Prerequisite – Effect Structure in Kripke Semantics

Katsuhiko Sano, Shingo Hagiwara, and Satoshi Tojo

School of Information Science, Japan Advanced Institute of Science and Technology,
1–1 Asahidai, Nomi, Ishikawa 923-1292, Japan
{v-sano,s-hagiwa,tojo}@jaist.ac.jp

Abstract. Although the formalization of legal documents is quite useful, they contain various kinds of if–then relations. In this paper, we aim at formalizing the prerequisite–effect structure in temporal/causal settings. We regard the progress of time as hereditary accessibility in temporal states, and thus we introduce Kripke semantics. Our ultimate objective is to construct a legal reasoning system, however, since those various kinds of logical relations may complicate the reasoning system we avoid to introduce multiple modal operators. We stay in simple intuitionistic logic, and we will extend it to include the prerequisite–effect structure. Then, the structure is defined in the augmentation of known facts, that is, the effect is immediately follows after the prerequisite is satisfied.

1 Introduction

To translate legal documents into formal language is quite useful; for example if they can be represented in HTML format, they can be utilized electrically on the web. Furthermore if they could be translated into logic, they could be applied to electric law consulting system and automatic deduction. This kind of trial has a long history and was actively studied as so-called expert systems. However, one of the most salient problems of the translation concerns if–then structure in legal documents. Since these if–then structures have various meanings, we cannot translate them into logical implication in predicate/first-order logic uniformly.

Law, or so-called 'normative knowledge,' is written in the prerequisite–effect structure. Roughly speaking, the prerequisite corresponds to 'if' part, and the conditions for the application of the normative knowledge is written here. The effect corresponds to 'then' part, and the expected result for the application of the law is mentioned. For example, in the following statement:

> *Penal Code Law number: Act No. 45 of 1907, Article 204 (Injury)*
> A person who causes another to suffer injury shall be punished by imprisonment with work for not more than 15 years or a fine of not more than 500,000 yen.

the law is to applied the person who caused another the injury, and the effect would be 'imprisonment' or 'fine.' However, if we are to apply this law to actual case, we need to implement many other relations concerning 'person' or 'injury,' i.e., we must declare that many instances or other hyponyms are subsumed by such words. Thus, such subsumption relations also come to appear as another kind of 'if–then' statements. If–then structures are not limited to these. We notice that the structure may represent multiple meanings as temporal order, conceptual subsumption relation, causal relations, and so on.

In this paper, we aim at formalizing the prerequisite–effect structure. We first consider the specification of the structure, looking back the history of study of causal relation. Then, we contend that the prerequisite–effect structure mentions that 'the effect appears immediately after the prerequisite is satisfied.'

To construct a formal reasoning system of law, we must mix various kinds of if–then relations. If we introduce many modal operators in accordance with each relation, the legal system would be a complicated product of polymodal logic, which is not realistic in implementation. We prefer to stay in simple Kripke semantics in which various relations are represented merely by accessibility among possible worlds. In the following section, we formalize 'prerequisite–effect structure' simply in Kripke semantics, avoiding introducing modal operators.

In Section 3, we survey the formalization of causal relation. In Section 4, we propose a formalism on 'prerequisite-effect structure.' In Section 5, we discuss its adequacy and conclude.

2 Logic in Multiple Worlds

When legal documents are translated into logical terms, they are actually multiple, various meanings. In the following we will roughly sketch the variety.

Lewis and Stalnaker's 'A causes B' does not mean 'A implies B' in classical logic, that is:

$$A \to B \iff \neg(A \wedge \neg B) \iff \neg A \vee \neg\neg B \iff \neg A \vee B.$$

Notice that de Morgan's law and double negation cancellation were applied in the above. Since classical implication assumes one and only one possible world, everything must happen at the same time and the same place. This strict implication causes many interesting paradoxes. For example, we, as parents, often complain that "My kids do not study if they are not scolded to do so." Let us take its contraposition: "My kids are told to study if they study. For the kids, this statement is unacceptable; even though they study, they are again told to study. Generally speaking, when we are talking about in a unique possible world like the classical logic, a proposition and its contraposition share a common truth value. But, if we refer to different possible worlds, the contraposition may not be valid, like intuitionistic logic. In the previous case, 'studying' and 'scolding' refer to different time points, i.e., different possible worlds. The correct contraposition is that "My kids are studying since they were told to so."

