# Revisiting Model Checking of Chandy-Lamport Distributed Snapshot Algorithm

By DOAN, Ha Thi Thu

A thesis submitted to
School of Information Science,
Japan Advanced Institute of Science and Technology,
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# Glossary

| | |
|---|---|
| $\mathcal{CLDSA}$ | Chandy-Lamport Distributed Snapshot Algorithm. |
| $\mathcal{UDS}$ | Underlying Distributed System. |
| $\mathcal{DSR}$ | Distributed Snapshot Reachability. |
| $M_{\mathcal{UDS}}$ | The State Machine for a $\mathcal{UDS}$. |
| $M_{\mathcal{CLDSA}}$ | The State Machine for a $\mathcal{UDS}$ Superimposed by $\mathcal{CLDSA}$. |

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# Chapter 1

# Introduction

## 1.1 Motivation

In recent decades, many applications have relied on distributed systems, which involve multiple processes connected by channels. Due to the fact that distributed systems should be fault tolerant because they need to run for a long time, keeping on providing services to human beings, other systems, etc. To make distributed systems fault tolerant, it is necessary to use many non-trivial distributed algorithms, such as snapshot algorithms and checkpointing algorithms. Since many problems on distributed systems, such as recovering from faulty states and detecting stable properties can be cast to taking global snapshots of distributed systems [8], distributed snapshot algorithms are crucial.

A distributed system consists of a finite set of processes and channels. The processes communicate by sending and receiving messages through channels. A global state of a distributed system includes the states of all processes and all channels in the system, where the state of a channel is a sequence of messages sent along the channel, but not been received. Distributed snapshot algorithms help to determine global states of a distributed system (called global snapshots) during a computation. One desired property distributed snapshot algorithms should enjoy is as follows: let $s_1$ be the state in which a distributed snapshot algorithm concerned initiates, $s_2$ be the state in which the distributed snapshot algorithm terminates, and $s_*$ be the snapshot taken, and then $s_*$ is reachable from $s_1$ and $s_2$ is reachable from $s_*$, whenever the distributed snapshot algorithm terminates. The property is called the distributed snapshot reachability ($\mathcal{DSR}$) property. Note that the snapshot may not appear in the actual computation from the start state to the finish state.

Taking global snapshots of a distributed system is not straightforward because distributed systems are asynchronous and processes do not share clocks and memory. These lead to that all processes cannot record their local states at exactly the same time. What we obtained might be inconsistent global states. Therefore, Chandy and Lamport have proposed a distributed snapshot algorithm [1] called $\mathcal{CLDSA}$ in this thesis, by which processes can record their own states and the states of communication channels such that the combination of process states and channel states form a consistent global state.

With the advancement of computer and model checking technologies, many model checkers, such as Spin [3] and NuSMV [4] have been developed and applied to formal verification of various kinds of software and hardware systems. To the best of our knowledge, however, application of model checking to formal verification of distributed snapshot algorithms have not been fully investigated. Since distributed snapshot algorithms are non-trivial, the problem to model check that the distributed snapshot algorithms enjoys the $\mathcal{DSR}$ property is not straightforward. There is an existing study [2] in which Maude [5] has been used to model check that $\mathcal{CLDSA}$ enjoys the $\mathcal{DSR}$ property. Maude is a specification and programming language system based on rewriting logic, and equipped with model checking facilities: the LTL model checker and the search command. The search command is used in the existing study. The authors of the existing study, however, do not discuss whether their definition of the $\mathcal{DSR}$ property faithfully expresses the property. In addition, we recognize that the informal description of the $\mathcal{DSR}$ property involves both an underlying distributed system ($\mathcal{UDS}$) and the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$, while the $\mathcal{DSR}$ property encoded in terms of the Maude search command involves only the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ in the existing study. Consequently, we do not think that the existing study provides the good foundation to guarantee that the $\mathcal{DSR}$ property has been sufficiently model checked.

Therefore, it is necessary to faithfully express the $\mathcal{DSR}$ property, and then consider the similarities between the new formal definition and the existing one. Moreover, one significant characteristic of distributed snapshot algorithms is that distributed snapshot algorithms should run concurrently with but not alter the $\mathcal{UDS}$. Since this property relate to computation and the behaviors of a $\mathcal{UDS}$, however, it is not straightforward to prove that a distributed snapshot algorithm satisfies this property manually. The authors in [1] asserts that $\mathcal{CLDSA}$ satisfy this property, but they have not yet proved it. Until now, we have not yet found out any reliable studies, in which this property is proven. In addition, this property is not mentioned in the existing study. But, we should prove that $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$.

## 1.2 The Purpose of the Research and Outcome

The purpose of our research is to revisit model checking of $\mathcal{CLDSA}$. Carefully investigating the informal description of the $\mathcal{DSR}$ property, the way to faithfully express the $\mathcal{DSR}$ property has been given in [6]. However, the study has not yet completed. The technical report [6] has not been reviewed. Our research complete [6] giving a formal definition of the $\mathcal{DSR}$ property, which is more likely to faithfully express the informal description. Then we prove that the new definition of the $\mathcal{DSR}$ property is equivalent to the definition in the existing study to confirm the validity of the model checking approach used in the existing study. Since the new definition involves both a $\mathcal{UDS}$ and the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$, it is not straightforward to directly model check the new definition with any existing model checker. Hence the equality of the two definitions also asserts that we can use the model checking approach used in the existing study to model check the new definition. Moreover, we prove that $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$ .

Since state machines are suitable to formalize concurrent systems including distributed systems, state machines are used in our research to formalize a $\mathcal{UDS}$ and the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$. To faithfully express the informal description of the $\mathcal{DSR}$ property, we first formalize a $\mathcal{UDS}$ and the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ as state machines $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$, respectively. The $\mathcal{DSR}$ property is formalized based on them. Maude notation is used to describe state machines. Then we prove Theorem 1 saying that our new definition is equivalent to the definition of the $\mathcal{DSR}$ property in the existing study. To prove the Theorem 1, we prove Proposition 1 and Lemma 1. Lemma 1 asserts that reachability is preserved between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$. Proposition 1 says that whenever $\mathcal{CLDSA}$ terminates in state $s$, there is no marker in the start state, the snapshot and the finish state. We prove as Lemma 2 and Lemma 3 that one-step reachability is preserved between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ to prove Lemma 1. The proof of Theorem 1 follows from Proposition 1 and Lemma 1. Furthermore, a binary simulation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ is used to prove that $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$. Since $\mathcal{CLDSA}$ works by using a special message called marker, $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$ means that excepting for putting markers in a $\mathcal{UDS}$, the algorithm does not change the state of all processes and channels. We propose the binary relation $\mathbf{r}$ between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ saying that for each $s1 \in S_{\mathcal{UDS}}$ and each $s2 \in S_{\mathcal{CLDSA}}$, $\mathbf{r}(s1, s2)$ if and only if $s1$ is the same as the state obtained by deleting all markers from $s2$. To guarantee that $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$, we prove Theorem 2 saying that $\mathbf{r}$ is a bi-simulation relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$.

Summarizing the research results, we have given a more faithful formal definition of the $\mathcal{DSR}$ property. Moreover, we have proved Theorem 1 saying that our formalization of the $\mathcal{DSR}$ property is equivalent to the existing one for each $M_{\mathcal{UDS}}$. Theorem 1 guarantees that it suffices to model check the definition used in the existing study for $\mathcal{CLDSA}$ and the existing model checking approach can be used for this end. We have also proved Theorem 2 asserting that $M_{\mathcal{CLDSA}}$ simulates $M_{\mathcal{UDS}}$ and vice versa to guarantee that $\mathcal{CLDSA}$ does not alter the behaviors of the $\mathcal{UDS}$. The research results are depicted in Fig. 1.1.

## 1.3 Related Work

$\mathcal{CLDSA}$ has been originally proposed in [1] as the first, in which the $\mathcal{DSR}$ property is described in the informal way. The $\mathcal{DSR}$ property is formalized as an invariant in [7], but their definition does not explicitly involve two state machines. $\mathcal{CLDSA}$ is modeled in Promela and model checked with Spin [10]. In which, the consistent snapshot is defined, but the $\mathcal{DSR}$ property is not mentioned directly.

Andriamiarina, Méry and Singh [9] investigate the correct-by-construction process through which some distributed snapshot algorithms including $\mathcal{CLDSA}$ are derived with Event-B and Rodin. Starting with an abstract model in which a consistent cut is instantaneously recorded, several refinement steps are repeated to construct some distributed snapshot algorithms that describe how to obtain consistent cuts by multiple transition steps. A consistent cut is a set of events of a distributed snapshot algorithm that satisfy some con-

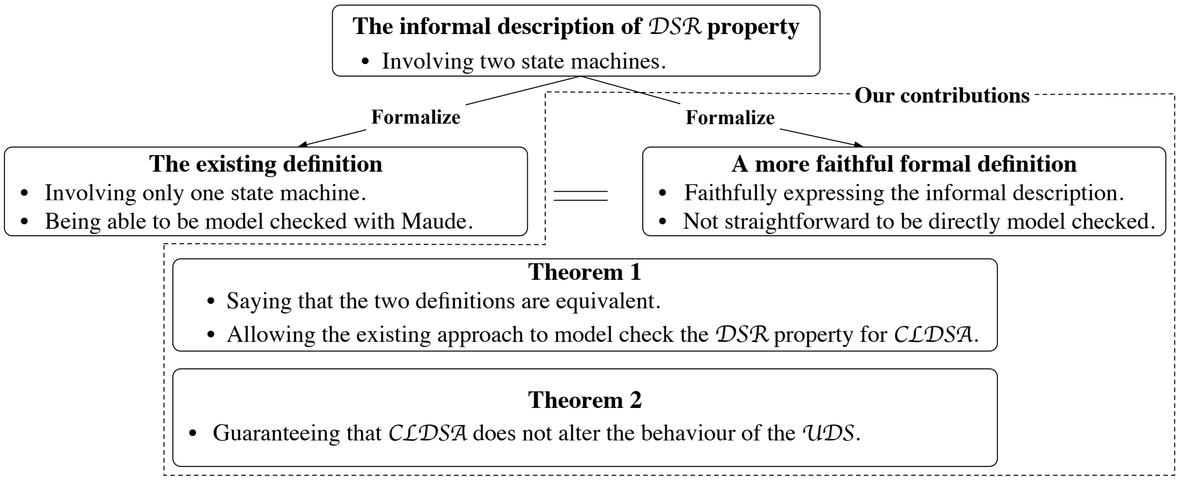

The informal description of $\mathcal{DSR}$ property
• Involving two state machines.
Formalize
Formalize
Our contributions
The existing definition
• Involving only one state machine.
• Being able to be model checked with Maude.
A more faithful formal definition
• Faithfully expressing the informal description.
• Not straightforward to be directly model checked.
Theorem 1
• Saying that the two definitions are equivalent.
• Allowing the existing approach to model check the $\mathcal{DSR}$ property for $\mathcal{CLDSA}$.
Theorem 2
• Guaranteeing that $\mathcal{CLDSA}$ does not alter the behaviour of the $\mathcal{UDS}$.

Figure 1.1: The contributions of our research

ditions, and can induce a snapshot. So, recording a consistent cut is regarded as taking a snapshot. Although they use multiple state machines described in Event-B, the relation between their state machines and ours ($M_{\mathcal{UDS}}$ and $CL(M_{\mathcal{UDS}})$) is not clear and must be worth investigating.

## 1.4 Organization of the Thesis

The thesis is organized as follows:

- Chapter 2 introduces some preliminaries, such as a $\mathcal{UDS}$, state machines, model checking and $\mathcal{CLDSA}$, and gives an informal description of the $\mathcal{DSR}$ property.
- Chapter 3 presents the specification and model checking of $\mathcal{CLDSA}$ in Maude.
- Chapter 4 describes how to formalize a $\mathcal{UDS}$ and the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ as state machines $M_{\mathcal{UDS}}$ and $CL(M_{\mathcal{UDS}})$, where $CL$ is a function that takes $M_{\mathcal{UDS}}$ and returns the state machine of the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$.
- Chapter 5 gives the more faithful formal definition of the $\mathcal{DSR}$ property.
- Chapter 6 proves the theorem that guarantees the validity of the model checking approach used in the existing study and we can use the model checking approach to model check the new definition of the $\mathcal{DSR}$ property.
- Chapter 7 proves another theorem asserting that $M_{\mathcal{CLDSA}}$ simulates $M_{\mathcal{UDS}}$ and vice versa to guarantee that $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$.
- Chapter 8 discusses some future work.
- Chapter 9 concludes the thesis.

# Chapter 2

# Preliminaries

## 2.1 Underlying Distributed Systems

As described in [1], a $\mathcal{UDS}$ consists of a finite set of processes and channels, which can be described as a labeled, directed graph in which the vertices represent the processes and the directed edges represent the channels. An example is shown in Fig. 2.1. The $\mathcal{UDS}$ consists of three processes p, q, r and four channels c1, c2, c3, c4. The channel c4 is used to directly send messages to the process p from the process r, but not vice versa. There may be more than one channel from one process to another. There are the two channels c1, c2 from the process p and the process q in the $\mathcal{UDS}$. In addition, channels are assumed to have infinite buffers, to be error-free and to deliver messages in the order sent. The delay experienced by a message is arbitrary but finite. The processes communicate by sending and receiving messages through channels.

The *global state* of a distributed system consists of the states of all processes and all channels in the system, where the state of a channel is characterized by the sequence of messages sent along the channel, excluding the messages received along the channel. Since many problems on distributed systems such as recovering from faulty states and detecting stable properties can be cast to taking global snapshots of distributed systems. Taking a global state of a distributed system is very important. A global state is a consistent global state iff it satisfies that: if in the local state of a sender process $p$, a message m is recorded as sent, it must be captured in the state of the channel $c$ connected the sender and the receiver or in the local state of the receiver process $q$ and in the collected global state, for every effect, its cause must be present, and if a message $m$ is not recorded as sent in the local state of process $p$, then it must neither be presented in the state of the channel $c$ nor in the local state of the receiver process $q$ [8]. To recording the meaning a global state of a distributed system, the states of all processes and all channels should be record at exactly the same instant. Due to the asynchrony of distributed systems, the lack of globally shared memory, global clock and unpredictable message delays, however, all processes cannot record their local states at exactly the same time. Then the global states we obtained may be inconsistent. This makes recording consistent global states non-trivial.

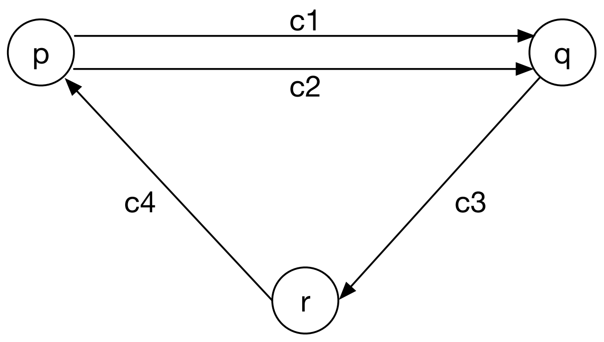


Figure 2.1: A distributed system with processes p, q, r and channels c1, c2, c3, c4.

## 2.2 $\mathcal{CLDSA}$ and the $\mathcal{DSR}$ Property

Chandy and Lamport have proposed a distributed snapshot algorithm [1] by which processes can record their own states and the states of incoming channels such that the combination of all process states and all channel states forms a consistent global state. $\mathcal{CLDSA}$ guides each process when it should record its own state and the state of each incoming channel by using a special message called marker. $\mathcal{CLDSA}$ runs concurrently with the computation but does not alter the underlying computation. Each process can record its state at any time when it has not yet received any markers from other processes. The following rules must be followed:

**Marker-Sending Rule for a process $p$:** for each its outgoing channel $c$, $p$ sends one marker along $c$ after recording its state and before sending further messages along $c$.

**Marker-Receiving Rule for a process $p$:** when the process $p$ gets a marker from one its incoming channel $c$,

> **if** $p$ has not yet recorded its state
>
> **then** $p$ records its state according to Marker-Sending Rule for $p$ and the state of channel $c$ as empty sequence
>
> **else** $p$ records the state of $c$ as the sequence of messages received along $c$ after recording $p$'s state and before receiving the marker along $c$.

$\mathcal{CLDSA}$ can be initiated by one or more processes in a distributed system by executing the *"Marker Sending Rule"* by which each of them records its local state spontaneously without receiving markers from other processes and sends one marker along each of its outgoing channels. A process executes the *"Marker Receiving Rule"* on receiving a marker. If the process has not yet recorded its local state, then it records its local state, records the state of the channel on which the marker is received as the empty sequence, and sends one marker along each of its outgoing channels. Otherwise, it records the state of the channel

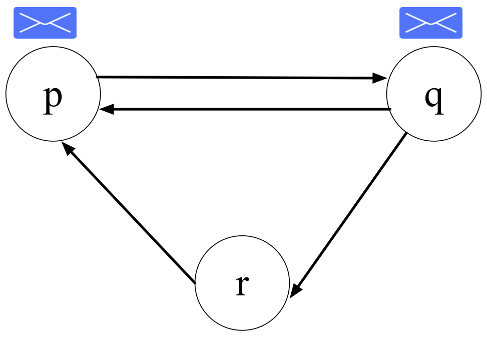


Figure 2.2: The $\mathcal{UDS}$ with processes p, q, r and 4 channels

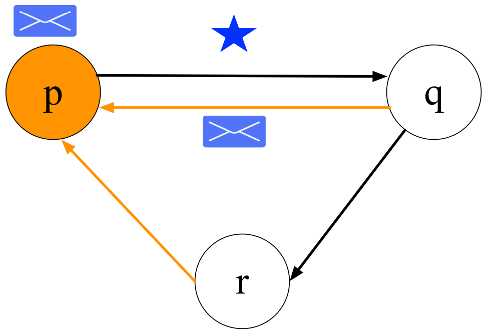


Figure 2.3: The process $p$ initially start the algorithm.

as the sequence of messages received along the channel after its state was recorded and before it received the marker along the channel. When the algorithm terminates, each process has already recorded its state and the states of all its incoming channels. The global snapshot is the combination of those records.

## One Scenario of $\mathcal{CLDSA}$.

Let us consider the $\mathcal{UDS}$ that consists of three processes $p, q, r$ and four channels as shown in Fig .2.2. We assume that there are two tokens in the system and the state of each process depends on the number of tokens it has. In the state shown in Fig .2.2, there are one token in $p$ and one token in $q$.

Now we assume that $p$ initially starts the algorithm. The start state is same the state of the system as shown in Fig .2.2. There are one token in $p$ and another in $q$. Since $p$ initially starts the algorithm, it records its state as the state in which there is one token and the state of two its incoming channels as empty sequence. Then, it sends one marker along one its outgoing channel from $p$ to $q$. Assume that in the same time $q$ sends one token to $p$. Let see the illustration in Fig .2.3.

