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Description	

Optimal Choices of Fare Collection Systems for Public Transportations: Barrier versus Barrier-free

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Abstract

The present study focuses on two major types of fare collection systems for public transportations, barrier and barrier-free, and provides a mathematical framework to evaluate optimal choices between them, i.e., which system can be more profitable for a transit agency. In particular, we consider game-theoretic interactions between the transit agency and passengers for the barrier-free system and suppose that frequencies of free rides of passengers as well as inspections of the transit agency are given as a Nash equilibrium. Then the optimal choice of fare collection system is described as a subgame perfection solution in an extensive form game. We also conduct a comparative static analysis and examine how each parameter can affect the choice. As an application, we use the framework to explain various choices of fare collection systems in our society depending on local circumstances or transportation types.

Keywords: Public transportation, Fare collection system, Proof of payment, Profit maximization, Game theory

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1. Introduction

There are various fare collection systems for public transportations in the world, depending on countries and regions, types of transportations, and so on. In this paper, we focus on two major types of them, barrier and barrier-free, and provide a mathematical framework to evaluate in a formal way optimal choices between them, i.e., which system can be more profitable for a transit agency. Hence, in particular, we think of transportations such that the choice between the two systems can potentially be worth serious consideration, such as heavy rail, commuter rail, LRT and BRT.

While the detail of each system may always differ in each case, the two systems we deal with in the study, the barrier system (henceforth BA) and the barrier-free system (or the proof-of-payment system, henceforth POP), can generally be described as follows. First, the BA system requires the transit agency to install ticket gates, or turnstiles, and establish a clearly defined paid area. Therefore, by its nature, all passengers need to pay for the tickets before going on board. On the other hand, the POP system allows the platform to be barrier-free. Passengers are required to pay for fares legally, but not forced to do so physically. In order to crack down on free rides, the transit agency randomly conducts inspections for the valid proof of payment and collects fines from passengers without it[1].

Since their different characteristics have different financial impacts, the choice of fare collection system becomes an important managerial issue for the transit agency. For example, a typical trade-off is that, in general, BA

[1] There is another type of fare collection system that allows on-board fare payments, where passengers are allowed to pay for fares via inspectors or conductors on board. We distinguish this system from POP and do not deal with it in this paper.

requires higher capital costs for ticket gates and facilities for enclosed platforms, while POP requires higher labor costs for inspections. In addition, fare evasion may become a serious problem particularly in POP. Although there are some reports and guidelines that discuss the issue (e.g. Toronto Transit Commission, 2000; and Transportation Research Board, 2002), any rigorous mathematical framework considering economic behaviors of the transit agency as well as passengers has not been provided[2]. Our contribution is to develop a formal and consistent perspective to deal with the issue of comparisons of fare collection systems.

For the purpose, assuming a simple public transportation, we model the transit agency's revenue and cost for each fare collection system. In particular, we assume game-theoretic interactions between the transit agency and passengers for POP. Under the system, as the transit agency's inspection is conducted randomly, fare evasions of passengers become an unavoidable issue. Indeed, according to Transportation Research Board (2002) that reports the fare evasion rates of 19 transit agencies using POP in the world, they are from 0.3% to 15.0%. There are some economic studies that discuss a passenger's fare evasion as her expected utility maximization (Polinsky and Shavell, 1979; Boyd et al., 1989; and Kooreman, 1993), but they do not consider such a game-theoretic interaction that POP can bring about, that is, the interdependency of the frequency of free rides of passengers and that of inspections by the transit agency. If a high volume of passengers is expected to go without tickets, the transit agency should conduct inspections

[2]Tirachini and Hensher (2011) discuss mathematically comparisons of fare collection systems for buses. However the alternatives are on-board or off-board, or payment by cash or contactless smart card etc., and they leave POP, which is of our main concern, out of consideration, for POP is uncommon for buses.

frequently, and on the other hand, if inspections are made so frequently, passengers would not take a risk of free ride. We formulate such a situation by applying inspection game (Avenhaus et al., 2002; and Avenhaus, 2004) and suppose their behaviors are given as a Nash equilibrium of the game.