The semantic difference of various causal relations are often ascribed to multiple possible worlds and accessibility relation in them, and this multiplicity bears rich semantic representation of natural language. In this paper, we first review Lewis' analysis of counterfactual conditionals, where the plausibility is explained by multiple possible worlds.

2.1 Causality

David Lewis (1973) gave an account of counterfactual conditionals as follows.

> "$P \Box\!\!\rightarrow Q$ if and only if the most plausible $P \wedge Q$ world is nearer to the reality than the most plausible $P \wedge \neg Q$ world."

where many possible worlds are arranged concentrically with regard to *plausibility*, the center of which the *reality* is located [1]. For example,

> 'If he had called an ambulance immediately, she would have been saved.'

'he has not called an ambulance.' and 'she was not saved.' are implicitly presumed.

Now we look back the branching temporal structure of subjunctive mood. Suppose a prototype of counterfactual conditionals:

If P were the case, Q would be the case. (1)

where $\neg P \wedge \neg Q$ in reality. In 'if P, then Q,' P must precede Q temporally / causally / epistemically [2], and thus Q's valuation should be postponed by the occurrence of P.

Let us see an example sentence. Suppose:

P: "He calls an ambulance immediately."
Q: "She is saved."

Now, in reality 'he did not call an ambulance' ($\neg P$) and 'She was not saved' ($\neg Q$). Getting back to the branching point, let us suppose a situation that 'he did call an ambulance' (P). Now two futures bifurcates; one is 'she is saved' (Q) and the other is that 'she was not saved' ($\neg Q$). Now we can judge that under the occurrence of P, Q is more plausible than $\neg Q$. Furthermore, we can consider that 'she is saved (Q) though he did not call an ambulance' ($\neg P$) is the most unlikely situation.

This setting also accommodates the distinction of '$\Box\!\!\rightarrow$' and '$\Diamond\!\!\rightarrow$'. Possible worlds may contain other propositions besides P and Q, and thus, $P \wedge Q$ worlds are multiple. Thus, If P were assumed to be the case, the following consequences are drawn.

Q *would* be the case.	$\Box\!\!\rightarrow$	in all the most plausible possible worlds
Q *might* be the case.	$\Diamond\!\!\rightarrow$	in some of the most plausible possible worlds

Also, the temporal structure fits Rescher's analysis (1964) in the style of belief revision; suppose M is a set of propositions in reality where $\neg P \wedge \neg Q$ and $M \cup \{P\} \vdash \bot$. We need to choose such maximal $M' \subset M$ that $M' \cup \{P\}$ is consistent and $M' \cup \{P\} \vdash Q$. Such M' lies nearest to the reality in concentric circles while such $M'' \subset M' \subset M$ lies rather far from the center. This explanation exactly matches the psychological aspect of our using past tense. In order to accept an unrealistic fact, we need to erase current facts as minimally as possible.

2.2 Priorian Temporal Logic

The most common way of representing time is one-dimensional time axis. Priorian temporal logic, with F, P, G, and H operators each of which represents *some future*, *some past*, *all the future*, and *all the past*, respectively, can represent this time axis by a sequence of linearly ordered possible worlds [3]. The minimal tense logic consists of

$$\begin{cases} G(\varphi \to \psi) \to (G\varphi \to G\psi),\ H(\varphi \to \psi) \to (H\varphi \to H\psi), \ \ (K) \\ \text{if } \vdash \varphi \text{ then } \vdash G\varphi \ \&\ H\varphi, \ \ (Nec) \\ PG\varphi \to \varphi, \ \ FH\varphi \to \psi. \end{cases}$$

Then, transitivity ($G\varphi \to GG\varphi$ etc.), density, and non-branching conditions are added as axioms.

2.3 Computational Tree Logic

As opposed to such linear (totally ordered) time, we can represent the bifurcation to the future, employing the *path* modalities.

A	in all the paths.
E	there exists a path.