Then $q$ receives the marker. Because this is the first time $q$ receives a marker, it records its state as the state in which there is no token and the state of the channel is as empty

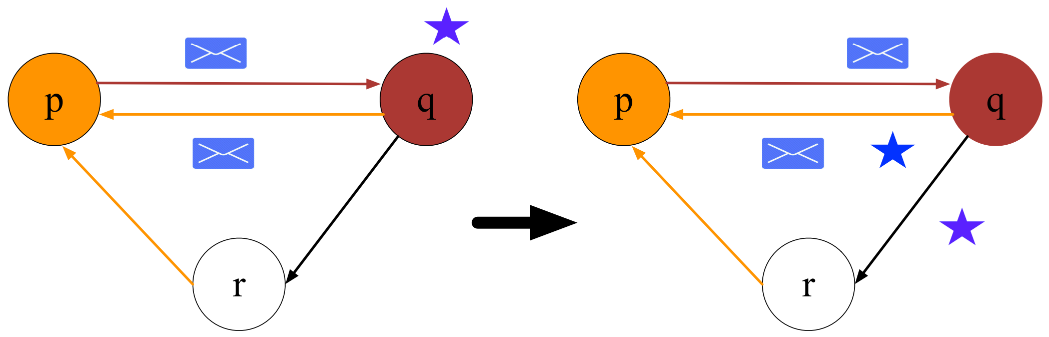


Figure 2.4: The process $q$ receive the marker from $p$.

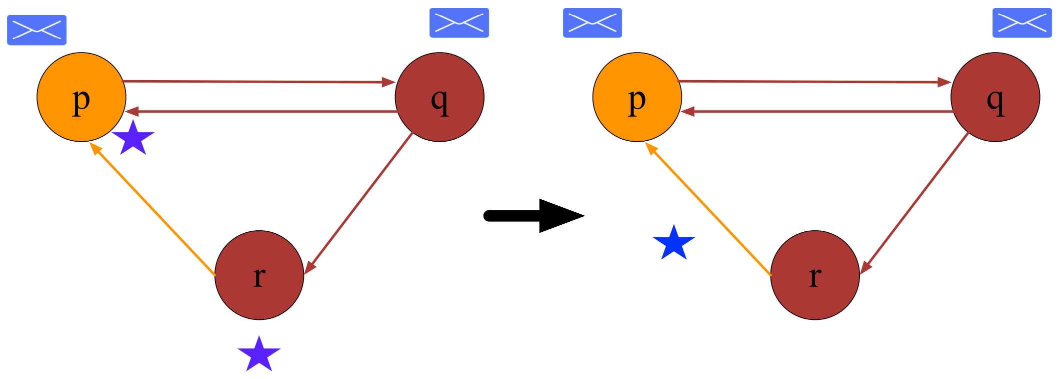


Figure 2.5: The process $p$ and $r$ receive the marker from $p$.

sequence. Then, it sends two markers along two its outgoing channels. Since $q$ has only one incoming channel, it locally completes the algorithm. Assuming that in the same time, $p$ sends one token to $q$. Let see the illustration in Fig .2.4.

Because of FIFO channel, $p$ receives the token before it receives the marker from $q$. When $p$ receives the marker from $q$, since it has already recored its state then it records the state of the channel from $q$ to $p$ as the sequence in which there is one token. Also, $r$ receives a marker from $q$. In the same way, $r$ records its state and the state of the channel and then it sends one marker to $p$. $r$ locally completes the algorithm. Let see the illustration in Fig .2.5.

The algorithm is globally completed when $p$ receives the marker from $r$. The finish state is shown in Fig .2.6. The snapshot is shown in Fig .2.7 in which there are one token in $p$ and another in the channel from $q$ to $p$.

## The $\mathcal{DSR}$ Property.

Let $s_1$, $s_*$ and $s_2$ be the state in which $\mathcal{CLDSA}$ initiates, the snapshot taken, and the state in which $\mathcal{CLDSA}$ terminates, respectively. Although the snapshot $s_*$ may not be identical to any of the global states that occur in the computation from $s_1$ to $s_2$, one

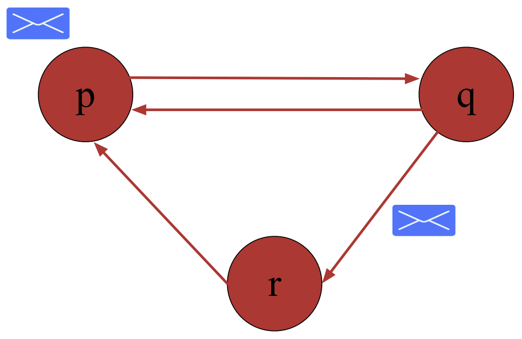


Figure 2.6: The finish state.

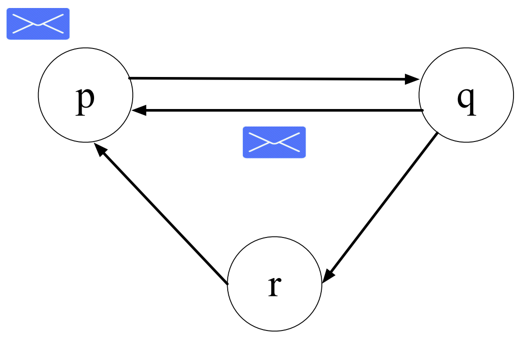


Figure 2.7: The snapshot taking by $\mathcal{CLDSA}$.

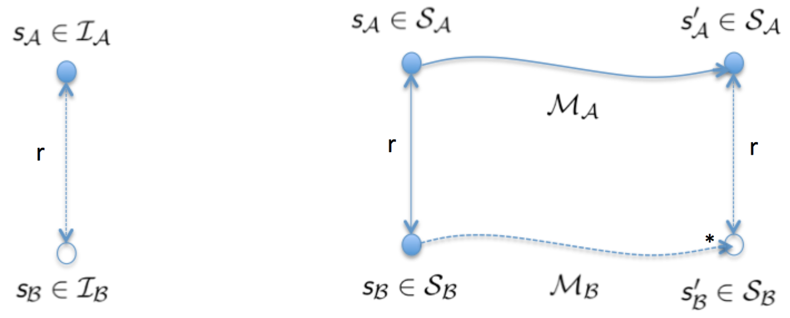

$s_{\mathcal{A}} \in \mathcal{I}_{\mathcal{A}}$
r
$s_{\mathcal{B}} \in \mathcal{I}_{\mathcal{B}}$
$s_{\mathcal{A}} \in \mathcal{S}_{\mathcal{A}}$
$s'_{\mathcal{A}} \in \mathcal{S}_{\mathcal{A}}$
$\mathcal{M}_{\mathcal{A}}$
r
r
*
$s_{\mathcal{B}} \in \mathcal{S}_{\mathcal{B}}$
$\mathcal{M}_{\mathcal{B}}$
$s'_{\mathcal{B}} \in \mathcal{S}_{\mathcal{B}}$

Figure 2.8: Simulation from $\mathcal{M}_{\mathcal{A}}$ to $\mathcal{M}_{\mathcal{B}}$.

desired property (called the $\mathcal{DSR}$ property) $\mathcal{CLDSA}$ should satisfy is that $s_*$ is reachable from $s_1$ and $s_2$ is reachable from $s_*$, whenever $\mathcal{CLDSA}$ terminates. Note that $s_1$, $s_2$ and $s_*$ are states of the $\mathcal{UDS}$ but not those of the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$.

## 2.3 State Machine

State machines can be used to formalize distributed systems. It consists of a set of states including a set of initial states and a total binary relation over states. The definition is as follows:

**Definition 1 (State Machine)** *A state machine* $M \triangleq \langle S, I, T \rangle$ *consists of*

1. *a set of states* $S$*;*
2. *a set of initial states* $I \subseteq S$*;*
3. *a total binary relation over states* $T \subseteq S \times S$*.*

*Each element* $(s, s') \in T$ *is called a (state) transition from* $s$ *to* $s'$ *(or just a transition). Since* $T$ *is total, for each state* $s \in S$ *there exists a state* $s' \in S$ *such that* $(s, s') \in T$*.*

Infinite sequences of states called paths are generated from a state machine. Paths are defined as follows:

**Definition 2 (*Path*)** *A path* $\pi$ *of a state machine* $M \triangleq \langle S, I, T \rangle$ *from a state* $s_0$ *is an infinite sequence of states* $\pi \triangleq (s_0, s_1, s_2, \dots)$*, where* $(\forall i \geq 0)((s_i, s_{i+1}) \in T)$*.* $\pi_i$ *denotes the i-th state (i.e.,* $s_i$*) in* $\pi$ *and* $\Pi$ *denotes the set of all paths of* $M$*.*

Simulation from one state machine to another state machine is defined as follows:

**Definition 3 (*Simulation from* $\mathcal{M}_{\mathcal{A}}$ *to* $\mathcal{M}_{\mathcal{B}}$)** *Given two state machines* $\mathcal{M}_{\mathcal{A}} \triangleq \langle \mathcal{S}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{T}_{\mathcal{A}} \rangle$ *and* $\mathcal{M}_{\mathcal{B}} \triangleq \langle \mathcal{S}_{\mathcal{B}}, \mathcal{I}_{\mathcal{B}}, \mathcal{T}_{\mathcal{B}} \rangle$ *,* $r : \mathcal{S}_{\mathcal{A}}\ \mathcal{S}_{\mathcal{B}} \to Bool$ *is called a simulation from* $\mathcal{M}_{\mathcal{A}}$ *to* $\mathcal{M}_{\mathcal{B}}$ *if it satisfies the following conditions:*

1. *For each $s_{\mathcal{A}} \in \mathcal{I}_{\mathcal{A}}$, there exists $s_{\mathcal{B}} \in \mathcal{I}_{\mathcal{B}}$ such that $r(s_{\mathcal{A}}, s_{\mathcal{B}})$.*

2. *For each $s_{\mathcal{A}}, s'_{\mathcal{A}} \in \mathcal{S}_{\mathcal{A}}$ and $s_{\mathcal{B}} \in \mathcal{S}_{\mathcal{B}}$ such that $r(s_{\mathcal{A}}, s_{\mathcal{B}})$ and $s_{\mathcal{A}} \rightsquigarrow_{\mathcal{M}_{\mathcal{A}}} s'_{\mathcal{A}}$, there exists $s'_{\mathcal{B}} \in \mathcal{S}_{\mathcal{B}}$ such that $r(s'_{\mathcal{A}}, s'_{\mathcal{B}})$ and $s_{\mathcal{B}} \rightsquigarrow^{*}_{\mathcal{M}_{\mathcal{B}}} s'_{\mathcal{B}}$.*

Note that $s_{\mathcal{A}} \rightsquigarrow_{\mathcal{M}_{\mathcal{A}}} s'_{\mathcal{A}}$ means that $s_{\mathcal{A}}$ goes to $s'_{\mathcal{A}}$ by one state transition in $\mathcal{M}_{\mathcal{A}}$ and $s_{\mathcal{B}} \rightsquigarrow^{*}_{\mathcal{M}_{\mathcal{B}}} s'_{\mathcal{B}}$ means that $s_{\mathcal{B}}$ goes to $s'_{\mathcal{B}}$ by zero or more state transitions in $\mathcal{M}_{\mathcal{B}}$.
Fig. 2.8 shows the diagrams corresponding to the two conditions in the Definition 3.

## 2.4 Model Cheking

Model checking is an automated technique for verifying finite state systems, such as sequential circuit designs and communication protocols [11]. Given a finite-state model of a system and a formal property, that technique systematically checks whether this property holds for that model. The principle of model checking is: generate all possible states of the system and check that whether its desired property holds in each state. In case, there are errors, the property does not hold, model checking will produce a counterexample that can be used to find the source of the errors. Model-checking problem can be described as follows:

*Given a transition system $M$ and a formula $f$, the model-checking problem is to decide whether $M \models f$ holds or not. If not, the model checker should provide an explanation why, in the form of counterexample.*

A model checker is the software tool that performs the model checking in which a model is described in a formal description language and a property is specified in formal way. A model checker generate all possible states of a system, then it automatically checks that whether the system satisfies its desired property. If the property is found to not hold, a counterexample is generated to help to find out why the property does not hold.
The process of model checking can be distinguished into the following different tasks:

- **Modeling**: is to model a system under consideration using the model description language. Models of systems describe the behavior of systems in an accurate and unambiguous way. They are often expressed by using finite-state automata, which consist of a finite set of states and a set of transitions. Transitions describe how a system moves from one state to another.

- **Specification**: is to state the property that a system must satisfy before model checking. The property is specified in a specification language, usually in a logic-based formalism. A model of the system and a formula of the property are considered as inputs to model checking. Completeness is one of the important issue in specification. Model checking provides means for checking that a model of the system satisfies a given specification. However, it is impossible to determine whether the given specification covers all the properties that the system should satisfy [11].

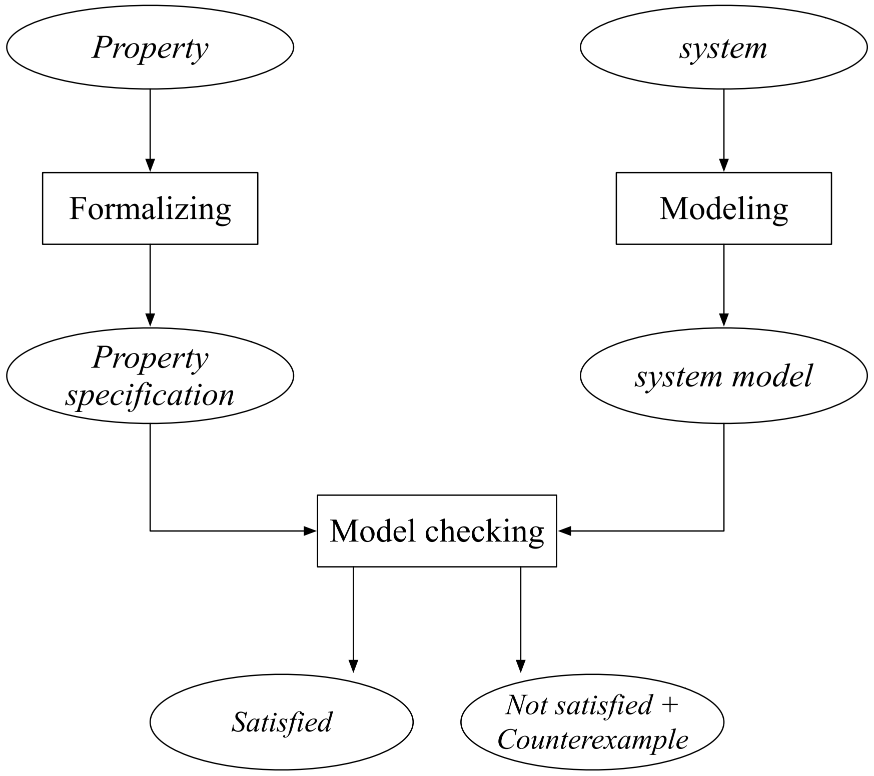


Figure 2.9: Schematic view of the model-checking approach.

- **Verification**: is to check the validity of the property. Running the model checker to automatic check the validity of the property in the system model. There are basically three possible outcomes: the specified property is either valid in the given model or not, or the model turns out to be too large to fit within the physical limits of the computer memory.

The entire model-checking process should be well organized, well structured, and well planned. One schematic view of the model-checking approach is shown in Fig .2.9.

Model checking has a number of advantages over verification techniques based on automated theorem proving. The main benefit of model checking is highly automatic. However, the main challenge in model checking is dealing with the state space explosion problem. This problem occurs in systems with many components that can interact with each other or systems with data structures that can assume many different values.

## 2.5 Maude

Maude, a specification and programming language, is based on rewriting logic, which contains membership equational logic as its sub-logic. State machines (or transition systems) and their data are specified in rewriting logic and membership equaltional logic, respectively. States of state machines are expressed as data, such as tuples and associative commutative collections, and state transition rules are described in rewriting rules. The specifications of state machines are called system specifications. Basic units of Maude specification and programming are modules. A module consists of syntax declaration, which declare for sorts, subsorts, kinds and operators. The syntax declaration part provides appropriate language to describe system and is called a signature. There are two kinds of modules in Core Maude: *functional modules* and *system modules*. Signatures are common for both of them, however,

- A *functional modules* is a equational theory in membership equational logic. functional modules admit equations and memberships. *functional modules* are declared with the keywork *fmod ... endfm*, where '...' corresponding to all declaration of submodule importation, sorts, subsorts, operators, variables, equations, and so on.

- A *system modules* is a rewrite theory. *system modules* admit rules, which are used to describe transitions between states. *system modules* are declared with the keywork *mod ... endm*, where '...' corresponding to all declaration of submodule importation, sorts, subsorts, operators, variables, equations, rules and so on.

Some built-in modules are provided in Maude such as BOOL and NAT for Boolean values and natural numbers. The boolean values are denoted as true and false, and natural numbers as 0, 1, 2, ... as usual. The corresponding sorts are Bool and Nat.

In addition, Maude is equipped with many useful functionalities. Among them are model checking facilities and meta-programming facilities. The model checking facilities mainly consists of an LTL model checker and a search command that can be used as a reachability checker. The search command is used in the existing study. the search command is used to model check invariant properties of concurrent systems specified as system modules in Maude.

Given a state $s$, a state pattern $p$ and an optional condition $c$, the search command searches the reachable state space from $s$ in a breadth-first manner for all states that match $p$ such that $c$ holds. The syntax of search command is as follows:

$$\textbf{search } [\ \boldsymbol{n, m}\ ] \textbf{ in } \boldsymbol{M} : \boldsymbol{s} \Rightarrow \boldsymbol{p} \textbf{ such that } \boldsymbol{c}\ .$$

where $M$ is a module in which the specification of the state machine concerned is described or available, $s$ is the start term, $p$ is the pattern that has to be reached, $\Rightarrow$ is an arrow indicating the form of the rewriting proof from $s$ until $p$. A rewrite expression $t \Rightarrow t'$ can be used in the optional condition $c$, that has to be satisfied by the reached state. This checks if $t'$ is reachable from $t$ by zero or more rewrite steps with rewrite rules. $\Rightarrow$ can be:

$\Rightarrow$1 : means a rewriting proof consisting of exactly one step,

$\Rightarrow$+ : means a rewriting proof consisting of exactly one or more steps.

$\Rightarrow$* : means a rewriting proof consisting of exactly none, one or more steps, and

$\Rightarrow$! : indicates that only canonical final states are allowed.

Although the reachable state space is bounded, the whole state space is unbounded. The search command can be given as options the maximum number of solutions and the maximum depth of search, namely $n$ is an optional argument proving a bound on the number of desired solutions and $m$ is an optional argument for the maximum depth of search.