On the other hand, we formulate the BA system simply by assuming that it entails no fare evasion. Then we consider the transit agency chooses, or should choose, the fare collection system that can be more profitable. Such an approach that compares some systems taking into account each system's incentive scheme can also be considered as a comparative institutional analysis (Aoki, 2001).

Following this introduction, Section 2 provides the analytical framework, namely, the transit agency's profit model of each fare collection system. Based on it, Section 3 conducts a comparative static analysis by which we examine how each parameter can affect the optimal choice. In addition, we specify the transit agency's optimal choice of fare collection system as a subgame perfection solution in an extensive form game. Then Section 4 applies the model to interpret our society's various fare collection choices. Finally Section 5 states concluding remarks and several open questions.

2. Analytical Framework

2.1. The Target and Basic Assumptions

As the target of our analysis, we suppose a simple public transportation that connects two stations. Then we discuss the optimal choice of its fare collection system, that is, which system, BA or POP, can be more profitable for the transit agency.

Let us denote the fare by $a(> 0)$. Then suppose n passengers board the transportation per operation, from one station to another. In this paper, we do not discuss pricing of the fare. It is also assumed that the demand of the transportation does not depend on the fare collection system, that is, the both systems have common n (see also Section 5 for the assumption). We investigate the optimal choice of fare collection system under given a and n.

Let the assessment period of the analysis be t-time operations. That is, we evaluate the profitability of the transportation while it is operated t times. As its revenue, we only consider revenues from fares, and fines in the case of POP[3]. With regard to the cost, since our interest is in comparisons of the two fare collection systems, we only specify additional costs that are required for introducing either of them: we leave out the description of costs common in the both systems such as the land cost, the construction and maintenance cost of the rail tracks, and so on. Then the profit is the difference between the revenue and the cost.

2.2. Barrier System

The BA system requires the transit agency to install ticket gates and establish a clearly defined paid area. Let us denote the initial costs for them by $c_s(> 0)$, and the operating and maintenance costs for them per operation by $c_m(> 0)$. We suppose, for simplicity, every passenger pays for the fare, i.e., no fare evasion under BA[4].

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[3]Some real-world transit agencies do not receive fines as their own revenues, and instead they are collected by the courts or other relevant organizations. We, in this paper, assume the transit agency can count all the amount of the collected fines as its own revenue.

[4]One may be inclined to consider the possibility of fare evasion under BA as well because there are transit agencies using the BA system facing some levels of fare evasions (e.g. Reddy et al., 2011). A simple way to incorporate this is assuming that the initial investment, c_s, is a decision variable for the transit agency and the fare evasion rate is a decreasing function with respect to it, that is, $\partial x/\partial c_s \leq 0$, where x is the fare evasion

Hence, the revenue and the cost of BA are defined as ant and $c_s + c_m t$, respectively, and thus the total profit during the assessment period, $\pi_{BA}(t)$, is determined as:

$$\pi_{BA}(t) = ant - (c_s + c_m t) \tag{1}$$

2.3. Proof-of-payment System

The POP system does not set up ticket gates but instead requires the transit agency to make occasional inspections for valid proof of payment. When the system is adopted, it is physically possible for passengers to go without tickets. Let the probability that a passenger properly buys the ticket be $p \in [0, 1]$, that is, the frequency of free ride is $1 - p$. On the other hand, the transit agency makes an inspection with the probability $q \in [0, 1]$. An inspector checks passengers on board with this frequency and collects fines from any passenger without a ticket.

Let us denote the fine of a fare evasion by b and suppose the fine level is determined by the transit agency. Then let us denote the inspection cost per passenger inspected by c_i. This can be understood as follows. Suppose an inspector can inspect $n'(\leq n)$ passengers in one operation, and let the labor cost for it be C_i. Then c_i is given as C_i/n'. While c_i is constant in the basic model, we will explicitly describe it as a function of the passenger volume as $c_i(n)$ in 4.2.

