] - 1)\epsilon_r \\ \Leftrightarrow & \\ & - E[f_{1,i-1}] - E[f_{2,i-1}]\frac{\epsilon_t(d_2) + \epsilon_r}{\epsilon_t(d_1) + \epsilon_r} + E[f_{1,i}] \\ & + E[f_{2,i}]\frac{\epsilon_t(d_2) + \epsilon_r}{\epsilon_t(d_1) + \epsilon_r} = 0 \end{aligned} \tag{3.5}$$

For the sake of writing, we denote

- $E[f_{1,1}]$ by x_1
- $E[f_{1,i}]$ by $x_{2i-2}, \quad \forall i = 2, 3, .., N$
- $E[f_{2,i}]$ by $x_{2i-1}, \quad \forall i = 2, 3, .., N$
- $\frac{\epsilon_t(d_2)+\epsilon_r}{\epsilon_t(d_1)+\epsilon_r}$ by C

With the new notations, (3.5) now becomes:

$$E[\varepsilon_i] = E[\varepsilon_{i-1}] \Leftrightarrow$$

$$-x_{2i-4} - Cx_{2i-3} + x_{2i-2} + Cx_{2i-1} = 0 \tag{3.6}$$

And then, $E[\varepsilon_i] = E[\varepsilon_{i-1}], \quad \forall i = 2, 3, ..., N \Leftrightarrow$

$$\begin{cases} -x_{2N-4} - Cx_{2N-3} + x_{2N-2} + Cx_{2N-1} = 0 \\ -x_{2N-6} - Cx_{2N-5} + x_{2N-4} + Cx_{2N-3} = 0 \\ ... \\ -x_2 - Cx_3 + x_4 + Cx_5 = 0 \\ -x_1 + x_2 + Cx_3 = 0 \end{cases} \tag{3.7}$$

We regard $x_{2N-1}, x_{2N-2}, ..., x_1$ as the variables of the system of simultaneous linear equations (3.7). There are a total of $2N - 1$ variables but (3.7) has only $N - 1$ equations. Therefore; to solve (3.7), we need N more equations.

As we can see from **Fig. 3.1**: $f_{1,i} + f_{2,i} = f_{1,i+1} + f_{2,i+2} + 1 \Rightarrow E[f_{1,i}] + E[f_{2,i}] = E[f_{1,i+1}] + E[f_{2,i+2}] + 1$, or $x_{2i-2} + x_{2i-1} = x_{2i} + x_{2i+3} + 1 \Leftrightarrow$

$$x_{2i-2} + x_{2i-1} - x_{2i} - x_{2i+3} = 1 \quad \forall i = 1, 2, ..., N \quad (x_0 = x_{2N} = x_{2N+3} = 0) \tag{3.8}$$

We can add N more equations (3.8) to (3.7) to get a system of simultaneous linear equations of $2N-1$ variables and $2N-1$ equations:

$$\begin{cases} -x_{2N-4} - Cx_{2N-3} + x_{2N-2} + Cx_{2N-1} = 0 \\ -x_{2N-6} - Cx_{2N-5} + x_{2N-4} + Cx_{2N-3} = 0 \\ ... \\ -x_2 - Cx_3 + x_4 + Cx_5 = 0 \\ -x_1 + x_2 + Cx_3 = 0 \\ x_{2N-2} + x_{2N-1} = 1 \\ x_{2N-4} + x_{2N-3} - x_{2N-2} = 1 \\ x_{2N-6} + x_{2N-5} - x_{2N-4} - x_{2N-1} = 1 \\ ... \\ x_2 + x_3 - x_4 - x_7 = 1 \\ x_1 - x_2 - x_5 = 1 \end{cases} \tag{3.9}$$

If (3.9) has a solution and all the values of x_i are non-negative numbers, then we can calculate p_i based on *Lemma 1*: $p_i = \frac{x_{2i-2}}{x_{2i-2}+x_{2i-1}} \quad \forall i = 2, 3, ..., N$.

3.1.3 Numerical Results and Analysis

In this section, we will find an optimal solution for a chain topology N sensors. Initial battery energy of each sensor is assumed to be $B = 30\ J$. *First-order radio model* with the following parameters is used to calculate energy consumption of each sensor.

* Maximum hop-by-hop transmission range, $d_1 = 20\ m$
* Maximum 2-hop transmission range, $d_2 = 2d_1 = 40\ m$
* Path loss exponent, $\alpha = 3.5$
* $\epsilon_{elec} = 50\ nJ/bit$
* $\epsilon_{amp} = 100\ pJ/bit/m^\alpha = 100\ pJ/bit/m^{3.5}$
* Packet size, $m = 1024\ bits$

Based on (2.1), with the above parameters, we can calculate:

* Energy to send one packet in hop-by-hop transmission, $\epsilon_t(d_1) = (\epsilon_{elec} + \epsilon_{amp} * d_1^\alpha) * m = 3.7147737743356557\ mJ$
* Energy to send one packet in 2-hop transmission, $\epsilon_t(d_2) = (\epsilon_{elec} + \epsilon_{amp} * d_2^\alpha) * m = 41.49980574735899\ mJ$
* Energy to receive one packet, $\epsilon_r = \epsilon_{elec} * m = 0.0512\ mJ$

Then $C = \frac{\epsilon_t(d_2)+\epsilon_r}{\epsilon_t(d_1)+\epsilon_r} = 11.033270074932712$.

- **$N = 3$ (The chain topology network consists of three sensors)**

  - **hop-by-hop scheme**

    In one DGC, S_3 forwards one packet to S_2 (the packet generated by itself), S_2 forwards two packets to S_1 (one packet generated by itself and one packet received from S_3), S_1 transmits three packets to the sink (one packet generated by itself and two packets received from S_3, S_2). Thus, the energy consumption of each sensor in one DGC is

    * $e_1 = 3\epsilon_t(d_1) + 2\epsilon_r = 11.246721323006968\ mJ$
    * $e_2 = 2\epsilon_t(d_1) + \epsilon_r = 7.480747548671312\ mJ$
    * $e_3 = \epsilon_t(d_1) = 3.7147737743356557\ mJ$

    Then the total number of DGCs that can be performed until S_1 runs out of battery is

$$T_{h2h} = \frac{B}{e_1} = \frac{30\ J}{11.246721323006968\ mJ} = 2667 \tag{3.10}$$

  - **2-hop scheme**

    (3.9) now becomes

$$\begin{cases} -x_2 - 11.033270074932712x_3 + x_4 + 11.033270074932712x_5 = 0 \\ -x_1 + x_2 + 11.033270074932712x_3 = 0 \\ x_4 + x_5 = 1 \\ x_2 + x_3 - x_4 = 1 \\ x_1 - x_2 - x_5 = 1 \end{cases} \tag{3.11}$$

    Solving (3.11), we get

    * $x_1 = E[f_{1,1}] = 2.892271316563032$
    * $x_2 = E[f_{1,2}] = 1.703671657386039$
    * $x_3 = E[f_{2,2}] = 0.107728683436968$
    * $x_4 = E[f_{1,3}] = 0.811400340823007$
    * $x_5 = E[f_{2,3}] = 0.188599659176993$

    The optimal transmission probabilities are:

    * $p_2 = \frac{x_2}{x_2+x_3} = 0.940527402469174$
    * $p_3 = \frac{x_4}{x_4+x_5} = 0.811400340823007$

    Using (3.3), with the optimal transmission probabilities above, we can calculate the expected energy consumption of each sensor in one DGC

    * $E[\varepsilon_1] = 10.841017926439638\ mJ$
    * $E[\varepsilon_2] = 10.841017926439638\ mJ$
    * $E[\varepsilon_3] = 10.841017926439638\ mJ$

We can see that, with the optimal transmission probabilities, the expected energy consumption has been balanced among all sensors in the network. The *expected* total number of DGCs that can be performed until one sensor runs out of battery is

$$E[T_{2hop}] = \frac{B}{E[\varepsilon_1]} = \frac{30\ J}{10.841017926439638\ mJ} = 2767 \tag{3.12}$$

From (3.10) and (3.12), with optimal transmission probabilities, the network lifetime can be increased $\frac{E[T_{2hop}]-T_{h2h}}{T_{h2h}} = \frac{2767-2667}{2667} \approx 3.75\%$

- $\boldsymbol{N = 4, 5, ..., 10, 11}$

  With similar calculation, we get

Table 3.1: Chain network – expected network lifetime increase with 2-hop scheme ($\alpha = 3.5$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme (optimal transmission probabilities)**	**Expected network lifetime increase**
$\boldsymbol{N = 4}$	$T_{h2h} = 1998$	$E[T_{2hop}] = 2055$	2.85%
$\boldsymbol{N = 5}$	$T_{h2h} = 1597$	$E[T_{2hop}] = 1633$	2.25%
$\boldsymbol{N = 6}$	$T_{h2h} = 1330$	$E[T_{2hop}] = 1355$	1.88%
$\boldsymbol{N = 7}$	$T_{h2h} = 1140$	$E[T_{2hop}] = 1158$	1.58%
$\boldsymbol{N = 8}$	$T_{h2h} = 997$	$E[T_{2hop}] = 1011$	1.4%
$\boldsymbol{N = 9}$	$T_{h2h} = 886$	$E[T_{2hop}] = 897$	1.24%
$\boldsymbol{N = 10}$	$T_{h2h} = 797$	$E[T_{2hop}] = 806$	1.13%
$\boldsymbol{N = 11}$	$T_{h2h} = 725$	$E[T_{2hop}] = 732$	0.97%

For $N = 3, 4, ..., 11$, see **Table 3.1**, we can solve (3.9) to find optimal transmission probabilities to achieve balanced energy consumption of all sensors throughout the network. Network lifetimes with hop-by-hop and 2-hop scheme for different levels are shown in **Fig. 3.2**.

- $\boldsymbol{N = 12}$ **(The chain topology network consists of 12 sensors)**

  Because $E[f_{1,i}] \geq 0$ and $E[f_{2,i}] \geq 0 \quad \forall i = 1, 2, ..., N$. Thus a valid solution of (3.9) should contain only non-negative numbers. However; solving (3.9) when $N = 12$ gives negative values for some of the variables. Therefore, it can be concluded that, when $N = 12$, balanced energy consumption throughout all sensors in the network cannot be achieved. This is an important characteristic of 2-hop scheme: *if the network consists of so many sensors, balanced energy consumption may not be achieved.*

  If it is impossible to balance energy consumption of all sensors in the network, a natural thought is to try to reduce the range of balancing. That is, if it is impossible

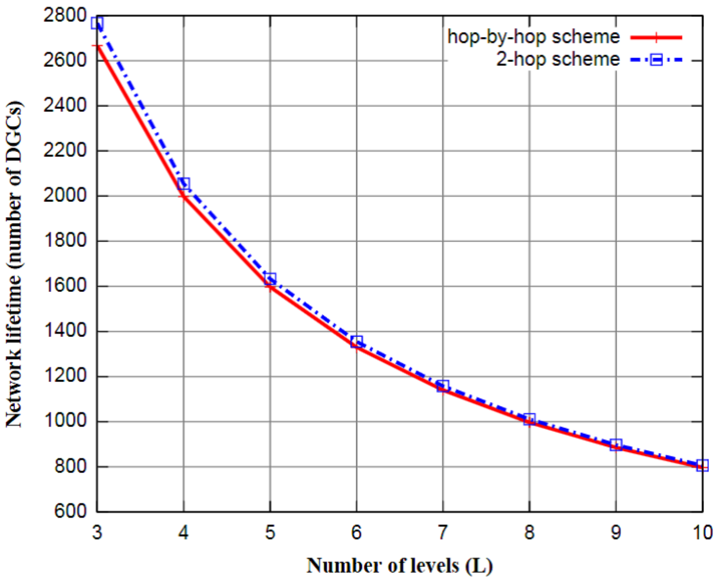

Figure 3.2: Chain network - network lifetime with 2-hop scheme and hop-by-hop scheme ($\alpha = 3.5$)

to find transmission probabilities to achieve balanced energy consumption for all 12 sensors, we will try to find transmission probabilities to balance energy consumption of 11 sensors. To do that, we first assign a random values for p_{12} and try to find $(p_{11}, p_{10}, ..., p_2)$ making $E[\varepsilon_{11}] = E[\varepsilon_{10}] = ... = E[\varepsilon_2]$.

The reason why we try to balance energy consumption of sensors $1, 2, ..., k-1$ after assigning initial values to $p_N, p_{N-1}, ..., p_k$ is based on *Theorem 3* below.

Theorem 3 *After assigning initial values to* $p_N, p_{N-1}, ..., p_k (2 \leq k \leq N)$*, for a set of transmission probabilities* $(p_{k-1}, p_{k-2}, ..., p_2)$*, if* $E[\varepsilon_{k-1}] = E[\varepsilon_{k-2}] = ... = E[\varepsilon_1]$ *then* $(p_{k-1}, p_{k-2}, ..., p_2)$ *is the best probabilities we can choose. That is, for other* $(p'_{k-1}, p'_{k-2}, ..., p'_2)$ *that does not make* $E[\varepsilon'_{k-1}] = E[\varepsilon'_{k-2}] = ... = E[\varepsilon'_1]$*, then* $\max_{1 \leq i \leq N} \{E[\varepsilon'_i]\} > \max_{1 \leq i \leq N} \{E[\varepsilon_i]\}$.

Proof. See *Appendix A*

Theorem 3 gives us a way to calculate a "good" set of transmission probabilities in cases when an optimal solution cannot be found. For $N = 12$, we assign $p_{12} = 0.9$, then $E[f_{1,12}] = x_{22} = 0.9$ and $E[f_{2,12}] = x_{23} = 0.1$. The system of equations to balance $E[\varepsilon_{11}] = E[\varepsilon_{10}] = ... = E[\varepsilon_1]$ is similar to (3.9):

$$\begin{cases} -x_{18} - 11.033270074932712x_{19} + x_{20} + 11.033270074932712x_{21} = 0 \\ ... \\ -x_2 - 11.033270074932712x_3 + x_4 + 11.033270074932712x_5 = 0 \\ -x_1 + x_2 + 11.033270074932712x_3 = 0 \\ x_{20} + x_{21} = 1.9 \\ x_{18} + x_{19} - x_{20} = 1.1 \\ ... \\ x_2 + x_3 - x_4 - x_7 = 1 \\ x_1 - x_2 - x_5 = 1 \end{cases} \tag{3.13}$$

Solving (3.13) and using *Lemma 1* to calculate the p_i, we have

* $p_2 = 0.989729471286986$
* $p_3 = 0.977099070899121$
* $p_4 = 0.961189884099742$
* $p_5 = 0.940534759388095$
* $p_6 = 0.912637971253011$
* $p_7 = 0.872885120911284$
* $p_8 = 0.811735034916467$
* $p_9 = 0.706609907927385$
* $p_{10} = 0.508470224398521$
* $p_{11} = 0.475990849849834$

With this set of transmission probabilities (they are not optimal ones, but according to *Theorem 3*, they are the best values if we assign $p_{12} = 0.9$), we have

* $E[\varepsilon_{12}] = 7.493276971637988\ mJ$
* $E[\varepsilon_{11}] = E[\varepsilon_{10}] = ... = E[\varepsilon_1] = 44.723584907141255\ mJ$

The expected network lifetime is now $E[T_{2hop}] = \frac{30\ J}{44.723584907141255\ mJ} = 670$. We can calculate the network lifetime with hop-by-hop scheme for a chain network of $N = 12$ sensors: $T_{h2h} = 664$. Then the expected network lifetime increase is approximately 0.9%.