A metaprogram is a program that takes programs as inputs and performs some useful computation. It may transform one program into another or my analyze such programs. In Maude, metaprogram has a logical, reflective semantics. This reflect the fact that both membership equational logic and rewriting are reflective logics. The Maude meta-programming functionalities can treat Maude specifications as data. The META-LEVEL module, in which key functionality of the universal theory has been implemented, is built. META-LEVEL module can be imported to easy write meta-programming in Maude. It includes the module META-MODULE and META-TERM. In the META-TERM module, Maude terms are metarepresented as elements of a data type Term of terms. In the META-MODULE module, Maude modules are metarepresented as elements of a data type Module of modules. In the META-LEVEL, there are operations, such as *upModule, upTerm* and *downTerm* to allow moving between reflection levels. The process of reducing and rewriting a term using Maude's command is metarepresented by built-in functions *metaReduce* and *metaRewrite*, respectively. The process of applying a rule is metarepresented by built-in function *metaApply.* The process of matching two terms is metarepresented by built-in function *metaApply* and the process of searching for a term satisfying some conditions is reified by built-in functions *metaSearch* and *metaSearchPath.* The functions *metaReduce, metaRewrite, metaApply, metaSearch* and *metaSearchPath* are called descent functions that allow us to descend levels in the reflective tower.

## 2.6 Chapter Summary

This chapter introduced some preliminaries that are very necessary. It first described a $\mathcal{UDS}$ and introduced the concept of the global state of a distributed system. Also, the problems of taking global snapshots of distributed systems were mentioned.

It then introduced $\mathcal{CLDSA}$ and the $\mathcal{DSR}$ property. One scenario of $\mathcal{CLDSA}$ was given to easily imagine how the algorithm works. The informal description of the $\mathcal{DSR}$ property was given based on the original paper [1] in which the $\mathcal{DSR}$ property is described as the first.

The chapter then gave the definitions of state machine. Then some other definitions on state machines, such as path and simulation from one state machine to another state machine were given.

The Model checking section presented about model checking in which the principle of model checking was presented and the model-checking problem was described. Also, this section described the process of model checking and the schematic view of the model-checking approach.

The last section in this chapter introduced Maude, in which Maude search command and metaprogram in Maude were introduced in detail.

# Chapter 3

# Specification and Model Checking of $\mathcal{CLDSA}$ in Maude

$\mathcal{CLDSA}$ has been formally analyzed in the existing study [2] in which Maude has been used to model check that $\mathcal{CLDSA}$ enjoys the $\mathcal{DSR}$ property. The authors describe how to specify $\mathcal{CLDSA}$ in Maude and how to model check that $\mathcal{CLDSA}$ enjoys the $\mathcal{DSR}$ properties with the Maude search command.

## 3.1 Specification of $\mathcal{CLDSA}$

$\mathcal{CLDSA}$ has been specified in Maude. What is modeled is actually the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$. A $\mathcal{UDS}$ consists one or more processes that are connected with directed channels that are unbounded queues. A system is assumed that it may be has only one process, and some processes have no outgoing channels, no incoming channels, or neither of them. There are no self-channel, which from one process to its self. Messages in the system are separated into token for non-marker message and marker for $\mathcal{CLDSA}$. Tokens can be consumed by processes.

To specify the algorithm, basic data used for identifying processes, tokens and makers are sorts Pid, Tokens and marker, respectively. Sorts EmpChan, NeChan and Chan are used for empty channels, non-empty channels and channels. The state of each process and the state of each channel are considered as observable components, in which the state of a process and the state of a channel are denoted as p-state and c-state, respectively. The state of a process depends on the number of token it has and the state of a channel is as the sequence of messages containing tokens and markers on the channel. Other observable components are for control information, which are used to control behaviour of the algorithm. The state of a $\mathcal{UDS}$, the start state, the finish state, and the snapshot are expressed as base-state(...), start-state(...), finish-state(...) and snapshot(...), respectively, where ‘ ... ’ is a soup of p-state and c-state observable components. Those are called meta configuration components and the sort is MCComp. Moreover, the control information is expressed as control(...) that is also meta configuration components, where ‘ ... ’ is also a soup of observable components of control information. A state of a $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ is expressed as a soup of meta configuration components, which is called a meta configuration. A meta configuration is a global view of the system (the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$). Although the system specification does not depend on the number of processes or the number of channels, they need to fix those number to model check the $\mathcal{DSR}$ property. Fixing those number is described in initial meta configurations.

Each process in the a $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ can do the following actions:

1. The process may consume a token owned by it and changes its state accordingly.

2. The process may put a token into one of its outgoing channels if it has some outgoing channels and changes its state accordingly.

3. The process may get a token from one of its nonempty incoming channels if it has some nonempty incoming channels and changes its state accordingly.

4. The process may start the $\mathcal{CLDSA}$when it has not yet received any markers. It records its state, initializes the states of its incoming channels as empty if any, and puts one marker into each of its outgoing channels if any.

5. The process may get a marker from one of its incoming channels if it has some incoming channel. If it has already started the $\mathcal{CLDSA}$, it has completed the record of the incoming channel. Moreover, if it has received markers from all the incoming channels, it has locally completed the $\mathcal{CLDSA}$. If it has not yet started, it records its state and the state of the incoming channel as empty, and initializes the states of the other incoming channels as empty if any. Then, it puts one marker into each of its outgoing channels if any. If it has only one incoming channel, it has locally completed the $\mathcal{CLDSA}$. Note that the first three describe the $\mathcal{UDS}$part.

Actions of processes in the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ are described by rewriting rules in Maude. Here is an example, in which the consumption of tokens is described by the following rewriting rule:

rl [chgStt] :
base-state((p-state[$P$] : ($T$ $PS$)) $BC$) finish-state(empConfig) control((consume : true) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) $BC$) finish-state(empConfig) control((consume : true) $CC$)

where $BC$ and $CC$ are variable of sort for a soup of observable components, $T$ is variable of sort for Token, $PS$ is variable of sort for state of processes (as the sequence of Tokens).

For each of basic actions of processes, there are multiple rewriting rules that have been obtained by case analyzes based on predicates that are not locally observable by any process. Case analyzes are to cover all possible situations.

## 3.2 Model Checking of $\mathcal{CLDSA}$

The Maude system is equipped with model checking facilities: the search command and the LTL model checker. In the existing study, the model checking invariants through search is used. They model check that $\mathcal{CLDSA}$ satisfies the $\mathcal{DSR}$ property by the Maude search command. The $\mathcal{DSR}$ property can be divided into $\mathcal{RP}1$ and $\mathcal{RP}2$ as follows:

1. $s_*$ is always reachable from $s_1$ ( $\mathcal{RP}1$ ), and
2. $s_2$ is always reachable from $s_*$ ( $\mathcal{RP}2$ ).

which can be checked with the Maude search command. The Maude search command is used to search states that satisfies some conditions. To model check $\mathcal{DSR}$ property, they do the following Maude search commands. The following part, $SC$, $FC$ and $SSC$ are variable of sort for a soup of observable components, $MC$ is variable of sort for meta configuration, $b$ is either true or false, and $n$ is the number of processes in the system.

- To find all states in which a snapshot has been taken by following Maude search command:

search in EXPERIMENT : imc ⇒* start-state($SC$) finish-state($FC$) snapshot($SSC$) $MC$
such that $FC$ =/= empConfig .

- To find all states in which a snapshot has been taken such that the snapshot $SSC$ is reachable from the start state $SC$ by following Maude search command:

search in EXPERIMENT :
imc ⇒* start-state($SC$) finish-state($FC$) snapshot($SSC$) $MC$
such that $FC$ =/= empConfig
∧ base-state($SC$) control((prog[p(0)]: notYet) ... (prog[p(n-1)]: notYet) (consume : b))
⇒
base-state($SSC$) control((prog[p(0)]: notYet) ... (prog[p(n-1)]: notYet) (consume : b)) .

- To find all states in which a snapshot has been taken such that the snapshot $FC$ is reachable from the snapshot $SSC$ by following Maude search command:

search in EXPERIMENT :
imc01 ⇒* start-state($SC$) finish-state($FC$) snapshot($SSC$) $MC$
such that $FC$ =/= empConfig
∧ base-state($SSC$) control((prog[p(0)]: notYet) ... (prog[p(n-1)]: notYet) (consume : b))
⇒
base-state($FC$) control((prog[p(0)]: notYet) ... (prog[p(1)]: notYet) (consume : b)) .

Let $m_0$, $m_1$ and $m_2$ be the number of the solutions to the search for snapshots, the search for $\mathcal{RP}1$ and the search for $\mathcal{RP}2$ . The search for snapshots finds all states in which a snapshot has been taken. The search for $\mathcal{RP}1$ finds all states in which a snapshot has been taken such that the snapshot is reachable from the start state under the $\mathcal{UDS}$, if $m_1$ equals $m_0$, $\mathcal{CLDSA}$ enjoys $\mathcal{RP}1$ . $\mathcal{RP}2$ can be checked likewise. The search for $\mathcal{RP}2$ finds all states in which a snapshot has been taken such that the finish state is reachable from the snapshot under the $\mathcal{UDS}$, if $m_2$ equals $m_0$, $\mathcal{CLDSA}$ enjoys $\mathcal{RP}2$ . The $\mathcal{DSR}$ property ($\mathcal{RP}1$ and $\mathcal{RP}2$ ) holds if and only if $((m_1 == m_0) \wedge (m_2 == m_0))$. And this is the key point of model checking the $\mathcal{DSR}$ property.

## 3.3 Chapter Summary

This chapter described how to specify $\mathcal{CLDSA}$ and how to model check that the algorithm enjoy the $\mathcal{DSR}$ property in the existing study. In detail, it described what the system is actually specified and presented how the states of the system is expressed. Then it described how the actions of processes in the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ are described by rewriting rules in Maude. The way to specify the algorithm in Maude is presented. However, please see fully the specification of $\mathcal{CLDSA}$ in Appendix A.

The chapter then showed how to model check that $\mathcal{CLDSA}$ enjoy the $\mathcal{DSR}$ property by Maude search command in the existing study. It also explained how the $\mathcal{DSR}$ property is encoded in Maude search command. The method to model check that $\mathcal{CLDSA}$ enjoy the $\mathcal{DSR}$ property was presented. However, more experiments conducted are shown in Appendix B.

# Chapter 4

# Formalizing a $\mathcal{UDS}$ and a $\mathcal{UDS}$ Superimposed by $\mathcal{CLDSA}$ as State Machines

## 4.1 Modeling a $\mathcal{UDS}$ as a State Machine

To model a $\mathcal{UDS}$ as a state machine $M_{\mathcal{UDS}}$, we consider how to express states and transitions of $M_{\mathcal{UDS}}$.

### 4.1.1 State Expression for a $\mathcal{UDS}$

Each process in a $\mathcal{UDS}$ has its own local state, and so does each channel. Each state of $M_{\mathcal{UDS}}$ should consist of the state of each process and the state of each channel in the $\mathcal{UDS}$. We use *name-value* pairs (called observable components) to express the states of processes and channels, where name may have parameters. For observable components for process states and channel states, the following operators that are constructors as specified with ctor are prepared[1]:

```
sort MsgQueue .
op empChan : → MsgQueue [ctor] .
op _|_ : Msg MsgSeq → MsgQueue [ctor] .
sort OCom .
op p-state[_]:_ : Pid PState → OCom [ctor] .
op c-state[_, _, _]:_ : Pid Pid Nat MsgQueue → OCom [ctor] .
```

where Pid is the sort for process identifiers, PState is the sort for process states, Nat is the sort for natural numbers, MsgQueue is the sort for queues of messages for which the sort Msg is used, and OCom is the sort for observable components. A sort is also used as the set of all ground constructor terms (or values) of the sort. Let $p$, $q \in$ Pid, $ps \in$ PState,

[1]Maude notation is used to describe state machines.

$n \in$ Nat and $ms \in$ MsgQueue. (p-state[$p$]: $ps$) is an observable component whose name is p-state[$p$] where $p$ is a parameter and whose value is $ps$, expressed as a term of OCom that says that the local state of a process $p$ is $ps$. (c-state[$p, q, n$]: $ms$) is an observable component whose name is c-state[$p, q, n$] where $p, q, n$ are parameters and whose value is $ms$, expressed as a term of OCom that says that the state of a channel from a process $p$ to a process $q$ is $ms$. Since there may be more than one channel from $p$ to $q$, a natural number $n$ is used in (c-state[$p, q, n$]: $ms$) to identify the channel.

We use a soup[2] of process and channel states to represent each state of $M_{\mathcal{UDS}}$. For this end, the following sorts and operators are prepared:

```
sort Config.
subsort OCom < Config .
op empConfig : → Config [ctor] .
op _ _ : Config Config → Config [ctor assoc comm id:empConfig] .
```

Config is the sort for soups of observable components and a super-sort of OCom, which means that each term of OCom is treated as the singleton soup only consisting of the term. empConfig denotes the empty soup of observable components, and the juxtaposition operator _ _ is used to construct soups of observable components. For $c_1, c_2 \in$ Config, $c_1\ c_2 \in$ Config. The juxtaposition operator is associative and commutative as specified with assoc and comm, and empConfig is an identity of the operator specified with id: empConfig.

We give the following functions on the states of a $\mathcal{UDS}$. In the following part, $P, Q \in$ Pid, $PS \in$ PState, $N \in$ Nat, $CS, MS \in$ MsgQueue, $CF, CF1 \in$ Config and $M \in$ Msg are variables of those sorts.

- Function chans : to get the set of all channels of a $\mathcal{UDS}$;

```
op chans : Config → Config .
eq chans(empConfig) = empConfig .
eq chans((p-state[P]: PS) CF) = chans(CF) .
eq chans((c-state[P,Q,N]: CS) CF) = (c-state[P,Q,N] : CS) chans(CF) .
```

- Function msg : to get the state of a channel;

```
op msg : Ocom → MsgSeq .
eq msg(c-state[P,Q,N] : CS) = CS .
eq msg(p-state[P]: PS) = empChan .
```

- Function enq : to put a message at the end of a sequence of messages;

```
op enq : MsgSeq Msg → MsgSeq .
```

[2]Associative-commutative collections may be called multisets or bags, but we use soups to refer to such collections according to the nomenclature of the Maude community.

eq enq(empChan, $M2$) = $M2$ | empChan .
eq enq($M1$ | $MS$, $M2$) = $M1$ | enq($MS, M2$) .

Function # : to count the number of occurrences of a process or channel in a state of a $\mathcal{UDS}$;

op # : Config Pid → Nat .
eq #(empConfig, $P$) = 0 .
eq #((p-state[$P1$]: $PS$) $CF$, $P$) = if $P = P1$ then 1 + #($CF, P$) else #($CF, P$) fi .
eq # ((c-state[$P1, Q1, N1$]: $CS$) $CF$, $P$) = #($CF, P$) .

op # : Config Pid Pid Nat → Nat .
eq # (empConfig, $P, Q, N$) = 0 .
eq # ((p-state[$P1$] : $PS$) $CF, P, Q, N$) = #($CF, P, Q, N$) .
eq # ((c-state[$P1, Q1, N1$]: $CS$) $CF, P, Q, N$) =
if ($P = P1$) and ($Q = Q1$) and ($N = N1$)
then 1 + #($CF, P, Q, N$) else #($CF, P, Q, N$) fi .

Function = : to check out the equivalence between the two states of a $\mathcal{UDS}$;

op _=_: Config Config → Bool .
eq (empConfig = empConfig) = true .
eq (((p-state[$P$]: $PS$) $CF$) = ((p-state[$P$]: $PS$) $CF1$)) = (CF = CF1) .
eq (((c-state[$P, Q, N$]: $CS$) $CF$) = ((c-state[$P, Q, N$] : $CS$) $CF1$)) = ($CF = CF1$) .
eq (($OC$ $CF$) = ($OC1$ $CF1$)) = false [owise] .

### 4.1.2 State Transitions for $M_{\mathcal{UDS}}$

Each process in a $\mathcal{UDS}$ may do three kinds of actions:

i it may change its state without putting any message into any of its outgoing channels nor getting any message from any of its incoming channels,

ii it may put a message into one of its outgoing channels and may change its state (may not change its state), and

iii it may get the top message from one of its incoming channels if the channel is not empty and may change its state (may not change its state).

The three kinds of actions i, ii and iii are called Change of Process State, Sending of Message and Receipt of Message, respectively. The actions are described as transition rules. A transition rule is described in form of rewrite rule. In the following part, $P, Q \in$ Pid, $PS1, PS2 \in$ PState, $N \in$ Nat, $CS \in$ MsgQueue and $M \in$ Msg are variables of those

sorts.
- Change of Process State is described as the following transition rule[3]:

$$(\text{p-state}[P] : PS1) \Rightarrow (\text{p-state}[P] : PS2)$$

- Sending of Message is described as the following transition rule:

$$(\text{p-state}[P] : PS1)\ (\text{c-state}[P, Q, N] : CS)$$

$$\Rightarrow$$

$$(\text{p-state}[P] : PS2)\ (\text{c-state}[P, Q, N] : \text{enq}(CS, M))$$

where $PS1$ may be the same as $PS2$ and enq is a standard function for queues, taking a queue $q$ and an element $e$ and putting $e$ into $q$ at bottom.
- Receipt of Message is described as the following transition rule:

$$(\text{p-state}[P] : PS1)\ (\text{c-state}[Q, P, N] : M \mid CS)$$

$$\Rightarrow$$

$$(\text{p-state}[P] : PS2)\ (\text{c-state}[Q, P, N] : CS)$$

where $PS1$ may be the same as $PS2$. The operator $\_|\_$ is used to construct queues of messages. For $m \in$ Msg and $q \in$ MsgQueue, $m \mid q \in$ MsgQueue where $m$ is the top message of the queue.

There are only three transition rules. Due to the number of states for each process, the number of channels, the number of states for each channel, etc., however, there may be more than three ground instances of the transition rules. Given a transition rule L ⇒ R, a ground instance of the transition rule is obtained by replacing each variable in L ⇒ R with a ground constructor term (or a value) of the sort of the variable.

**Definition 4** **($TR_{\mathcal{UDS}}$)** *Let $TR_{\mathcal{UDS}}$ be the set of all ground instances of the three transition rules.*

[3]The rewrite rule is not executable because the variable $PS2$ does not appear in the left-hand side. Each rewrite rule (and each equation) should be executable and the rewrite rule should be split into multiple executable ones so that model checking can be doable with Maude. Since the main purpose of the paper is to give a more faithful definition of the $\mathcal{DSR}$ property and confirm the validity of the model checking approach used in the existing study, we make each rewrite rule as general as possible to cover all possible situations.