In the subsequent analysis, we assume $b \geq a$ because otherwise a pas-

rate. Then the revenue is expressed as $a(1 - x)nt$. We note that this does not affect our result in Section 3 significantly, but of course can make BA less attractive for the transit agency. In reality, it is virtually impossible to predict accurately the relation of x and c_s, therefore the fare evasion rate would be estimated based on similar existing cases, for instance.

senger has no monetary incentive to buy a ticket. We also assume $b \geq c_i$ because otherwise the transit agency has no monetary incentive to make inspections. Furthermore, let us suppose there is an upper limit of the fine that the transit agency can set and denote it by b_{max}, which would depend on local circumstances such as laws, cultures and so on (see also 4.1). Consequently, $b \in [\max\{a, c_i\}, b_{max}]$.

Hence, the revenue of POP is defined as the sum of the fares and fines collected, namely, $apnt + bq(1-p)nt$, while its cost is the total inspection cost, namely, $c_i qnt$. Therefore, the total profit during the assessment period under given b, $\pi_{POP}(b, t)$, is determined as:

$$\pi_{POP}(b, t) = apnt + bq(1-p)nt - c_i qnt \tag{2}$$

Here p and q are supposed to be interdependent. That is, if a high volume of passengers is expected to go without tickets, the transit agency should conduct inspections frequently, while otherwise it should decrease inspections to save the cost. On the other hand, if the inspections are made so frequently, passengers would not take a risk of free ride, while otherwise the proportion of free riders would increase.

We formulate such a situation as a two-player normal form game played by a passenger and the transit agency by applying the idea of inspection game (Avenhaus et al., 2002; and Avenhaus, 2004) as shown in Table 1. Here we assume that all passengers are homogenous and each passenger plays the game with the transit agency.

a passenger \ the transit agency	Inspect	Not inspect
Buy ticket	$-a,\ a - c_i$	$-a,\ a$
Free ride	$-b - m,\ b - c_i$	$-m,\ 0$

Table 1: the inspection game

In the inspection game, a passenger has two alternatives, or strategies: to buy a ticket properly (Buy ticket) or to take a free ride (Free ride). Likewise, the transit agency has two alternatives: to inspect (Inspect) or not to inspect (Not inspect). The payoffs are defined as their monetary gains and losses at each outcome, i.e., each combination of each player's choice. In addition, we take into account a passenger's morality expressed as m in the payoff matrix, for one may argue that a passenger's behavior might be determined not only by the monetary gain or loss for such a legal issue. Here $m \geq 0$, and $-m$ indicates the passenger's disutility of taking an illegal action, which is expressed in the monetary term. Then p is the probability that the passenger chooses "Buy ticket," while q is the probability that the transit agency chooses "Inspect."

Let us suppose these probabilities are given as a Nash equilibrium of the game. The game always has the unique Nash equilibrium given as follows:

$$(p, q) = \begin{cases} (1, 0) & \text{if } a \leq m \\ (1 - \frac{c_i}{b}, \frac{a - m}{b}) & \text{otherwise} \end{cases} \tag{3}$$

When a passenger's morality is high enough ($a \leq m$), the game has the unique pure strategy Nash equilibrium where the passenger always chooses "Buy ticket," while the transit agency always chooses "Not inspect." Otherwise, there exists the unique strict mixed strategy Nash equilibrium, where free rides and inspections are made stochastically. As stated in Section 1, since the fare evasion rates in real-world transportations with POP are not zero, it might not be realistic to assume $a \leq m$. Anyway, by using the values of p and q, the profit of the transit agency, $\pi_{POP}(b, t)$, can be described as:

$$\pi_{POP}(b, t) = \begin{cases} ant & \text{if } a \leq m \\ a(1 - \frac{c_i}{b})nt & \text{otherwise} \end{cases} \tag{4}$$

As a mathematical consequence of the inspection game, we note that the total amounts of fines taken in and the costs for inspections, i.e. $bq(1-p)nt$ and $c_i qnt$, always cancel each other out[5]. Furthermore, a passenger's expected payoff at the equilibrium is calculated as $-a$. This means that, under POP, the expected travel cost for a passenger to use the transportation depends neither on the fine level nor on the morality, and is just equal to the regular fare. Hence the difference of fare collection system does not provide any benefits, positive or negative, to passengers on average in our settings (see also Section 5). On the other hand, the transit agency's expected payoff at the equilibrium is calculated as $a(1-c_i/b)$, which depends on how much the fine is.