From the *First-order radio model* section above (equation (2.1)), we can see that the energy for transmitting a packet depends on the path loss exponent α. ($E_{Tx}(m, d) = (\epsilon_{elec} + \epsilon_{amp} * d^{\alpha}) * m$). We have also known that the value of α depends on the environment where the sensors are deployed.

Until now, we have analyzed the 2-hop scheme for chain topology networks consisting of 3, 4, ..., 12 sensors deployed in an environment where $\alpha = 3.5$. Now, we will analyze the 2-hop scheme if the sensors are placed in different environments with different α. More specifically, with the same maximum hop-by-hop transmission range $d_1 = 20\ m$, the same maximum 2-hop transmission range $d_2 = 40\ m$, the same $\epsilon_{elec} = 50\ nJ/bit$ and $\epsilon_{amp} = 100\ pJ/bit/m^{\alpha}$, the same packet size $m = 1024\ bits$; we will analyze how much network lifetime could be increased with 2-hop scheme with different values of α.

- $\boldsymbol{\alpha = 2}$ (See **Table 3.2**)

  After solving (3.9) for $N = 3$ (the chain network consists of only three sensors), the variables are invalid (some of them have negative values), this means that the energy consumption cannot be achieved. However, after assigning $p_3 = 0.9$, we can find p_2 to make $E[\varepsilon_2] = E[\varepsilon_1] = 0.29396676923076925\ mJ$. Then $E[T_{2hop}] = \frac{30\ J}{0.29396676923076925\ mJ} = 102052$. We can also calculate $T_{h2h} = 79180$. Then the network lifetime increase is approximately 28.89%.

  For $N = 4, 5, .., 10$, although we cannot find optimal transmission probabilities because solving (3.9) gives invalid solution (negative values). However, based on *Theorem 3*, we can find "good" transmission probabilities for 2-hop scheme. See **Table 3.2** for more details. In **Table 3.2**, we denote by K the number of sensors that their energy consumption can be balanced after assigning $p_N = p_{N-1} = ... = p_{K+1} = 0.9$. For example, $N = 4$ and $K = 3$ means that, although we cannot find optimal transmission probabilities to balance energy consumption of all four sensors in the network, by assigning $p_4 = 0.9$ we can find p_3 and p_2 to balance energy consumption of the first three sensors S_3, S_2, S_1.

Table 3.2: Chain network – expected network lifetime increase with 2-hop scheme ($\alpha = 2$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme**	**Expected network lifetime increase**
$\boldsymbol{N = 4}$	$T_{h2h} = 57444$	$E[T_{2hop}] = 83641\ (K = 3)$	45.60%
$\boldsymbol{N = 5}$	$T_{h2h} = 45072$	$E[T_{2hop}] = 60355\ (K = 3)$	33.91%
$\boldsymbol{N = 6}$	$T_{h2h} = 37084$	$E[T_{2hop}] = 53760\ (K = 4)$	44.97%
$\boldsymbol{N = 7}$	$T_{h2h} = 31502$	$E[T_{2hop}] = 43020\ (K = 4)$	36.56%
$\boldsymbol{N = 8}$	$T_{h2h} = 27380$	$E[T_{2hop}] = 35857\ (K = 4)$	30.96%
$\boldsymbol{N = 9}$	$T_{h2h} = 24212$	$E[T_{2hop}] = 33542\ (K = 5)$	38.53%
$\boldsymbol{N = 10}$	$T_{h2h} = 21701$	$E[T_{2hop}] = 29005\ (K = 5)$	33.66%

- $\boldsymbol{\alpha = 2.5}$ (See **Table 3.3**)

- $\boldsymbol{\alpha = 3}$ (See **Table 3.4**)

We can see that (**Fig. 3.3**), if α is small, which means that 2-hop transmission does not consume too much energy, 2-hop scheme can increase network lifetime quite large;

Table 3.3: Chain network – expected network lifetime increase with 2-hop scheme ($\alpha = 2.5$)

	Network lifetime with hop-by-hop scheme	Expected network lifetime with 2-hop scheme	Expected network lifetime increase
$N = 3$	$T_{h2h} = 37242$	$E[T_{2hop}] = 43152$ ($K = 3$)	15.87%
$N = 4$	$T_{h2h} = 27494$	$E[T_{2hop}] = 31187$ ($K = 4$)	13.43%
$N = 5$	$T_{h2h} = 21791$	$E[T_{2hop}] = 24061$ ($K = 4$)	10.42%
$N = 6$	$T_{h2h} = 18047$	$E[T_{2hop}] = 19680$ ($K = 5$)	9.05%
$N = 7$	$T_{h2h} = 15401$	$E[T_{2hop}] = 16577$ ($K = 5$)	7.64%
$N = 8$	$T_{h2h} = 13432$	$E[T_{2hop}] = 14340$ ($K = 6$)	6.76%
$N = 9$	$T_{h2h} = 11909$	$E[T_{2hop}] = 12618$ ($K = 6$)	5.95%
$N = 10$	$T_{h2h} = 10696$	$E[T_{2hop}] = 11270$ ($K = 7$)	5.37%

Table 3.4: Chain network – expected network lifetime increase with 2-hop scheme ($\alpha = 3$)

	Network lifetime with hop-by-hop scheme	Expected network lifetime with 2-hop scheme	Expected network lifetime increase
$N = 3$	$T_{h2h} = 11055$	$E[T_{2hop}] = 11770$ ($K = 3$)	6.47%
$N = 4$	$T_{h2h} = 8252$	$E[T_{2hop}] = 8667$ ($K = 4$)	5.03%
$N = 5$	$T_{h2h} = 6583$	$E[T_{2hop}] = 6848$ ($K = 5$)	4.03%
$N = 6$	$T_{h2h} = 5476$	$E[T_{2hop}] = 5658$ ($K = 6$)	3.32%
$N = 7$	$T_{h2h} = 4687$	$E[T_{2hop}] = 4820$ ($K = 7$)	2.84%
$N = 8$	$T_{h2h} = 4097$	$E[T_{2hop}] = 4198$ ($K = 7$)	2.47%
$N = 9$	$T_{h2h} = 3639$	$E[T_{2hop}] = 3718$ ($K = 8$)	2.17%
$N = 10$	$T_{h2h} = 3273$	$E[T_{2hop}] = 3337$ ($K = 9$)	1.96%

however, balanced energy consumption cannot be achieved when N is large. For example, when $\alpha = 2$, even though energy consumption cannot be balanced for the network consisting of three sensors, the increase in network lifetime can be quite large, 28.89%.

3.1.4 Conclusion

There are some main results in this section:

- In some cases, we cannot find the optimal transmission probabilities to achieve balanced energy consumption throughout all sensors in the network. For example, when ($\alpha = 3.5, d_1 = 20\ m, d_2 = 40\ m, \epsilon_{elec} = 50\ nJ/bit, \epsilon_{amp} = 100\ pJ/bit/m^{3.5}$), we can find optimal transmission probabilities for chain network consisting of up to eleven sensors. In that cases, we can find "good" transmission probabilities by

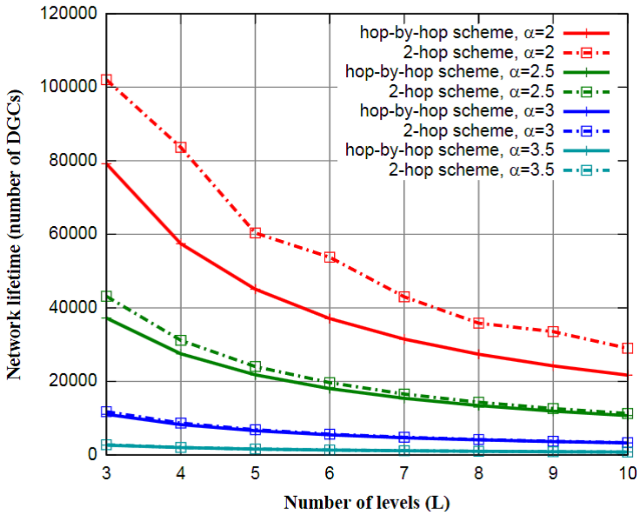

Figure 3.3: Chain network – network lifetime with hop-by-hop scheme and expected network lifetime increase with 2-hop scheme for different path loss exponents

assigning some values for $p_N, p_{N-1}, ..., p_{K+1}$ and find $(p_K, p_{K-1}, ..., p_2)$ to balance energy consumption of $S_K, S_{K-1}, ..., S_1$.

- Network lifetime depends on the number of sensors (N) in the network. When N is small, the lifetime and increase are both large. When N is large, it becomes small.
- Network lifetime and network lifetime increase depends on the environment where the network operates, this environment is expressed by the path loss exponent (α). When α is small, the lifetime and increase are both large. When α is large, they become small.

3.2 Network Lifetime Increase with 2-hop Scheme in Binary Tree Topology Networks

3.2.1 Binary Tree Topology Networks

Let us consider a binary tree network as shown in **Fig. 3.4**, where all sensors form a tree topology of L levels. A sensor in level i has exactly two children in level $i+1$, $i = 1, 2, ..., L-1$. We denote by $S_{i,j}$ the j^{th} sensor in level i (if $j = 1$, then the sensor is the leftmost one in level i). We also denote by $n_{i,j}$ the number of packets transmitted in one DGC by sensor $S_{i,j}$; denote by $f_{1,i,j}$ and $f_{2,i,j}$ the number of packets sent in hop-by-hop and 2-hop transmission, respectively, by sensor $S_{i,j}$.

Let $p_{i,j}$ be the transmission probability of $S_{i,j}$. Because all sensors in a same level are completely identical to each other in terms of the number of children and the initial

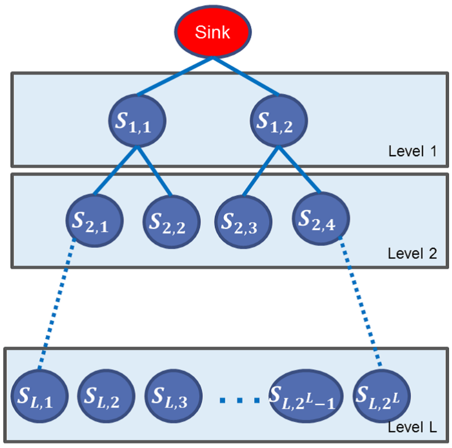

Figure 3.4: A binary tree topology network of L levels of sensors

energy level; therefore, it is reasonable to assume that all the sensors in level i have the same transmission probability p_i. That is, $p_{i,1} = p_{i,2} = ... = p_{i,2^i} = p_i$.

Lemma 3 *In a binary tree network,* $p_i = \frac{E[f_{1,i,j}]}{E[f_{1,i,j}]+E[f_{2,i,j}]}, i = 1, 2, ..., L, j = 1, 2, ..., 2^i$.

Proof. See *Appendix B*

Theorem 4 *In a binary tree network,* $E[f_{1,i,j}] = E[f_{1,i,k}]$ *and* $E[f_{2,i,j}] = E[f_{2,i,k}], i = 1, 2, ..., L; j, k = 1, 2, ..., 2^i$. *This means that, the expected number of packets sent in hop-by-hop and 2-hop transmission of all sensors in the same level are the same.*

Proof. See *Appendix A*

From *Theorem 4*, for simplicity, for now on let us denote by $E[f_{1,i}]$ and $E[f_{2,i}]$ the expected number of packets sent in hop-by-hop and 2-hop transmission of any sensor in level i. We have

$$E[f_{1,i}] = E[f_{1,i,1}] = E[f_{1,i,2}] = ... = E[f_{1,i,2^i}] \tag{3.14}$$

and

$$E[f_{2,i}] = E[f_{2,i,1}] = E[f_{2,i,2}] = ... = E[f_{2,i,2^i}] \tag{3.15}$$

From *Theorem 4* and (3.3), it is obvious that the expected energy consumption of all sensors in the same level is also the same. We denote by $E[\varepsilon_i]$ the expected energy consumption of sensors in level i.

3.2.2 Optimal Transmission Probabilities

For binary tree topology networks, like for chain networks, it can be proved that, if we can find a set of transmission probabilities $(p_L, p_{L-1}, ..., p_3, p_2)$ such that $E[\varepsilon_i] = E[\varepsilon_j] \forall i, j = 1, 2, ..., L$, then the network lifetime is maximized.

Similar to the previous section, we also have $E[\varepsilon_i] = E[\varepsilon_j] \quad \forall i, j = 1, 2, ..., L \Leftrightarrow E[\varepsilon_i] = E[\varepsilon_{i-1}] \quad \forall i = 2, 3, ..., L \Leftrightarrow$

$$\begin{cases} -x_{2L-4} - Cx_{2L-3} + x_{2L-2} + Cx_{2L-1} = 0 \\ -x_{2L-6} - Cx_{2L-5} + x_{2L-4} + Cx_{2L-3} = 0 \\ ... \\ -x_2 - Cx_3 + x_4 + Cx_5 = 0 \\ -x_1 + x_2 + Cx_3 = 0 \end{cases} \tag{3.16}$$

In (3.16), like the previous section, for the sake of writing:

- $E[f_{1,1}]$ is denoted by x_1
- $E[f_{1,i}]$ is denoted by $x_{2i-2}, \quad \forall i = 2, 3, .., L$
- $E[f_{2,i}]$ by $x_{2i-1}, \quad \forall i = 2, 3, .., L$
- $\frac{\epsilon_t(d_2)+\epsilon_r}{\epsilon_t(d_1)+\epsilon_r}$ by C

There are a total of $2L - 1$ variables $x_1, x_2, ..., x_{2L-1}$; however, (3.16) consists of only $L - 1$ equations. Therefore, to solve (3.16), we need L more equations.

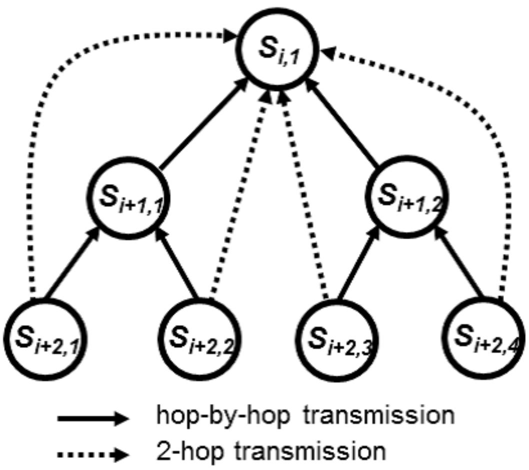

Figure 3.5: The relationship between sensors in levels $i, i + 1$ and $i + 2$

From **Fig. 3.5**, we can see that

$$\begin{gathered} E[f_{1,i}] + E[f_{2,i}] = 2E[f_{1,i+1}] + 4E[f_{2,i+2}] + 1 \\ \Leftrightarrow \\ x_{2i-2} + x_{2i-1} - 2x_{2i} - 4x_{2i+3} = 1 \end{gathered} \tag{3.17}$$

From (3.17), for $i = 2, 3, ..., L$, we have L more equations:

$$\begin{cases} x_{2L-2} + x_{2L-1} = 1 \\ x_{2L-4} + x_{2L-3} - 2x_{2L-2} = 1 \\ x_{2L-6} + x_{2N-5} - 2x_{2N-4} - 4x_{2L-1} = 1 \\ ... \\ x_2 + x_3 - 2x_4 - 4x_7 = 1 \\ x_1 - 2x_2 - 4x_5 = 1 \end{cases} \tag{3.18}$$

Combining (3.16) and (3.18), we get the final system of equations:

$$\begin{cases} -x_{2L-4} - Cx_{2L-3} + x_{2L-2} + Cx_{2L-1} = 0 \\ -x_{2L-6} - Cx_{2L-5} + x_{2L-4} + Cx_{2L-3} = 0 \\ ... \\ -x_2 - Cx_3 + x_4 + Cx_5 = 0 \\ -x_1 + x_2 + Cx_3 = 0 \\ x_{2L-2} + x_{2L-1} = 1 \\ x_{2L-4} + x_{2L-3} - 2x_{2L-2} = 1 \\ x_{2L-6} + x_{2L-5} - 2x_{2L-4} - 4x_{2L-1} = 1 \\ ... \\ x_2 + x_3 - 2x_4 - 4x_7 = 1 \\ x_1 - 2x_2 - 4x_5 = 1 \end{cases} \tag{3.19}$$

If (3.19) has a solution and all the values of x_i are non-negative numbers, then we can calculate p_i based on *Lemma 3*: $p_i = \frac{x_{2i-2}}{x_{2i-2}+x_{2i-1}} \quad \forall i = 2, 3, ..., L$.