### 4.1.3 State Machine $M_{\mathcal{UDS}}$

A $\mathcal{UDS}$ is formalized as $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}},\ I_{\mathcal{UDS}},\ T_{\mathcal{UDS}}\rangle$. $S_{\mathcal{UDS}}$ is the set of all ground constructor terms whose sorts are Config. $I_{\mathcal{UDS}}$ is a subset of $S_{\mathcal{UDS}}$ such that for each state $s \in I_{\mathcal{UDS}}$, for each channel $c$ in $s$, the message queue in $c$ is empty, for each process $p \in$ Pid, there exists at most one p-state[$p$] observable component in $s$, for each $(p, q, n)$ where $p, q \in$ Pid and $n \in$ Nat, there exists at most one c-state[$p, q, n$] observable component in $s$, and there is no dangling channel in $s$. $T_{\mathcal{UDS}}$ is the binary relation over $S_{\mathcal{UDS}}$ made from $TR_{\mathcal{UDS}}$.

**Definition 5** **($M_{\mathcal{UDS}}$)** *The state machine formalizing a $\mathcal{UDS}$ is $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}},\ I_{\mathcal{UDS}},\ T_{\mathcal{UDS}}\rangle$, where*

1. *$S_{\mathcal{UDS}}$ is the set of all ground constructor terms whose sorts are Config;*
2. *$I_{\mathcal{UDS}}$ is a subset of $S_{\mathcal{UDS}}$ such that $(\forall s \in I_{\mathcal{UDS}})(\forall c \in chans(s))(msg(c) = empChan)$, $(\forall\ s \in I_{\mathcal{UDS}})$ $(\forall\ p \in Pid)$ $(\#(s,\ p) \leq 1)$, $(\forall\ s \in I_{\mathcal{UDS}})$ $(\forall\ p,\ q \in Pid)$ $(\forall\ n \in Nat)$ $(\#(s,\ p,\ q,\ n) \leq 1)$ and $(\forall\ s \in I_{\mathcal{UDS}})$ $(\forall\ (c\text{-}state[p,\ q,\ n] : cs) \in s)$ $((\#(s,\ p) = 1) \wedge (\#(s,\ q) = 1))$;*
3. *$T_{\mathcal{UDS}}$ is the binary relation over $S_{\mathcal{UDS}}$ defined as follows:*
   $\{(L\ CF,\ R\ CF) \mid L \Rightarrow R \in TR_{\mathcal{UDS}},\ CF \in Config\}$.

Function chans gets all channels of a state $s \in S_{\mathcal{UDS}}$, msg gets the state of a channel $c$ in $s$, and # counts the number of occurrences of a process or a channel in $s$.

## 4.2 Modeling a $\mathcal{UDS}$ Superimposed by $\mathcal{CLDSA}$ as a State Machine

This part considers how to model a $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ as a state machine $CL(M_{\mathcal{UDS}})$. Let us consider the state expression and the state transition for $CL(M_{\mathcal{UDS}})$.

### 4.2.1 State Expression for $CL(M_{\mathcal{UDS}})$

Taking into account a $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$, each state of $CL(M_{\mathcal{UDS}})$ consists of the local states of all processes and channels, the state (called the start state) when $\mathcal{CLDSA}$ initiates, the snapshot, the state (called the finish state) when $\mathcal{CLDSA}$ terminates and the information to control behaviour of $\mathcal{CLDSA}$. The local states of all processes and channels, the start state, the snapshot, the finish state and the control information are expressed as base-state($bc$), start-state($sc$), snapshot($ssc$), finish-state($fc$) and control($ctl$) meta configuration components, respectively, where:
- $bc$ is a soup of process and channel states whose sorts are BOCom and the corresponding sort is BConfig. Note that there may be makers in channels of $bc$. MMsg is the sort for ordinary messages and a marker. MMsgQueue is the sort for queues of MMsg. We

prepare the following sorts and operators for them:

```
sort MMsg .
subsort Msg < MMsg .
op marker : → MMsg [ctor] .
op empChan : → MMsgQueue [ctor] .
op _|_ : MMsg MMsgQueue → MMsgQueue [ctor] .
op p-state[_]:_ : Pid PState → BOCom [ctor] .
op c-state[_, _, _]:_ : Pid Pid Nat MMsgQueue → BOCom [ctor].
subsort BOCom < BConfig .
op empBConfig : → BConfig [ctor] .
op _ _ : BConfig BConfig → BConfig [ctor assoc comm id:empBConfig] .
```

- $sc$, $ssc$, $fc$ are ground constructor terms of BConfig .
- $ctl$ is a soup of cnt, prog, #ms, and done control observable components that will be described. CtlOCom is the sort for those control observable components, and CtlConfig is the sort for soups of CtlOCom. We prepare the following sorts and operators for them:

```
sort Prog .
ops notYet, started, completed : → Prog [ctor] .
op cnt: _ : Nat → CtlOCom [ctor] .
op prog[_]:_ : Pid Prog → CtlOCom [ctor] .
op #ms[_]:_ : Pid Nat → CtlOCom [ctor] .
op done[_, _, _]:_ : Pid Pid Nat Bool → CtlOCom [ctor] .
subsort CtlOCom < CtlConfig .
op empCtlConfig : → CtlConfig [ctor] .
op _ _ : CtlConfig CtlConfig → CtlConfig [ctor assoc comm id: empCtlConfig] .
```

We need the following control observable components to specify the behaviour of $\mathcal{CLDSA}$, where $p$, $q \in$ Pid, $n \in$ Nat, $pg \in$ Prog, $b \in$ Bool:

- (cnt : $n$): $n$ is the number of processes that have not yet completed $\mathcal{CLDSA}$.
- (prog[$p$] : $pg$): $pg$ is the progress of a process $p$, indicating that the process has not yet started, has started, or completed $\mathcal{CLDSA}$.
- (#ms[$p$] : $n$): $n$ is the number of incoming channels to a process $p$ from which markers have not yet been received.
- (done[$p, q, n$] : $b$): $b$ is either $true$ or $false$. If $b$ is $true$, $q$ has received a marker from the incoming channel identified by $n$ from $q$. Otherwise, $q$ has not.

Each state of $CL(M_{\mathcal{UDS}})$ is expressed as the soup of the meta configuration components whose sorts are MBCom, which is typically in the form:

base-state($bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$)

control($ctl$)

which is called a meta configuration and the corresponding sort is MBConfig. We prepare the following operators for expressing a state of $CL(M_{\mathcal{UDS}})$:

```
sort MBCom .
op base-state: BConfig → MBCom [ctor] .
ops start-state, snapshot, finish-state : BConfig → MBCom [ctor].
op control : CtlConfig → MBCom [ctor] .
sort MBConfig .
subsort MBCom < MBConfig [ctor] .
op _ _ : MBConfig MBConfig → MBConfig [ctor assoc comm].
```

Initially, $bc$ is an initial state of $M_{\mathcal{UDS}}$ and all of $sc$, $ssc$ and $fc$ are empBConfig. The number of processes that have not yet completed $\mathcal{CLDSA}$ is equal to the number of processes in the system, the progress of all processes are notYet and all processes have not yet received any markers. If $fc$ is not empBConfig, a distributed snapshot has been taken and then $ssc$ is the snapshot.

We give the following functions on the state of a $M_{\mathcal{CLDSA}}$. In the following part, $P, Q \in$ Pid, $PS \in$ PState, $N \in$ Nat, $CS, MS \in$ MMsgQueue, $CF, CF1 \in$ Config, $BCF \in$ BConfig and $M \in$ MMsg are variables of those sorts.

- Function cntN: to get the number of processes in a $\mathcal{UDS}$;

```
op cntN : BConfig → Nat .
eq cntN(empConfig) = 0 .
eq cntN((p-state[P]: PS) CF) = 1 + cntN(CF) .
eq cntN((c-state[P, Q, N]: CS) CF) = cntN(CF) .
```

- Function msN: to get the number of incoming channel of a process;

```
op msN : Pid BConfig → Nat .
eq msN(P, empConfig) = 0 .
eq msN(P, (p-state[P1]: PS) CF) = msN(P, CF) .
eq msN(P, (c-state[P1, Q1, N]: CS) CF)) =
if (P = Q1) then 1 + msN(P, CF) else msN(P, CF) fi.
```

- Function mkCnt: the number of processes that have not yet completed $\mathcal{CLDSA}$ is equal to the number of processes in the system;

```
op mkCnt : BConfig → CtlConfig .
eq mkCnt(CF) = (cnt: cntN(CF)) .
```

- Function mkProg : the progress of all processes in a system are notYet;

op mkProg : BConfig $\rightarrow$ CtlConfig .
eq mkProg(empConfig) = empCtlConfig .
eq mkProg((p-state[$P$]: $PS$) $CF$) = (prog[$P$]: notYet) mkProg($CF$) .
eq mkProg((c-state[$P, Q, N$]: $MS$) $CF$) = mkProg($CF$) .

- Function mkDone : all processes have not yet received any markers from any their incoming channels;

op mkDone : BConfig $\rightarrow$ CtlConfig .
eq mkDone(empConfig) = empCtlConfig .
eq mkDone((p-state[$P$]: $PS$) $CF$) = mkDone($CF$) .
eq mkDone((c-state[$P, Q, N$]: $CS$) $CF$) = (done[$P, Q, N$]: false) mkDone($CF$) .

- Function mkMs : the number of channels of a process that have not yet received a marker is equal to the number of channels of the process;

op mkMs : BConfig BConfig $\rightarrow$ CtlConfig .
eq mkMs(empConfig, $CF1$) = empCtlConfig .
eq mkMs((p-state[$P$]: $PS$) $CF, CF1$) = (#ms[$P$]: msN($P, CF1$)) mkMs($CF, CF1$) .
eq mkMs((c-state[$P, Q, N$]: $CS$) $CF, CF1$) = mkMs($CF, CF1$) .

- Function InitCtlConfig;

op InitCtlConfig : BConfig $\rightarrow$ CtlConfig .
eq InitCtlConfig($BC$) = mkCnt($BC$) mkMs($BC$, $BC$)
mkDone($BC$) mkDone($BC$) mkProg($BC$) .

Functions InitCtlConfig, which call sub-functions mkCnt, mkProg, mkDone and mkMs, is to initial control meta configuration component of an initial state of $M_{\mathcal{UDS}}$ such that the number of processes that have not yet completed $\mathcal{CLDSA}$ is equal to the number of processes in the system, the progress of all processes are notYet and all processes have not yet received any markers.

- Function delMchan: to delete all markers in the sequence of message. Note that there is almost one marker in a channel in a state of a $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$, but this function is defined for general case, means that there may have more than one marker in a channel.

op delMchan : MMsgSeq $\rightarrow$ MsgSeq .
eq delMchan(empChan) = empChan .
eq delMchan($M$ | $MMS$ ) = if $M$ = marker then delMchan($MMS$)
else $M$ | delMchan($MMS$) fi.

- Function delM: to delete all markers in all channels in a soup of process and channel states in a $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$;

op delM: BConfig → Config .
eq delM(empBConfig) = empConfig .
eq delM((p-state[$P$]: $PS$) $BCF$) = (p-state[$P$]: $PS$) delM($BCF$) .
eq delM((c-state[$P, Q, N$] : $MMS$) $BCF$) = (c-state[$P, Q, N$] :
delMchan($MMS$)) delM($BCF$) .

- Function bcast: to puts markers in all the outgoing channels from a process $P$;

op bcast : BConfig Pid MMsg → BConfig .
eq bcast(empBConfig, $P$, marker) = empBConfig .
eq bcast((c-state[$P, Q, N$] : $MMS$) $BCF$, $P$, marker) =
(c-state[$P, Q, N$]: put($MMS$, marker)) bcast($BCF, P$, marker) .
eq bcast($OCBCF, P, M1$) = $OC$ bcast($BCF, P$, marker) [owise] .

- Function inchans : to initialize the states of a process's all incoming channels as empty channel;

op inchans : Config Pid → Config .
eq inchans(empConfig, $P$) = empConfig .
eq inchans((c-state[$P, Q, N$]: $CS$) $CF, P$) = (c-state[$P, Q, N$]: empChan) inchans($CF, P$) .
eq inchans($OCCF, P$) = inchans($CF, P$) [owise] .

### 4.2.2 State Transitions for $CL(M_{\mathcal{UDS}})$

Each process in the system may do five kinds of actions. The three kinds of actions i, ii and iii are almost the same as those for a $\mathcal{UDS}$ and the two more kinds of actions iv and v are as follows:

iv it may record its state and put markers into its outgoing channels, and

v it may get a marker from one of its incoming channels.

The two kinds of actions iv and v are called Record of Process State and Receipt of Marker, respectively. In the following part, $P$, $Q \in$ Pid, $PS, PS1, PS2 \in$ PState, $BC$, $SC$, $FC$, $SSC \in$ BConfig, $MMS$, $MMS' \in$ MMsgQueue, $CC \in$ CtlConfig, $N$, $NzN$, $NzN' \in$ NzNat and $M \in$ Msg are variables of those sorts, where NzNat is the sort for non-zero natural numbers and a subsort of Nat.
- Change of Process State is described as the following transition rule:

base-state((p-state[$P$] : $PS1$) $BC$)
$\Rightarrow$

base-state((p-state$[P]$ : $PS2$) $BC$)

- Sending of Message is described as the following transition rule:

base-state((p-state$[P]$ : $PS1$) (c-state$[P, Q, N]$ : $MMS$)
$BC$)
$\Rightarrow$
base-state((p-state$[P]$ : $PS2$) (c-state$[P, Q, N]$ : enq($MMS, M$))
$BC$)

- Receipt of Message is split into four subcases:

1. The process has not yet started $\mathcal{CLDSA}$.

   base-state((p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M \mid MMS$) $BC$)
   control((prog$[P]$ : notYet) $CC$)
   $\Rightarrow$
   base-state((p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $MMS$) $BC$)
   control((prog$[P]$ : notYet) $CC$)

2. The process has completed $\mathcal{CLDSA}$.

   base-state((p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M \mid MMS$) $BC$)
   control((prog$[P]$ : completed) $CC$)
   $\Rightarrow$
   base-state((p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $MMS$) $BC$)
   control((prog$[P]$ : completed) $CC$)

3. The process has started $\mathcal{CLDSA}$, not yet completed it, and not yet received a marker from the incoming channel.

   base-state((p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M \mid MMS$) $BC$)
   snapshot((c-state$[Q, P, N]$ : $MMS'$) $SSC$)
   control((prog$[P]$ : started) (done$[Q, P, N]$ : false) $CC$)
   $\Rightarrow$
   base-state((p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $MMS$) $BC$)
   snapshot((c-state$[Q, P, N]$ : enq($MMS', M$)) $SSC$)
   control((prog$[P]$ : started) (done$[Q, P, N]$ : false) $CC$)

4. The process has started $\mathcal{CLDSA}$, but not yet completed it and it has already received a marker from the incoming channel.

   base-state((p-state[$P$] : $PS1$) (c-state[$Q, P, N$] : $M$ | $MMS$) $BC$)

   control((prog[$P$] : started) (done[$Q, P, N$] : true) $CC$)

   $\Rightarrow$

   base-state((p-state[$P$] : $PS2$) (c-state[$Q, P, N$] : $MMS$) $BC$)

   control((prog[$P$] : started) (done[$Q, P, N$] : true) $CC$)

- Record of Process State is split into two subcases:

1. The process globally initiates $\mathcal{CLDSA}$. This case is further split into three subcases:

   (a) The $\mathcal{UDS}$ only consists of the process.

   base-state((p-state[$P$] : $PS$))
   start-state(empBConfig)
   snapshot(empBConfig)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 0) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$))
   start-state((p-state[$P$] : $PS$))
   snapshot((p-state[$P$] : $PS$))
   finish-state((p-state[$P$] : $PS$))
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) $CC$)

   (b) The system consists of more than one process, and the process does not have any incoming channels.

   base-state((p-state[$P$] : $PS$) $BC$)
   start-state(empBConfig)
   snapshot(empBConfig)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : 0) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
   start-state((p-state[$P$] : $PS$) $BC$)
   snapshot((p-state[$P$] : $PS$))
   control((prog[$P$] : completed) (cnt : sd($NzN$,1)) (#ms[$P$] : 0) $CC$)
   if $NzN > 1$

where bcast is a function, putting makers in all outgoing channels from process $P$ and sd is a function for natural number, taking two natural numbers $x$ and $y$ and then return $x - y$ if $x > y$ and $y - x$ otherwise.

(c) The system consists of more than one process, and the process has one or more incoming channels.

base-state((p-state[$P$] : $PS$) $BC$)
start-state(empBConfig)
snapshot(empBConfig)
finish-state(empBConfig)
control((prog[$P$] : notYet) (#ms[$P$] : $NzN'$) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state((p-state[$P$] : $PS$) $BC$)
snapshot((p-state[$P$] : $PS$) inchans($BC$, $P$))
control((prog[$P$] : started) (#ms[$P$] : $NzN'$) $CC$)

where inchans$(BC, P)$ initialises the states of all incoming channels of $P$ as empChan.

2. The process does not globally initiates $\mathcal{CLDSA}$. This case is further split into three subcases:

(a) The process does not have any incoming channels, and there are no processes except for the process that has not completed $\mathcal{CLDSA}$.

base-state((p-state[$P$] : $PS$) $BC$)
start-state($SC$)
snapshot($SSC$)
finish-state(empBConfig)
control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 0) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$))
start-state($SC$)
snapshot((p-state[$P$] : $PS$) $SSC$)
finish-state((p-state[$P$] : $PS$) $BC$)
control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) $CC$)
if ($SC \neq$ empBConfig) .

(b) The process does not have any incoming channels, and there are some other processes that have not completed $\mathcal{CLDSA}$.

base-state((p-state[$P$] : $PS$) $BC$)
start-state($SC$)

snapshot($SSC$)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : 0) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state($SC$)
snapshot($SSC$)
control((prog[$P$] : completed) (cnt : sd($NzN$,1)) (#ms[$P$] : 0) $CC$)
if ($SC \neq$ empBConfig) $\wedge$ ($NzN > 1$)

(c) The process has some incoming channels.

base-state((p-state[$P$] : $PS$) $BC$)
start-state($SC$)
snapshot($SSC$)
control((prog[$P$] : notYet) (#ms[$P$] : $NzN'$) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state($SC$)
snapshot((p-state[$P$] : $PS$) inchans($BC$, $P$) $SSC$)
control((prog[$P$] : started) (#ms[$P$] : $NzN'$) $CC$)
if ($SC \neq$ empBConfig)

- Receipt of Marker is split into two subcases:

1. The process has not yet started $\mathcal{CLDSA}$. This case is further split into three subcases:

   (a) The process has only one incoming channel, and there are no processes that have not yet completed $\mathcal{CLDSA}$ except for the process.

