Title	時間性を表現する含意を持つ論理
Author(s)	菊池，健太郎
Citation	
Issue Date	1999-03
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/1237
Rights	
Description	Supervisor:石原 哉，情報科学研究科，修士

A Logic with Implication Expressing Temporality

Kentaro Kikuchi

School of Information Science,
Japan Advanced Institute of Science and Technology

February 15, 1999

Keywords: Kripke semantics, intuitionistic logic, completeness, embedding.

In Kripke semantics which is well-known as semantics for modal logics, each formula is interpreted on triples consisting of a set of possible worlds, a binary relation (called an accessibility relation) on the set, and a valuation which assigns to each possible world the basic propositions that hold at the world. In Kripke semantics for non-modal propositional logics such as intuitionistic logic, intermediate logics, and weak logics with strict implication, a formula $A \to B$ is true at a possible world if and only if at every possible world accessible from there, whenever A is true, also B is true. In the case of intuitionistic logic an accessibility relation is a partial order, so regarding it as the order of time we may consider that $A \to B$ is true if and only if at any future world (including the present), whenever A is true, also B is true.

In this paper, we propose new Kripke semantics in which interpretation of implication is different from that in intuitionistic logic etc. In that semantics, an accessibility relation is a partial order as in intuitionistic logic, but the informal meaning of $A \to B$ is "B holds as long as A holds (in the future)." This interpretation of implication enables us to express temporality by comparing how long each formula remains true.

This paper consists of comparative studies between intuitionistic logic and the logic corresponding to the new Kripke semantics. We will first give some informal motivation of Kripke semantics for intuitionistic logic. Think of an idealized mathematician (in this context traditionally called the creative subject), who extends his knowledge in the course of time. At each moment x he has a stock of sentences, which he, by some means, has recognized as true. Since at every moment x the idealized mathematician has various choices for his future activities (he may even stop altogether), the stages of his activity must be thought of as being partially ordered, and not necessarily linearly ordered. How will the idealized mathematician interpret the logical connectives? Evidently the interpretation of a composite statement must depend on the interpretation of its parts, e.g. the idealized mathematician has established A or (and) B at stage x if he has established A

at stage x or (and) B at stage x. The implication is more cumbersome, since $A \to B$ may be known at stage x without A or B being known. Clearly, the idealized mathematician knows $A \to B$ at stage x if he knows that if at any future stage (including x) A is established, also B is established.

Now we will sketch new Kripke semantics we propose, which is different from that for intuitionistic logic in the following two respects. First, we make no assumption that if a propositional variable p is true at stage x, then p is true at any future stage (including x). Furthermore, we modify the interpretation of implication in intuitionistic logic, so that we can express temporality by comparing how long each formula remains true. Namely, the informal meaning of $A \to B$ is "B holds as long as A holds (in the future)." In this interpretation, there exist Kripke models with possible worlds at which formulas $A \to (B \to A)$ or $(A \to C) \wedge (B \to C) \to (A \vee B \to C)$ is not true, while those are valid in any Kripke model of intuitionistic logic. Conversely, in this interpretation, $A \vee \neg A$ and $\neg\neg A \to A$ are valid in any Kripke model, while there exist Kripke models of intuitionistic logic such that those formula is not valid. Therefore, neither of intuitionistic logic and the new logic contains the other in Kripke validity.

Next, we summarize formal systems for each Kripke semantics. In this paper, we present Hilbert-type formal systems of intuitionistic logic and the new logic. The Hilbert-type system of intuitionistic logic contains nine axioms and one inference rule (modus ponens). Soundness Theorem which says that any formula derivable in this system is intuitionistically Kripke valid, is proved as usual by induction on derivation. Every axiom is intuitionistically Kripke valid and the inference rule (modus ponens) preserves the validity. Conversely, for the Completeness Theorem which says that any intuitionistically Kripke valid formula is derivable in the system, we need some notions and a few lemma's. To prove this theorem, we construct so-called canonical model whose possible worlds are the sets of all maximal consistent sets, whose accessibility relation is including relation on those sets, and whose valuation is defined such that a basic proposition is a member of a maximal consistent set if and only if the proposition is true at the maximal consistent set. Now if a formula is not derivable in the system above, then there exists a maximal consistent set such that the formula is not an element in it. Therefore the formula is not true at the possible world, so it is not valid in the canonical model. In this way, the Completeness Theorem is proved in the case of intuitionistic logic. On the other hand, the Hilbert-type formal system of the new logic is based on eleven axioms and four inference rules. We can prove that the system is also sound for the new Kripke semantics in the same way as that in intuitionistic logic. To prove the Completeness Theorem we try to construct the canonical model as in intuitionistic logic, but a problem rise as follow. A Kripke model in which $(p \to q) \wedge (q \to p) \to (p \vee q \to q)$ is not valid requires infinitely many elements at which p is true and q is not true as the example above. This implies that one set of formulas have to be recognized as different elements in the canonical model, so we cannot construct the canonical model in the same way as that in intuitionistic logic.

As we see in the examples of formulas above, neither intuitionistic logic nor the new logic contains the other in Kripke validity. We can, however, consider some connections between intuitionistic logic and the new logic. Namely, we can try to embed intuitionistic logic into the new logic in the sense that there is a translation Tr such that for every formula A, A is intuitionistically Kripke valid if and only if $Tr(A)$ is Kripke valid in the new logic. Such a relation holds between classical propositional logic and intuitionistic propositional logic, where the translation is a simple one from a formula A of classical propositional logic to $\neg\neg A$ of intuitionistic propositional logic. The translation presented in this paper is one of such translations, and makes it possible to interpret logical connectives in intuitionistic logic in terms of those in the new logic.

Finally, we arrange our results of this paper. We proposed new Kripke semantics in which implication is interpreted to express temporality, and provided a formal system which is sound for the semantics, while completeness remains a problem. We also showed that intuitionistic logic can be embedded into the logic corresponding to the new semantics. Thus, in that logic it is certainly possible to express further details than in intuitionistic logic.