3.2.3 Numerical Results and Analysis

In this section, we calculate theoretically the expected increase in network lifetime with 2-hop scheme in a binary tree topology network consisting of L levels. Similar to the previous section, the maximum hop-by-hop transmission range is 20 m, maximum 2-hop transmission range is 40 m, $\epsilon_{elec} = 50\ nJ/bit, \epsilon_{amp} = 100\ pJ/bit/m^{3.5}$, the path loss exponent is $\alpha = 3.5$, the packet size is $m = 1024\ bits$.

Table 3.5 below displays the network lifetime with hop-by-hop scheme and 2-hop scheme in many binary tree networks. Here we denote by K the number of levels that can be balanced energy consumption. For example, $L = 4$ and $K = 3$ means that, even though we can find optimal transmission probabilities to balance all sensors in all four levels, if we assign $p_4 = 0.9$ (all sensors in level four are assigned a transmission probability of 0.9) then we can find p_2 and p_3 to get a balanced energy consumption of all sensors in the first three levels.

From **Fig. 3.6** and **Fig. 3.2** we can see that, the network lifetime depends on the topology. In binary tree topology networks, the lifetime is smaller than that in chain networks. However, the increase in network lifetime is better with 2-hop scheme (As shown in **Table 3.5** (binary tree networks), we can see that the increase is about 12%; In **Table 3.1** (chain networks), the increase is not greater than 4%).

Table 3.5: Binary tree network – expected network lifetime increase with 2-hop scheme ($\alpha = 3.5$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme**	**Expected network lifetime increase**
$\boldsymbol{L = 3}$	$T_{h2h} = 1140$	$E[T_{2hop}] = 1295$ $(K = 3)$	13.59%
$\boldsymbol{L = 4}$	$T_{h2h} = 531$	$E[T_{2hop}] = 598$ $(K = 3)$	12.62%
$\boldsymbol{L = 5}$	$T_{h2h} = 257$	$E[T_{2hop}] = 288$ $(K = 3)$	12.06%
$\boldsymbol{L = 6}$	$T_{h2h} = 126$	$E[T_{2hop}] = 142$ $(K = 4)$	12.70%
$\boldsymbol{L = 7}$	$T_{h2h} = 62$	$E[T_{2hop}] = 70$ $(K = 4)$	12.90%
$\boldsymbol{L = 8}$	$T_{h2h} = 31$	$E[T_{2hop}] = 35$ $(K = 4)$	12.90%
$\boldsymbol{L = 9}$	$T_{h2h} = 15$	$E[T_{2hop}] = 17$ $(K = 4)$	13.33%
$\boldsymbol{L = 10}$	$T_{h2h} = 7$	$E[T_{2hop}] = 8$ $(K = 4)$	14.29%

Now, we come to the analysis of how the path loss exponent α affects the network lifetime and network lifetime increase.

- $\boldsymbol{\alpha = 2}$ (See **Table 3.6**)

Table 3.6: Binary tree network – expected network lifetime increase with 2-hop scheme ($\alpha = 2$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme**	**Expected network lifetime increase**
$\boldsymbol{L = 3}$	$T_{h2h} = 31502$	$E[T_{2hop}] = 56514$ $(K = 2)$	79.40%
$\boldsymbol{L = 4}$	$T_{h2h} = 14291$	$E[T_{2hop}] = 24189$ $(K = 2)$	69.26%
$\boldsymbol{L = 5}$	$T_{h2h} = 6829$	$E[T_{2hop}] = 11286$ $(K = 2)$	65.27%
$\boldsymbol{L = 6}$	$T_{h2h} = 3340$	$E[T_{2hop}] = 5460$ $(K = 2)$	63.47%
$\boldsymbol{L = 7}$	$T_{h2h} = 1652$	$E[T_{2hop}] = 2686$ $(K = 2)$	62.59%
$\boldsymbol{L = 8}$	$T_{h2h} = 821$	$E[T_{2hop}] = 1332$ $(K = 2)$	62.24%
$\boldsymbol{L = 9}$	$T_{h2h} = 409$	$E[T_{2hop}] = 663$ $(K = 2)$	62.10%
$\boldsymbol{L = 10}$	$T_{h2h} = 204$	$E[T_{2hop}] = 331$ $(K = 2)$	62.25%

- $\boldsymbol{\alpha = 2.5}$ (See **Table 3.7**)
- $\boldsymbol{\alpha = 3}$ (See **Table 3.8**)

We can see that (**Fig. 3.7**), if α is small, which means that 2-hop transmission does not consume too much energy, 2-hop scheme can increase network lifetime quite large. For example, when $\alpha = 2$, even though energy consumption cannot be balanced for all sensors in the binary tree network consisting of three levels sensors, the increase in network lifetime can be quite large, 79.4%.

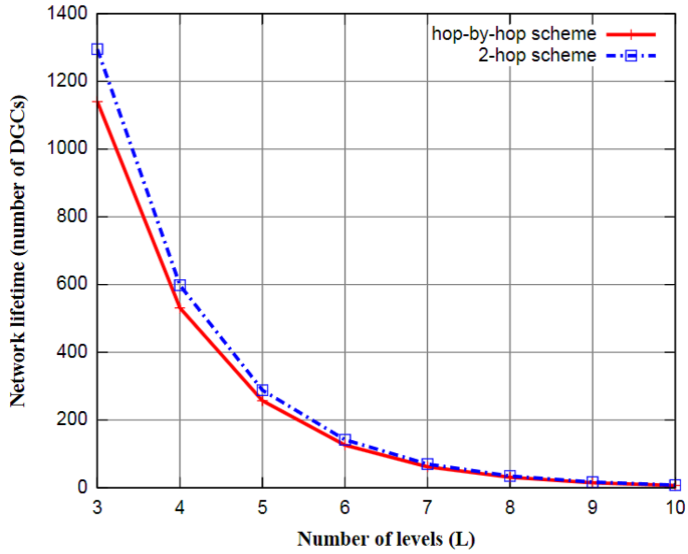

Figure 3.6: Binary tree network – expected network lifetime increase with 2-hop scheme ($\alpha = 3.5$)

3.2.4 Conclusion

There are some main results in this section, some of them are similar to what stated in the previous section.

- In some cases, we cannot find the optimal transmission probabilities to achieve balanced energy consumption throughout all sensors in the network. For example, when ($\alpha = 3.5, d_1 = 20\ m, d_2 = 40\ m, \epsilon_{elec} = 50\ nJ/bit, \epsilon_{amp} = 100\ pJ/bit/m^{3.5}$), we can find optimal transmission probabilities for chain network consisting of up to only three levels of sensors. In that cases, we can find "good" transmission probabilities by assigning some values for $p_L, p_{L-1}, ..., p_{K+1}$ for sensors in levels $L, L-1, ..., K+1$ and find $(p_K, p_{K-1}, ..., p_2)$ to balance energy consumption of sensors in levels $K, K-1, ..., 1$.

- Network lifetime depends on the number of levels of sensors (L) in the network. When L is small, the lifetime is large. When L is large, it becomes small.

- Network lifetime and network lifetime increase depends on the environment where the network operates, this environment is expressed by the path loss exponent (α). When α is small, the lifetime and increase are both large. When α is large, they become small.

- Network lifetime increase depends on the topology of the network. For example, with $\alpha = 3.5$, in chain network, the increase is not greater than 4.0%; however, in binary tree network, the increase could be more than 12%.

Table 3.7: Binary tree network – expected network lifetime increase with 2-hop scheme ($\alpha = 2.5$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme**	**Expected network lifetime increase**
$\boldsymbol{L=3}$	$T_{h2h} = 15401$	$E[T_{2hop}] = 20452$ $(K = 2)$	32.80%
$\boldsymbol{L=4}$	$T_{h2h} = 7088$	$E[T_{2hop}] = 9224$ $(K = 2)$	30.14%
$\boldsymbol{L=5}$	$T_{h2h} = 3408$	$E[T_{2hop}] = 5108$ $(K = 3)$	49.88%
$\boldsymbol{L=6}$	$T_{h2h} = 1672$	$E[T_{2hop}] = 2485$ $(K = 3)$	48.62%
$\boldsymbol{L=7}$	$T_{h2h} = 828$	$E[T_{2hop}] = 1200$ $(K = 3)$	44.93%
$\boldsymbol{L=8}$	$T_{h2h} = 412$	$E[T_{2hop}] = 608$ $(K = 3)$	47.57%
$\boldsymbol{L=9}$	$T_{h2h} = 205$	$E[T_{2hop}] = 303$ $(K = 3)$	47.80%
$\boldsymbol{L=10}$	$T_{h2h} = 102$	$E[T_{2hop}] = 151$ $(K = 3)$	48.04%

Table 3.8: Binary tree network – expected network lifetime increase with 2-hop scheme ($\alpha = 3$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme**	**Expected network lifetime increase**
$\boldsymbol{L=3}$	$T_{h2h} = 4687$	$E[T_{2hop}] = 5805$ $(K = 3)$	23.85%
$\boldsymbol{L=4}$	$T_{h2h} = 2178$	$E[T_{2hop}] = 2656$ $(K = 3)$	21.95%
$\boldsymbol{L=5}$	$T_{h2h} = 1051$	$E[T_{2hop}] = 1273$ $(K = 3)$	21.12%
$\boldsymbol{L=6}$	$T_{h2h} = 517$	$E[T_{2hop}] = 624$ $(K = 3)$	20.70%
$\boldsymbol{L=7}$	$T_{h2h} = 256$	$E[T_{2hop}] = 309$ $(K = 3)$	20.70%
$\boldsymbol{L=8}$	$T_{h2h} = 127$	$E[T_{2hop}] = 153$ $(K = 3)$	20.47%
$\boldsymbol{L=9}$	$T_{h2h} = 63$	$E[T_{2hop}] = 76$ $(K = 3)$	20.63%
$\boldsymbol{L=10}$	$T_{h2h} = 31$	$E[T_{2hop}] = 38$ $(K = 3)$	22.58%

3.3 Network Lifetime Increase with Direct Scheme and 2-hop Scheme

We now compare the performance of direct and 2-hop schemes in terms of network lifetime and data collection delay. First of all, let us explain briefly about the direct scheme proposed by Zhang *et al.* [10].

3.3.1 Direct Scheme

The direct scheme is the combination of direct transmission (data is sent directly to the sink) and hop-by-hop transmission to balance energy consumption among sensors in the network. More specifically, see **Fig. 3.8**, a packet is forwarded to the next hop with transmission probability p or directly transmitted to the sink with probability $1 - p$.

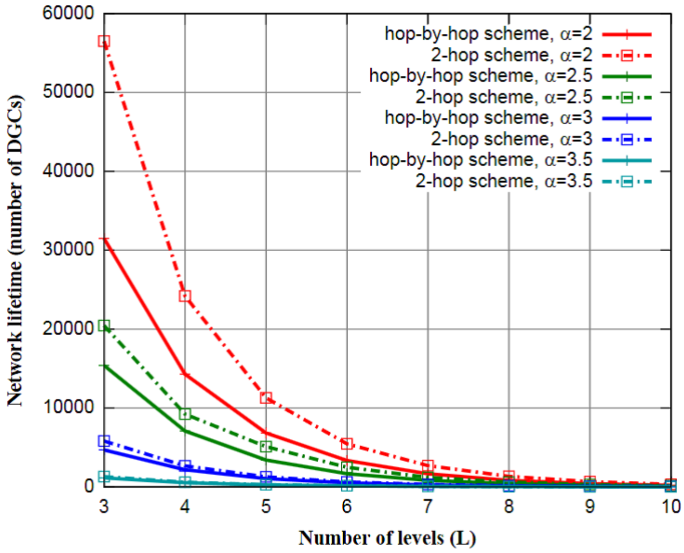

Figure 3.7: Binary tree network – network lifetime with hop-by-hop scheme and expected network lifetime with 2-hop scheme for different path loss exponents

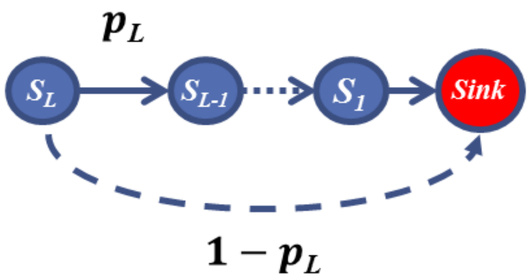

Figure 3.8: Direct scheme

By assigning each sensor an optimal transmission probability, balanced network consumption among all sensors throughout the network can be achieved.

3.3.2 Network Lifetime with Direct Scheme and 2-hop Scheme

We analyze the two schemes in an environment where path loss exponent $\alpha = 3.5$, maximum hop-by-hop transmission range is $d_1 = 20\ m$, maximum 2-hop transmission range is $d_2 = 2d_1 = 40\ m$, the direct transmission range for sensor S_i is $d_i = id_1 = 20i\ m$. $\epsilon_{elec} = 50\ nJ/bit$ and $\epsilon_{amp} = 100\ pJ/bit/m^{3.5}$.

- In Chain Topology Networks We have known that the optimal transmission probabilities to balance energy consumption exists for chain network consisting of up to eleven sensors. On the other hand, in [10], Zhang *et al.* has proved that the optimal transmission probabilities to achieve balanced energy consumption always exist regardless how many sensors in the network. This means that direct scheme has a better capability to balance energy consumption than 2-hop scheme.

The network lifetime with direct scheme and hop-by-hop scheme is shown in **Table 3.9**

Table 3.9: Chain network – network lifetime with 2-hop scheme and direct scheme

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with direct scheme (optimal transmission probabilities)**	**Expected increase in network lifetime**
$\boldsymbol{L = 3}$	$T_{h2h} = 2667$	$E[T_{direct}] = 2791$	4.65%
$\boldsymbol{L = 4}$	$T_{h2h} = 1998$	$E[T_{direct}] = 2079$	4.05%
$\boldsymbol{L = 5}$	$T_{h2h} = 1597$	$E[T_{direct}] = 1653$	3.51%
$\boldsymbol{L = 6}$	$T_{h2h} = 1330$	$E[T_{direct}] = 1371$	3.08%
$\boldsymbol{L = 7}$	$T_{h2h} = 1140$	$E[T_{direct}] = 1171$	2.72%
$\boldsymbol{L = 8}$	$T_{h2h} = 997$	$E[T_{direct}] = 1021$	2.41%
$\boldsymbol{L = 9}$	$T_{h2h} = 886$	$E[T_{direct}] = 906$	2.26%
$\boldsymbol{L = 10}$	$T_{h2h} = 797$	$E[T_{direct}] = 813$	2.01%

For networks of $N = 3, 4, ..., 10$ sensors, we can find optimal transmission probabilities for 2-hop scheme. Then, we compare the network lifetime between 2-hop scheme (using 2-hop optimal transmission probabilities) and direct scheme (using direct optimal transmission probabilities, those optimal probabilities are found based on an algorithm in [10]). The result is shown in following **Fig. 3.9**.