   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
   snapshot($SSC$)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) $SSC$)
   finish-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)

   (b) The process has only one incoming channel, and there are some other processes that have not yet completed $\mathcal{CLDSA}$.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
snapshot($SSC$)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) bcast($BC$, $P$, maker))
snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) $SSC$)
control((prog[$P$] : completed) (cnt : sd($NzN$,1)) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)
if $NzN > 1$

(c) The process has more than one incoming channel.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
snapshot($SSC$)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : $NzN'$) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) bcast($BC$, $P$, maker))
snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) inchans($BC, P$) $SSC$)
control((prog[$P$] : started) (cnt : sd($NzN$,1)) (#ms[$P$] : sd($NzN'$,1)) (done[$Q$, $P$, $N$] : true) $CC$)
if $NzN' > 1$

2. The process has already started $\mathcal{CLDSA}$. This case is further split into three sub-cases:

   (a) There are no incoming channels from which markers have not been received except for the incoming channel, and there are no processes that have not yet completed $\mathcal{CLDSA}$ except for the process.

   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
   finish-state(empBConfig)
   control((prog[$P$] : started) (cnt : 1) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   finish-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)

   (b) There are no incoming channels from which markers have not been received except for the incoming channel, and there are some other processes that have not yet completed $\mathcal{CLDSA}$.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
control((prog[$P$] : started) (cnt : $NzN$) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
control((prog[$P$] : completed) (cnt : sd($NzN$,1) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)
if $NzN > 1$

(c) There are some other incoming channels from which markers have not been received.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
control((prog[$P$] : started) (cnt : $NzN$) (#ms[$P$] : $NzN'$) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
control((prog[$P$] : started) (cnt : $NzN$)) (#ms[$P$] : sd($NzN'$,1)) (done[$Q$, $P$, $N$] : true) $CC$)
if $NzN' > 1$

The 18 transition rules can be classified into three parts: $\mathcal{UDS}$, $\mathcal{UDS}$&$\mathcal{CLDSA}$, and $\mathcal{CLDSA}$. The $\mathcal{UDS}$ part consists of the transition rules describing actions i, ii and iii-1. The $\mathcal{UDS}$ part depends on the $\mathcal{UDS}$ concerned, can be constructed from the three transition rules of the $\mathcal{UDS}$ and changes the base-state meta configuration component of a state of $CL(M_{\mathcal{UDS}})$. The $\mathcal{UDS}$&$\mathcal{CLDSA}$ part also depends on the $\mathcal{UDS}$ concerned and can be constructed from the three transition rules of the $\mathcal{UDS}$, but changes the other meta configuration components of a state of $CL(M_{\mathcal{UDS}})$ as well. Three transition rules describing actions iii-2, iii-3 and iii-4 are in the $\mathcal{UDS}$&$\mathcal{CLDSA}$ part. The $\mathcal{CLDSA}$ part is independent from the $\mathcal{UDS}$ concerned, can be constructed regardless of any $\mathcal{UDS}$s, and does not change the base-state meta configuration component of a state of $CL(M_{\mathcal{UDS}})$. The transition rules describing two kinds of actions iv and v are in the $\mathcal{CLDSA}$ part.

**Definition 6** **($TR_{\mathcal{CLDSA}}$)** *Let $TR_{\mathcal{CLDSA}}$ be the set of all ground instances of the 18 transition rules.*

### 4.2.3 State Machine $CL(M_{\mathcal{UDS}})$

In this part, we propose the function $CL$ that takes a state machine $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}}$, $I_{\mathcal{UDS}}$, $T_{\mathcal{UDS}}\rangle$ and returns another state machine $M_{\mathcal{CLDSA}} \triangleq \langle S_{\mathcal{CLDSA}}$, $I_{\mathcal{CLDSA}}$, $T_{\mathcal{CLDSA}}\rangle$. Note that $M_{\mathcal{UDS}}$ is a state machine of a $\mathcal{UDS}$ and $M_{\mathcal{CLDSA}}$ is a state machine of the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$. The definition of the function $CL$ is as follows:

**Definition 7** **($CL(M_{\mathcal{UDS}})$)** *For a state machine $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}}$, $I_{\mathcal{UDS}}$, $T_{\mathcal{UDS}}\rangle$ formalizing a $\mathcal{UDS}$, CL is the function that takes $M_{\mathcal{UDS}}$ and returns $CL(M_{\mathcal{UDS}}) \triangleq \langle CL_{State}(S_{\mathcal{UDS}})$,*

*$CL_{Init}(I_{\mathcal{UDS}})$, $CL_{Trans}(T_{\mathcal{UDS}})\rangle$, where*

1. *$CL_{State}(S_{\mathcal{UDS}})$ is the set of all ground constructor terms of sort MBConfig;*
2. *$CL_{Init}(I_{\mathcal{UDS}})$ is*
   *$\{$base-state(bc) start-state(empBConfig)*
   *snapshot(empBConfig) finish-state(empBConfig)*
   *control(ctl) $\mid bc \in I_{\mathcal{UDS}}$, clt = InitCtlConfig(bc)$\}$;*
3. *$CL_{Trans}(T_{\mathcal{UDS}}) \subseteq CL_{State}(S_{\mathcal{UDS}}) \times CL_{State}(S_{\mathcal{UDS}})$ is $\{$(L MCF, R MCF) $\mid L \Rightarrow R \in TR_{\mathcal{CLDSA}}$, MCF $\in$ MBConfig$\}$.*

Function InitCtlConfig($bc$) initializes values for all control information components. Let $M_{\mathcal{CLDSA}}$ be $CL(M_{\mathcal{UDS}})$. Note that $S_{\mathcal{CLDSA}}$ is the set of all ground constructor terms of sort MBConfig, although each reachable state from an initial state in $I_{\mathcal{CLDSA}}$ is in the following form:

base-state($bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$)

## 4.3 Chapter Summary

This chapter described how to formalize a $\mathcal{UDS}$ and the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ as state machines $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$.

The first section of the chapter described how to formalize a $\mathcal{UDS}$ as state machines $M_{\mathcal{UDS}}$. It first showed how to express a state of a $\mathcal{UDS}$. The sorts Pid, PState, Msg and MsgQueue are used for process identifiers, process states, messages and queues of messages, respectively. The *name-value* pairs (called observable components) are used to express the states of processes and channels, where name may have parameters. The corresponding operators for observable components for process states and channel states were given. The sort OCom is used for observable components. A soup of process and channel states is used to represent each state of $M_{\mathcal{UDS}}$. The corresponding sort is Config. Also, the functions on the states of a $\mathcal{UDS}$ were given. It then showed how the actions of a $\mathcal{UDS}$ are described as transition rules and a transition rule is described in form of rewrite rule. At the end of this section, the definition of state machine $M_{\mathcal{UDS}}$ was given.

The chapter then described how to formalize a $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ as a state machines $CL(M_{\mathcal{UDS}})$. Each state of $CL(M_{\mathcal{UDS}})$ consists of the local states of all processes and channels, the start state, the snapshot, the finish state and the information to control behaviour of $\mathcal{CLDSA}$. The local states of all processes and channels, the start state, the snapshot, the finish state and the control information are expressed as base-state($bc$), start-state($sc$), snapshot($ssc$), finish-state($fc$) and control($ctl$) meta configuration components, respectively. Each state of $CL(M_{\mathcal{UDS}})$ is expressed as the soup of the meta configuration components whose sorts are MBCom, which is typically in the form: base-state($bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$), which is called a meta configuration and the corresponding sort is MBConfig. The functions on the states of a $CL(M_{\mathcal{UDS}})$ were presented. Then state transitions for $CL(M_{\mathcal{UDS}})$ was described.

The chapter then defined the function $CL$ that takes a state machine $M_{\mathcal{UDS}}$ and returns the state machine $CL(M_{\mathcal{UDS}})$. Let $M_{\mathcal{CLDSA}}$ is $CL(M_{\mathcal{UDS}})$. $M_{\mathcal{CLDSA}}$ is the state machine of the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$.

# Chapter 5

# A More Faithful Definition of the $\mathcal{DSR}$ Property

## 5.1 Some Definitions and Proposition 1

### 5.1.1 Some Definitions on $S_{\mathcal{CLDSA}}$

Some functions on $S_{\mathcal{CLDSA}}$ are defined as follows:

**Definition 8 (*b-state*, *s-state*, *snapshot*, *f-state*, *finished*)** *For each $s \in S_{\mathcal{CLDSA}}$, b-state(s) is bc if there exists exactly one occurrence of the base-state(bc) meta configuration component in s and empBConfig otherwise,*
*s-state(s) is sc if there exists exactly one occurrence of the start-state(sc) meta configuration component in s and empBConfig otherwise,*
*snapshot(s) is ssc if there exists exactly one occurrence of the snapshot(ssc) meta configuration component in s and empBConfig otherwise,*
*f-state(s) is fc if there exists exactly one occurrence of the finish-state(fc) meta configuration component in s and empBConfig otherwise, and*
*finished(s) is false if f-state(s) is empBConfig and true otherwise.*

The following is the definition that $\mathcal{CLDSA}$ has terminated in a state $s$ in $M_{\mathcal{CLDSA}}$:

**Definition 9 ($M_{\mathcal{CLDSA}} \models terminated(s)$)** *For a state machine $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}}, I_{\mathcal{UDS}}, T_{\mathcal{UDS}} \rangle$, for each $s \in S_{\mathcal{CLDSA}}$, $M_{\mathcal{CLDSA}} \models terminated(s)$ iff finished(s).*

### 5.1.2 Proposition 1

We have the following proposition on $M_{\mathcal{CLDSA}}$:

**Proposition 1 (No marker in s-state, snapshot and f-state)** *For each $s \in S_{\mathcal{CLDSA}}$, if $M_{\mathcal{CLDSA}} \models terminated(s)$, then there is no maker in s-state(s), snapshot(s) and f-state(s), equivalently that the sorts of s-state(s), snapshot(s) and f-state(s) are Config.*

Note that, whenever $\mathcal{CLDSA}$ has terminated in a state $s$, the function s-state($s$), snapshot($s$) and f-state($s$) return start state, snapshot and finish state, respectively.

**Proof of Proposition 1.** We will separate Proposition 1 into three parts, then prove them independently.

**1. For each $s \in S_{\mathcal{CLDSA}}$, if $M_{\mathcal{CLDSA}} \models$ terminated($s$), there is no maker in s-state($s$).**

It suffices to consider $s \in S_{\mathcal{CLDSA}}$ that can be reach from any initial state in $S_{\mathcal{CLDSA}}$. Then $s$ is in form base-state($bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$). if $M_{\mathcal{CLDSA}} \models$ terminated($s$), s-state($s$) is $sc$. We will prove that there is no marker in $sc$. To prove, we consider all transition rules that change the start-state meta configuration component of a state of $M_{\mathcal{CLDSA}}$. Our proof shows that there is no marker in any the start-state meta configuration component in the right hand side of any those transition rules. This means that there is no marker in $sc$.
All transition rules that change the start-state meta configuration component of a state of $M_{\mathcal{CLDSA}}$ are the following transition rules:

**- Record of Process State:** the process globally initiates $\mathcal{CLDSA}$. This case is further split into three subcases:

1. The $\mathcal{UDS}$ only consists of the process.

   base-state((p-state[$P$] : $PS$))
   start-state(empBConfig)
   snapshot(empBConfig)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 0) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$))
   start-state((p-state[$P$] : $PS$))
   snapshot((p-state[$P$] : $PS$))
   finish-state((p-state[$P$] : $PS$))
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) $CC$)

   Since the process globally initiates $\mathcal{CLDSA}$, there is no-marker in (p-state[$P$] : $PS$). Therefore, there is no marker in start-state((p-state[$P$] : $PS$)).

2. The system consists of more than one process, and the process does not have any incoming channels.

   base-state((p-state[$P$] : $PS$) $BC$)

start-state(empBConfig)
snapshot(empBConfig)
finish-state(empBConfig)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : 0) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state((p-state[$P$] : $PS$) $BC$)
snapshot((p-state[$P$] : $PS$))
control((prog[$P$] : completed) (cnt : sd($NzN$,1)) (#ms[$P$] : 0) $CC$)
if $NzN > 1$

Since the process globally initiates $\mathcal{CLDSA}$, there is no-marker in (p-state[$P$] : $PS$) $BC$. Therefore, there is no marker in start-state((p-state[$P$] : $PS$) $BC$).

3. The system consists of more than one process, and the process has one or more incoming channels.

   base-state((p-state[$P$] : $PS$) $BC$)
   start-state(empBConfig)
   snapshot(empBConfig)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (#ms[$P$] : $NzN'$) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
   start-state((p-state[$P$] : $PS$) $BC$)
   snapshot((p-state[$P$] : $PS$) inchans($BC$, $P$))
   control((prog[$P$] : started) (#ms[$P$] : $NzN'$) $CC$)

   Since the process globally initiates $\mathcal{CLDSA}$, there is no-marker in (p-state[$P$] : $PS$) $BC$. Therefore, there is no marker in start-state((p-state[$P$] : $PS$) $BC$).

Our proof have considered all transition rules that change the start-state meta configuration component of a state of $M_{\mathcal{CLDSA}}$ and showed that there is no marker in the start-state meta configuration component after apply those transition rules. This case is dischanged.

**2. For each $s \in S_{\mathcal{CLDSA}}$, if $M_{\mathcal{CLDSA}} \models$ terminated($s$), there is no maker in snapshot($s$).**

It suffices to consider $s \in S_{\mathcal{CLDSA}}$ that can be reach from any initial state in $S_{\mathcal{CLDSA}}$. Then $s$ is in form base-state($bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$). if $M_{\mathcal{CLDSA}} \models$ terminated($s$), snapshot($s$) is $ssc$. We will prove that there is no marker in $ssc$. To prove, we consider all transition rules that change the snapshot meta configuration component of a state of $M_{\mathcal{CLDSA}}$. Our proof shows that those transition rules do not put any marker into snapshot meta configuration component of a state of $M_{\mathcal{CLDSA}}$ when they change the snapshot meta configuration component, this means that there is no marker in the snapshot.

All the transition rules that change the snapshot meta configuration component of a state of $M_{\mathcal{CLDSA}}$ are the following transition rules:

**- Receipt of Message** is split into four subcases. The subcase that change the snapshot meta configuration component of a state of $M_{\mathcal{CLDSA}}$ is as follows:

1. The process has started $\mathcal{CLDSA}$, not yet completed it, and not yet received a marker from the incoming channel.

   base-state((p-state[$P$] : $PS1$) (c-state[$Q, P, N$] : $M$ | $MMS$) $BC$)

   snapshot((c-state[$Q, P, N$] : $MMS'$) $SSC$)

   control((prog[$P$] : started) (done[$Q, P, N$] : false) $CC$)

   $\Rightarrow$

   base-state((p-state[$P$] : $PS2$) (c-state[$Q, P, N$] : $MMS$) $BC$)

   snapshot((c-state[$Q, P, N$] : enq($MMS'$, $M$)) $SSC$)

   control((prog[$P$] : started) (done[$Q, P, N$] : false) $CC$)

   where $M \in$ Msg. Because $M$ is a message, the transition rule does not put any marker in the the snapshot meta configuration component: snapshot((c-state[$Q, P, N$] : $MMS'$) $SSC$) $\Rightarrow$ snapshot((c-state[$Q, P, N$] : enq($MMS'$, $M$)) $SSC$)

**- Record of Process State** is split into two subcases:

1. The process globally initiates $\mathcal{CLDSA}$. This case is further split into three subcases:

   (a) The $\mathcal{UDS}$ only consists of the process.

   base-state((p-state[$P$] : $PS$))
   start-state(empBConfig)
   snapshot(empBConfig)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 0) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$))
   start-state((p-state[$P$] : $PS$))
   snapshot((p-state[$P$] : $PS$))
   finish-state((p-state[$P$] : $PS$))
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) $CC$)

   Since the process globally initiates $\mathcal{CLDSA}$, there is no marker in (p-state[$P$] :

$PS$). The transition rule does not put any marker in the the snapshot meta configuration component: snapshot(empBConfig) $\Rightarrow$ snapshot((p-state[$P$] : $PS$)).

(b) The system consists of more than one process, and the process does not have any incoming channels.

base-state((p-state[$P$] : $PS$) $BC$)
start-state(empBConfig)
snapshot(empBConfig)
finish-state(empBConfig)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : 0) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state((p-state[$P$] : $PS$) $BC$)
snapshot((p-state[$P$] : $PS$))
control((prog[$P$] : completed) (cnt : sd($NzN$,1)) (#ms[$P$] : 0) $CC$)
if $NzN > 1$

Since the process globally initiates $\mathcal{CLDSA}$, there is no marker in (p-state[$P$] : $PS$) $BC$. The transition rule does not put any marker in the the snapshot meta configuration component: snapshot(empBConfig) $\Rightarrow$ snapshot((p-state[$P$] : $PS$) $BC$).

(c) The system consists of more than one process, and the process has one or more incoming channels.

base-state((p-state[$P$] : $PS$) $BC$)
start-state(empBConfig)
snapshot(empBConfig)
finish-state(empBConfig)
control((prog[$P$] : notYet) (#ms[$P$] : $NzN'$) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state((p-state[$P$] : $PS$) $BC$)
snapshot((p-state[$P$] : $PS$) inchans($BC$, $P$))
control((prog[$P$] : started) (#ms[$P$] : $NzN'$) $CC$)

where inchans($BC, P$) initialises the states of all incoming channels of $P$ as empChan. The transition rule does not put any marker in the the snapshot meta configuration component: snapshot(empBConfig) $\Rightarrow$ snapshot((p-state[$P$] : $PS$) inchans($BC$, $P$)).

2. The process does not globally initiates $\mathcal{CLDSA}$. This case is further split into three subcases:

(a) The process does not have any incoming channels, and there are no processes except for the process that has not completed $\mathcal{CLDSA}$.

base-state((p-state[$P$] : $PS$) $BC$)
start-state($SC$)
snapshot($SSC$)
finish-state(empBConfig)
control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 0) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$))
start-state($SC$)
snapshot((p-state[$P$] : $PS$) $SSC$)
finish-state((p-state[$P$] : $PS$) $BC$)
control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) $CC$)
if ($SC \neq$ empBConfig) .

It is clear to see that the transition rule does not put any marker in the the snapshot meta configuration component: snapshot($SSC$) $\Rightarrow$ snapshot((p-state[$P$] : $PS$) $SSC$).

(b) The process has some incoming channels.

base-state((p-state[$P$] : $PS$) $BC$)
start-state($SC$)
snapshot($SSC$)
control((prog[$P$] : notYet) (#ms[$P$] : $NzN'$) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state($SC$)
snapshot((p-state[$P$] : $PS$) inchans($BC$, $P$) $SSC$)
control((prog[$P$] : started) (#ms[$P$] : $NzN'$) $CC$)
if ($SC \neq$ empBConfig)

It is clear to see that the transition rule does not put any marker in the the snapshot meta configuration component: snapshot($SSC$) $\Rightarrow$ snapshot((p-state[$P$] : $PS$) inchans($BC$, $P$) $SSC$).

- **Receipt of Marker** is split into two subcases:

1. The process has not yet started $\mathcal{CLDSA}$. This case is further split into three subcases:

   (a) The process has only one incoming channel, and there are no processes that have not yet completed $\mathcal{CLDSA}$ except for the process.