3. Results

3.1. Comparative Static Analysis

Let us denote the difference of profits of each fare collection system, $\pi_{BA}(t)-\pi_{POP}(b,t)$, by $\bar{\pi}(b,t)$, under given b and t. Therefore, if it is positive, BA is more profitable, while if it is negative, POP is more profitable. If it is equal to zero, the two systems are indifferent in terms of profits. $\bar{\pi}(b,t)$ is calculated as follows:

$$\bar{\pi}(b,t) = \begin{cases} -(c_s + c_m t) & \text{if } a \leq m \\ -(c_s + c_m t) + \dfrac{c_i}{b} ant & \text{otherwise} \end{cases} \tag{5}$$

First, when $a \leq m$, that is, a passenger's morality is high enough, $\bar{\pi}(b,t)$ is always negative, and thus POP is always better. This would be quite an intuitive result. In this case, a passenger always buys the ticket even in the

[5]This has been suggested by Avenhaus (2004).

case of POP because it is her dominant strategy. Given that no fare evasion occurs, it is the transit agency's optimal strategy not to inspect at all. Then, by bringing in POP, it can just save the costs required for BA.

On the other hand, when $a > m$, the optimal choice depends on values of each variable. Among them, c_s, c_m and b are such parameters that make POP more competitive when those values are big, that is,

$$\frac{\partial \bar{\pi}(b,t)}{\partial c_s}, \frac{\partial \bar{\pi}(b,t)}{\partial c_m}, \frac{\partial \bar{\pi}(b,t)}{\partial b} < 0, \tag{6}$$

while a, c_i and n are such parameters that make BA more competitive when those values are big, that is,

$$\frac{\partial \bar{\pi}(b,t)}{\partial a}, \frac{\partial \bar{\pi}(b,t)}{\partial c_i}, \frac{\partial \bar{\pi}(b,t)}{\partial n} > 0. \tag{7}$$

Whether $\partial \bar{\pi}(b,t)/\partial t$ is positive or negative depends on other parameters.

Some of these results would be non-trivial. With respect to the fine level, the result above, $\partial \bar{\pi}(b,t)/\partial b < 0$, is derived from $\partial \pi_{POP}(b,t)/\partial b > 0$. This is because the bigger b is, the higher the transit agency's expected payoff, which determines its profit per passenger, is at the Nash equilibrium of the inspection game. This implies that if the transit agency introduces POP, it should set the fine level as high as possible.

The result also tells about the impact of the fare as $\partial \bar{\pi}(b,t)/\partial a > 0$. As mentioned in 2.3, under the POP system, the total amounts of fines taken in and the costs for inspections cancel each other out, so the profit becomes equal to $apnt$. Here, p, the probability of non-free-ride, is irrespective of the fare as the Nash equilibrium indicates. Hence, the rate of decrease in revenue from regular fare collections due to fare evasions is constant at $1-p$, but the amount of it becomes big, and this makes POP less attractive, as the fare becomes high. The same thing holds for the impact of the passenger volume and explains $\partial \bar{\pi}(b,t)/\partial n > 0$.

3.2. Profit Maximization

Next let us consider the procedure of the transit agency's decision making that maximizes the total profit. When a, n, c_s, c_m, c_i, m and t are given, the optimal choice can be described as a subgame perfection solution in an extensive form game depicted in Figure 1.

(Figure 1 is inserted here.)

Figure 1: Profit maximization of the transit agency: the subgame perfection

It is a two-player extensive form game played by the transit agency and a (homogenous) passenger. In this game, the transit agency first chooses the fare collection system, BA or POP. If the former is taken, the game ends. Otherwise, then it decides the fine level, $b \in [\max\{a, c_i\}, b_{max}]$. Then under the given b, the transit agency and a passenger play the inspection game of Table 1, which is indicated as $G(b)$ in the figure, where they decide the frequency of inspections or free rides. Thus the transit agency's payoff should be given as its profit per passenger in this extensive form game, while it is omitted in the figure.