We also employ $X\varphi$ (in neXt situation φ holds) and $\varphi U\psi$ (φ Until ψ in some future) in addition to Priorian F (some future) and G (all the future) in Computational Tree Logic (CTL) [5].

The linearly ordered possible worlds are convenient to represent situational changes, where the truth-value of P may change from false to true, or true to false chronologically. However, this linear time is inconvenient to represent the augmentation of knowledge, where if once a proposition is known to be either true or false then it becomes a firm knowledge and its truth value persists. In order to represent such knowledge expansion, we assume the *heredity* of truth-value in the branching time.

2.4 Intuitionistic Logic

This branching time naturally leads us to *Hasse* diagram of intuitionistic logic [3]. Classical logic assumes only one single world and all the proposition must refer to the same time, and thus the 'studying' time shares the 'being scolded' time in the previous example. Since the accessibility of possible worlds in intuitionistic logic is transitive and reflexive, if a proposition is true in the current world its veridicality is inherited to an eternity as well as the fallacy is . This condition is represented by $\varphi \mapsto \Box\varphi$ and $\neg\varphi \mapsto \Box\neg\varphi$ (McKinsey–Tarski Transformation). Note that in the branching time each possible world (time) is not *maximal*, i.e., truth value of every proposition is not necessarily fixed.

In intuitionistic logic, logical implication $\varphi \to \psi$ is defined as:

$w \models \varphi \to \psi$ if and only if for all w' such that $w \leq w'$ $w' \not\models \varphi$ or $w' \models \psi$.

where 'R' is a transitive and reflexive accessibility relation in possible worlds. Note that neither φ nor $\neg\varphi$ may hold in some worlds. Thus, we cannot utilize the law of excluded-middle, double negation cancellation, and *reductio ad absurdum*. Also, we cannot prove a contraposition if its antecedent is headed by negation, and one of de-Morgan's laws. As the negation is hereditary, i.e.,

$w \models \neg\varphi$ if and only if for all w' such that $w \leq w'$ $w' \not\models \varphi$,

$\varphi \rightarrow \psi$ does not imply $\neg\varphi \vee \psi$. Still, the hereditary *Hasse* diagram and the partial valuation in intuitionistic logic are worth applying to natural language semantics. For example, the lack of the law of excluded middle is convenient for us to represent the distinction between 'if not' and 'unless.' Geis (1973) showed that 'P unless Q' is stronger than 'P if not Q'. The former implies the latter, but not vice versa [2].

Now, we specify how we should represent the prerequisite–effect structure in temporal/causal settings. We introduce the notion of multiple worlds, each of which corresponds to temporal state, and we regard the accessibility in worlds as progress of time, which is hereditary; namely, once a known fact appears it will be kept true in the later states. Then, the prerequisite–effect structure would be defined in the augmentation of known facts; no sooner the knowledge increases to the point the prerequisite is satisfied, than the effect follows. Then, the relation between the prerequisite and the effect should be Dedekind cut in Fig. 1.

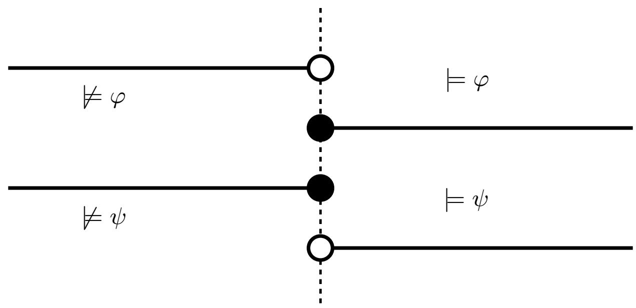

Fig. 1. Dedekind cut for prerequisite–effect structure

3 Prerequisite-Effect Structure in Intuitionistic Kripke Model

Let us take an example from the Penal Code of Japan:

> Article 118 (Leakage of Gas) (1) A person who causes gas, electricity, or steam to leak or flow out or to be cut off and thereby endangers the life, body or property of another person shall be punished by imprisonment with work for not more than 3 years or a fine of not more than 100,000 yen.

We can rewrite this article by using ‘If’ and ‘then’ as follows:

If a person X causes [...], *then* X shall be punished [...].