   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
   snapshot($SSC$)

finish-state(empBConfig)
control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) $SSC$)
finish-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)

Since there is no marker in (c-state[$Q$, $P$, $N$] : empChan). The transition rule does not put any marker in the the snapshot meta configuration component: snapshot($SSC$) $\Rightarrow$ snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) $SSC$).

(b) The process has only one incoming channel, and there are some other processes that have not yet completed $\mathcal{CLDSA}$.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
snapshot($SSC$)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) bcast($BC$, $P$, maker))
snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) $SSC$)
control((prog[$P$] : completed) (cnt : sd($NzN$,1)) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)
if $NzN > 1$

Since there is no marker in (c-state[$Q$, $P$, $N$] : empChan). The transition rule does not put any marker in the the snapshot meta configuration component: snapshot($SSC$) $\Rightarrow$ snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) $SSC$) .

(c) The process has more than one incoming channel.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
snapshot($SSC$)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : $NzN'$) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) bcast($BC$, $P$, maker))
snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) inchans($BC, P$) $SSC$)
control((prog[$P$] : started) (cnt : sd($NzN$,1)) (#ms[$P$] : sd($NzN'$,1)) (done[$Q$, $P$, $N$] : true) $CC$)
if $NzN' > 1$

Since there is no marker in (c-state[$Q$, $P$, $N$] : empChan) inchans($BC, P$).

The transition rule does not put any marker in the the snapshot meta configuration component: snapshot($SSC$) $\Rightarrow$ snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) inchans($BC, P$) $SSC$).

**3. For each $s \in S_{\mathcal{CLDSA}}$, if $M_{\mathcal{CLDSA}} \models$ terminated(s), there is no maker in f-state(s).**

It suffices to consider $s \in S_{\mathcal{CLDSA}}$ that can be reach from any initial state in $S_{\mathcal{CLDSA}}$. Then $s$ is in form base-state($bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$). if $M_{\mathcal{CLDSA}} \models$ terminated($s$), f-state($s$) is $fc$. We will prove that there is no marker in $fc$. To prove, we consider all transition rules that change the finsh-state meta configuration component of a state of $M_{\mathcal{CLDSA}}$. Our proof shows that there is no marker in any the finsh-state meta configuration component in the right hand side of any those transition rules. This means that there is no marker in $fc$.
All transition rules that change the finish state meta configuration component of a state of $M_{\mathcal{CLDSA}}$ are the following transition rules:

**- Record of Process State** is split into two subcases:

1. The process globally initiates $\mathcal{CLDSA}$. This case is further split into three subcases. The subcase that change the snapshot meta configuration component of a state of $M_{\mathcal{CLDSA}}$ is as follows:

   (a) The $\mathcal{UDS}$ only consists of the process.

   base-state((p-state[$P$] : $PS$))
   start-state(empBConfig)
   snapshot(empBConfig)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 0) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$))
   start-state((p-state[$P$] : $PS$))
   snapshot((p-state[$P$] : $PS$))
   finish-state((p-state[$P$] : $PS$))
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) $CC$)

   Since there is no marker in (p-state[$P$] : $PS$). There is no marker in finish-state((p-state[$P$] : $PS$)). Therefore, there is no marker in the finsh-state meta configuration component in the right hand side of any the transition rule: finish-state(empBConfig) $\Rightarrow$ finish-state((p-state[$P$] : $PS$)).

2. The process does not globally initiates $\mathcal{CLDSA}$. This case is further split into three subcases:

(a) The process does not have any incoming channels, and there are no processes except for the process that has not completed $\mathcal{CLDSA}$.

base-state((p-state[$P$] : $PS$) $BC$)
start-state($SC$)
snapshot($SSC$)
finish-state(empBConfig)
control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 0) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$))
start-state($SC$)
snapshot((p-state[$P$] : $PS$) $SSC$)
finish-state((p-state[$P$] : $PS$) $BC$)
control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) $CC$)
if ($SC \neq$ empBConfig) .

Since there is no marker in (p-state[$P$] : $PS$) $BC$. There is no marker in finish-state((p-state[$P$] : $PS$)). Therefore, there is no marker in the finsh-state meta configuration component in the right hand side of any the transition rule: finish-state(empBConfig) $\Rightarrow$ finish-state((p-state[$P$] : $PS$) $BC$).

- **Receipt of Marker** is split into two subcases:

(a) The process has not yet started $\mathcal{CLDSA}$. This case is further split into three subcases:

i. The process has only one incoming channel, and there are no processes that have not yet completed $\mathcal{CLDSA}$ except for the process.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
snapshot($SSC$)
finish-state(empBConfig)
control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) $SSC$)
finish-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)

Because the process has only one incoming channel, and there are no processes that have not yet completed $\mathcal{CLDSA}$ except for the process, there is no marker in $BC$. Then there is no marker in finish-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$).

(b) The process has already started $\mathcal{CLDSA}$. This case is further split into three subcases:

i. There are no incoming channels from which markers have not been received except for the incoming channel, and there are no processes that have not yet completed $\mathcal{CLDSA}$ except for the process.

   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
   finish-state(empBConfig)
   control((prog[$P$] : started) (cnt : 1) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   finish-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)

   Because There are no incoming channels from which markers have not been received except for the incoming channel, and there is no process that has not yet completed $\mathcal{CLDSA}$ except for the process, there is no marker in $BC$. Then there is no marker in finish-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$).

The case has been discharged.

From the three parts are proven, Proposition 1 is proven. QED.

We have already given the proof of Proposition 1.

## 5.2 A More Faithful Definition of the $\mathcal{DSR}$ Property

For a state machine $M$, $M$, $\pi \models$ isReachable$(s_2, s_1)$ that $s_2$ is reachable from $s_1$ in a path $\pi$ in $M$ is defined and then $M \models$ isReachable$(s_2, s_1)$ that $s_2$ is reachable from $s_1$ in M is defined.

**Definition 10 (Reachabilty in M)** *For a state machine $M \triangleq \langle S, I, T \rangle$, for each $\pi \in \Pi$ and each $s_1, s_2 \in S$,*
*$M, \pi \models isReachable(s_2, s_1)$ iff $(\exists i, j \in Nat)\,(i \leq j \wedge s_1 = \pi_i \wedge s_2 = \pi_j)$, and $M \models isReachable(s_2, s_1)$ iff $(\exists \pi \in \Pi)\,(M, \pi \models isReachable(s_2, s_1))$.*

In other words, a state $s_2$ is said to be reachable from a state $s_1$ if and only if $s_1$ can go to $s_2$ by zero or more state transition steps in the state machine $M$.

In the informal description of the $\mathcal{DSR}$ property, it is checked that $\mathcal{CLDSA}$ terminates, and it is checked that some states of a $\mathcal{UDS}$ are reachable from some others in the $\mathcal{UDS}$ but not the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$. Accordingly, the property involves two systems, a $\mathcal{UDS}$ and the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$, and hence we need to use two state machines $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ to faithfully define the $\mathcal{DSR}$ property. Our definition of the $\mathcal{DSR}$ property is as follows:

**Definition 11** (**$\mathcal{DSR}$ *Property***) *For a state machine* $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}}, I_{\mathcal{UDS}}, T_{\mathcal{UDS}} \rangle$, $(\forall s \in S_{\mathcal{CLDSA}})$
$(M_{\mathcal{CLDSA}} \models \textit{terminated(s)} \Rightarrow M_{\mathcal{UDS}} \models \textit{isReachable}(s_*, s_1) \wedge M_{\mathcal{UDS}} \models \textit{isReachable}(s_2, s_*))$, *where* $s_1 =$ *s-state(s),* $s_* =$ *snapshot(s) and* $s_2 =$ *f-state(s).*

## 5.3 Chapter Summary

This chapter gave the more faithful formal definition of the $\mathcal{DSR}$ property. It first gave the definitions of functions b-state, s-state, snapshot and f-state corresponding to the local states of all processes and channels, the start-state, the snapshot, the finish-state of $M_{\mathcal{CLDSA}}$, and function finished for termination of $\mathcal{CLDSA}$. It then gave Proposition1 saying that whenever $\mathcal{CLDSA}$ terminates in state $s$, there is no marker in the start state, the snapshot and the finish state.

The chapter then gave the proof of Proposition 1, which is separated into three parts: whenever $\mathcal{CLDSA}$ terminates in state $s$, there is no marker in the start state, whenever $\mathcal{CLDSA}$ terminates in state $s$, there is no marker in the snapshot and whenever $\mathcal{CLDSA}$ terminates in state $s$, there is no marker in the finish state.

At the end of the chapter, the more faithful formal definition of the $\mathcal{DSR}$ property was given, which involes two state machines $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$. The definition more faithfully expresses the informal description of the $\mathcal{DSR}$ property.

# Chapter 6

# Equivalence of Two Definitions of the $\mathcal{DSR}$ Property

## 6.1 The Theorem on Equivalence of the Two Definitions and Some Lemmas

Since our new definition of the $\mathcal{DSR}$ property involves two state machines, it is not straightforward to directly model check the new definition of the $\mathcal{DSR}$ property for $\mathcal{CLDSA}$ with any existing model checker. If the new definition is equivalent to the definition used in the existing study for each $\mathcal{UDS}$, we can use the model checking approach used in the existing study to model check the new definition of the $\mathcal{DSR}$ property for $\mathcal{CLDSA}$. Therefore, we prove a theorem saying that our new definition is equivalent to the definition in the existing study, which also guarantees the validity of the model checking approach used in the existing study.
Let us suppose that there are $n$ processes in a $\mathcal{UDS}$ and let $p_1, ..., p_n$ be their identifications, namely that Pid is $\{p_1, ..., p_n\}$, where $n \geq 1$. Let *ctl* be (prog[$p_1$] : notYet) ... (prog[$p_n$] : notYet) in the rest of the paper.
Although the $\mathcal{DSR}$ property is encoded in terms of the Maude search command, the definition of $\mathcal{DSR}$ property in the existing study can be expressed as follows:

*For a state machine $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}}, I_{\mathcal{UDS}}, T_{\mathcal{UDS}} \rangle$, ($\forall$ $s \in S_{\mathcal{CLDSA}}$) ($M_{\mathcal{CLDSA}} \models$ terminated(s) $\Rightarrow$ $M_{\mathcal{CLDSA}} \models$ isReachable(base-state($s_*$) control(ctl), base-state($s_1$) control(ctl)) $\wedge$ $M_{\mathcal{CLDSA}} \models$ isReachable(base-state($s_2$) control(ctl), base-state($s_*$) control(ctl))), where $s_1$ = s-state(s), $s_*$ = snapshot(s) and $s_2$ = f-state(s).*

Checked are the termination of $\mathcal{CLDSA}$ and then the reachability from the snapshot and the final state to the start state and the snapshot, respectively, in a $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ but not in the $\mathcal{UDS}$. Their definition of the $\mathcal{DSR}$ property only involves $M_{\mathcal{CLDSA}}$, while the new definition involves $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$. Hence, both definitions are seemingly different. However, we reveal that the new definition coincides with theirs by proving a theorem in this section. The theorem on the equivalence of two definitions is as follows:

**Theorem 1 (Equivalence of the Two Definitions)** *For a state machine* $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}},\, I_{\mathcal{UDS}},\, T_{\mathcal{UDS}}\rangle$, $(\forall s \in S_{\mathcal{CLDSA}})$ $(M_{\mathcal{CLDSA}} \models \mathit{terminated}(s) \Rightarrow$
$M_{\mathcal{UDS}} \models \mathit{isReachable}(s_*, s_1) \wedge M_{\mathcal{UDS}} \models \mathit{isReachable}(s_2, s_*))$
$\Leftrightarrow$
$(M_{\mathcal{CLDSA}} \models \mathit{terminated}(s) \Rightarrow M_{\mathcal{CLDSA}} \models$ *isReachable(base-state*$(s_*)$ *control*$(\mathit{ctl})$*,*
*base-state*$(s_1)$ *control*$(\mathit{ctl})$*))* $\wedge$ $M_{\mathcal{CLDSA}} \models$ *isReachable(base-state*$(s_2)$ *control*$(\mathit{ctl})$*,*
*base-state*$(s_*)$ *control*$(\mathit{ctl})$*))),*
*where* $s_1$ = *s-state(s),* $s_*$ = *snapshot(s) and* $s_2$ = *f-state(s).*

Only the difference between the new definition of the $\mathcal{DSR}$ property and the definition used in the existing study is the conclusion parts of the implications. If the conclusion parts are equivalent, then the two definitions are equivalent. The equivalence of the conclusion parts means that reachability is preserved between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$. The lemma on reachability preservation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ is as follows:

**Lemma 1 (Reachability Preservation)** *For a state machine* $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}},\, I_{\mathcal{UDS}},\, T_{\mathcal{UDS}}\rangle$, $(\forall s_1, s_2 \in S_{\mathcal{UDS}})$ $(M_{\mathcal{UDS}} \models \mathit{isReachable}(s_2, s_1) \Leftrightarrow M_{\mathcal{CLDSA}} \models$
*isReachable(base-state*$(s_2)$ *control*$(\mathit{ctl})$*, base-state*$(s_1)$ *control*$(\mathit{ctl})$*)).*

We first prove two more lemmas, which say that one-step reachability is preserved between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$, to prove Lemma 1. The two lemmas are as follows:

**Lemma 2 (One-step Reachability Preservation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$)** $\forall s_1, s_2 \in S_{\mathcal{UDS}}$ *such that* $s_1$ *goes to* $s_2$ *with one state transition step in* $M_{\mathcal{UDS}}$*, base-state*$(s_1)$ *control(ctl) goes to base-state*$(s_2)$ *control(ctl) with one state transition step in* $M_{\mathcal{CLDSA}}$.

**Lemma 3 (One-step Reachability Preservation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$)** $\forall s_1, s_2 \in S_{\mathcal{UDS}}$ *such that base-state*$(s_1)$ *control(ctl) goes to base-state*$(s_2)$ *control(ctl) with one state transition step in* $M_{\mathcal{CLDSA}}$*,* $s_1$ *goes to* $s_2$ *with one state transition step in* $M_{\mathcal{UDS}}$.

For each $\mathcal{UDS}$, $T_{\mathcal{UDS}}$ is constructed from the three transition rules and $T_{\mathcal{CLDSA}}$ is constructed from the 18 transition rules. Therefore, all we have to do is to take into account the three transition rules and the 18 transition rules to discuss $T_{\mathcal{UDS}}$ and $T_{\mathcal{CLDSA}}$, respectively. In the following proofs, $p, q \in$ Pid, $ps_1, ps_2 \in$ PState, $cs \in$ MsgQueue, $m \in$ Msg, $bc \in$ Config and $n \in$ Nat are fresh constants of those sorts.

## 6.2 Proof of Lemma 2.

Assume that $s_1$ goes to $s_2$ by a state transition $t$ in $M_{\mathcal{UDS}}$. Our proof shows that there exists a state transition $t'$ in $M_{\mathcal{CLDSA}}$ that moves base-state$(s_1)$ control$(\mathit{ctl})$ to base-state$(s_2)$ control$(\mathit{ctl})$. In our proof, we consider all possible state transition $t$ in $M_{\mathcal{UDS}}$ that can move $s_1$ to $s_2$. As we mentioned above, it suffices to take into account the three transition rules that describes Change of Process State, Sending of Message and Receipt of Message in $M_{\mathcal{UDS}}$.

**1. Change of Process State.** Let us consider the case in which $t$ is constructed from the transition rule that describes Change of Process State in $M_{\mathcal{UDS}}$.

(p-state[$P$] : $PS1$) $\Rightarrow$ (p-state[$P$] : $PS2$)

It suffices to consider $s_1$ as an arbitrary state (p-state[$p$]: $ps_1$) $bc$ in $S_{\mathcal{UDS}}$ to which the transition rule can be applied. Therefore, $s_2$ is (p-state[$p$]: $ps_2$) $bc$. Then, base-state($s_1$) control($ctl$) is base-state((p-state[$p$] : $ps_1$) $bc$) control($ctl$), and base-state($s_2$) control($ctl$) is base-state((p-state[$p$]: $ps_2$) $bc$) control($ctl$). The transition rule that describes Change of Process State in $M_{\mathcal{CLDSA}}$ can be applied to base-state($s_1$) control($ctl$) and obtains base-state($s_2$) control($ctl$).

base-state((p-state[$P$] : $PS1$) $BC$) $\Rightarrow$ base-state((p-state[$P$] : $PS2$) $BC$)

Hence, there exists $t'$. The case has been discharged.

**2. Sending of Message.** Let us consider the case in which $t$ is constructed from the transition rule that describes Sending of Message in $M_{\mathcal{UDS}}$.

(p-state[$P$] : $PS1$) (c-state[$P, Q, N$] : $CS$)

$\Rightarrow$

(p-state[$P$] : $PS2$) (c-state[$P, Q, N$] : enq($CS, M$))

It suffices to consider $s_1$ as an arbitrary state (p-state[$p$]: $ps_1$) (c-state[$p, q, n$]: $cs$) $bc$ in $S_{\mathcal{UDS}}$ to which the transition rule can be applied. Therefore, $s_2$ is (p-state[$p$]: $ps_2$) (c-state[$p, q, n$]: enq($cs, m$)) $bc$. Then, base-state($s_1$) control($ctl$) is base-state((p-state[$p$] : $ps_1$) (c-state[$p, q, n$]: $cs$) $bc$) control($ctl$), and base-state($s_2$) control($ctl$) is base-state((p-state[$p$]: $ps_2$) (c-state[$p, q, n$]: enq($cs, m$)) $bc$) control($ctl$). The transition rule that describes Sending of Message in $M_{\mathcal{CLDSA}}$ can be applied to base-state($s_1$) control($ctl$) and obtains base-state($s_2$) control($ctl$).

base-state((p-state[$P$] : $PS1$) (c-state[$P, Q, N$] : $MMS$) $BC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS2$) (c-state[$P, Q, N$] : enq($MMS, M$)) $BC$)

Hence, there exists $t'$. The case has been discharged.

**3. Receipt of Message.** Let us consider the case in which $t$ is constructed from the transition rule that describes Receipt of Message in $M_{\mathcal{UDS}}$.

(p-state[$P$] : $PS1$) (c-state[$Q, P, N$] : $M \mid CS$)

$\Rightarrow$

(p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $CS$)

It suffices to consider $s_1$ as an arbitrary state (p-state$[p]$: $ps_1$) (c-state$[q, p, n]$: $m \mid cs$) $bc$ in $S_{\mathcal{UDS}}$ to which the transition rule can be applied. Therefore, $s_2$ is (p-state$[p]$: $ps_2$) (c-state$[q, p, n]$: $cs$) $bc$. Then, base-state($s_1$) control($ctl$) is base-state((p-state$[p]$ : $ps_1$) (c-state$[p, q, n]$: $m \mid cs$) $bc$) control($ctl$), and base-state($s_2$) control($ctl$) is base-state((p-state$[p]$: $ps_2$) (c-state$[p, q, n]$: $cs$) $bc$) control($ctl$). The transition rule that describes Receipt of Message in $M_{\mathcal{CLDSA}}$ can be applied to base-state($s_1$) control($ctl$) and obtains base-state($s_2$) control($ctl$).

base-state((p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M \mid MMS$) $BC$)
control((prog$[P]$ : notYet) $CC$)
$\Rightarrow$
base-state((p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $MMS$) $BC$)
control((prog$[P]$ : notYet) $CC$)

Hence, there exists $t'$. The case has been discharged.

