Title	WAVELETを用いたナビエ・ストークス方程式の並列解法
Author(s)	福井，直人
Citation	
Issue Date	1998-03
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/1115
Rights	
Description	Supervisor:松澤 照男，情報科学研究科，修士

Parallel Scheme of Navier-Stokes Equation using Wavelets

Fukui Naoto

School of Information Science,
Japan Advanced Institute of Science and Technology

February 13, 1998

Keywords: wavelet, preconditioning, conjugate gradient method, ,incomplete wavelet transform, matrix solver, differential equation.

The numerical simulation in CFD(computational fluid dynamics) is a large-scale calculation which needs degree of freedom that most is large. In such a simulation, the partial differential equation which rules the phenomenon of the object is calculated by discreting by the method like the finite element method and the finite difference method, etc. It is a part to have the majority of computing time of the calculation where the linear system is solved.

In general, if the problem becomes large-scale, the distribution of the eigenvalue of the coefficient matrix is made ill-conditioned, and when the scale of the equation is large, the increase of computing time will be invited though iterative method is used. The reason for this is that the distribution of the eigenvalue of coefficient matrix is made ill-conditioned, and the iterative number increases. The increase of such a computing time is one of factors to make a detailed numerical analysis difficult.

On the other hand, in recent years, the wavelet analysis came to be used by the signal analysis and image processing, etc. as a technique which took the place of the Fourier analysis. In addition, the range extended by wavelet with a compact support having been proposed by I.Daubechies, too and the application became possible also in the field of the numerical analysis. The methods using wavelet for the numerical solution of the partial differential equation are methods by which the grid at each level in the adaptive grid scheme, as the basis function in the Galerkin method, using the scaling function of wavelet, the matrix methods by the wavelet transform, etc.

When Tanaka take advantage of the fact that the partial differential equation discreted and used wavelet for coefficient matrix of the obtained linear system, the condition number do not depend on the number grid poits and calculated the Poisson's equation as elliptic problem, using wavelet for preconditioning the conjugate gradient method.

In general, if we calculate linear system obtained discreting the differential equation using the iterative methods, as the problem becomes large-scale, the distribution of the eigenvalue of the coefficient matrix is made ill-conditioned, the number of iteration is increased, and, in a word, the increase of computing time is invited. However, we transforms the coefficient matrix by discrete wavelet transform, and the coefficient matrix with discrete wavelet transform is rescaled suitably, so that the conditon number of the coefficient matrix do not depned on the number of grid points. So if the number of grid points is increased, the number of iteration do not depend on the number of grid points. However, as usual, the matrix solver taking advantage of this merit has the following problems.

(a) It is necessary to convertion into the one that the problem was periodic.

(b) DWT should be made periodic.

(c) The eigenvalue which becomes 0 appears.

(d) The method becomes very complex.

Then, Tanaka proposed an easily executable, imcomplete wavelet transform to solve these problems takeing advantage of the merit of wavelet, and applied to not the matrix solver but the preconditioning. The reason for this is that, the matrix solver requires strictness, but as preconditioning of the matrix solver, approximating technique which is efficient, and which procedure is simple, is more suitable than the solver which is strict and complex. Moreover, because the merit of wavelet is the locality of data, we only treat local data for discrete wavelet transform. and this methods is suitable to parallelize and to vectorize.

Navier-Stokes equation represent a phenomenon of flow. It is combined the elliptic problem and the parabolic problem. The elliptic problem is Poisson's equation, and the parabolic problem is nonlinear.

In this study, we calculate basic partial differential equations by the method using imcomplete wavelet transform and verify the effect of this method, and examine the efficiency of this method. The method used in this study is, for solving the linear system obtained that the partial differential equation is discreted by the finite difference methods, the preconditioning conjugate gradient method using the imcomplete discrete wavelet transform. As a result, for the Poisson's equation, the diffusion equation, and the Burger's equation, we obtained the number of the iteration and accuracy, and verified the performance of this methods.