Then, we can regard the antecedent ‘a person X causes [...]’ as the *prerequisite-part* of this article and the consequent ‘X shall be punished [...]’ as the *effect-part*. Let us denote the prerequisite-part by φ and the effect-part by ψ. Our goal is to give a formalization of the prerequisite-effect structures in general. So, let us denote this by $\varphi \rightsquigarrow \psi$.

Our first approximation is to read ‘$\varphi \rightsquigarrow \psi$’ as ‘$\psi$ becomes effective as soon as φ is satisfied’ or ‘Immediately after φ is satisfied, ψ will become effective’. Then, we cannot formalize ‘$\varphi \rightsquigarrow \psi$’ as the material implication, because it allows the possibility that ψ hold at the same time as φ is satisfied and we should prohibit such a possibility. Here, we cannot employ the intuitionistic implication, either. If we formalize ‘$\varphi \rightsquigarrow \psi$’ as the intuitionistic $\varphi \rightsquigarrow \psi$, then we may consider the situation that ψ becomes effective, e.g., *two years after* when φ is satisfied. We, however, should exclude such a situation in the case of the prerequisite-effect structures. These considerations lead us to the following semantic formulation of ‘$\varphi \rightsquigarrow \psi$’.

Definition 1 (prerequisite-effect structure). *Given any Kripke frame $(W, \leq, V)$ for intuitionistic logic, a prerequisite-effect structure $\varphi \rightsquigarrow \psi$ holds at w iff, for any future state w' of w, if w' is the first state satisfying the prerequisite φ, then the effect ψ does not hold at w' but ψ hold immediately after w', i.e.,*

$$\forall\, w' \geq w.\, \Big(\big(w' \models \varphi \text{ and } \forall\, z < w'.\, z \not\models \varphi\big) \text{ implies } (w' \not\models \psi \text{ and } \forall\, y > w'.\, y \models \psi)\Big),$$

where $x < y$ is defined as $x \leq y$ and $x \neq y$ and $y > x$ means $x < y$.

We, however, cannot define $\varphi \rightsquigarrow \psi$ in the syntax of intuitionistic logic.

Proposition 1. *$\rightsquigarrow$ is undefinable in the syntax of intuitionistic logic.*

Proof. Suppose for contradiction that $p \rightsquigarrow q$ is definable in the syntax of intuitionistic logic. Consider the following two Kripke models:

- $\mathfrak{M} = \langle \{0, 1, 2\}, \leq, V \rangle$, where $\leq$ is the restriction of the ordinary partial ordering on $\mathbb{N}$ and $V(p) = \{0, 1, 2\}$ and $V(q) = \{2\}$.
- $\mathfrak{M}' = \langle \{a, b\}, \leq', V' \rangle$, where $\leq'$ is the partial ordering satisfying $a \leq' b$, $V'(p) = \{a, b\}$ and $V'(q) = \{b\}$.

Consider the mapping $f : \{0, 1, 2\} \to \{a, b\}$ such that it sends 0 and 1 to a and 2 to b. Then, f is a (subjective) *p-morphism* from $\mathfrak{M}$ to $\mathfrak{M}'$ (In [6, p.30], a subjective p-morphism is called *reduction*). It is well-known that, for any formula φ of the syntax of intuitionistic logic and any w from $\mathfrak{M}$, we have $\mathfrak{M}, w \models \varphi$ iff $\mathfrak{M}', f(w) \models \varphi$ (see, e.g., [6, p.31, Theorem 2.15]). It is clear that $\mathfrak{M}, 0 \not\models p \rightsquigarrow q$ but $\mathfrak{M}', f(0) \models p \rightsquigarrow q$ (recall $f(0) = a$). A contradiction. □

Therefore, we need to expand the syntax of intuitionistic logic (with $\perp$) with an additional symbol $\rightsquigarrow$. Let us denote this expanded syntax by $\mathcal{L}_{\rightsquigarrow}$. Our addition of $\rightsquigarrow$ does not break the following *hereditary condition* over Kripke models.

