A Study on Discrete Abstraction of Stochastic Hybrid Systems

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Along with the development of science and technology, higher performance is required for production processes and industrial robots. Accordingly, it is necessary develop methods to control complex systems. The modeling method using a hybrid systems is known as a method to express the behavior of complex systems. Hybrid systems are dynamical systems composed of continuous dynamics such as differential/difference equations and discrete dynamics such as finite automata. Recently, analysis and control of hybrid systems have been extensively studied in the control theory community and the computer science community. Furthermore, the framework on analysis and control of hybrid systems has been extended to stochastic hybrid systems. A stochastic hybrid systems is well known as a model of communication networks and biological systems, and developing analysis and control methods is one of the significant works from theoretical and practical viewpoints. Although a general class of stochastic hybrid systems has been proposed in, it is difficult to solve control problems, and special classes are frequently considered. One of the typical classes is to

assume that continuous dynamics are deterministic. Even if the system is limited to such a class, then there are several applications such as failure-prone systems .

However, in order to apply theory of stochastic hybrid systems to practical plants, there are several technical issues. As one of the issues, we can consider the computation time for solving the problem such as the optimal control problem and the reachability problem. The computation time exponentially grows for the number of discrete states (modes). In many cases, it is difficult to compute the optimal control input and the reachability set. As a method to overcome this technical issue, the discrete abstraction method has been focused. In this method, the state space of a given system is partitioned to a finite set of state regions, and the behavior of the system is expressed by the transition between obtained state regions. In some methods, the representative point is given for each state region, and the behavior is expressed as a transition between representative points. Thus, by the discrete abstraction method, a given system can be transformed into a finite-state system. So we can easily consider several problems such as the reachability problem.

As a method of discrete abstraction methods, three methods have been obtained so far.. First, the predicate abstraction is explained. In this method, the partition of the state space is given in advance, and the transitions between regions implies an over-approximation of the original system. So we cannot consider the liveness verification problem, but can consider the safety verification problem. Next, the method based on the notion of bisimulation is explained. Using this method, a finite-state system that is equivalent to the original system in the sense of bisimulation can be obtained. By using the obtained finite-state system, we can verify not only safety but also reachability. In addition, we can apply this finite-state system to control synthesis. However, in the procedure for deriving a finite-state system, termination is in general not guaranteed. Finally, the

concept of approximate bisimulation has been proposed so far. For stochastic hybrid systems, both bisimulation and approximate bisimulation have been proposed so far.

In this paper, the notion of bounded bisimulation is proposed for a class of stochastic hybrid systems in which continuous dynamics are deterministic. In bounded bisimulation, the behavior of a given is considered in a given finite time interval. So the obtained finite-state system can be applied to bounded model checking. For deterministic/non-deterministic hybrid systems, the notion of bounded bisimulation and the algorithm for deriving a finite-state system have been proposed so far. In this approach, termination of the algorithm is guaranteed. The finite-state state system obtained by discrete abstraction based on bounded bisimulation preserves reachability and transition probabilities in the original system. By using the obtained finite-state system, the state region that can be reached from a given state and its transition probability can be obtained. Therefore, we can solve the control problem considering the probability that the state reaches the unsafe state region and the the probability that the state reaches the desired state region.

This paper is organized as follows. First, a class of stochastic hybrid systems studied in this paper is explained. Next, a notion of bounded bisimulation is defined. Third, an algorithm of discrete abstraction based on bounded bisimulation is proposed. The proposed algorithm consists of four procedures, and details of each procedure are explained by illustrations. Finally, the effectiveness of the proposed method and the implementation is explained by numerical examples.

Next, details of numerical examples are explained. In numerical examples, the effectiveness of the proposed is shown by four examples. In the first example, a deterministic hybrid system is considered as a special case of stochastic hybrid systems. By the derivation of a finite-state system for this system, the procedure and the implementation of discrete abstraction

are explained. In addition, the obtained state transitions and a set of linear inequalities expressing the state partition are shown, and the discrete dynamics obtained by discrete abstraction are explained in detail. In the second example, the stochastic hybrid system, which is derived by modifying the above example of deterministic hybrid systems, is considered. Comparing the first example with this example, the stochastic behavior of discrete dynamics obtained by discrete abstraction is explained. In the third example, for some stochastic hybrid systems, bounded bisimulation is derived, and the number of regions in the state partition is discussed. From this example, we see that depending on the dynamics in a given stochastic hybrid system, the number of regions is extensively different. In the final example, a model of a genetic toggle switch is considered, and an application to control synthesis of a finite-state system obtained by discrete abstraction is explained. From the state transitions in the obtained finite-state system, the state and the control input that realize a given control purpose is derived. From this result, we see that bounded bisimulation is effective for analysis and control of stochastic hybrid systems.

In the proposed discrete abstraction method based on bounded bisimulation, the behavior in the finite time interval is focused, and termination of the algorithm is guaranteed. However, depending on a given time interval, the number of regions in the state partition may explosively increase. In future works, it is important to consider the development of an approximate algorithm.