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Inductionless induction and rewriting induction

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The properties of systems based on equation such as functional programming and algebraic specification are often treated as inductive theorems in equational reasoning. Therefore, automated theorem proving of inductive theorems in equational reasoning is indispensable to the automated verification of functional programming and algebraic specifications.

Automated inductive theorem proving is classified into explicit induction and implicit induction. Explicit induction is the method using inductive scheme directly. The Nqthm system, developed within this framework, is considered as one of the most successful theorem provers. On the other hand, implicit induction is the methods proving inductive theorems without the explicit use of induction scheme. There exist inductionless induction and rewriting induction as implicit induction. Inductionless induction was proposed by Musser(1980) and was refined in Huet and Hullot(1982). Rewriting induction was proposed by Reddy(1990). RRL and SPIKE are well-known automated theorem provers based on inductionless induction and rewriting induction, respectively.

Implicit induction methods prove inductive theorems as follows: Let E be an axiom and $e = e'$ the equation to be proven. By adding $e = e'$ to E, we can obtain another system $E' = E \cup \{e = e'\}$. The next step is to show that two congruence relations generated by E and E' are the same on ground terms. If this step succeeds, $e = e'$ is an inductive theorem in E, since obviously $e = e'$ is an inductive theorem in E'. Thus, it is shown that $e = e'$ is an inductive theorem in E without the explicit use of mathematical induction.

Knuth-Bendix completion, cover set and test set techniques are used in automated theorem provers based on inductionless induction or rewriting induction. Therefore, inductionless induction and rewriting induction are often studied as an extension of Knuth-Bendix completion techniques or induction based on cover set (test set). However, essence of implicit induction is to show equivalence of two systems. Either Knuth-Bendix completion technique or cover set (test set) techniques are not essential. In fact, Toyama(1991) removes the concept of completion and reconstructs the inductionless induction in the framework of abstract reduction systems. Consequently, it was shown that the Church-Rosser property and the weak normalization property are essential in inductionless induction.

In this paper, we study the relation between inductionless induction and rewriting induction within an abstract framework as follows:

- Formalization of rewriting induction

  We reconstruct the rewriting induction methods in the flame work of abstract reduction systems. We clearly show that the regression property and the strong normalization property are essential in rewriting induction. The principle of rewriting induction can be concisely explained by our result.

- Comparison between inductionless induction and rewriting induction

  From the above result, it is possible to compare inductionless induction and rewriting induction in a uniform abstract framework. We make it clear that the Church-Rosser property and the reachability are essential in inductionless induction while the regression property and the strong normalization property are essential in rewriting induction. Thus, two methods are incompatible.

- Comparison with refutationaly theorem proving

  In the similar way, we also study refutationaly proving that is widely used in automated theorem provers. Consequently, we show that under the combination with refutaional proving inductionless induction and rewriting induction coincide.

- Relationship with automated theorem provers

  We consider the relation between automated theorem provers and our abstract framework. In this paper, we deal with Kapur's procedure as inductionless induction and Fribourg's procedure as rewriting induction. Both procedures prove as follows: Let a TRS R_1 be an axiom and $e \to e'$ a rule that corresponds to an equation to be proven. In the first step, by adding $e \to e'$ to R_1, we obtain $R_2 = R_1 \cup \{e \to e'\}$. Next, we transform R_2 into the TRS that meets suitable conditions with procedures based on Knuth-Bendix completion. When the procedure halts, the resulted TRS has properties in our abstract framework. Thus, mechanisms of automated theorem provers are explained concisely.

We generalize the methods to show equivalence of two systems within abstract reduction systems. Our approach is effective for designing new automated theorem provers. Furthermore, our methods can apply not only to inductive theorems but also to various automated theorem provers.