Proposition 2. *Let $\langle W, \leq, V \rangle$ be a Kripke model for intuitionistic logic. For any formula φ of $\mathcal{L}_{\rightsquigarrow}$, if $w \leq u$ and $w \models \varphi$, then $u \models \varphi$.*

This hereditary condition implies the *reverse hereditary condition*, i.e., $w \leq v$ and $v \not\models \varphi$ implies $w \not\models \varphi$. By this, we can demonstrate that our formalization $\varphi \rightsquigarrow \psi$ of prerequisite-effect structures can exclude the situation that the effect ψ holds *before* the prerequisite φ is satisfied, since we prohibit, in $w \models \varphi \rightsquigarrow \psi$, the possibility that the effect ψ hold at the same time as the prerequisite φ is satisfied.

Proposition 3. *Let $\langle W, \leq, V \rangle$ be a Kripke model for intuitionistic logic. If $w \models \varphi \rightsquigarrow \psi$, then, for any future state w' of w, if w' is the first state satisfying the prerequisite φ, then the effect ψ does not hold in any past state of w', i.e.,*

$$\forall w' \geq w. \Big((w' \models \varphi \textit{ and } \forall z < w'. z \not\models \varphi) \textit{ implies } (\forall y \leq w'. y \not\models \psi) \Big).$$

Define the *semantic consequence relation* $\varphi_1, .., \varphi_n \models \psi$ as follows: for any Kripke model $\mathfrak{M}$ and any w in $\mathfrak{M}$, $w \models \varphi_i$ ($1 \leq i \leq n$) implies $w \models \psi$. Then, we have the following logical properties about $\rightsquigarrow$.

Proposition 4. *(i)* $(\varphi_1 \rightsquigarrow \psi) \wedge (\varphi_2 \rightsquigarrow \psi) \models (\varphi_1 \vee \varphi_2) \rightsquigarrow \psi$.
(ii) $((\varphi_1 \vee \varphi_2) \rightsquigarrow \psi) \wedge \neg\varphi_1 \models \varphi_2 \rightsquigarrow \psi$.
(iii) $(\varphi \rightsquigarrow \psi_1) \wedge (\varphi \rightsquigarrow \psi_2) \models \varphi \rightsquigarrow (\psi_1 \wedge \psi_2)$.
(iv) $(\varphi \rightsquigarrow (\psi_1 \vee \psi_2)) \wedge \neg\psi_1 \models \varphi \rightsquigarrow \psi_2$.
(v) $(\varphi \rightsquigarrow (\psi_1 \vee \psi_2)) \wedge (\varphi \rightarrow \neg\psi_1) \models \varphi \rightsquigarrow \psi_2$.
(vi) $(\varphi \rightsquigarrow (\psi \rightarrow \gamma)) \models \varphi \rightsquigarrow \gamma$.

Proof. (i) Assume that $w \models \varphi_1 \rightsquigarrow \psi$ and $w \models \varphi_2 \rightsquigarrow \psi$. Consider any $w' \geq w$ with $w' \models \varphi_1 \vee \varphi_2$ and $\forall y < w'. y \not\models \varphi_1 \vee \varphi_2$. We divide our argument into the following cases: a) $w' \models \varphi_1$, b) $w' \models \varphi_2$. Let us show a) alone. By assumption, it is clear that $\forall y < w'. y \not\models \varphi_1$. Then, w' is also the first state satisfying φ_1. By $w \models \varphi_1 \rightsquigarrow \psi$, we can conclude that $w' \not\models \psi$ and $\forall z > w. z \models \psi$.

(ii) Assume that $w \models (\varphi_1 \vee \varphi_2) \rightsquigarrow \psi$ and $w \models \neg\varphi_1$. Consider any $w' \geq w$ with $w' \models \varphi_2$ and $\forall y < w'. y \not\models \varphi_2$. We show that w' is the first state satisfying $\varphi_1 \vee \varphi_2$. It is clear that $w' \models \varphi_1 \vee \varphi_2$. So, let us establish $\forall y < w'. y \not\models \varphi_1 \vee \varphi_2$. Fix any $y < w'$. We can assume that $w \leq y$, because $w \not\models \varphi_1 \vee \varphi_2$ implies $y' \models \varphi_1 \vee \varphi_2$ for any $y' < w$ by the reverse hereditary condition. Trivially, $y \not\models \varphi_2$. Moreover, we deduce from $w \models \neg\varphi_1$ that $y \not\models \varphi_1$, which implies $y \not\models \varphi_1 \vee \varphi_2$. We have shown that w' is the first state satisfying $\varphi_1 \vee \varphi_2$. Then, we can demonstrate the desired conclusion by $w \models (\varphi_1 \vee \varphi_2) \rightsquigarrow \psi$.