As we can see from **Fig. 3.9**, the expected network lifetime with 2-hop scheme is a little bit smaller than that with direct scheme.

- In Binary Tree Topology Networks From **Table 3.5**, we can see that, when the path loss exponent $\alpha = 3.5$, 2-hop scheme optimal transmission probabilities only exist for a binary tree network consisting of $L = 3$ levels. On the other hand, direct scheme optimal transmission probabilities exist for any binary tree with arbitrary levels. Therefore, we can say that, direct scheme has a better capability to balance energy consumption than 2-hop scheme.

  The network lifetime with direct scheme and hop-by-hop scheme is shown in **Table 3.10**

  For a binary tree of $L = 3$ levels of sensors, we now compare network lifetime between 2-hop scheme (using 2-hop optimal transmission probabilities) and direct scheme (using direct optimal transmission probabilities): with 2-hop scheme, network lifetime is increased about 13.59% compared to hop-by-hop scheme; with direct scheme, network lifetime is increased about 18.07%. This means that the lifetime is better with direct scheme than 2-hop scheme.

  For $L \geq 4$, because 2-hop scheme optimal transmission probabilities do not exist, the network lifetime comparison between 2-hop scheme and direct scheme is not fair.

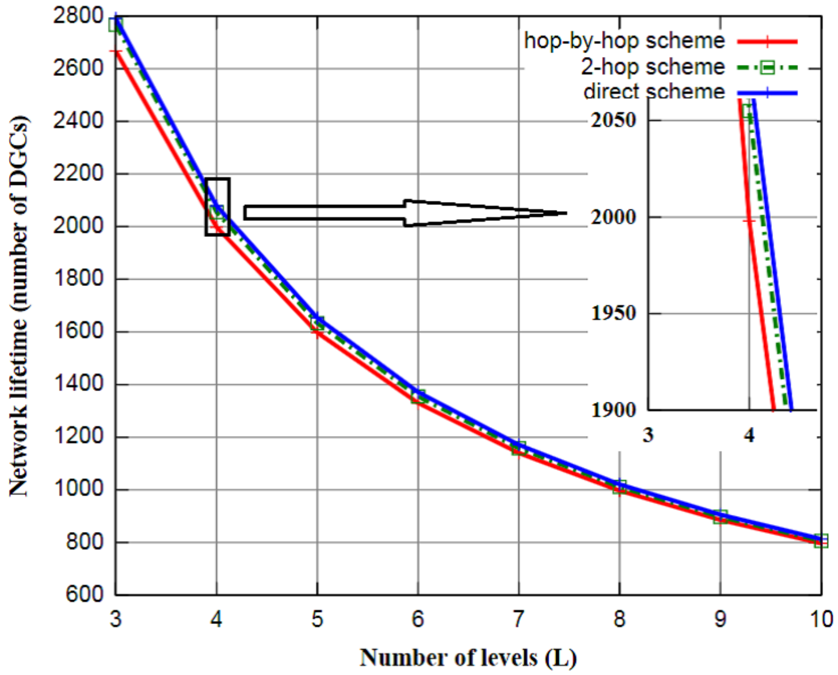

Figure 3.9: Chain network – network lifetime with hop-by-hop scheme, direct scheme and 2-hop scheme

However, until now we can see that, when 2-hop optimal transmission probabilities exist, the lifetime with 2-hop scheme is smaller than that with direct scheme.

3.3.3 Conclusion

There are some main points in this section:

- Direct scheme has better capability to balance energy consumption throughout the whole network (direct optimal transmission probabilities always exist). On the other hand, with 2-hop scheme, optimal transmission probabilities do not always exist.
- With optimal transmission probabilities, for the same topology, network lifetime with direct scheme is better than that with 2-hop scheme.

3.4 Data Collection Delay with Hop-by-hop Scheme and 2-hop Scheme

The data collection delay in one DGC is defined as the time for all packets from all sensors to be received by the sink. In this research, to avoid signal interference, at one time, only one sensor transmits data.

Table 3.10: Binary tree network – network lifetime with 2-hop scheme and direct scheme

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with direct scheme (optimal transmission probabilities)**	**Expected increase in network lifetime**
$\boldsymbol{L=3}$	$T_{h2h} = 1140$	$E[T_{direct}] = 1346$	18.07%
$\boldsymbol{L=4}$	$T_{h2h} = 531$	$E[T_{direct}] = 652$	22.79%
$\boldsymbol{L=5}$	$T_{h2h} = 257$	$E[T_{direct}] = 327$	27.24%
$\boldsymbol{L=6}$	$T_{h2h} = 126$	$E[T_{direct}] = 168$	33.33%
$\boldsymbol{L=7}$	$T_{h2h} = 62$	$E[T_{direct}] = 87$	40.32%
$\boldsymbol{L=8}$	$T_{h2h} = 31$	$E[T_{direct}] = 46$	48.39%
$\boldsymbol{L=9}$	$T_{h2h} = 15$	$E[T_{direct}] = 25$	66.67%
$\boldsymbol{L=10}$	$T_{h2h} = 7$	$E[T_{direct}] = 13$	85.71%

3.4.1 Data Collection Delay with Hop-by-hop Scheme

Let us consider a chain network consisting of four sensors S_4, S_3, S_2 and S_1 (see **Fig. 3.10**). In one DGC, S_i generates one packet pkt_i, $i = 1, 2, 3, 4$. With hop-by-hop scheme, let us assume that the order of transmission is like this:

1. S_4 forwards pkt_4 to S_3
2. S_3 forwards pkt_3 and pkt_4 to S_2
3. S_2 forwards pkt_2, pkt_3 and pkt_4 to S_1
4. S_2 forwards pkt_1, pkt_2, pkt_3 and pkt_4 to the sink

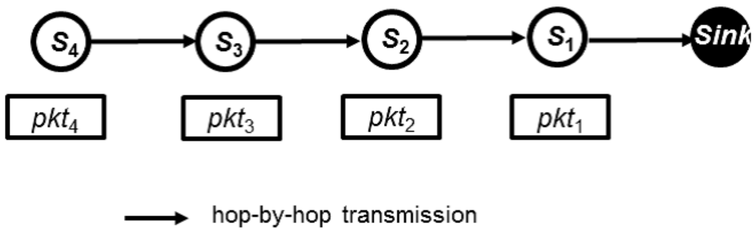

Figure 3.10: A chain topology network consisting of 4 sensors

We denote by τ the transmission time for one packet. The data collection delay, denoted by D, in this case is

$$
\begin{aligned}
D = {} & \tau\ (pkt_4,\ S_4 \to S_3) + \\
& \tau\ (pkt_3,\ S_3 \to S_2) + \tau\ (pkt_4,\ S_3 \to S_2) + \\
& \tau\ (pkt_2,\ S_2 \to S_1) + \tau\ (pkt_3,\ S_2 \to S_1) + \tau\ (pkt_4,\ S_2 \to S_1) \\
& \tau\ (pkt_1,\ S_1 \to sink) + \tau\ (pkt_2,\ S_1 \to sink) + \tau\ (pkt_3,\ S_1 \to sink) + \\
& \tau\ (pkt_4,\ S_1 \to sink) \\
= {} & \tau \sum (number\ of\ transmissions\ of\ each\ sensor) \\
= {} & \tau \sum_{i=1}^{4} n_{i,h2h}
\end{aligned}
\tag{3.20}
$$

In (3.20), $n_{i,h2h}$ denotes the total number of packets sensor S_i transmits in one DGC with hop-by-hop scheme. We now come to a general equation for data collection delay in chain network.

$$
D_{h2h} = \tau \sum_{i=1}^{N} n_{i,h2h} \tag{3.21}
$$

where N is the total number of sensors in the network.

3.4.2 Data Collection Delay with 2-hop Scheme

With 2-hop scheme the data collection delay may be different between different DGC. This is because sometimes a packet is transmitted in 2-hop transmission but sometimes in hop-by-hop transmission. To illustrate this, let us consider the following order of transmission in two different DGCs:

- DGC 1

  1. S_4 forwards pkt_4 to S_3 in hop-by-hop transmission
  2. S_3 forwards pkt_3 and pkt_4 to S_2 in hop-by-hop transmission
  3. S_2 forwards pkt_2, pkt_3 and pkt_4 to S_1 in hop-by-hop transmission
  4. S_2 forwards pkt_1, pkt_2, pkt_3 and pkt_4 to the sink in hop-by-hop transmission

  Data collection delay in DGC 1 is:

$$
\begin{aligned}
D_1 &= \tau \sum (number\ of\ transmissions\ of\ each\ sensor) \\
&= \tau\,[1\ (of\ S_4) + 2\ (of\ S_3) + 3\ (of\ S_2) + 4\ (of\ S_1)] \\
&= 10\tau
\end{aligned}
\tag{3.22}
$$

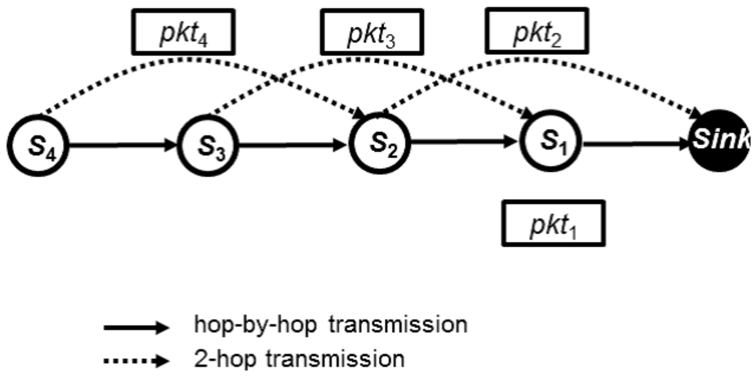

Figure 3.11: A chain topology network consisting of 4 sensors

- DGC 2 (see **Fig. 3.11**)
  1. S_4 forwards pkt_4 to S_2 in 2-hop transmission
  2. S_3 forwards pkt_3 to S_1 in 2-hop transmission
  3. S_2 forwards pkt_2 to the sink in 2-hop transmission, and pkt_4 to S_1 in hop-by-hop transmission
  4. S_2 forwards pkt_1, pkt_3 and pkt_4 to the sink in hop-by-hop transmission

  We assume that the transmission time for one packet in 2-hop transmission and hop-by-hop are the same. Data collection delay in DGC 2 is:

$$\begin{aligned} D_2 &= \tau \sum (number\ of\ transmissions\ of\ each\ sensor) \\ &= \tau \left[1\ (of\ S_4) + 1\ (of\ S_3) + 2\ (of\ S_2) + 3\ (of\ S_1)\right] \\ &= 7\tau \end{aligned} \tag{3.23}$$

We can see that, the more 2-hop transmissions, the smaller the data collection delay. The *expected* data collection delay in one DGC with 2-hop scheme is:

$$E[D_{2hop}] = \tau \sum_{i=1}^{N} E[n_{i,2hop}] \tag{3.24}$$

where $n_{i,2hop}$ is the number of packets S_i transmits in one DGC with 2-hop scheme.

3.4.3 Data Collection Delay Comparison between Hop-by-hop Scheme and 2-hop Scheme

We can see that, with hop-by-hop scheme, all the packets from $S_{i+1}, S_{i+2}, ..., S_N$ are forwarded to S_i; however, with 2-hop scheme, some of the packets are forwarded to S_{i-1}

by S_{i+1}, thus the number of packets S_i transmits is expected to be reduced compared to hop-by-hop scheme. To illustrate this, let us take a look at sensors S_3 and S_4 in **Fig. 3.11**. In a DGC:

- With hop-by-hop scheme, $n_{3,h2h} = 2$
- With 2-hop scheme
  - if S_4 forwards pkt_4 to S_3 in hop-by-hop transmission, then $n_{3,2hop} = 2$ (pkt_3 and pkt_4)
  - if S_4 does not forward pkt_4 to S_3, but forwards to S_2 in 2-hop transmission, then $n_{3,2hop} = 1$ (pkt_3 only)

  We can see that $n_{3,2hop} \leq n_{3,h2h}$, hence

$$E[n_{3,2hop}] < n_{3,h2h} \tag{3.25}$$

Generalizing (3.25), we have

$$E[n_{i,2hop}] < n_{i,h2h} \quad \forall i = 1, 2, ..., N \tag{3.26}$$

Therefore, from (3.21), (3.24) and (3.26), we have

$$D_{2hop} < D_{h2h} \tag{3.27}$$

(3.27) shows us that, the delay in data collection can be decreased with 2-hop scheme compared to that with hop-by-hop scheme.

3.4.4 Numerical Results for Chain Topology Networks

Let us consider a WSN where each sensor transmits a packet of 1024 *bits* in hop-by-hop and 2-hop transmission at a transmission rate of $R = 250\ kbps$. The transmission time of one packet is $\tau = \frac{1024\ bits}{250\ kbps} = 4.096\ ms$.

The other configuration settings is similar to the previous section. More specifically:

* Maximum hop-by-hop transmission range, $d_1 = 20\ m$
* Maximum 2-hop transmission range, $d_2 = 2d_1 = 40\ m$
* Path loss exponent, $\alpha = 3.5$
* $\epsilon_{elec} = 50\ nJ/bit$
* $\epsilon_{amp} = 100\ pJ/bit/m^{\alpha} = 100\ pJ/bit/m^{3.5}$

- $\boldsymbol{N = 3}$ **(A chain network consisting of three sensors)**

- **hop-by-hop scheme**

  In one DGC, it is easy to see that

  * $n_{3,h2h} = 1$
  * $n_{2,h2h} = 2$
  * $n_{1,h2h} = 3$

  Then

  $$D_{h2h} = \tau \sum_{i=1}^{3} n_{i,h2h} = 4.096(1 + 2 + 3) = 24.576 \ ms \tag{3.28}$$

- **2-hop scheme**

  In the previous section, the optimal transmission probabilities have been found to be

  * $p_2 = 0.940527402469174$
  * $p_3 = 0.811400340823007$

  Then the expected number of packets each sensor transmits in one DGC is:

  * $E[n_{3,2hop}] = 1$
  * $E[n_{2,2hop}] = E[f_{1,3}] + 1 = p_3 E[n_{3,2hop}] + 1 \approx 1.8114$
  * $E[n_{1,2hop}] = E[f_{1,2}] + E[f_{2,3}] + 1 = p_2 E[n_{2,2hop}] + (1 - p_3) E[n_{3,2hop}] + 1 \approx 2.8923$

  Then

  $$E[D_{2hop}] = \tau \sum_{i=1}^{3} E[n_{i,2hop}] \approx 4.096(1 + 1.8114 + 2.8923) = 23.362 \ ms \tag{3.29}$$

  From (3.28) and (3.29), the expected decrease in data collection delay is $\frac{D_{h2h} - E[D_{2hop}]}{D_{h2h}} = \frac{24.576 - 23.362}{24.576} \approx 4.94\%$

- ***N* = 4, 5, ..., 11**

  With similar calculations, we can get the following results: (see **Table 3.11**). In 2-hop scheme, we analyze the data collection delay with optimal transmission probabilities. We want to know how much the delay of data collection could be decreased when the network lifetime is maximized.