We have considerd all possible state transition $t$ in $M_{\mathcal{UDS}}$ that can move $s_1$ to $s_2$. All cases has been discharged. QED

## 6.3 Proof of Lemma 3.

Assume that base-state($s_1$) control($ctl$) goes to base-state($s_2$) control($ctl$) by a state transition $t$ in $M_{\mathcal{CLDSA}}$. Because $s_1 \in S_{\mathcal{UDS}}$, there is no marker in $s_1$. This is why any of the transition rules that describe Record of Process State and Receipt of Marker in $M_{\mathcal{CLDSA}}$ cannot be applied to base-state($s_1$) control($ctl$). Therefore, $t$ is not a state transition constructed from those transition rules. Any of the transition rules that describe the 2nd, 3rd and 4th sub-cases of Receipt of Message in $M_{\mathcal{CLDSA}}$ cannot be applied to base-state($s_1$) control($ctl$), either. Therefore, $t$ is not a state transition constructed from those transition rules, either. Then, all we have to do is to consider the transition rules that describe Change of Process State, Sending of Message and the 1st part of Receipt of Message in $M_{\mathcal{CLDSA}}$. The same proof strategy used in the proof of Lemma 2 can be used to show that there exists a state transition that moves $s_1$ to $s_2$ in $M_{\mathcal{UDS}}$ for each state transition that moves base-state($s_1$) control($ctl$) to base-state($s_2$) control($ctl$) in $M_{\mathcal{CLDSA}}$. The following proof considers all the possible transition rules as what we mentioned above.

**1. Change of Process State.** Let us consider the case in which $t$ is constructed from the transition rule that describes Change of Process State in $M_{\mathcal{CLDSA}}$.

base-state((p-state$[P]$ : $PS1$) $BC$) $\Rightarrow$ base-state((p-state$[P]$ : $PS2$) $BC$)

It suffices to consider base-state($s_1$) control($ctl$) as an arbitrary state base-state((p-state$[p]$

: $ps_1$) $bc$) control($ctl$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied. Therefore, base-state($s_2$) control($ctl$) is base-state((p-state[$p$]: $ps_2$) $bc$) control($ctl$). Then, $s_1$ is (p-state[$p$]: $ps_1$) $bc$ , and $s_2$ is (p-state[$p$]: $ps_2$) $bc$. The transition rule that describes Change of Process State in $M_{\mathcal{UDS}}$ can be applied to $s_1$ and obtains $s_2$.

(p-state[$P$] : $PS1$) $\Rightarrow$ (p-state[$P$] : $PS2$)

Hence, there exists $t'$. The case has been discharged.

**2. Sending of Message.** Let us consider the case in which $t$ is constructed from the transition rule that describes Sending of Message in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS1$) (c-state[$P, Q, N$] : $MMS$) $BC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS2$) (c-state[$P, Q, N$] : enq($MMS, M$)) $BC$)

It suffices to consider base-state($s_1$) control($ctl$) as an arbitrary state base-state((p-state[$p$] : $ps_1$) (c-state[$p, q, n$]: $cs$) $bc$) control($ctl$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied. Therefore, base-state($s_2$) control($ctl$) is base-state((p-state[$p$]: $ps_2$) (c-state[$p, q, n$]: enq($cs, m$)) $bc$) control($ctl$). Then, $s_1$ is (p-state[$p$]: $ps_1$) (c-state[$p, q, n$]: $cs$) $bc$, and $s_2$ is (p-state[$p$]: $ps_2$) (c-state[$p, q, n$]: enq($cs, m$)) $bc$. The transition rule that describes Sending of Message in $M_{\mathcal{UDS}}$ can be applied to $s_1$ and obtains $s_2$.

(p-state[$P$] : $PS1$) (c-state[$P, Q, N$] : $CS$)

$\Rightarrow$

(p-state[$P$] : $PS2$) (c-state[$P, Q, N$] : enq($CS, M$))

Hence, there exists $t'$. The case has been discharged.

**3. Receipt of Message.**
**a. The process has not yet started the CLDSA.** Let us consider the case in which $t$ is constructed from the transition rule that describes 1st case of Receipt of Message in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS1$) (c-state[$Q, P, N$] : $M$ | $MMS$) $BC$)
control((prog[$P$] : notYet) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS2$) (c-state[$Q, P, N$] : $MMS$) $BC$)
control((prog[$P$] : notYet) $CC$)

It suffices to consider base-state($s_1$) control($ctl$) as an arbitrary state base-state((p-state[$p$] : $ps_1$) (c-state[$p, q, n$]: $m$ | $cs$) $bc$) control($ctl$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied. Therefore, base-state($s_2$) control($ctl$) is base-state((p-state[$p$]: $ps_2$) (c-state[$p, q, n$]: $cs$) $bc$) control($ctl$). Note that because $ctl$ is (prog[$p_1$] : notYet) ... (prog[$p_n$] : notYet), $\mathcal{CLDSA}$ has not yet started. There is no marker in $cs$. Then, $s_1$ is (p-state[$p$]:

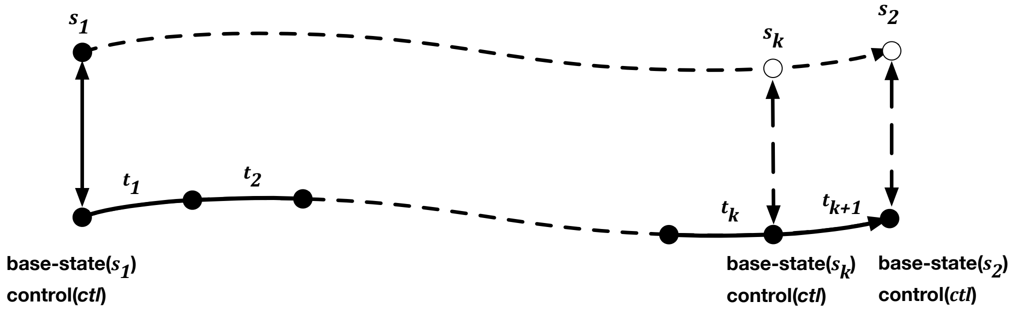

$s_1$
$s_k$
$s_2$
$t_1$
$t_2$
$t_k$
$t_{k+1}$
base-state($s_1$) control($ctl$)
base-state($s_k$) control($ctl$)
base-state($s_2$) control($ctl$)

Figure 6.1: The correspondence from the transitions in $M_{\mathcal{CLDSA}}$ to the transitions in $M_{\mathcal{UDS}}$.

$ps_1$) (c-state[$q,p,n$]: $m$ | $cs$) $bc$, and $s_2$ is (p-state[$p$]: $ps_2$) (c-state[$q,p,n$]: $cs$) $bc$. The transition rule that describes Receipt of Message in $M_{\mathcal{CLDSA}}$ can be applied to $s_1$ and obtains $s_2$.

(p-state[$P$] : $PS1$) (c-state[$Q,P,N$] : $M$ | $CS$)

$\Rightarrow$

(p-state[$P$] : $PS2$) (c-state[$Q,P,N$] : $CS$)

Hence, there exists $t'$. The case has been discharged.

We have considerd all possible state transition $t$ in $M_{\mathcal{UDS}}$ that can move base-state($s_1$) control($ctl$) to base-state($s_2$) control($ctl$). All cases has been discharged. QED

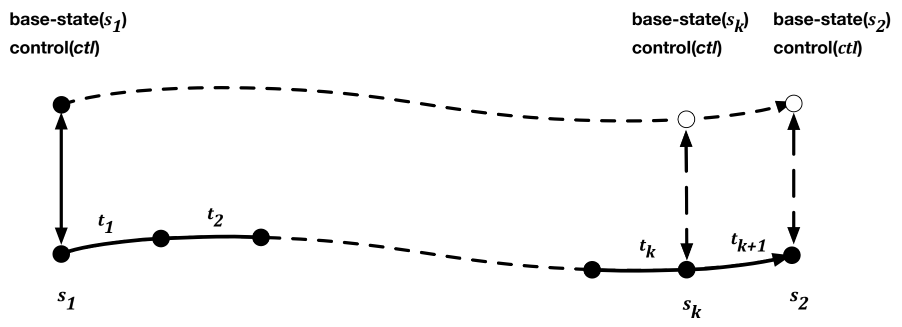
## 6.4 Proof of Lemma 1

### 6.4.1 Proof of the "if" part of Lemma 1

We prove that $\forall s_1, s_2 \in S_{\mathcal{UDS}}$, if $M_{\mathcal{CLDSA}} \models$ isReachable(base-state($s_2$) control($ctl$), base-state($s_1$) control($ctl$)), then $M_{\mathcal{UDS}} \models$ isReachable($s_2, s_1$).

Assume that $M_{\mathcal{CLDSA}} \models$ isReachable(base-state($s_2$) control($ctl$), base-state($s_1$) control($ctl$)) and then there must be a natural number $k$ such that base-state($s_1$) control($ctl$) goes to base-state($s_2$) control($ctl$) by $k$ state transition steps in $M_{\mathcal{CLDSA}}$. The proof is done by induction on $k$.

- Base case: since base-state($s_1$) control($ctl$) is the same as base-state($s_2$) control($ctl$) in this case, $s_1$ is the same as $s_2$. So, this case is discharged.
- Induction case: suppose that base-state($s_1$) control($ctl$) moves to base-state($s_2$) control($ctl$) by $k$+1 transition steps and the $k$+1 transitions taken are $t_1$, ..., $t_{k+1}$. As shown in Fig. 3, base-state($s_k$) control(ctl) is the state to which base-state($s_1$) control($ctl$) moves by the first $k$ transition steps, namely that $M_{\mathcal{CLDSA}} \models$ isReachable(base-state($s_k$) control($ctl$), base-state($s_1$) control($ctl$)). From the induction hypothesis, $M_{\mathcal{UDS}} \models$ isReachable($s_k, s_1$).



base-state($s_1$) control($ctl$)
base-state($s_k$) control($ctl$)
base-state($s_2$) control($ctl$)
$t_1$
$t_2$
$t_k$
$t_{k+1}$
$s_1$
$s_k$
$s_2$

Figure 6.2: The correspondence from the transitions in $M_{\mathcal{UDS}}$ to the transitions in $M_{\mathcal{CLDSA}}$.

Since base-state($s_k$) control($ctl$) moves to base-state($s_2$) control($ctl$) by one transition step in $M_{\mathcal{CLDSA}}$, $s_k$ also moves to $s_2$ by one transition step in $M_{\mathcal{UDS}}$ from Lemma 3. Then, this case is also discharged. Fig. 6.1 shows the correspondence from the transitions in $M_{\mathcal{CLDSA}}$ totransitions in $M_{\mathcal{UDS}}$. QED

### 6.4.2 Proof of the "only if" part of Lemma 1

We prove that $\forall\ s_1,\ s_2 \in S_{\mathcal{UDS}}$, if $M_{\mathcal{UDS}} \models$ isReachable($s_2, s_1$), then ($M_{\mathcal{CLDSA}} \models$ isReachable(base-state($s_2$) control($ctl$), base-state($s_1$) control($ctl$)).
Assume that $M_{\mathcal{UDS}} \models$ isReachable($s_2, s_1$) and then there must be a natural number $k$ such that $s_1$ goes to $s_2$ by $k$ state transition steps in $M_{\mathcal{UDS}}$. The proof is done by induction on $k$.
- Base case: since $s_1$ is the same as $s_2$ in this case, base-state($s_1$) control($ctl$) is the same as base-state($s_2$) control($ctl$). So, this case is discharged.
- Induction case: suppose that $s_1$ moves to $s_2$ by $k$+1 transition steps and the $k+1$ transitions taken are $t_1$, ..., $t_{k+1}$. As shown in Fig. 4, $s_k$ is the state to which $s_1$ moves by the first $k$ transition steps, namely that $M_{\mathcal{UDS}} \models$ isReachable($s_k$, $s_1$). From the induction hypothesis, $M_{\mathcal{CLDSA}} \models$ isReachable(base-state($s_k$) control($ctl$), base-state($s_2$) control($ctl$)). Since $s_k$ moves to $s_2$ by one transition step in $M_{\mathcal{CLDSA}}$, base-state($s_k$) control($ctl$) also moves to base-state($s_2$) control($ctl$) by one transition step in $M_{\mathcal{CLDSA}}$ from Lemma 2. Then, this case is also discharged. Fig. 6.2 shows the correspondence from the transitions in $M_{\mathcal{UDS}}$ to transitions in $M_{\mathcal{CLDSA}}$. QED

## 6.5 Proof of Theorem 1

Since Proposition 1 says that whenever $\mathcal{CLDSA}$ terminates in state $s$, there is no marker in the start state, the snapshot and the finish state, and Lemma 1 asserts that reachability is preserved between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$. Proof of Theorem 1 follows from Proposition 1 and Lemma 1. QED

## 6.6 Chapter Summary

This chapter gave the theorem on equivalence of the two definitions, the more faithful formal definition and the definition in the existing study, and some lemmas. Also, the proofs of them were given.

The chapter first represented the definition of $\mathcal{DSR}$ property in the existing study, which is encoded in terms of the Maude search command in the existing study. Then the analysis of the similarities between the new definition of the $\mathcal{DSR}$ property and the existing definition was conducted.

The chapter then gave Theorem 1 saying that new definition is equivalent to the definition of the $\mathcal{DSR}$ property in the existing study. To prove the Theorem 1, it the proved Proposition 1 and Lemma 1. Lemma 1 asserts that reachability is preserved between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$. Proposition 1 says that whenever $\mathcal{CLDSA}$ terminates in state $s$, there is no marker in the start state, the snapshot and the finish state. We prove as Lemma 2 and Lemma 3 that one-step reachability is preserved between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ to prove Lemma 1.

At the end of the chapter, The proof of Theorem 1 was given following from Proposition 1 and Lemma 1.

# Chapter 7

# $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$

## 7.1 A Binary Relation Between Two State Machines $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$

Since $\mathcal{CLDSA}$ works by using a special message called marker, $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$ means that excepting for putting markers in a $\mathcal{UDS}$, the algorithm does not change the state of all processes and channels. We propose a binary relation **r** between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ saying that for each $s1 \in S_{\mathcal{UDS}}$ and each $s2 \in S_{\mathcal{CLDSA}}$, r$(s1, s2)$ if and only if $s1$ is the same as the state obtained by deleting all markers from $s2$.

**Definition 12 (Binary relation r)** *Given two state machines* $M_{\mathcal{UDS}} \triangleq \langle S_{\mathcal{UDS}}, I_{\mathcal{UDS}}, T_{\mathcal{UDS}}\rangle$ *and* $M_{\mathcal{CLDSA}} \triangleq \langle S_{\mathcal{CLDSA}}, I_{\mathcal{CLDSA}}, T_{\mathcal{CLDSA}}\rangle$, $\forall\ s_{\mathcal{UDS}} \in S_{\mathcal{UDS}}$ *and* $\forall\ s_{\mathcal{CLDSA}} \in S_{\mathcal{CLDSA}}$*: A binary relation between* $M_{\mathcal{UDS}}$ *and* $M_{\mathcal{CLDSA}}$ $\boldsymbol{r} : S_{\mathcal{UDS}}\ S_{\mathcal{CLDSA}} \rightarrow\ Bool$ *.*

$r(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}}) = (s_{\mathcal{UDS}} = delM(\ b\text{-}state(s_{\mathcal{CLDSA}})))$

**Note that:** For $\forall\ s_{\mathcal{UDS}} \in S_{\mathcal{UDS}}$, $s_{\mathcal{UDS}} =$ delM$(s_{\mathcal{UDS}})$ because there is no marker in $s_{\mathcal{UDS}}$.

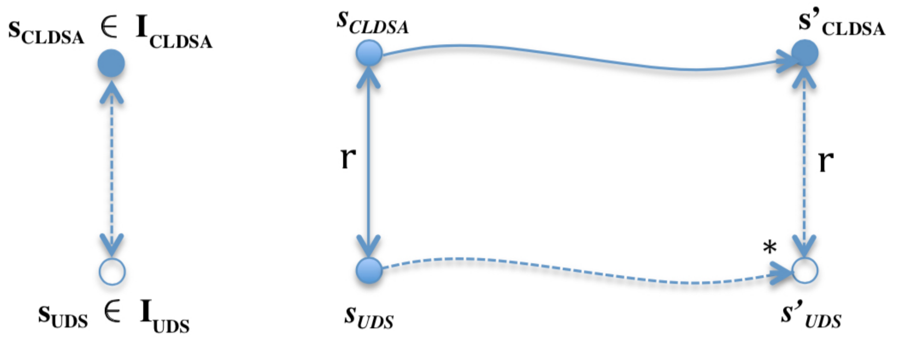


Figure 7.1: The binary relation **r** is a simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$.

## 7.2 Theorem 2 and the Proof of It

To guarantee that $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$, we prove that $M_{\mathcal{CLDSA}}$ simulates $M_{\mathcal{UDS}}$ and vice versa. To prove that we prove that exists a bi-simulation relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$. Therefore, we prove Theorem 2 saying that **r** is a bi-simulation relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$.

**Theorem 2 (Bi-simulation relation r)** *Binary relation **r** is a bi-simulation relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$.*

To prove Theorem 2, we shall prove that **r** is a simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$ and it is also a simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$.

Since, binary relation **r** is considered as a simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$ ($M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$). For two state machines, we only consider states what can be reached from any initial states, which has specific configuration and the configuration is never changed by any state transition. This is used to assume the configuration of states of $M_{\mathcal{CLDSA}}$ and states of $M_{\mathcal{UDS}}$ in the following proof.

### 7.2.1 Simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$.

To prove that **r** is a simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$, We will prove that r satisfies the following two conditions. Fig. 7.1 shows the diagrams corresponding to the two conditions.

**1. For each $s_{\mathcal{CLDSA}} \in I_{\mathcal{CLDSA}}$ there exists $s_{\mathcal{UDS}} \in I_{\mathcal{UDS}}$, such that r($s_{\mathcal{UDS}}$, $s_{\mathcal{CLDSA}}$).**

**Prove:**

For each $s_{\mathcal{CLDSA}} \in CL_{Init}(I_{\mathcal{UDS}})$, from the definition of $CL$, we have:

$s_{\mathcal{CLDSA}}$= base-state($bc$) start-state(empConfig) snapshot(empConfig) finsh-state(empConfig) control(ctl), where $bc \in I_{\mathcal{UDS}}$, ctl = InitCtlConfig(bc).