(iii) Assume that $w \models \varphi \rightsquigarrow \psi_1$ and $w \models \varphi \rightsquigarrow \psi_2$. Consider any $w' \geq w$ with $w' \models \varphi$ and $\forall y < w'. y \not\models \varphi$. What we need to show is: a) $w' \not\models \psi_1 \wedge \psi_2$ and b) $\forall z > w'. z \models \psi_1 \wedge \psi_2$. We can easily establish b) by our assumptions. As for a), our assumptions imply that $w' \not\models \psi_1$ and $w' \not\models \psi_1$. Therefore, $w' \not\models \psi_1 \wedge \psi_2$ holds.

(iv) Assume that a) $w \models \varphi \rightsquigarrow (\psi_1 \vee \psi_2)$ and b) $w \models \neg\psi_1$. Consider any $w' \geq w$ with $w' \models \varphi$ and $\forall y < w'. y \not\models \varphi$. Our goal is to show that c) $w' \not\models \psi_2$ and d) $\forall z > w'. z \models \psi_2$. First, let us show c). It follow from a) that $w' \not\models \psi_1 \vee \psi_2$, which implies c). Second, we show d). Fix any $z > w'$. By a), we can obtain $z \models \psi_1 \vee \psi_2$. Since b) implies $z \not\models \psi_1$, we get $z \models \psi_2$, as desired.

(v) Assume that a) $w \models \varphi \rightsquigarrow (\psi_1 \vee \psi_2)$ and b) $w \models \varphi \to \neg\psi_1$. Consider any $w' \geq w$ with $w' \models \varphi$ and $\forall\, y < w'.\, y \not\models \varphi$. Our goal is to show that c) $w' \not\models \psi_2$ and d) $\forall\, z > w'.\, z \models \psi_2$. We can establish c), similarly to the proof of the previous item. So, let us show d) here. Fix any $z > w'$. By a), we can obtain $z \models \psi_1 \vee \psi_2$. Since $w' \geq w$ forces φ, we deduce from b) that $w' \models \neg\psi_1$, which implies $z \not\models \psi_1$. It follows from $z \models \psi_1 \vee \psi_2$ that $z \models \psi_2$, as required.

(vi) Assume that a) $w \models \varphi \rightsquigarrow (\psi \to \gamma)$. Consider any $w' \geq w$ with $w' \models \varphi$ and $\forall\, y < w'.\, y \not\models \varphi$. Our goal is to show that c) $w' \not\models \gamma$ and d) $\forall\, z > w'.\, z \models \gamma$. First, let us establish c). By a), $w' \not\models \psi \to \gamma$ and $\forall\, z > w'.\, z \models \psi \to \gamma$. This means that $w' \not\models \psi \to \gamma$ can be rewritten as $w' \models \psi$ and $w' \not\models \gamma$. Therefore, c) holds. Second, we move to the proof of d). Fix any $z > w'$. By a), we can get $z \models \psi \to \gamma$. Since $w' \models \psi$, we conclude $z \models \gamma$, as desired. □

Example 1. Recall our motivating example of the first item of Article 118 in the beginning of this section. Both of the prerequisite- and effect-parts contain the disjunction ‘or’. Define the meaning of propositional variables as follows (remark that we do not fully rewrite all the disjunctions in the original prerequisite part, for simplicity):

– $\mathsf{Endanger}(n, v)$:= ‘X causes n to v and thereby endangers the life, body or property of another person’, where $n \in \{\,\mathrm{gas}, \mathrm{electricity}, \mathrm{steam}\,\}$ and $v \in \{\mathrm{flow}, \mathrm{leak}, \mathrm{cut\ off}\}$.
– $\mathsf{Imprisonment}$:= ‘X shall be punished by imprisonment with work for not more than 3 years’
– Fine := ‘X shall be punished by a fine of not more than 100,000 yen’

