# Abstract

**Title:** The independent set reconfiguration problem on some restricted graphs.
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Recently, *reconfiguration problems* attract the attention in the field of theoretical computer science. The problem arises when we wish to find a step-by-step transformation between two feasible solutions of a problem such that all intermediate results are also feasible and each step abides by a fixed reconfiguration rule. A well-known example is that given two specified satisfiable assignments (assignments which return the TRUE value) A and B to a Boolean formula, one might ask whether A can be transformed into B by changing the assignment of one variable at a time such that each intermediate assignment is also satisfiable. Readers may also remember Rubik's cube and its relatives as examples of reconfiguration puzzles. This kind of reconfiguration problems has been studied extensively for several well-known problems, including the so-called *independent set reconfiguration problem* (ISRECONF).

Recall that an *independent set* in a graph $G$ is a set of pairwise non-adjacent vertices. Given a graph $G$, and two independent sets $\mathbf{I}_b$ and $\mathbf{I}_r$ of $G$, imagine that a token (coin) is put at each vertex of $\mathbf{I}_b$, the ISRECONF problem asks whether we can transform $\mathbf{I}_b$ to $\mathbf{I}_r$ via a sequence of independent sets of $G$, each of which results from the previous one by moving a token under some given *reconfiguration rules*, namely token sliding (TS), token jumping (TJ), and token addition and removal (TAR).

- *Token Sliding* (TS rule): A single token can be slid only along an edge of a graph. The ISRECONF problem under TS rule is also known as the SLIDING TOKEN problem.
- *Token Jumping* (TJ rule): A single token can "jump" to any vertex (including non-adjacent one).
- *Token Addition and Removal* (TAR rule): We can either add or remove a single token at a time if it results in an independent set of cardinality at least a given threshold.

The ISReconf problem is PSPACE-complete under any of the three reconfiguration rules for general graphs, for planar graphs, for perfect graphs, and even for bounded bandwidth graphs.

The ISReconf problem under TS rule, in which tokens may only be moved to adjacent vertices, is called the sliding token problem and is of particular theoretical interest. Given two independent sets $\mathbf{I}_b$ and $\mathbf{I}_r$ of a graph $G = (V, E)$ such that $|\mathbf{I}_b| = |\mathbf{I}_r|$, and imagine that a token (coin) is placed on each vertex in $\mathbf{I}_b$, the sliding token problem asks whether there exists a sequence $\langle \mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_\ell \rangle$ of independent sets of $G$ such that:

(a) $\mathbf{I}_1 = \mathbf{I}_b$, $\mathbf{I}_\ell = \mathbf{I}_r$, and $|\mathbf{I}_i| = |\mathbf{I}_b| = |\mathbf{I}_r|$ for all $i$, $1 \leq i \leq \ell$; and

(b) for each $i$, $2 \leq i \leq \ell$, there is an edge $\{u, v\}$ in $G$ such that $\mathbf{I}_{i-1} \setminus \mathbf{I}_i = \{u\}$ and $\mathbf{I}_i \setminus \mathbf{I}_{i-1} = \{v\}$, that is, $\mathbf{I}_i$ can be obtained from $\mathbf{I}_{i-1}$ by sliding exactly one token on a vertex $u \in \mathbf{I}_{i-1}$ to its adjacent vertex $v$ along $\{u, v\} \in E$.

Such a sequence is called a *reconfiguration sequence* between $\mathbf{I}_b$ and $\mathbf{I}_r$. In computational complexity theory, several PSPACE-hardness results have been proved using reduction from the sliding token problem. sliding token is known to be PSPACE-complete even for planar graphs, and also for bounded treewidth graphs.

In this thesis, we mainly focus on the sliding token problem (i.e. ISReconf under TS rule) restricted to trees. In 2012, **?** gave a linear-time algorithm for solving ISReconf for even-hole-free graphs (which include trees) under TJ and TAR rules. Indeed, the answer is always yes under the two rules when restricted to even-hole-free graphs (as long as two given independent sets have the same cardinality for the TJ rule.) Furthermore, tokens never make detours in even-hole-free graphs under the TJ and TAR rules. On the other hand, under TS rule, tokens are required to make detours even in trees. In addition, there are no-instances for trees under TS rule. These are the reasons why the problem for trees under TS rule is much more complicated and was still open, despite the intensive algorithmic research on ISReconf. In this thesis, we show that sliding token for trees can be solved in linear time. This result was also presented at the $25^{th}$ International Symposium on Algorithms and Computation (ISAAC 2014, Jeonju, Korea).