Then we consider its profit maximization as follows. When POP is taken and a particular b is given, the transit agency and a passenger behave according to the Nash equilibrium of the corresponding inspection game as we have supposed. Therefore, at the transit agency's second decision node, where it decides b, such a fine level that maximizes its own expected payoff is chosen, given that they play the Nash equilibrium in $G(b)$ for any b. Then, given that they act in this way when POP is taken, the transit agency compares the payoff of BA and the expected payoff of POP, and chooses the better one at the first decision node. This way of solving a game is called subgame perfection in game theory.

This is equivalent to say as follows: the transit agency should choose BA if $\bar{\pi}(b^*, t) > 0$, where b^* is the optimal fine level when POP is taken, while, if $\bar{\pi}(b^*, t) < 0$, it should choose POP with setting b^* as the fine. If $\bar{\pi}(b^*, t) = 0$, the both systems are indifferent, if the fine is set as b^* when POP is taken. Based on the discussion in 3.1, b^* is always determined as b_{max}, that is, it is the fine level chosen at the second decision node if reached in the game of Figure 1. As mentioned above, POP is always better when $a \leq m$, which implies $\bar{\pi}(b^*, t) < 0$. On the other hand, if $a > m$, it depends on values of each parameter whether $\bar{\pi}(b^*, t)$ is positive or negative.

4. Applications: Interpretations of Various Fare Collection Systems

The framework and the results can be applied to interpret various choices of fare collection systems in the world depending on various factors. We here show two examples. In this section, we assume $a > m$.

4.1. Influence of the Maximum Fine Level

First, let us consider the influence of the upper limit of the fine the transit agency can set, b_{max}. This is important because it has been found in the previous section that setting the fine as high as possible can maximize the profit in the case of POP.

In those countries or cities where POP is common, the fine is usually as much as tens of the fare. On the other hand, in Japan, where POP is very uncommon, it is specified by law that the fine of a fare evasion must be less than three times as much as the fare. Therefore it has been pointed out that the restriction is one of reasons that POP is not popular in Japan (Nishikawa, 2007). Moreover, a very high fine may not be acceptable, or workable, in low income countries either because, for example, a free rider's income may be too low to pay for such a high fine. In those countries, if, in contrast, POP is introduced with setting a relatively low fine, transport users whose marginal utility of income is high might possibly be more prone to take free rides[6]. In fact, POP is not very common in regions other than Europe and North America.

Although these claims would be intuitive, we shall discuss here how our framework can support them. Based on the discussion above, when $b_{max} = ka$, that is, the maximum fine level is k times as much as the fare, $\bar{\pi}(b^*, t) = -(c_s + c_m t) + c_i nt/k$. Hence we have $\partial \bar{\pi}(b^*, t)/\partial k < 0$. That is, if the other conditions are same, then the bigger k is, the more competitive POP is.

Let us illustrate this graphically. In order for the transit agency's choice of POP to be optimal, $\bar{\pi}(b^*, t)$ must be negative, therefore, $c_i nt < k(c_s + c_m t)$ must be satisfied. This means that the total inspection cost when the transit agency makes inspections to all passengers in every operation are less than k times as much as the total cost of BA. In Figure 2, two lines, l_1 and l_2, express $\bar{\pi}(b^*, t) = 0$, where b^* is $k_1 a$ or $k_2 a$, respectively, with $k_1 > k_2$. Thus, when $b^* = k_1 a$, the area between l_1 and the horizontal axis indicates where POP is superior to BA, while the area between l_1 and the vertical axis indicates where BA is better. The similar thing can be said to l_2 for the case of $b^* = k_2 a$. Hence, when, for example, $c_s + c_m t = c_1$ and $c_i nt = c_2$, BA is optimal when $b^* = k_2 a$ while POP becomes optimal when $b^* = k_1 a$.

Thus the optimal fare collection system highly depends on the maximum

[6]Furthermore, the rate of household consumption expenditure for transportation is usually higher in developing countries, than in developed countries (United Nations Statistics Devision, 2013).

fine level. This would explain one aspect of the fact that POP is uncommon in Japan where $b_{max} = 3a$ while it is often adopted in countries where b_{max} is much bigger than it.

(Figure 2 is inserted here.)

