Discussion on: "Theory and Computation of Discrete State Space Decompositions for Hybrid Systems"

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I. Discussion

In the paper by E. De Santis and M. D. Di Benedetto, a new method of discrete state space decompositions is proposed for a class of hybrid systems, which is called an H-system. By using the proposed method, the problem of verifying the property such as stabilizability is decomposed to some problems for a set of linear systems and/or hybrid systems with small size. These problems can be relatively solved easier than the original problem. Thus computational burden is decreased. The obtained result is very interesting from both theoretical and practical viewpoints, and is one of the basic results in analysis and control of hybrid systems.

In this discussion, first, the main idea in this paper is summarized. Next, two topics are discussed.

A. Main Idea

First, an H-system is defined. Let $\mathbf{Q} = \{1, 2, \ldots, N\}$ denote the finite set of discrete states (modes). For each discrete state, the dynamical system $S(i)$ defined by

$$\dot{x}(t) = \phi_i(x(t), u(t)) \tag{1}$$

is assigned, where $x(t) \in \mathbb{R}^{n_i}$ is the state, $u(t) \in \mathbb{R}^m$ is the control input. A switch from mode i to mode j is decided by an event $\sigma = \mathbf{W} = \mathbf{U} \cup \mathbf{V}$, where $\mathbf{U}$ is the finite set of discrete controls, $\mathbf{V}$ is the finite set of discrete disturbances. The hybrid state space is defined by $\Xi := \cup_{i\in\mathbf{Q}}\{i\} \times \mathbb{R}^{n_i}$. In addition, the reset mapping is defined by $R : \mathbf{W} \times \Xi \to 2^{\Xi}$. More formally, an H-system is given as the following tuple:

$$\mathcal{S} = (\Xi, \mathbf{W}, S, E, g, f, R).$$

See the paper [3] for further details. In particular, if the system (1) is given as a linear state equation, then $\mathcal{S}$ is called an LH-system.

Next, for a given set $\mathbf{Q}' \subset \mathbf{Q}$, let $Reach^{-1}(\mathbf{Q}')$ denote the set of all initial discrete states such that for some real $\mathbf{t} \in \mathbb{R}$, some integer $\mathbf{j}$, and some control input, the relation $\xi(\mathbf{t}, \mathbf{j}) \in \Xi' := \cup_{i\in\mathbf{Q}'}\{i\} \times \mathbb{R}^{n_i}$ holds ($\xi(\mathbf{t}, \mathbf{j}) \in \Xi$ is the hybrid state), despite the action of the discrete disturbances. In addition, for a given set $\mathbf{Q}' \subset \mathbf{Q}$, let $\mathcal{S}|_{\mathbf{Q}'}$ denote the subsystem of $\mathcal{S}$ associated to the $\mathbf{Q}'$.

The set $Reach^{-1}(\mathbf{Q}')$ plays an important role in the proposed method. As one of the examples, I consider the stabilizability problem for LH-systems. Let $\mathbf{Q}_0$ denote the set of discrete states whose associated linear dynamics are controllable. Then, $\mathcal{S}$ is asymptotically stabilizable if and only if $\mathcal{S}|_{\overline{Reach^{-1}(\mathbf{Q}_0)}}$ is asymptotically stabilizable, where $\overline{Reach^{-1}(\mathbf{Q}_0)}$ is the complement in $\mathbf{Q}$ of $Reach^{-1}(\mathbf{Q}_0)$ (see Section 5.1 in the paper [3] for further details). Furthermore, traps are computed from $\mathcal{S}|_{\overline{Reach^{-1}(\mathbf{Q}_0)}}$. Thus the stabilizability problem for LH-systems can be decomposed to the stabilizability problems for LH-systems and/or linear systems, where the number of problems are given by the number of traps. In the example of the paper, the stabilizability problem for an LH-system with 11 modes is decomposed to the stabilizability problems for three linear systems and two bimodal LH-systems.

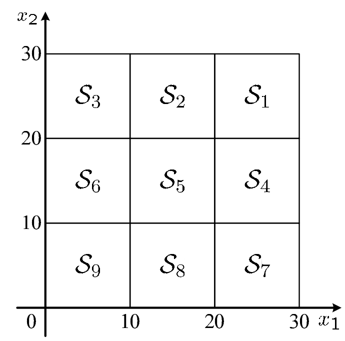

Fig. 1. State partition.

B. Discussion on Results

To decrease the computation time for verifying several properties, the decomposition method proposed in the paper is effective. Here, to support the effectiveness of the proposed method, I focus on the following two topics:

(i) Extension to piecewise affine (PWA) systems,
(ii) Graph structure that the proposed method efficiently works.

First, (i) is discussed. Consider the following PWA system

$$\dot{x}(t) = A_i x(t) + B_i u(t) + a_i, \quad \text{if } x(t) \in \mathcal{X}_i, \tag{2}$$

where $i \in \mathbf{Q}$. In PWA systems, events, i.e., discrete controls and disturbances are not given in advance. So these events must be defined. If the dynamics assigned to each mode are controllable, then the directed graph expressing a finite state machine will be obtained. For example, in the case that the state space is partitioned as Fig. 1, the directed graph in Fig.

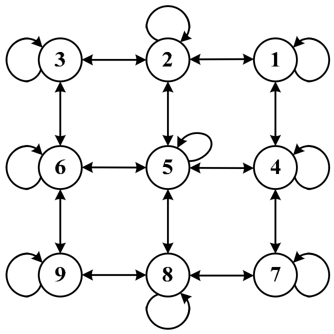

Fig. 2. Directed graph.

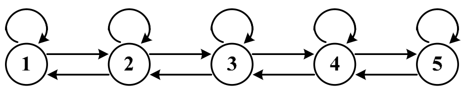

Fig. 3. Directed graph expressing gear shift logic.

2 is obtained. Depending on the dynamics, all events may become discrete controls.

However, when input constraints are imposed, several situations must be considered. Of course, both discrete controls and disturbances are included in a finite state machine. Furthermore, it will be necessary to decompose each partitioned region $\mathcal{X}_i$ in (2) to some regions. To discuss this topic, assume that there exist discrete controls and disturbances from some mode j. Then, the following situation may be appeared: discrete controls and disturbances are allowed in $\mathcal{Y}_1 \subset \mathcal{X}_j$ and $\mathcal{Y}_2 \subset \mathcal{X}_j$, respectively. If $\mathcal{Y}_1 \cap \mathcal{Y}_2 = \emptyset$ is satisfied, and the state does not reach from any element in $\mathcal{Y}_1$ ($\mathcal{Y}_2$) to some element in $\mathcal{Y}_2$ ($\mathcal{Y}_1$), then $\mathcal{X}_j$ must be decomposed to $\mathcal{Y}_1$ and $\mathcal{Y}_2$. In other words, a new mode must be added.

Thus in an extension to PWA systems, it is necessary to overcome several technical issues. In addition, it is very interesting to clarify the relation between the proposed method and the discrete abstraction technique [1].

Next, (ii) is discussed. Characterization of the class of directed graphs that the proposed method efficiently works is important in analysis of computational complexity. As a first step, it will be desirable to analyze typical graphs appearing in hybrid systems. One of the typical graphs is given by Fig. 2. Another example is shown in Fig. 3, which expresses gear shift logic [2], [4].

The above two topics are important for further understanding the proposed method, and show one of the directions in future works.

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