  We can see that, when the number of sensors in the network increases, the delay in data collection also increases (with both hop-by-hop scheme and 2-hop scheme), see **Fig. 3.12**. This is because the more sensors in the network, the more number of transmission in one DGC

Table 3.11: Chain network – expected data collection delay with 2-hop scheme

	Data collection delay with hop-by-hop scheme (ms)	**Expected data collection delay with 2-hop scheme, with optimal transmission probabilities (ms)**	**Expected decrease in data collection delay**
$N = 4$	$D_{h2h} = 40.96$	$E[D_{2hop}] = 38.439$	6.15%
$N = 5$	$D_{h2h} = 61.44$	$E[D_{2hop}] = 57.157$	6.97%
$N = 6$	$D_{h2h} = 86.016$	$E[D_{2hop}] = 79.516$	7.56%
$N = 7$	$D_{h2h} = 114.688$	$E[D_{2hop}] = 105.517$	8.00%
$N = 8$	$D_{h2h} = 147.456$	$E[D_{2hop}] = 135.161$	8.34%
$N = 9$	$D_{h2h} = 184.32$	$E[D_{2hop}] = 168.448$	8.61%
$N = 10$	$D_{h2h} = 225.28$	$E[D_{2hop}] = 205.377$	8.83%
$N = 11$	$D_{h2h} = 270.336$	$E[D_{2hop}] = 245.949$	9.02%

3.4.5 Numerical Results for Binary Tree Topology Networks

We assume that the transmission rate for 2-hop transmission is the same as section *Numerical Results for Chain Topology Networks* above. Thus the transmission time for one packet is also $\tau = 4.096$ ms. The other settings like maximum hop-by-hop transmission range, maximum 2-hop transmission range, path loss exponent, etc. are also assumed to be the same as in the previous section.

In the previous section, we analyzed the increase in network lifetime with "good" optimal transmission probabilities. Now, also with those "good" transmission probabilities, we analyze the delay in data collection (**Table 3.12**).

Table 3.12: Binary tree network – expected data collection delay with 2-hop scheme

	Data collection delay with hop-by-hop scheme (ms)	**Expected data collection delay with 2-hop scheme, with "good" transmission probabilities (ms)**	**Expected decrease in data collection delay**
$L = 3$	$D_{h2h} = 139.264$	$E[D_{2hop}] = 115.55$	17.03%
$L = 4$	$D_{h2h} = 401.408$	$E[D_{2hop}] = 346.743$	13.62%
$L = 5$	$D_{h2h} = 1056.768$	$E[D_{2hop}] = 928.540$	12.13%
$L = 6$	$D_{h2h} = 2629.632$	$E[D_{2hop}] = 2019.495$	23.20%
$L = 7$	$D_{h2h} = 6299.648$	$E[D_{2hop}] = 4993.76$	20.73%
$L = 8$	$D_{h2h} = 14688.256$	$E[D_{2hop}] = 11895.561$	19.01%
$L = 9$	$D_{h2h} = 33562.624$	$E[D_{2hop}] = 27605.662$	17.75%
$L = 10$	$D_{h2h} = 75505.664$	$E[D_{2hop}] = 69334.166$	8.17%

The delay in data collection is illustrated in **Fig. 3.13** below. We can see that, similar to chain network, the more levels the tree has, the more delay in data collection. This is because the more levels, the more sensors in the network and the more number of transmission in one DGC.

3.4.6 Data Collection Delay with Hop-by-hop, 2-hop and Direct Scheme as a Function of Transmission Probabilities

In this section, we analyze with different transmission probabilities, how data collection delay varies. All sensors are assigned the *same* transmission probabilities. The result is shown in **Fig. 3.14** (for chain network consisting of ten sensors) and **Fig. 3.15** (for binary tree network of ten levels of sensors).

From **Fig. 3.14** and **Fig. 3.15**, we can see that when the transmission probability p increases, which means that the more hop-by-hop transmission, data collection delay with 2-hop scheme also increases (data collection is slower). On the other hand, when p is small (the more 2-hop transmission), data collection delay with 2-hop scheme becomes smaller (data collection is faster).

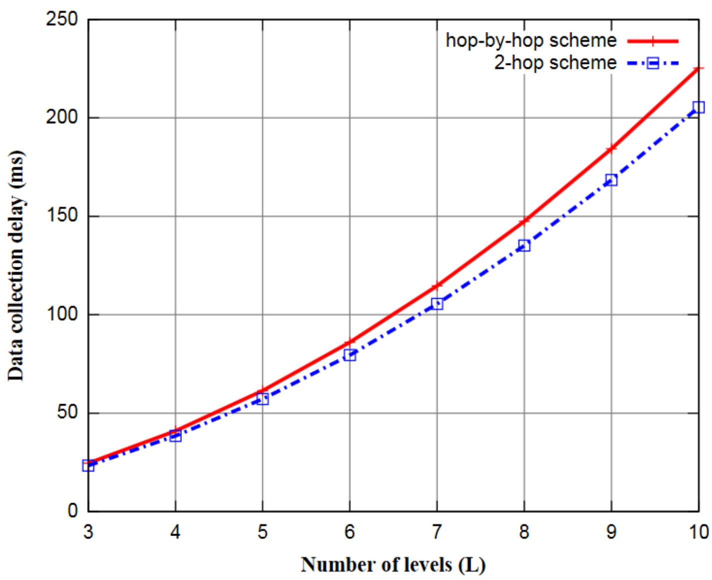

Figure 3.12: Chain network – data collection delay with 2-hop scheme and hop-by-hop scheme

3.4.7 Conclusion

There are some main results in this section:

- If the 2-hop transmission rate is assumed to be the same as hop-by-hop transmission rate, then data collection delay with 2-hop scheme can be proved to be smaller than that with hop-by-hop scheme.

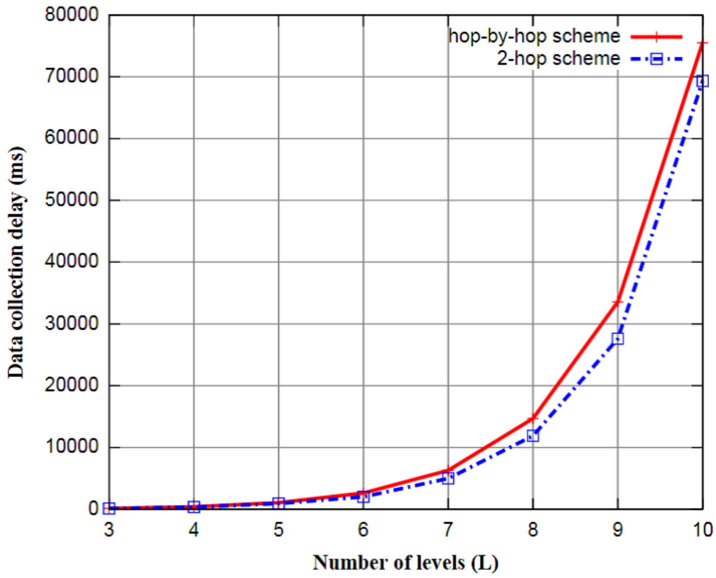

Figure 3.13: Binary tree network – expected decrease in data collection delay with 2-hop scheme over hop-by-hop scheme

- With the same transmission probabilities, data collection with direct scheme is fastest (smallest delay) compared to 2-hop scheme and hop-by-hop scheme.

3.5 2-hop Scheme when Initial Battery Levels are Different

We have already discussed about how to compute the optimal transmission probabilities for sensors in the network when the initial battery levels are the same. The condition for finding optimal probabilities then was:

$$E[\varepsilon_1] = E[\varepsilon_2] = ... = E[\varepsilon_N] \tag{3.30}$$

If we can find $p_i, i = 2, 3, ..., N$ satisfying (3.30), then the network lifetime is maximized with 2-hop scheme (if the initial battery levels are the same).

However; when the initial battery levels are different, probabilities satisfying (3.30) may not be optimal. For example, let us consider a chain network consisting of three sensors S_1, S_2 and S_3. The initial battery level in each sensor is (here we denote by B_i the initial battery level of S_i, $i = 1, 2, 3$):

- $B_1 = 3.806\ J$
- $B_2 = 41.708\ J$
- $B_3 = 5.308\ J$

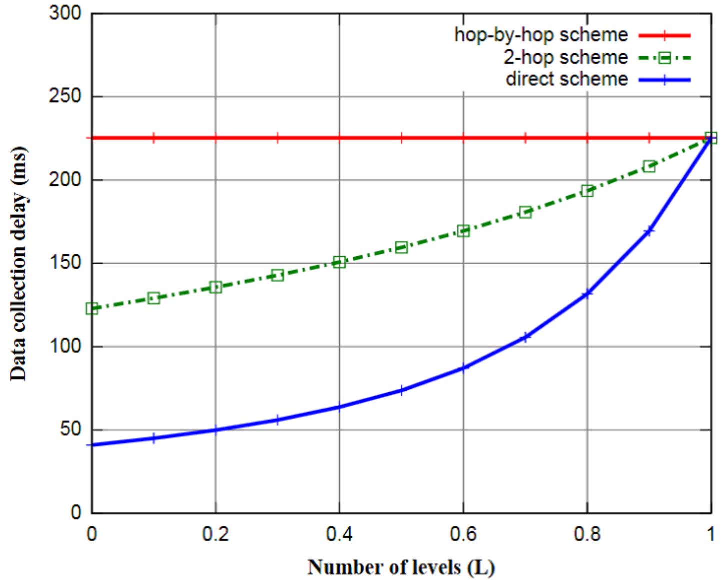

Figure 3.14: Chain network of 10 sensors – data collection delay with hop-by-hop scheme, 2-hop scheme and direct scheme as a function of transmission probabilities

The other parameters, for example, maximum hop-by-hop transmission range, maximum 2-hop transmission range, $\epsilon_{elec}, \epsilon_{amp}$, etc. are the same as in the previous section.

- **hop-by-hop scheme**

  As shown above, energy consumption of each sensor is:

  * $E[\varepsilon_1] = 11.246721323006968\ mJ$
  * $E[\varepsilon_2] = 7.480747548671312\ mJ$
  * $E[\varepsilon_3] = 3.7147737743356557\ mJ$

  Because the initial battery levels are different, then the number of DGCs can be done by each sensor until it runs out of battery may also be different.

  * For S_1, $\frac{3806\ mJ}{11.246721323006968\ mJ} = 338$
  * For S_2, $\frac{41708\ mJ}{7.480747548671312\ mJ} = 5575$
  * For S_3, $\frac{5308\ mJ}{3.7147737743356557\ mJ} = 1428$

  Because the network lifetime is represented by the number of DGCs done until one sensor runs out of battery, and as we can see above, S_1 runs out of battery after 351 DGCs, then the network lifetime is 338.

- **2-hop scheme**

  The optimal probabilities for a chain network of three sensors when the initial battery levels are the same are:

  * $p_2 = 0.940527402469174$

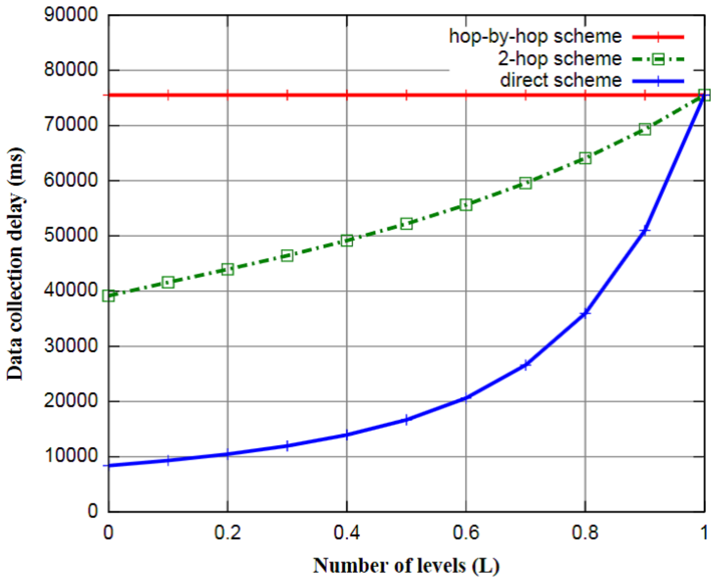

Figure 3.15: Binary tree network of 10 levels – data collection delay with hop-by-hop scheme, 2-hop scheme and direct scheme as a function of transmission probabilities

* $p_3 = 0.811400340823007$

With ($p_2 = 0.940527402469174, p_3 = 0.811400340823007$), the energy consumption in one DGC is balanced throughout three sensors: $E[\varepsilon_1] = E[\varepsilon_2] = E[\varepsilon_3] = 10.841017926439638$.

Because the initial battery levels are different, then the number of DGCs can be done by each sensor until it runs out of battery may also be different.

* For S_1, $\frac{3806\ mJ}{10.841017926439638\ mJ} = 351$
* For S_2, $\frac{41708\ mJ}{10.841017926439638\ mJ} = 3847$
* For S_3, $\frac{5308\ mJ}{10.841017926439638\ mJ} = 489$

Network lifetime is represented by the number of DGCs done until one sensor runs out of battery, and as we can see above, S_1 runs out of battery after 351 DGCs, then the network lifetime is 351. Compared to the lifetime of 338 with hop-by-hop scheme, the increase in network lifetime is 3.85%.

However, for another set of probabilities:

* $p_2 = 0.221279726078279$
* $p_3 = 0.887479063586813$

using similar computation like above, the network lifetime is now increased to 666 (number of DGCs done until one sensor runs out of battery) and the increase in network lifetime compared to hop-by-hop scheme is now 97.04%

3.5.1 Computation of Optimal Transmission Probabilities

Using the following *Theorem 5*, we can compute optimal probabilities for each sensor in the network:

Theorem 5 *For a network of N sensors $S_1, S_2, ..., S_N$, the initial battery level of sensor S_i is denoted by B_i. The expected energy consumption of S_i in one DGC is denoted by $E[\varepsilon_i]$, $i = 1, 2, ..., N$. With 2-hop scheme, if we can find $(p_N, p_{N-1}, ..., p_3, p_2)$ such that $\frac{B_1}{E[\varepsilon_1]} = \frac{B_2}{E[\varepsilon_2]} = ... = \frac{B_N}{E[\varepsilon_N]}$, then the network lifetime is maximized.*

Proof See *Appendix A*

We can see that the condition for finding optimal transmission probabilities is

$$\frac{B_1}{E[\varepsilon_1]} = \frac{B_2}{E[\varepsilon_2]} = ... = \frac{B_N}{E[\varepsilon_N]} \tag{3.31}$$

To find p_i satisfying (3.31), we also solve a system of equations similar to the one shown above. If the solved solution is valid (contains only non-negative values), then we the optimal probabilities exist and with that However, similar to the case when initial battery levels are the same, the probabilities satisfying (3.31) do not always exist. In those cases, we propose a method for finding "good" probabilities. The idea behind the method is very simple: as we have known, the network lifetime is constrained to the sensor that has minimum lifetime in the network, so if we can prolong the lifetime of that sensor, then the overall lifetime of the network can also be increased. The detailed algorithm is described as following:

Algorithm: Finding "good" transmission probabilities when the initial battery levels are different

1. Finding sensor S_i that has minimum lifetime with hop-by-hop scheme, that is $\frac{B_i}{\varepsilon_i} = \min_{1 \le k \le N} \{\frac{B_k}{\varepsilon_k}\}$

2. If $(i = N)$ then we conclude that, sensor S_N is always the first one running out of battery. There is no way to prolong network lifetime with 2-hop scheme.

3. If $(i < N)$ then we assign $p_N = p_{N-1} = ... = p_{i+2} = p_{i-1} = p_{i-2} = ... = p_2 = p_1 = 1$, then find p_{i+1} and p_i making $\frac{B_{i+1}}{E[\varepsilon_{i+1}]} = \frac{B_i}{E[\varepsilon_i]}$

3.5.2 Theoretical Results

Now we will analyze the network lifetime when initial battery levels are different with hop-by-hop and 2-hop scheme. We will analyze for chain networks and binary networks. For each network, we generate random initial battery level for a sensor using the following formula:

$$B_i = 60 * rand(0, 1) \tag{3.32}$$

where $rand(0, 1)$ represents a random numbers in the range $(0, 1)$. If we use the C++ randomly generating function, then the average of B_i is about 0.5, which means the average battery level of the network is about 30 J. After generating randomly initial battery levels, we find optimal transmission probabilities for each sensor; in cases when optimal probabilities do not exist, we find "good" probabilities based on the algorithm above.