With the help of the prerequisite-effect structure $\rightsquigarrow$, we can represent the first item of Article 118 as follows:

$$\left(\bigvee_{n,v} \mathsf{Endanger}(n, v)\right) \rightsquigarrow (\mathsf{Imprisonment} \vee \mathsf{Fine}).$$

Proposition 5. *(i)* $\not\models \varphi \rightsquigarrow \varphi$.
(ii) $\varphi \to \psi \not\models \varphi \rightsquigarrow \psi$.
(iii) $\varphi \rightsquigarrow \psi \not\models \varphi \to \psi$.
(iv) $(\varphi \rightsquigarrow \psi) \wedge (\psi \rightsquigarrow \gamma) \not\models \varphi \rightsquigarrow \gamma$.
(v) $\varphi \rightsquigarrow \psi \not\models (\varphi \wedge \varphi') \rightsquigarrow \psi$.
(vi) $(\varphi_1 \vee \varphi_2) \rightsquigarrow \psi \not\models (\varphi_1 \rightsquigarrow \psi) \wedge (\varphi_2 \rightsquigarrow \psi)$.
(vii) $\varphi \rightsquigarrow (\psi_1 \wedge \psi_2) \not\models (\varphi \rightsquigarrow \psi_1) \wedge (\varphi \rightsquigarrow \psi_2)$.

Proof. Let us consider the following three Kripke models:

– $\mathfrak{M}_1 = \langle\, \{\,0, 1\,\}, \leq, V_1\,\rangle$, where $\{\,0, 1\,\}$ is ordered as $0 \leq 1$ and $V_1(p) = V_1(q) = \{\,0, 1\,\}$.
– $\mathfrak{M}_2 = \langle\, \{\,0, 1\,\}, \leq, V_2\,\rangle$, where $\{\,0, 1\,\}$ is ordered as $0 \leq 1$, $V_2(p) = \{\,0, 1\,\}$ and $V_2(q) = \{\,1\,\}$.
– $\mathfrak{M}_3 = \langle\, \{\,0, 1, 2\,\}, \leq, V_3\,\rangle$, where $\{\,0, 1, 2\,\}$ is ordered as $0 \leq 1 \leq 2$, $V_3(p) = \{\,0, 1, 2\,\}$, $V_3(q) = \{\,1, 2\,\}$, and $V_3(r) = \{\,2\,\}$.

(i) Take $\mathfrak{M}_1$. Then, $0 \not\models p \rightsquigarrow p$.
(ii) In $\mathfrak{M}_1$, $0 \models p \to q$ but $0 \not\models p \rightsquigarrow q$.
(iii) Take $\mathfrak{M}_2$. Then, $0 \models p \rightsquigarrow q$ but $0 \not\models p \to q$.
(iv) Take $\mathfrak{M}_3$. $0 \models p \rightsquigarrow q$, $0 \models q \rightsquigarrow r$, but $0 \not\models p \rightsquigarrow r$.
(v) Take $\mathfrak{M}_2$. $0 \models p \rightsquigarrow q$ but $0 \not\models (p \wedge q) \to q$.
(vi) Take $\mathfrak{M}_3$. Then, $0 \models (p \vee q) \rightsquigarrow r$. But $w \not\models p \rightsquigarrow r$, which implies $w \not\models (q \rightsquigarrow r) \wedge (p \rightsquigarrow r)$.
(vii) In $\mathfrak{M}_3$, $0 \models q \rightsquigarrow (p \wedge r)$. But $w \not\models q \rightsquigarrow p$, which implies $w \not\models (q \rightsquigarrow p) \wedge (q \rightsquigarrow r)$.

□

4 Representation of legal sentences with prerequisite-effect

The prerequisite-effect requires that if a requirement holds, then the effect holds at the immediately following moment. We illustrate this situation by CISG [7] Article 15 as follows:

> *CISG Article 15*
> (1) An offer becomes effective when it reaches the offeree.
> (2) An offer, even if it is irrevocable, may be withdrawn if the withdrawal reaches the offeree before or at the same time as the offer.

To formally represent the above, we provide the following propositions.