Figure 2: Influence of the upper limit of the fine level on optimal choices of fare collection systems ($k_1 > k_2$)

4.2. Dependence on Types of Transportation

Next, let us consider how the optimal fare collection system may differ depending on types of transportation. According to Transportation Research Board (2002, henceforth TRB), while there are always exceptions depending on local circumstances, heavy rail systems are typically better suited to BA, while LRT and BRT are usually better off with POP. We here discuss how our framework can support the claim.

It says there are mainly two reasons for this: the station or platform configuration and the expected passenger volume. First, with respect to the platform configuration, POP, TRB says, is usually more appropriate for on-street platforms, and thus LRT and BRT typically featuring them will be better off with POP. This is because, for these platforms, the setup cost for facilities to bring in the BA system becomes relatively high. For the transit agency needs not only to install ticket gates but also to establish a clearly defined paid area, which requires additional costs compared to adopting BA at an enclosed station.

In our framework, this means c_s is relatively big in these cases. Now suppose $c_s = k'c_i$, that is, the setup cost of the BA system is k' times as much as the inspection cost per passenger inspected. Since c_i is basically determined by the labor cost of the region and thus we can naturally assume it does not depend on the platform type, the high setup cost of BA for on-street platforms can be considered as k''s being relatively big. Then $\bar{\pi}(b^*, t) = -(k'c_i + c_m t) + c_i ant/b^*$, and hence $\partial\bar{\pi}(b^*, t)/\partial k' < 0$. That is, the bigger k' is, the more competitive POP becomes. This result supports the TRB's claim that LRT and BRT that are typically operated with on-street platforms, which make k' relatively big, will usually be better off with POP.

Second, with regard to the expected passenger volume, TRB claims that it is often the case that heavy rail systems have a higher volume than LRT and BRT, and typically BA is better for the former. We have already shown in 3.1 that the bigger n is, the more competitive BA is. But TRB's discussion in this regard is based on congestion in the car, which our framework does not consider explicitly. It says that an on-board inspection requires sufficient spaces for an inspector to walk through the car and check each traveler, and thus, when the passenger volume is high and the car is crowded, it becomes increasingly difficult to conduct inspections.

In order to consider such an effect explicitly, let us suppose the inspection cost per passenger inspected is a function of the passenger volume as $c_i(n)$, and increases with respect to it, namely $\partial c_i(n)/\partial n \geq 0$, for sufficiently big n such that the congestion can affect the capability of inspectors. This can be understood as n''s becoming small due to crowding with constant C_i. Then $\bar{\pi}(b^*, t) = -(c_s + c_m t) + c_i(n)ant/b^*$, and hence still $\partial\bar{\pi}(b^*, t)/\partial n > 0$. It supports the TRB's claim that heavy rail systems with a high ridership are typically better suited to BA, while LRT and BRT with a relatively small passenger volume will usually be better off with POP.

However, we, in this respect, also note that c_i can decrease with demand for sufficiently small n due to economies of scale. For example, suppose the passenger volume is small enough for an inspector to inspect all the passengers on board. Then, $c_i(n) = C_i/n$, and hence $\partial c_i(n)/\partial n = -C_i/n^2 < 0$. In this case, $\partial\bar{\pi}(b^*, t)/\partial n = at(c_i(n) + n \cdot \partial c_i(n)/\partial n)/b^* = 0$. This implies that, when the demand is such small, change in it cannot affect the attractiveness of a particular system.

The discussion in this section is based on comparative static analyses, so we note that optimal fare collection systems are determined also by other parameters, which always depend on local circumstances.

5. Concluding Remarks

The present study has discussed optimal fare collection systems by comparing BA and POP. By using a mathematical framework, the procedure of a transit agency's profit maximization has been specified as a subgame perfection solution in a game played by the transit agency and passengers, and we have examined how each parameter can affect the optimal choice by a comparative static analysis.

Travel demand forecasting is widely used to estimate ridership of a transit agency under its given level of service, and often one of its purposes is to provide basic information for evaluation of its profitability. But, as we have seen, the profitability may depend on the fare collection system the transit agency uses. Therefore we consider this kind of approach to discuss impacts of choices of fare collection systems in a formal way is also required for rigorous feasibility studies.