The network lifetime is shown in **Fig. 3.16** (chain networks) and **Fig. 3.17** (binary tree networks). We can see that, the increase in network lifetime is better when the initial battery levels are different than that when initial battery levels are the same.

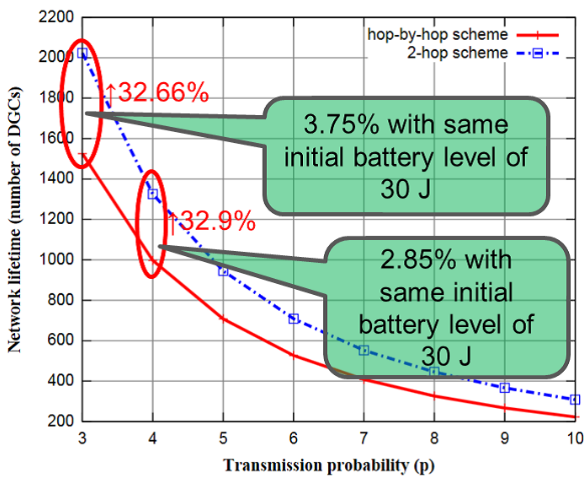

Figure 3.16: Chain network – network lifetime with 2-hop scheme and hop-by-hop scheme as a function of number of levels when initial battery levels are different

3.5.3 Conclusion

In this section, we proposed a method to find optimal transmission probabilities when the initial battery levels are different, and an algorithm to find "good" probabilities when optimal solutions do not exist for chain and binary tree networks.

Theoretical results show that, the increase in network lifetime is better when initial battery levels are different (e.g., for chain network consisting of three sensors, the increase is about 30%, compared to just about 4% when initial battery levels are the same).

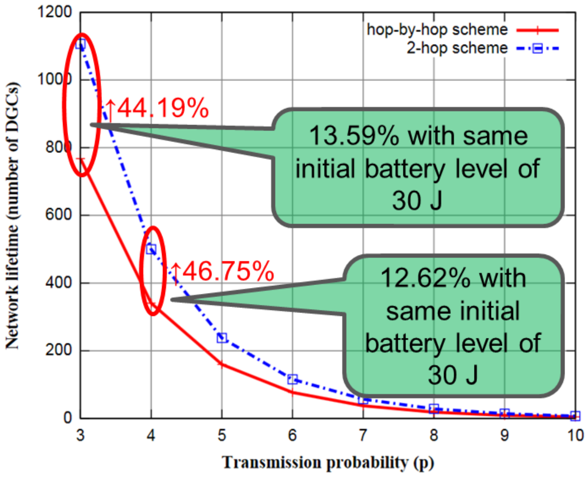

Figure 3.17: Binary tree network – network lifetime with 2-hop scheme and hop-by-hop scheme as a function of number of levels when initial battery levels are different

Chapter 4

Simulation Studies on Network Lifetime and Data Collection Delay with 2-hop Scheme

4.1 Simulation Studies for Chain and Binary Tree Topology Networks

In this section, we use simulation to verify the theoretical computation results for network lifetime in *Chapter 3* for chain and binary tree network. More specifically, for a chain or binary tree network, we assign the optimal or "good" transmission probabilities found by theoretical calculation in *Chapter 3* and do simulation to get the result of how many data gathering cycles done until one sensor in the network runs out of battery. The path loss exponent is assumed to be 3.5 for the simulation.

4.1.1 Simulation Results for Chain Networks

- **Theoretical Results**
- **Simulation Results**

4.1.2 Simulation Results for Binary Tree Networks

In **Table 4.3** and **Table 4.4** below, the meaning of K is the same as in **Table 3.5**.

- **Theoretical Results**
- **Simulation Results** (see **Table 4.4**)

Table 4.1: Chain network – Theoretical calculation results for expected network lifetime with 2-hop scheme ($\alpha = 3.5$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme (optimal transmission probabilities)**	**Expected network lifetime increase**
$\boldsymbol{N = 3}$	$T_{h2h} = 2667$	$E[T_{2hop}] = 2767$	3.75%
$\boldsymbol{N = 4}$	$T_{h2h} = 1998$	$E[T_{2hop}] = 2055$	2.85%
$\boldsymbol{N = 5}$	$T_{h2h} = 1597$	$E[T_{2hop}] = 1633$	2.25%
$\boldsymbol{N = 6}$	$T_{h2h} = 1330$	$E[T_{2hop}] = 1355$	1.88%
$\boldsymbol{N = 7}$	$T_{h2h} = 1140$	$E[T_{2hop}] = 1158$	1.58%
$\boldsymbol{N = 8}$	$T_{h2h} = 997$	$E[T_{2hop}] = 1011$	1.40%
$\boldsymbol{N = 9}$	$T_{h2h} = 886$	$E[T_{2hop}] = 897$	1.24%
$\boldsymbol{N = 10}$	$T_{h2h} = 797$	$E[T_{2hop}] = 806$	1.13%

Table 4.2: Chain network – Simulation results for expected network lifetime with 2-hop scheme ($\alpha = 3.5$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme (optimal transmission probabilities)**	**Expected network lifetime increase**
$\boldsymbol{N = 3}$	$T_{h2h} = 2667$	$E[T_{2hop}] = 2766$	3.71%
$\boldsymbol{N = 4}$	$T_{h2h} = 1998$	$E[T_{2hop}] = 2054$	2.80%
$\boldsymbol{N = 5}$	$T_{h2h} = 1597$	$E[T_{2hop}] = 1632$	2.19%
$\boldsymbol{N = 6}$	$T_{h2h} = 1330$	$E[T_{2hop}] = 1355$	1.88%
$\boldsymbol{N = 7}$	$T_{h2h} = 1140$	$E[T_{2hop}] = 1157$	1.49%
$\boldsymbol{N = 8}$	$T_{h2h} = 997$	$E[T_{2hop}] = 1010$	1.30%
$\boldsymbol{N = 9}$	$T_{h2h} = 886$	$E[T_{2hop}] = 896$	1.13%
$\boldsymbol{N = 10}$	$T_{h2h} = 797$	$E[T_{2hop}] = 805$	1.00%

4.1.3 Theoretical Verification and Conclusion

Comparing results in **Table 4.1** and **Table 4.2**; **Table 4.3** and **Table 4.4**, we can see that the simulation results is nearly the same as theoretical results. Theoretical results have been verified to be correct.

4.2 Simulation Studies for General Tree Topology Networks

We now estimate how much network lifetime could be increased with 2-hop scheme in a random network. Because of the random network topologies, it is impossible for us to compute theoretically the optimal transmission probabilities or the network lifetime

Table 4.3: Binary tree network – Theoretical calculation results for expected network lifetime with 2-hop scheme ($\alpha = 3.5$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme**	**Expected network lifetime increase**
$\boldsymbol{L = 3}$	$T_{h2h} = 1140$	$E[T_{2hop}] = 1295$ $(K = 3)$	13.59%
$\boldsymbol{L = 4}$	$T_{h2h} = 531$	$E[T_{2hop}] = 598$ $(K = 3)$	12.62%
$\boldsymbol{L = 5}$	$T_{h2h} = 257$	$E[T_{2hop}] = 288$ $(K = 3)$	12.06%
$\boldsymbol{L = 6}$	$T_{h2h} = 126$	$E[T_{2hop}] = 142$ $(K = 4)$	12.70%
$\boldsymbol{L = 7}$	$T_{h2h} = 62$	$E[T_{2hop}] = 70$ $(K = 4)$	12.90%
$\boldsymbol{L = 8}$	$T_{h2h} = 31$	$E[T_{2hop}] = 35$ $(K = 4)$	12.90%
$\boldsymbol{L = 9}$	$T_{h2h} = 15$	$E[T_{2hop}] = 17$ $(K = 4)$	13.33%
$\boldsymbol{L = 10}$	$T_{h2h} = 7$	$E[T_{2hop}] = 8$ $(K = 4)$	14.29%

Table 4.4: Binary tree network – Simulation results for expected network lifetime with 2-hop scheme ($\alpha = 3.5$)

	Network lifetime with hop-by-hop scheme	**Expected network lifetime with 2-hop scheme**	**Expected network lifetime increase**
$\boldsymbol{L = 3}$	$T_{h2h} = 1140$	$E[T_{2hop}] = 1294$ $(K = 3)$	13.51%
$\boldsymbol{L = 4}$	$T_{h2h} = 531$	$E[T_{2hop}] = 598$ $(K = 3)$	12.62%
$\boldsymbol{L = 5}$	$T_{h2h} = 257$	$E[T_{2hop}] = 287$ $(K = 3)$	11.67%
$\boldsymbol{L = 6}$	$T_{h2h} = 126$	$E[T_{2hop}] = 142$ $(K = 4)$	12.70%
$\boldsymbol{L = 7}$	$T_{h2h} = 62$	$E[T_{2hop}] = 70$ $(K = 4)$	12.90%
$\boldsymbol{L = 8}$	$T_{h2h} = 31$	$E[T_{2hop}] = 35$ $(K = 4)$	12.90%
$\boldsymbol{L = 9}$	$T_{h2h} = 15$	$E[T_{2hop}] = 17$ $(K = 4)$	13.33%
$\boldsymbol{L = 10}$	$T_{h2h} = 7$	$E[T_{2hop}] = 8$ $(K = 4)$	14.29%

increase like the previous chapters. We will evaluate 2-hop scheme by simulating the operation of the network.

4.2.1 Simulation Setup and Environment

The simulation parameters and settings for the simulator are shown in **Table 4.5**. We assume that the physical and MAC conditions of the IEEE802.15.4 are used in the simulations. Our simulations are written in the C language based on time-driven program. We also assume that all the sensors are identical, uniformly, and independently distributed in a two-dimensional square are and no sensor moves throughout the simulation. The sink node is located at the center of the simulation area. Since all the sensors are identical, they have the same transmitting and receiving power. We also use the first-order radio model in our simulation by assuming the path loss exponent (α) of 3.5.

Table 4.5: Simulation parameters and settings

Number of sensor nodes	100
Number of sink nodes	1
Network coverage	200 m x 200 m
Transmission range	20 m
Network protocol	Tree-based routing protocol [17]
RANN packet size	20 *bytes*
RANN broadcast interval	15 s
Traffic type	Constant bit rate
Data gathering cycle	10 s
Data payload size	104 *bytes*
MAC header size	24 *bytes*
Hardware specification	IEEE 802.15.4
MAC protocol	Beacon-enabled access method
Transmission rate	250 *kbps*
Number of channels	1
Energy model	First-order radio model
Path loss exponent (α)	3.5
ϵ_{elec}	50 nJ/bit
ϵ_{amp}	100 $pJ/bit/m^{\alpha}$
Initial battery capacity	30 J
Processing time	1 ms

4.2.2 Simulation Scenario

We apply the tree-based routing (TBR) protocol as proposed in [17] to form a tree topology for the sensors in the network. The RANN packet size is 20 bytes and we set the RANN broadcast interval is 15 seconds. We model our traffic based on constant bit rate (CBR). The CBR traffic consists of 104-byte payload size, which sends at the data packet is sent at every 10 seconds.

We run the simulator for 30 different topologies. In each topologies, the position of 100 sensors are randomly generated. The position of the sink is fixed at the center of the network. For one topology:

- hop-by-hop scheme simulation

  After running hop-by-hop simulation, we can get the network lifetime with hop-by-hop scheme. We denote by $T_{h2h,i}$ the network lifetime with hop-by-hop scheme for topology i ($i = 1, 2, ..., 30$).

- 2-hop scheme simulation

  For random networks, we cannot compute the optimal transmission probability for each sensor but we assign the same transmission probability p for all the sensors in the network. The range of p is $p = 0.5, 0.51, 0.52, ...0.98, 0.99$. We denote by

$T_{2hop,i}(p)$ the network lifetime with 2-hop scheme for topology i ($i = 1, 2, ..., 30$) when each sensor is assigned a transmission probability p.

After running the simulator for 30 topologies, we compute:

- Average lifetime with hop-by-hop scheme: $\bar{T}_{h2h} = \frac{\sum_{i=1}^{30} T_{h2h,i}}{30}$
- Average lifetime with 2-hop scheme
  - when $p = 0.5$: $\bar{T}_{2hop}(0.5) = \frac{\sum_{i=1}^{30} T_{2hop,i}(0.5)}{30}$
  - when $p = 0.51$: $\bar{T}_{2hop}(0.51) = \frac{\sum_{i=1}^{30} T_{2hop,i}(0.51)}{30}$

    ...
  - when $p = 0.98$: $\bar{T}_{2hop}(0.98) = \frac{\sum_{i=1}^{30} T_{2hop,i}(0.98)}{30}$
  - when $p = 0.99$: $\bar{T}_{2hop}(0.99) = \frac{\sum_{i=1}^{30} T_{2hop,i}(0.99)}{30}$

The whole simulation process is illustrated by **Fig. 4.1**

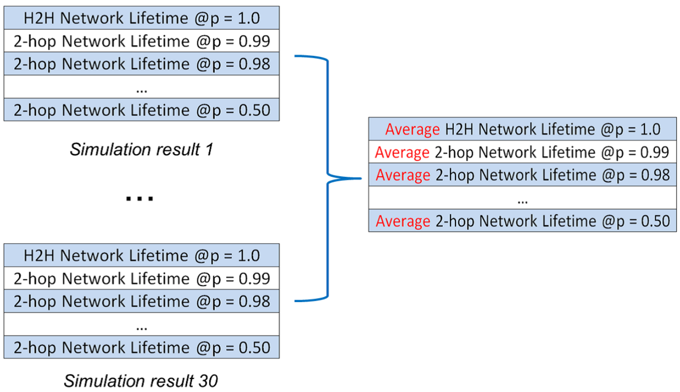

Figure 4.1: General tree network – simulation process

4.2.3 Simulation Results for Network Lifetime without Packet Loss

The simulation result based on the simulation scenario above is shown in **Fig. 4.2**

As we can see from the graph in **Fig. 4.2**, the network lifetime increases when p is from 0.99 to 0.76. After reaching the top with $p = 0.76$, network lifetime starts to decrease when p decreases. This is because when p is small, the sensors tend to transmit more packets in 2-hop transmission, if the number of packets transmitted in 2-hop transmission, the lifetime with 2-hop transmission starts to decrease. and and if there are so packets transmitted in 2-hop transmission (p is so small), the network lifetime may be less than that with hop-by-hop scheme.

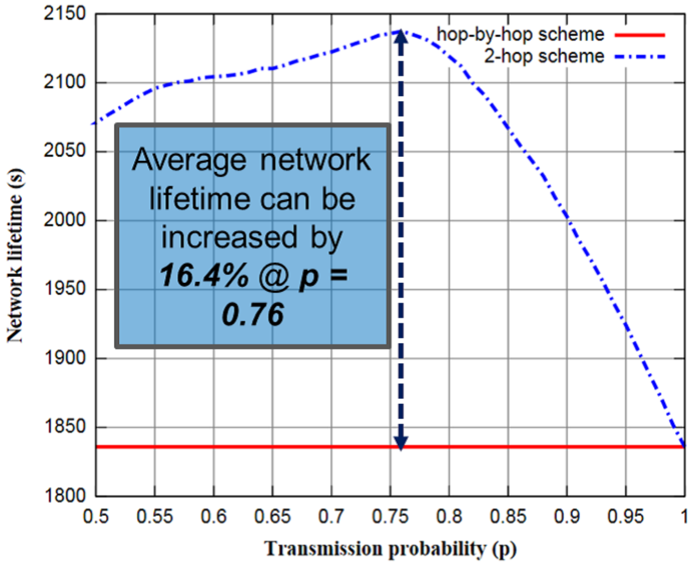

Figure 4.2: General tree network – network lifetime with hop-by-hop and 2-hop scheme

At $p = 0.76$, the average network lifetime with 2-hop scheme is maximum. Thus, for any random network, although we cannot optimal transmission probabilities for each sensor, it is reasonable to assign all sensors with transmission probability of 0.76 so that we can expect the increase in network lifetime to be maximum.