α : the offer reached the offeree.
β : the offer became effective.
γ : the withdrawal reached the offeree.
δ : the offer was withdrawn.

The item (1) in the article can be put into if–then structure as ‘if the offer reaches the offeree, then the offer becomes effective.’ We can directly translate this relation into the prerequisite–effect structure as follows.

$$\alpha \rightsquigarrow \beta.$$

The item (2) defines the condition of withdrawal for the offer. We detail the temporal structure of this condition as follows:

> ‘If the withdrawal reaches the offeree before or at the same time as the offer, then the offer may be withdrawn.’

Namely, the withdrawal must reach the offeree no later than the offer reaches th offeree. In intuitionistic logic, the time where the withdrawal reaches the offeree must precede, or be at the same time as, the time of the offer reaches the offeree, that is,

$$\alpha \to \gamma.$$

Therefore, the offer may be immediately cancelled even though the offer reached the offeree.

$$(\alpha \to \gamma) \to (\gamma \rightsquigarrow \delta).$$

With the item (2), we rewrite the item (1) as follows.

$$(\alpha \wedge \neg\delta) \rightsquigarrow \beta.$$

Finally, CISG Article 15 is formalized as follows.

$$\begin{cases} (\alpha \wedge \neg\delta) \rightsquigarrow \beta. \\ (\alpha \to \gamma) \to (\gamma \rightsquigarrow \delta). \end{cases}$$

And the situation is depicted as in Fig. 2. Fig. 2:(a) means the offer becomes effective, and Fig. 2:(b) means the offer was withdrawn.

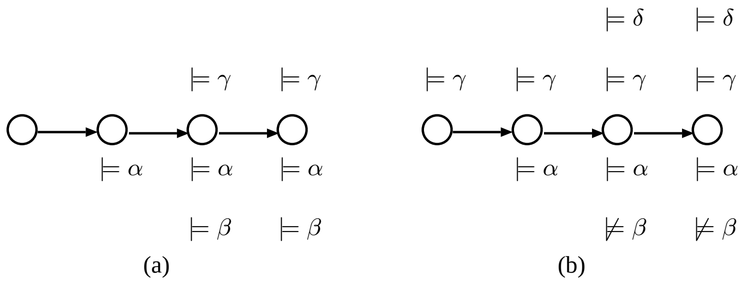

Fig. 2. The model of CISG Article 15

5 Conclusion

In this paper, we have formalized the prerequisite–effect structure in terms of extended intuitionistic logic. To construct a formal reasoning system of law, we must mix various kinds of if–then relations, including classical implication, subsumption relation, temporal relation, and other causal relations. Since our ultimate objective is to represent legal documents in logic programming, those various kinds of logical relations should be represented in one and unique logic formalism. If we adopt modal operators for each relation such as temporal, deontic, and epistemic operators, the legal system would be a complicated product of polymodal logic, which is not realistic in implementation. Thus, we avoided introducing modality; instead, we preferred to stay in simple Kripke semantics in which various relations are represented merely by accessibility among possible worlds. In the hereditary progress of temporal states, we formalized the relation as that 'the effect appears immediately after the prerequisite is satisfied' denoted by '$\rightsquigarrow$.' We have investigated the logical features of '$\rightsquigarrow$,' and have shown an example of CISG article 15.

References

[1] D. R. Dowty. *Word Meaning and Montague Grammar*, D. Reidel (1979)

[2] J. D. McCawley. *Everything That Linguists Have Always Wanted to Know about Logic* (2nd edition), The University of Chicago Press (1993)

[3] F. Landman. *Structures for Semantics*, Kluwer Academic Press (1991)

[4] L. T. F. Gamut. *Logic, Language, and Meaning*, The University of Chicago Press (1991)

[5] M. Wooldridge. Reasoning about Rational Agents, The MIT Press (2000)

[6] Chagrov, A., Zakharyaschev, M.: Modal Logic. Number 35 in Oxford Logic Guides. Oxford Science Publications, Oxford (1997)

[7] UNCITRAL: (UNITED NATIONS CONVENTION ON CONTRACTS FOR THE INTERNATIONAL SALE OF GOODS (1980)) ‘http://www.uncitral.org/’