Of course, transit agencies in real-world notice well the importance of this managerial issue. For instance, Metropolitan Atlanta Rapid Transit Authority (1993) conducts a case study that compares the two fare collection systems when the possibility of converting from BA to POP was considered. But it does not take into account effects of strategic interactions between passengers and the transit agency discussed in this paper: in the case study, the total inspection cost is just given without considering them, and also the financial impact of setting the fine level is not discussed rigorously. Hence our framework can be useful at least potentially to improve precision of such studies, and thus to help a transit agency design its transportation system.

Since our framework is based on simplified settings and assumptions, extending or relaxing some of them should be given considerations to develop it. We point out some open questions:

Extensions to more complicated transportation systems. Real transportations are much more complicated in terms of routes, fare systems, fine schemes (for POP), and so on, thus additional parameters would become required for practical use of our framework. In order to avoid ad hoc applications, we need to establish a general framework to discuss this issue. For example, when the transportation network is complex, it becomes an increasingly strategic task for the transit agency to decide the schedule of inspections such as the timing, places, etc. This kind of problem has been studied as game-theoretic randomized patrolling on graphs (e.g. Yin et al., 2012), and the ideas of such studies would be useful in generalization of our approach.

Impacts on travel demands. We assumed the passenger volume, n, is given and does not depend on the choice of fare collection system. Since the expected travel cost for a passenger under POP is equal to the regular fare as shown in 2.3, the assumption would be innocuous in our simple formulation. However, if one targets a more complicated transportation system, the expected travel cost might become different from the fare. This means the "expected level of service" may vary according to the fare collection system. Therefore, particularly when there exist competing transportation modes, it can affect the demand of the transit agency. If such an effect is considered as significant, the choice of fare collection system should be evaluated in accordance with travel demand forecasting for each system.

We also note that we have ignored the influence of different boarding time that each fare collection system may have. In general, the BA system may cause a slowdown of passenger flow particularly when its volume is high. This might be perceived as a decrease in the level of service by travelers and hence affect the travel demand. Tirachini and Hensher (2011) study such a relation of boarding time of each fare collection system and travel demands for buses.

Behaviors in inspection games. As typical in the standard game theory, for a Nash equilibrium to be played for sure in an inspection game, it is required that both the transit agency and a passenger to know each other's payoff correctly[7]. The assumption, however, might be unrealistic for some parameters in the payoff matrix. For example, a passenger may not know correctly the transit agency's inspection cost, while the transit agency may not recognize the degree of the morality of passengers. For such situations, Sasaki et al. (2007) study inspection games considering the possibility that players may have misperceptions about payoffs of other players. Although it would be virtually impossible to identify how they misperceive each other, such an approach can be useful to conduct a sensitivity analysis with regard to how such misperceptions can affect the equilibrium, and hence the profitability of the transit agency.

[7] For epistemic conditions for Nash equilibrium, see Aumann and Brandenburger (1995).

Other objective functions. Our analysis has been conducted from the viewpoint of the transit agency's profit maximization. The reader may think of the possibility of other objectives such as social welfare, i.e. the summation of the profit and the user benefit. It would also be an issue to be addressed to investigate how the difference of objective functions can influence the result.

If we assume the transit agency is still a profit maximizer, a passenger's expected travel cost (the amount of money she pays) for POP can be calculated as her expected payoff in the inspection game (Table 1) that reflects the fare evasion rate, the frequency of inspections and payments of fines. Then the user benefit can be calculated based on it[8]. While it has been shown in 2.3 that the difference of fare collection system does not bring about any user benefits in our simple settings, this might not be the case when a more complicated situations is of interest, e.g., when each fare collection system may have significantly different expected level of service.

Furthermore, if the transit agency is not a profit maximizer, we need another incentive scheme of it. For example, when the transportation is highly public, it might be designed so as to maximize the social welfare. Then its payoff in the game of Figure 1, or possibly the game structure itself, needs to be changed.

[8]We note that, from a legal or moral point of view, it might be questionable to say a passenger's saved money due to fare evasions is her "benefit." In addition, if one considers the possibility that designing a fare collection system can make a difference in a passenger's travel time as discussed above, the user benefit calculated here should incorporate the effect of change in it.

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