4.2.4 Simulation Results for Data Collection Delay without Packet Loss

We now analyze the delay in data collection by doing simulation. The parameters settings are the same as in the previous section.

For the simulation scenario, we also run the simulator for 30 different topologies to get the average data collection delay. In the simulation, all sensors are assigned the same transmission probability p.

For each simulation, instead of measuring the network lifetime, we measure the data collection delay with hop-by-hop scheme and the *average* delay with 2-hop transmission. (We cannot measure the exact delay with 2-hop scheme, because in different DGCs, the delay may be different, as mentioned previously in this chapter)

The result is shown in **Fig. 4.3**. We can see that, if the transmission probability p is large, meaning that the number of hop-by-hop transmission is also large the delay is also large (slower data collection). On the other hand, if p is small, meaning that the number of 2-hop transmission is large, the delay decreases (faster data collection). However, if p is too small, even though the delay in one DGC is small, the overall network lifetime may be decreased. When $p = 0.76$, which is a possibly "good" transmission probability for random networks (as discussed in *Chapter 5*), delay in data collection is reduced about

16.9%.

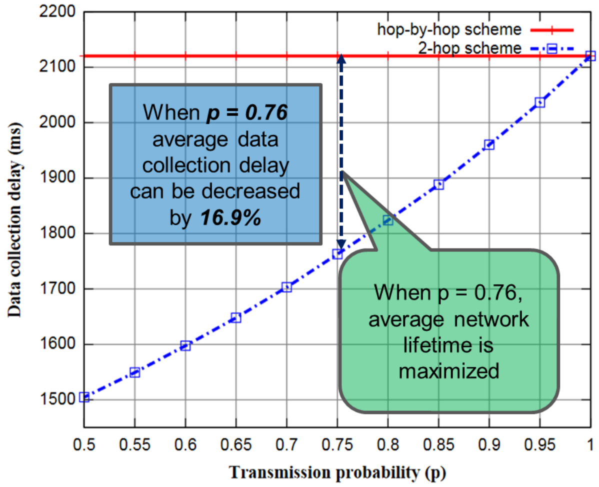

Figure 4.3: General tree network – average data collection delay with 2-hop scheme over hop-by-hop scheme as a function of transmission probabilities

4.2.5 Simulation Results for Network Lifetime with Packet Loss

The packet loss of, for example, 25% means that for all sensors in the network, 25% of its transmission needs to be retransmitted due to error in transmission. In this simulation, when a packet sent in hop-by-hop or 2-hop transmission is lost due to error, it will be retransmitted using that kind of transmission. For example, if a packet sent in hop-by-hop transmission needs to be retransmitted, it will be sent again also using hop-by-hop transmission (the similar thing for 2-hop transmission).

Parameter settings are the same as above, All sensors are assigned the same transmission probability p, $p = 0.50, 0.55, ..., 0.95, 1.0$. The simulator is run for 30 different general tree topologies when the packet loss is 0%, 25%, 50% and 75%. The results are shown in **Fig. 4.4**. We can see that, even though with the presence of packet loss, 2-hop scheme can prolong the network lifetime about 17% when $p = 0.76$.

4.2.6 Conclusion

For general tree networks, even with the presence of packet loss, average network lifetime can be increased about 17% when the transmission probability for all sensors is $p = 0.76$. In addition, when $p = 0.76$, average data collection delay can be reduced about 17%.

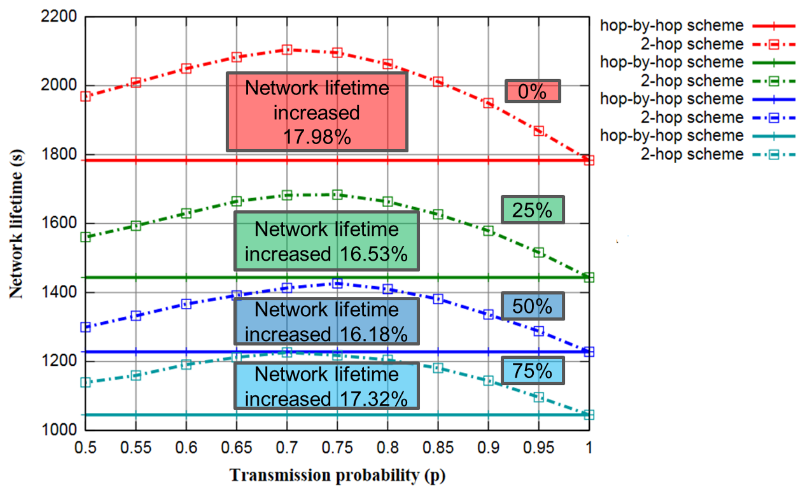

Figure 4.4: General tree network – network lifetime with hop-by-hop and 2-hop scheme with the presence of packet loss

4.3 Simulation Studies on Network Lifetime with 2-hop Scheme for General Tree Topology Networks when Initial Battery Levels are Different

We now do the simulation to study how the network lifetime varies when initial battery levels are different. Simulation parameters and settings are the same as in the previous section. We also run the simulator for 30 different general tree networks. For each network:

- (3.32) is used to generate initial battery level for each sensor
- After generating randomly initial battery levels for the sensors, we run the simulator for five times, in each time:
  - All sensors are assigned the same probability p, $p = 0.50, 0.55, ..., 0.95, 1.0$
  - Network lifetime with hop-by-hop scheme and expected network lifetime with 2-hop scheme is recorded
- The average network lifetimes with hop-by-hop and 2-hop scheme are then computed

Finally, a graph is drawn based on those average network lifetimes (see **Fig. 4.5**). We can see that, when the initial battery levels are different, we should not assign the same probability for all sensors; otherwise, the network lifetime with 2-hop scheme is even worse than that with hop-by-hop scheme. Therefore, when the initial battery levels are different, in order for the network lifetime with 2-hop scheme to be longer than that with hop-by-hop scheme, it is necessary to find optimal or "good" transmission probabilities for the sensors. However, until now, we can find those solutions for chain and binary

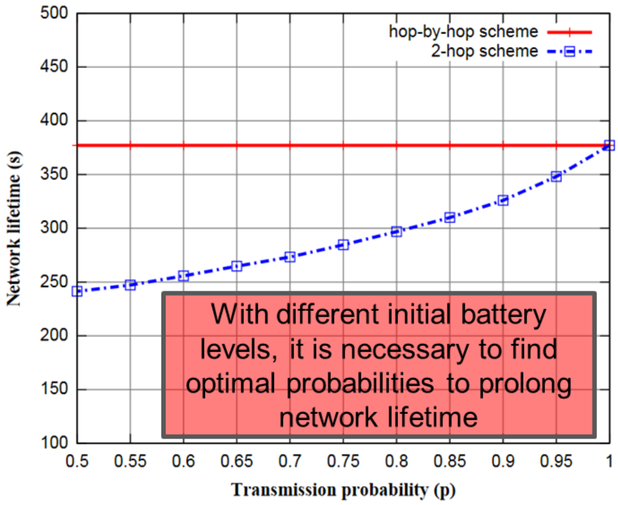

Figure 4.5: General tree network – network lifetime with 2-hop scheme and hop-by-hop scheme as a function of number of levels when initial battery levels are different

tree networks, optimal or "good" transmission probabilities for general tree networks is considered as future work.

For general tree networks, if we assign all sensors with the same probability, the network lifetime with 2-hop scheme is even smaller than that with hop-by-hop scheme. Therefore, it is necessary to find optimal or "good" transmission probabilities for sensors in the network, in order to prolong the network lifetime. However, until now, we can find solutions for chain and binary tree networks only. How to find optimal probabilities for general tree networks is considered as future work.

Chapter 5

Concluding Remarks

5.1 Summary

In this research, the proposed 2-hop scheme has been analyzed in chain topology networks, binary tree topology networks and random tree topology networks. In chain and binary tree networks, based on theoretical computation, we showed that 2-hop scheme can be used to increase network lifetime compared to hop-by-hop scheme by balancing energy consumption of all sensors throughout the network (if optimal transmission probabilities exist) or reducing energy consumption unbalance in the network (if optimal transmission probabilities do not exist) by finding "good" transmission probability to balance energy consumption of some top levels sensors (sensors near the sink in the topology). In random networks, by doing simulation, we showed that on average, the network lifetime could be increased about 16.4%

The results showed that the increase in network lifetime depends on the kinds of the topologies. For chain networks, even with optimal transmission probabilities, the increase is smaller than that for binary tree networks (although optimal transmission probabilities usually do not exist for binary tree networks).

In this research, *First-order radio model* is used to compute energy for transmitting and receiving packets. In this energy model, the energy for transmitting depends on the environment in which the network operates. We analyzed the effect of environments on the network lifetime increase and saw that, the same network, in different environments, the increase may be very large or very small.

We also analyzed the data collection delay between hop-by-hop and 2-hop scheme. With the assumption that 2-hop transmission rate is the same as hop-by-hop transmission rate, delay with 2-hop scheme can be mathematically proved to be smaller than the delay with hop-by-hop scheme, which means data collection is faster with 2-hop scheme than with hop-by-hop scheme.

We also compared 2-hop scheme with direct scheme. Although 2-hop scheme do not outperform direct scheme in terms of network lifetime and data collection delay, However, in practical, there are some cases when direct scheme cannot be used:

- In large scale network, where the sensor is too far away from the sink to perform direct transmission.

- Each sensor use different power level to transmit data directly to the sink. For example, in a chain network consisting of ten sensors, S_4 and S_5 use different power for direct transmission. Because all sensors in the network are homogeneous regardless of its position (near the sink or not), all of them must have capability to transmit data with different ten power levels. .In practical, producing sensors having many different power levels to transmit data may be expensive. On the other hand, 2-hop scheme only requires two power levels for transmitting (corresponding to hop-by-hop and 2-hop transmission), which means that 2-hop scheme is more economy than direct scheme.

5.2 Contributions

There are two main contributions in this research

- We proposed 2-hop scheme, which
  - Can operate in large-scale networks where direct scheme cannot be well-deployed
  - Increases network lifetime (even with the presence of packet loss)
  - Decreases delay in data collection compared to hop-by-hop scheme
- Solved how to find optimal probabilities
  - Not only for chain but also for binary tree networks
  - With same or different initial battery levels

5.3 Future Works

- As mentioned in *Chapter 5*, a solution to find optimal or "good" transmission probabilities is necessary to prolong network lifetime when initial battery levels are different.
- We know that, because the duty cycle of each sensor is short compared to the interval between one duty cycle to the next, a periodical sleeping scheme is usually used to conserve energy. A synchronized sleep/wake up scheme for 2-hop scheme is also an interesting research in the future. One of the problem of the sleep/wake up scheme for 2-hop scheme should be the sleep/wake up schedule for when the next hop wakes up to receive packets sent in hop-by-hop transmission, and when the next 2-hop wakes up to receive packets sent in 2-hop transmission.
- 3-hop or 4-hop scheme could also be an interesting research.

Appendix A

Proof of Theorems

A.1 Proof of Theorem 1

Theorem 1 $E[\xi_i] = E[\xi_j]$ *if and only if* $E[\varepsilon_i] = E[\varepsilon_j] \quad \forall i, j = 1, 2, ..., N$

Proof Because the transmission probabilities are not changed once the network starts to operate, then the expected energy consumption in a DGC is equaled to those in other DGCs. Let T be the total number of DGCs in the whole lifetime, then $E[\xi_i] = TE[\varepsilon_i]$. Thus, $E[\xi_i] = E[\xi_j] \Leftrightarrow TE[\varepsilon_i] = TE[\varepsilon_j] \Leftrightarrow E[\varepsilon_i] = E[\varepsilon_j]$. *Theorem 1* has been proved.

A.2 Proof of Theorem 2

Theorem 2 *For a set of transmission probabilities* $(p_N, p_{N-1}, ..., p_3, p_2)$*, if* $E[\varepsilon_N] = E[\varepsilon_{N-1}] = ... = E[\varepsilon_2] = E[\varepsilon_1]$*; then with* $(p_N, p_{N-1}, ..., p_3, p_2)$*, the network lifetime is also maximized.*

Proof If with $(p_N, p_{N-1}, ..., p_3, p_2)$, the network lifetime is not maximized, then from *Lemma 2*, there exists another set of transmission probabilities $(p'_N, p'_{N-1}, ..., p'_3, p'_2)$ where $\max_{1 \le i \le N}\{E[\varepsilon'_i]\} < \max_{1 \le i \le N}\{E[\varepsilon_i]\}$.

In this proof, we denote by $\boldsymbol{\varepsilon'_i, n'_i, f'_{1,i}, f'_{2,i}}$the energy consumption of S_i in one DGC, the total number of packets transmitted by S_i in one DGC, the number of packets transmitted by S_i in hop-by-hop transmission in one DGC, the number of packets transmitted by S_i in 2-hop transmission in one DGC, respectively, with the set of transmission probabilities $\boldsymbol{(p'_N, p'_{N-1}, ..., p'_3, p'_2)}$.

From the precondition of the theorem, with $(p_N, p_{N-1}, ..., p_3, p_2)$, $E[\varepsilon_N] = E[\varepsilon_{N-1}] = ... = E[\varepsilon_2] = E[\varepsilon_1]$. Thus, $\max_{1 \le i \le N}\{E[\varepsilon_i]\} = E[\varepsilon_N] = E[\varepsilon_{N-1}] = ... = E[\varepsilon_2] = E[\varepsilon_1]$. Then, $\max_{1 \le i \le N}\{E[\varepsilon'_i]\} < \max_{1 \le i \le N}\{E[\varepsilon_i]\} \Rightarrow \boldsymbol{E[\varepsilon'_i] < E[\varepsilon_i], 1 \le i \le N \quad (*)}$.

For sensor S_N (the farthest one from the sink), in one DGC, it generates only one packet and forwards that packet to S_{N-1}. Because S_N is the farthest node from the sink, it does not receive any packets from other sensors. Therefore, $n_N = n'_N = 1$. From (3.4), we have:

$$
\begin{aligned}
E[\varepsilon_N] &= p_n E[n_N]\epsilon_t(d_1) + (1-p_N)E[n_N]\epsilon_t(d_2) + (E[n_N]-1)\epsilon_r \\
&= p_N\epsilon_t(d_1) + (1-p_N)\epsilon_t(d_2) \\
&= \epsilon_t(d_2) - p_N\left[\epsilon_t(d_2) - \epsilon_t(d_1)\right]
\end{aligned} \tag{A.1}
$$

and

$$
E[\varepsilon'_N] = \epsilon_t(d_2) - p'_N\left[\epsilon_t(d_2) - \epsilon_t(d_1)\right] \tag{A.2}
$$

From $(*)$, (A.1) and (A.2), we have $\epsilon_t(d_2) - p'_N\left[\epsilon_t(d_2) - \epsilon_t(d_1)\right] < \epsilon_t(d_2) - p_N\left[\epsilon_t(d_2) - \epsilon_t(d_1)\right] \Rightarrow p'_N > p_N$. We also have $E[f_{2,N}] = (1-p_N)E[n_N] = 1-p_N$ and $E[f'_{2,N}] = (1-p'_N)E[n'_N] = 1-p'_N$. Because $p'_N > p_N$, then $\boldsymbol{E[f'_{2,N}] < E[f_{2,N}] \quad (**)}$.

From (B.4) and (B.6), we have

$$
E[n_{i-1}] = N - i + 2 - E[f_{2,i}] \tag{A.3}
$$

From (A.3), $E[n_{N-1}] = 2 - E[f_{2,N}]$ and $E[n'_{N-1}] = 2 - E[f'_{2,N}]$. From $(**)$, $\boldsymbol{E[n'_{N-1}] > E[n_{N-1}] \quad (***)}$.

We have

$$
\begin{aligned}
E[\varepsilon_{N-1}] &= E[f_{1,N-1}]\epsilon_t(d_1) + E[f_{2,N-1}]\epsilon_t(d_2) + (E[n_{N-1}]-1)\epsilon_r \\
&= (E[n_{N-1}] - E[f_{2,N-1}])\epsilon_t(d_1) + E[f_{2,N-1}]\epsilon_t(d_2) + (E[n_{N-1}]-1)\epsilon_r \\
&= E[n_{N-1}]\left[\epsilon_t(d_1) + \epsilon_r\right] + E[f_{2,N-1}]\left[\epsilon_t(d_2) - \epsilon_t(d_1)\right] - \epsilon_r
\end{aligned} \tag{A.4}
$$

and

$$
E[\varepsilon'_{N-1}] = E[n'_{N-1}]\left[\epsilon_t(d_1) + \epsilon_r\right] + E[f'_{2,N-1}]\left[\epsilon_t(d_2) - \epsilon_t(d_1)\right] - \epsilon_r \tag{A.5}
$$

From (A.4) and (A.5), we have

$$
\begin{aligned}
E[\varepsilon_{N-1}] - E[\varepsilon'_{N-1}] = &\left(E[n_{N-1}] - E[n'_{N-1}]\right)\left[\epsilon_t(d_1) + \epsilon_r\right] \\
&+ \left(E[f_{2,N-1}] - E[f'_{2,N-1}]\right)\left[\epsilon_t(d_2) - \epsilon_t(d_1)\right]
\end{aligned} \tag{A.6}
$$

From $(*), E[\varepsilon_{N-1}] - E[\varepsilon'_{N-1}] > 0$ then

$$
\left(E[n_{N-1}] - E[n'_{N-1}]\right)\left[\epsilon_t(d_1) + \epsilon_r\right] + \left(E[f_{2,N-1}] - E[f'_{2,N-1}]\right)\left[\epsilon_t(d_2) - \epsilon_t(d_1)\right] > 0 \tag{A.7}
$$

From $(***), \left(E[n_{N-1}] - E[n'_{N-1}]\right)\left[\epsilon_t(d_1) + \epsilon_r\right] < 0$. Therefore, in order for the left-hand side of (A.7) to be larger than 0, we must have $\left(E[f_{2,N-1}] - E[f'_{2,N-1}]\right)\left[\epsilon_t(d_2) - \epsilon_t(d_1)\right] > 0 \Leftrightarrow \boldsymbol{E[f'_{2,N-1}] < E[f_{2,N-1}]}$.

Continuing this reasoning, $E[f'_{2,N-1}] < E[f_{2,N-1}] \Rightarrow E[n'_{N-2}] > E[n_{N-2}] \Rightarrow ... \Rightarrow \boldsymbol{E[n'_1] > E[n_1] \quad (****)}$.

We have

$$
\begin{aligned}
E[\varepsilon_1] &= E[n_1]\epsilon_t(d_1) + (E[n_1]-1)\epsilon_r \\
&= E[n_1]\left[\epsilon_t(d_1)+\epsilon_r\right] - \epsilon_r
\end{aligned} \tag{A.8}
$$

and

$$
\begin{aligned}
E[\varepsilon'_1] &= E[n'_1]\epsilon_t(d_1) + (E[n'_1]-1)\epsilon_r \\
&= E[n'_1]\left[\epsilon_t(d_1)+\epsilon_r\right] - \epsilon_r
\end{aligned} \tag{A.9}
$$

From $(\ast\ast\ast\ast)$, (A.8) and (A.9), we get $E[\varepsilon_1] < E[\varepsilon'_1]$. This contradicts with $(\ast)$. Therefore, $(p_N, p_{N-1}, ..., p_2)$ must be an optimal solution. *Theorem 2* has been proved.

A.3 Proof of Theorem 3

Theorem 3 *After assigning initial values to $p_N, p_{N-1}, ..., p_k (2 \le k \le N)$, for a set of transmission probabilities $(p_{k-1}, p_{k-2}, ..., p_2)$, if $E[\varepsilon_{k-1}] = E[\varepsilon_{k-2}] = ... = E[\varepsilon_1]$ then $(p_{k-1}, p_{k-2}, ..., p_2)$ is the best probabilities we can choose. That is, for other $(p'_{k-1}, p'_{k-2}, ..., p'_2)$ that does not make $E[\varepsilon'_{k-1}] = E[\varepsilon'_{k-2}] = ... = E[\varepsilon'_1]$, then $\max_{1\le i\le N}\{E[\varepsilon'_i]\} > \max_{1\le i\le N}\{E[\varepsilon_i]\}$.*

Proof This theorem can be considered as a generalization of *Theorem 2*. Its proof is similar to the proof of *Theorem 2* and we omit it here.

A.4 Proof of Theorem 4

Theorem 4 *In a binary tree network, $E[f_{1,i,j}] = E[f_{1,i,k}]$ and $E[f_{2,i,j}] = E[f_{2,i,k}], i = 1, 2, ..., L; j, k = 1, 2, ..., 2^i$. This means that, the expected number of packets sent in hop-by-hop and 2-hop transmission of all sensors in the same level are the same.*

Proof. For all sensors in level L, in one DGC, they all generate and send one packet. Then, from *Lemma 3*, $E[f_{1,L-1,j}] = p_L$ and $E[f_{2,L,j}] = 1 - p_L,\ j = 1, 2, ..., 2^L$. Thus, *Theorem 4* is true for all sensors in level L. Let $E[f_{1,L}] = E[f_{1,L,j}]$ and $E[f_{2,L}] = E[f_{2,L,j}],\ j = 1, 2, ..., 2^L$.

For a sensor $S_{L-1,j}$ in level $L-1$. Let $S_{L,k}$ and $S_{L,k+1}$ be the two children of $S_{L-1,j}$. From **Fig. 3.2.2**, we can see that $n_{L-1,j} = f_{1,L,k} + f_{1,L,k+1} + 1 \Rightarrow E[n_{L-1,j}] = E[f_{1,L,k}] + E[f_{1,L,k+1}] + 1 = 2E[f_{1,L}] + 1 \Rightarrow E[f_{1,L-1,j}] = p_{L-1}(2E[f_{1,L}] + 1)$ and $E[f_{2,L-1,j}] = (1 - p_{L-1})(2E[f_{1,L}] + 1)$. It is easy to see that $E[f_{1,L-1,j}]$ and $E[f_{2,L-1,j}]$ do not depend on j. Therefore, *Theorem 4* is true for all sensors in level $L-1$.

Continuing this reasoning, *Theorem 4* is true for all levels in the network.

A.5 Proof of Theorem 5

Theorem 5 *For a network of N sensors $S_1, S_2, ..., S_N$, the initial battery level of sensor S_i is denoted by B_i. The expected energy consumption of S_i in one DGC is denoted by*

$E[\varepsilon_i]$, $i = 1, 2, ..., N$. *With 2-hop scheme, if we can find* $(p_N, p_{N-1}, ..., p_3, p_2)$ *such that* $\frac{B_1}{E[\varepsilon_1]} = \frac{B_2}{E[\varepsilon_2]} = ... = \frac{B_N}{E[\varepsilon_N]}$, *then the network lifetime is maximized.*

Proof It is obvious that, the network lifetime is the minimum lifetime of a sensor in the network. Here we denote by T the network lifetime.

$$T = \min_{1 \leq i \leq N} \{\frac{B_i}{E[\varepsilon_i]}\} \tag{A.10}$$

If $(p_N, p_{N-1}, ..., p_2)$ is not optimal probabilities, then there exists another set of probabilities $(p'_N, p'_{N-1}, ..., p'_2)$ such that $T' > T$.

Because with $(p_N, p_{N-1}, ..., p2)$, $\frac{B_1}{E[\varepsilon_1]} = \frac{B_2}{E[\varepsilon_2]} = ... = \frac{B_N}{E[\varepsilon_N]}$, we have

$$T' = \min_{1 \leq i \leq N} \{\frac{B_i}{E[\varepsilon'_i]}\} > \frac{B_1}{E[\varepsilon_1]} = \frac{B_2}{E[\varepsilon_2]} = ... = \frac{B_N}{E[\varepsilon_N]} \tag{A.11}$$

Because $\dfrac{B_k}{E[\varepsilon'_k]} \geq \min_{1 \leq i \leq N} \{\dfrac{B_i}{E[\varepsilon'_i]}\}, \forall k = 1, 2, ..., N$ then from (A.11), we have

$$\frac{B_k}{E[\varepsilon'_k]} > \frac{B_k}{E[\varepsilon_k]} \Leftrightarrow E[\varepsilon'_k] < E[\varepsilon_k], \forall k = 1, 2, 3, ..., N \tag{A.12}$$

From (A.12), the rest of the proof is similar to the proof of *Theorem 2* and we omit it here.

Appendix B

Proof of Lemmas

B.1 Proof of Lemma 1

Lemma 1 $p_i = \frac{E[f_{1,i}]}{E[f_{1,i}]+E[f_{2,i}]}, \quad \forall i = 1, 2, ..., N$

Proof Let $P(j,i)$ $(1 \leq i < j \leq N)$ be the probability that S_i receives a packet from S_j. Then $1 - P(j,i)$ is the probability that S_i does not receive a packet from S_j, this occurs if and only if the packet is forwarded to S_{i+1} and then forwarded to S_{i-1}. Thus,

$$1 - P(j,i) = P(j,i+1)(1 - p_{i+1}) \Leftrightarrow P(j,i) = 1 - P(j,i+1)(1 - p_{i+1}) \tag{B.1}$$

Let $r_k, 1 \leq k \leq N$, be the number of packets received by S_k in one DGC. We have

$$\begin{aligned}
E[r_{i-1}] &= \sum_{k=i}^{N} P(k, i-1) \\
&= P(i, i-1) + \sum_{k=i+1}^{N} P(k, i-1) \\
&= p_i + \sum_{k=i+1}^{N} [1 - P(k,i)(1 - p_i)] \\
&= p_i + \sum_{k=i+1}^{N} [1] - (1 - p_i) \sum_{k=i+1}^{N} P(k,i) \\
&= p_i + N - i - (1 - p_i)E[r_i] \\
&= N - i - E[r_i] + p_i(E[r_i] + 1)
\end{aligned} \tag{B.2}$$

The number of packets transmitted by S_i in one DGC, denoted by n_i, consists of $f_{1,i}$ packets transmitted in hop-by-hop transmission, and $f_{2,i}$ packets transmitted in 2-hop transmission. Thus,

$$n_i = f_{1,i} + f_{2,i} \tag{B.3}$$

Because each sensor transmits all packets forwarded to each by other sensors, together with one packet generated by itself, then we also have

$$n_i = r_i + 1 \tag{B.4}$$

From (B.3) and (B.4), we have: $f_{1,i} + f_{2,i} = r_i + 1 \Leftrightarrow r_i = f_{1,i} + f_{2,i} - 1$, then

$$E[r_i] = E[f_{1,i}] + E[f_{2,i}] - 1 \tag{B.5}$$

We can see that, all the packets generated by S_N, S_{N-1}, ..., S_i will be gradually received by S_{i-1}, except $f_{2,i}$ packets transmitted by S_i to S_{i-2}. Since each sensor only generates one packet per DGC, then the total number of packets generated by S_N, S_{N-1}, ..., S_i is $N - i + 1$. Thus $r_{i-1} = N - i + 1 - f_{2,i}$, then

$$E[r_{i-1}] = N - i + 1 - E[f_{2,i}] \tag{B.6}$$

Substituting (B.5) and (B.6) into (B.2), we have

$$\begin{aligned} N - i + 1 - E[f_{2,i}] &= N - i - (E[f_{1,i}] + E[f_{2,i}] - 1) + p_i(E[f_{1,i}] + E[f_{2,i}] - 1 + 1) \\ &= N - i + 1 - E[f_{1,i}] - E[f_{2,i}] + p_i(E[f_{1,i}] + E[f_{2,i}]) \end{aligned} \tag{B.7}$$

Then, $E[f_{1,i}] = p_i(E[f_{1,i}] + E[f_{2,i}]) \Rightarrow p_i = \frac{E[f_{1,i}]}{E[f_{1,i}]+E[f_{2,i}]}$. *Lemma 1* has been proved.

B.2 Proof of Lemma 2

Lemma 2 *Assuming all sensors in the network have the same amount of initial battery; then the network lifetime is maximized if and only if* $\max\limits_{1 \le i \le N}\{E[\varepsilon_i]\}$ *is minimized.*

Proof The network lifetime is defined as the time until at least one sensor drains out battery. As mentioned in *Data Gathering Model*, sensors periodically transmits their sensed data to the sink; therefore, if we know the number of DGCs completed until at least one sensor runs out of battery, it is possible for us to calculate the operating time of the network (by multiplying with the time for each cycle). Thus, network lifetime T can be represented by the total number of DGCs completed until at least one sensor drains out energy.

Let B be the initial battery energy in each sensor. Because the expected energy consumption in S_i in one DGC is $E[\varepsilon_i]$, then the expected number of DGC, denoted by T_i, that S_i can perform until it runs out of battery is $T_i = \frac{B}{E[\varepsilon_i]}$. It is obvious that

$$T = \min_{1 \le i \le N}\{T_i\} = \min_{1 \le i \le N}\{\frac{B}{E[\varepsilon_i]}\} = \frac{B}{\max\limits_{1 \le i \le N}\{E[\varepsilon_i]\}} \tag{B.8}$$

From (B.8), we can easily see that, the network lifetime T is maximized if and only if $\max\limits_{1 \le i \le N}\{E[\varepsilon_i]\}$ is minimized. *Lemma 2* has been proved.

B.3 Proof of Lemma 3

Lemma 3 *In a binary tree network,* $p_i = \frac{E[f_{1,i,j}]}{E[f_{1,i,j}]+E[f_{2,i,j}]}, i = 1, 2, ..., L, j = 1, 2, ..., 2^i.$

Proof. The proof of this lemma is similar to the proof of *Lemma 2* and we omit it here.

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List of Publications

[1] A.H. Vuong, Y. Tan, and A.O. Lim, "2-hop scheme for data collection in wireless sensor networks," *IPSJ SIG Mobile Computing and Ubiquitous Communications (SIG-MBL 62)*, Okinawa, Japan, May 2012.

[2] A.H. Vuong, Y. Tan, and A.O. Lim, "2-hop scheme for data collection in wireless sensor networks," *IEICE Society Conference*, Toyama, Japan, September 2012.

[3] A.O. Lim, A.H. Vuong, Z. Chen, Y. Tan, "2-hop scheme for maximum lifetime in wireless sensor networks," to be appeared in *IEEE SENSORS*, Taipei, Taiwan, October 2012.

List of Awards

[1] Encouragement award (奨励発表賞) for presentation at the *IPSJ SIG Mobile Computing and Ubiquitous Communications (MBL)* conference, Okinawa, Japan, May 2012. The awards ceremony was held on August 30, 2012, at Tokyo University of Science (東京理科大).