We have b-state($s_{\mathcal{CLDSA}}$) = $bc$. Let $s_{\mathcal{UDS}}$= $bc \in I_{\mathcal{UDS}}$. Since, $s_{\mathcal{UDS}} = bc$ and delM(b-state($s_{\mathcal{CLDSA}}$)) = delM($s_{\mathcal{UDS}}$) = $bc$, $s_{\mathcal{UDS}}$= delM(b-state($s_{\mathcal{CLDSA}}$) ). Then **r**($s_{\mathcal{UDS}}$, $s_{\mathcal{CLDSA}}$).

Therefore, for each $s_{\mathcal{CLDSA}} \in I_{\mathcal{CLDSA}}$ there exists $s_{\mathcal{UDS}} \in I_{\mathcal{UDS}}$, such that **r**($s_{\mathcal{UDS}}$, $s_{\mathcal{CLDSA}}$). This condition is satisfied.

**2. For each $s_{\mathcal{CLDSA}}$, $s'_{\mathcal{CLDSA}} \in S_{\mathcal{CLDSA}}$ and $s_{\mathcal{UDS}} \in S_{\mathcal{UDS}}$ such that r($s_{\mathcal{UDS}}$, $s_{\mathcal{CLDSA}}$) and $s_{\mathcal{CLDSA}} \rightsquigarrow_{M_{\mathcal{CLDSA}}} s'_{\mathcal{CLDSA}}$ by one state transition of the $M_{\mathcal{CLDSA}}$, there exists $s'_{\mathcal{UDS}}$ such that r($s'_{\mathcal{UDS}}$, $s'_{\mathcal{CLDSA}}$) and $s_{\mathcal{UDS}} \rightsquigarrow^{*}_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by zero or more state transitions of the $M_{\mathcal{UDS}}$.**

**Prove:**

In the following proofs, $p, q \in$ Pid, $ps_1, ps_2 \in$ PState, $cs \in$ MMsgQueue, $m \in$ Msg, $bc, sc, ssc, fc \in$ BConfig, $ctl \in$ CtlConfig and $n \in$ Nat are fresh constants of those sorts. As we said before, for two state machines, we only consider states what can be reached from any initial states. The configuration of a state of $M_{\mathcal{CLDSA}}$ is as following:

$s_{\mathcal{CLDSA}}$= base-state(bc) start-state(sc) snapshot(ssc) finish-state(fc) control(ctl), where $bc, sc, ssc, fc \in$ BConfig and $ctl \in$ CtlConfig.

Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$= delM(b-state($s_{\mathcal{CLDSA}}$)) = delM(bc).

Assume that $s_{\mathcal{CLDSA}} \rightsquigarrow_{M_{\mathcal{CLDSA}}} s'_{\mathcal{CLDSA}}$ by state transition $t$.

The same as what we have mentioned before. For each $\mathcal{UDS}$, $T_{\mathcal{UDS}}$ is constructed from the three transition rules and $T_{\mathcal{CLDSA}}$ is constructed from the 18 transition rules. Therefore, all we have to do is to take into account the three transition rules and the 18 transition rules to discuss $T_{\mathcal{UDS}}$ and $T_{\mathcal{CLDSA}}$, respectively. Since, the 18 transition rules can be classified into three parts: $\mathcal{UDS}$, $\mathcal{UDS}$&$\mathcal{CLDSA}$, and $\mathcal{CLDSA}$. The $\mathcal{UDS}$ part consists of the transition rules describing actions i, ii and iii-1. Three transition rules describing actions iii-2, iii-3 and iii-4 are in the $\mathcal{UDS}$&$\mathcal{CLDSA}$ part. The transition rules describing two kinds of actions iv and v are in the $\mathcal{CLDSA}$ part. No loss of generality, in our proof, state transition $t$ is separated into three parts: $\mathcal{UDS}$, $\mathcal{UDS}$&$\mathcal{CLDSA}$, and $\mathcal{CLDSA}$. In the following proofs, $p, q \in$ Pid, $ps_1, ps_2 \in$ PState, $cs \in$ MsgQueue, $m \in$ Msg, $bc, sc, ssc \in$ BConfig, $ctl \in$ CtlConfig and $n \in$ Nat are fresh constants of those sorts. To prove that this condition is satisfied, we will separate the proof in two parts depended on the transition rules.

**The $\mathcal{UDS}$ and $\mathcal{UDS}$&$\mathcal{CLDSA}$ part: the transition $t$ constructed from the transition rules in the $\mathcal{UDS}$ part and the $\mathcal{UDS}$&$\mathcal{CLDSA}$ part.**

Since, the $\mathcal{UDS}$ part depends on the $\mathcal{UDS}$ concerned, can be constructed from the three transition rules of the $\mathcal{UDS}$ and changes the base-state meta configuration component of a state of $M_{\mathcal{CLDSA}}$. The $\mathcal{UDS}$&$\mathcal{CLDSA}$ part also depends on the $\mathcal{UDS}$ concerned and can be constructed from the three transition rules of the $\mathcal{UDS}$, but changes the other meta configuration components of a state of $CL(M_{\mathcal{UDS}})$ as well. For any state transition $t$ in $\mathcal{UDS}$ and $\mathcal{UDS}$&$\mathcal{CLDSA}$ part that moves state $s_{\mathcal{CLDSA}}$ to state $s'_{\mathcal{CLDSA}}$ in $M_{\mathcal{CLDSA}}$. We can find state transition $t'$ in $M_{\mathcal{UDS}}$ that can moves state $s_{\mathcal{UDS}}$ =delM(b-state($s_{\mathcal{CLDSA}}$)) to state $s'_{\mathcal{UDS}}$ = delM(b-state($s'_{\mathcal{CLDSA}}$)) in $M_{\mathcal{UDS}}$. This will be proven by the following. Existing $t'$ in $M_{\mathcal{UDS}}$ corresponding $t$ is shown in Fig .7.2.

1. **Change of Process State.** Let us consider the case in which $t$ is constructed from the transition rule that describes Change of Process State in $M_{\mathcal{CLDSA}}$.

   base-state((p-state[$P$] : $PS1$) $BC$)

   $\Rightarrow$

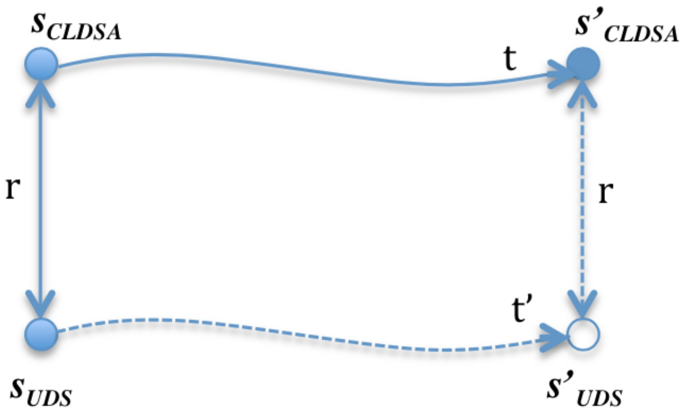


Figure 7.2: Existing $t'$ in $M_{\mathcal{UDS}}$ corresponding $t$.

base-state((p-state$[P]$ : $PS2$) $BC$)

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state$[p]$ : $ps1$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.
Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state$[p]$ : $ps1$) $bc$) = (p-state$[p]$ : $ps1$) delM($bc$) from definition of function delM. $s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, then $s'_{\mathcal{CLDSA}}$ is base-state((p-state$[p]$ : $ps2$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$).
Let us choose the state transition $t'$ is constructed from the transition rule that describes Change of Process State in $M_{\mathcal{UDS}}$.

(p-state$[P]$ : $PS1$) $\Rightarrow$ (p-state$[P]$ : $PS2$)

Let $s'_{\mathcal{UDS}}$ be (p-state$[p]$ : $ps2$) delM($bc$). Then $s_{\mathcal{UDS}}$ can goes to $s'_{\mathcal{UDS}}$ by state transition $t'$.

We have:
$s'_{\mathcal{UDS}}$= (p-state$[p]$ : $ps2$) delM($bc$) (*) and,
delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state$[p]$ : $ps2$) $bc$) = (p-state$[p]$ : $ps2$) delM($bc$) (**),
From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$ and $s_{\mathcal{UDS}} \leadsto_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by state transition $t'$.

2. **Sending of Message.** Let us consider the case in which $t$ is constructed from the transition rule that describes Sending of Message in $M_{\mathcal{CLDSA}}$.

base-state((p-state$[P]$ : $PS1$) (c-state$[P, Q, N]$ : $MMS$)

$BC)$

$\Rightarrow$

base-state((p-state$[P]$ : $PS2$) (c-state$[P, Q, N]$ : enq$(MMS, M)$)

$BC)$

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state$[p]$ : $ps1$) (c-state$[p, q, n]$ : $mms$) $bc$) start-state$(sc)$ snapshot$(ssc)$ finish-state$(fc)$ control$(ctl)$ in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.
Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}} =$ delM((p-state$[p]$ : $ps1$) (c-state$[p, q, n]$ : $mms$) $bc$) = (p-state$[p]$ : $ps1$) (c-state$[p, q, n]$ : delChan$(mms)$) delM$(bc)$ from definition of function delM.
$s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition t, then $s'_{\mathcal{CLDSA}}$ is base-state((p-state$[p]$: $ps2$) (c-state$[p, q, n]$ : enq$(mms, m)$) $bc$)) start-state$(sc)$ snapshot$(ssc)$ finish-state$(fc)$ control$(ctl)$.
Let us choose the state transition $t'$ is constructed from the transition rule that describes Sending of Message in $M_{\mathcal{UDS}}$.

(p-state$[P]$ : $PS1$) (c-state$[P, Q, N]$ : $CS$)

$\Rightarrow$

(p-state$[P]$ : $PS2$) (c-state$[P, Q, N]$ : enq$(CS, M)$)

Let $s'_{\mathcal{UDS}}$ be (p-state$[p]$ : $ps2$) (c-state$[p, q, n]$ : enq(delChan$(mms)$, $m$)) delM$(bc)$.
Then $s_{\mathcal{UDS}}$ can goes to $s'_{\mathcal{UDS}}$ by state transition $t'$.

We have:
$s'_{\mathcal{UDS}}$= (p-state$[p]$ : $ps2$) (c-state$[p, q, n]$ : enq(delChan$(mms)$, $m$)) delM$(bc)$ (*) and, because $m$ is a message, delChan(enq$(mms, m)$) = enq(delChan$(mms)$, $m$).
Then, delM(base-sate($s'_{\mathcal{CLDSA}}$)) = (p-state$[p]$ : $ps2$)
(c-state$[p, q, n]$ : delChan(enq$(mms, m)$)) delM$(bc)$) = (p-state$[p]$ : $ps2$) (c-state$[p, q, n]$ : enq(delChan$(mms)$,$m$)) delM$(bc)$) (**).

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) .

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$ and $s_{\mathcal{UDS}} \rightsquigarrow_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by state transition $t'$.

3. **Receipt of Message:** can be separated into four subcases

   (a) **The process has not yet started the $\mathcal{CLDSA}$.** Let us consider the case in which $t$ is constructed from the transition rule that describes 1st case of Receipt of Message in $M_{\mathcal{CLDSA}}$.

   base-state((p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M \mid MMS$) $BC$)
   control((prog$[P]$ : notYet) $CC$)

$\Rightarrow$

base-state((p-state$[P]$ : $PS2$) (c-state$[Q,P,N]$ : $MMS$) $BC$)

control((prog$[P]$ : notYet) $CC$)

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state$[p]$ : $ps1$) (c-state$[q,p,n]$ : $m$ | $mms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog$[p]$ : notYet) $cc$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.
Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM(b-state($s_{\mathcal{CLDSA}}$)) = delM((p-state$[p]$ : $ps1$) (c-state$[q,p,n]$ : $m$ | $mms$) $bc$)) = (p-state$[p]$ : $ps1$) (c-state$[q,p,n]$ : delChan($m$ | $mms$)) delM($bc$) = (p-state$[p]$ : $ps1$) (c-state$[q,p,n]$ : $m$ | delChan($mms$)) delM($bc$) from definition of function delM.
$s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, then $s'_{\mathcal{CLDSA}}$ is base-state((p-state$[p]$ : $ps2$) (c-state$[q,p,n]$ : $mms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog$[p]$ : notYet) $cc$).
Let us choose the state transition $t'$ is constructed from the transition rule that describes Receipt of Message in $M_{\mathcal{UDS}}$.

(p-state$[P]$ : $PS1$) (c-state$[Q,P,N]$ : $M$ | $CS$)

$\Rightarrow$

(p-state$[P]$ : $PS2$) (c-state$[Q,P,N]$ : $CS$)

Let $s'_{\mathcal{UDS}}$ is (p-state$[p]$ : $ps2$) (c-state$[q,p,n]$ : delChan($mms$)) delM($bc$). Then $s_{\mathcal{UDS}}$ can goes to $s'_{\mathcal{UDS}}$ by state transition $t'$.

We have:
$s'_{\mathcal{UDS}}$ is (p-state$[p]$ : $ps2$) (c-state$[q,p,n]$ : delChan($mms$)) delM($bc$) (*) and,

delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state$[p]$ : $ps2$) (c-state$[q,p,n]$ : $mms$) $bc$))= (p-state$[p]$ : $ps1$) (c-state$[q,p,n]$ : delChan($mms$)) delM($bc$) from definition of function delM(**),

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) .

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$ and $s_{\mathcal{UDS}} \rightsquigarrow_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by state transition $t'$.

(b) **The process has completed $\mathcal{CLDSA}$.** Let us consider the case in which $t$ is constructed from the transition rule that describes 2st case of Receipt of Message in $M_{\mathcal{CLDSA}}$.

base-state((p-state$[P]$ : $PS1$) (c-state$[Q,P,N]$ : $M$ | $MMS$) $BC$)

control((prog$[P]$ : completed) $CC$)

$\Rightarrow$

base-state((p-state$[P]$ : $PS2$) (c-state$[Q,P,N]$ : $MMS$) $BC$)

control((prog[$P$] : completed) $CC$)

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$] : $ps1$) (c-state[$q,p,n$] : $m$ | $mms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog[$p$] : completed) $cc$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.
Because of $\mathbf{r}$($s_{\mathcal{UDS}}$, $s_{\mathcal{CLDSA}}$), $s_{\mathcal{UDS}}$ is delM(b-state($s_{\mathcal{CLDSA}}$)) = delM((p-state[$p$] : $ps1$) (c-state[$q,p,n$] : $m$ | $mms$) $bc$)) = (p-state[$p$] : $ps1$) (c-state[$q,p,n$] : delChan($m$ | $mms$)) delM(bc) = (p-state[$p$] : $ps1$) (c-state[$q,p,n$] : $m$ | delChan($mms$)) delM($bc$) from definition of function delM.
$s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition t, then $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps2$ (c-state[$q,p,n$] : $mms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog[$p$] : (prog[$p$] : completed)) $cc$).
Let us choose the state transition $t'$ is constructed from the transition rule that describes Receipt of Message in $M_{\mathcal{UDS}}$.

> (p-state[$P$] : $PS1$) (c-state[$Q,P,N$] : $M$ | $CS$)
> $\Rightarrow$
> (p-state[$P$] : $PS2$) (c-state[$Q,P,N$] : $CS$)

Let $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps2$) (c-state[$q,p,n$] : delChan($mms$)) delM($bc$). Then $s_{\mathcal{UDS}}$ can goes to $s'_{\mathcal{UDS}}$ by state transition $t'$.

We have:
$s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps2$) (c-state[$q,p,n$] : delChan($mms$)) delM($bc$) (*) and,

delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$] : $ps2$) (c-state[$q,p,n$] : $mms$) $bc$))= (p-state[$p$] : $ps1$) (c-state[$q,p,n$] : delChan($mms$)) delM($bc$) from definition of function delM(**),

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) .

Therefore, $\mathbf{r}$($s'_{\mathcal{UDS}}$, $s'_{\mathcal{CLDSA}}$) and $s_{\mathcal{UDS}} \rightsquigarrow_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by state transition $t'$.

(c) **The process has started $\mathcal{CLDSA}$, not yet completed it, and has not yet received a marker from the incoming channel.** Let us consider the case in which $t$ is constructed from the transition rule that describes 3 st case of Receipt of Message in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS1$) (c-state[$Q,P,N$] : $M$ | $MMS$) $BC$)
snapshot((c-state[$Q,P,N$] : $MMS'$) $SSC$)
control((prog[$P$] : started) (done[$Q,P,N$] : false) $CC$)
$\Rightarrow$

base-state((p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $MMS$) $BC$)
snapshot((c-state$[Q, P, N]$ : enq($MMS'$, $M$)) $SSC$)
control((prog$[P]$ : started) (done$[Q, P, N]$ : false) $CC$)

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state$[p]$ : $ps1$) (c-state$[q, p, n]$ : $m$ | $mms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog$[p]$ : started) (done$[q, p, n]$ : false) $cc$). in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.
Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM(b-state($s_{\mathcal{CLDSA}}$)) = delM((p-state$[p]$ : $ps1$) (c-state$[q, p, n]$ : $m$ | $mms$) $bc$)) = (p-state$[p]$ : $ps1$) (c-state$[q, p, n]$ : delChan($m$ | $mms$)) delM($bc$) = (p-state$[p]$ : $ps1$) (c-state$[q, p, n]$ : $m$ | delChan($mms$)) delM($bc$) from definition of function delM.
$s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition t, then $s'_{\mathcal{CLDSA}}$ is base-state((p-state$[p]$ : $ps2$) (c-state$[q, p, n]$ : $mms$) $bc$) start-state($sc$) snapshot((c-state$[q, p, n]$ : enq($ms$, $m$)) $ssc$) finish-state($fc$) control((prog$[p]$ : started) (done$[q, p, n]$ : false) $cc$).
Let us choose the state transition $t'$ is constructed from the transition rule that describes Receipt of Message in $M_{\mathcal{UDS}}$.

(p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M$ | $CS$)
$\Rightarrow$
(p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $CS$)

Let $s'_{\mathcal{UDS}}$ is (p-state[P] : ps2) (c-state[q, p, n] : delChan(mms)) delM(bc). Then $s_{\mathcal{UDS}}$ can goes to $s'_{\mathcal{UDS}}$ by state transition $t'$.

We have:
$s'_{\mathcal{UDS}}$ is (p-state[p] : ps2) (c-state[q, p, n] : delChan(mms)) delM(bc) (*) and,

delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[p] : ps2) (c-state[q, p, n] : mms) bc))= (p-state[p] : ps1) (c-state[q, p, n] : delChan(mms)) delM(bc) from definition of function delM(**),

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) .

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$ and $s_{\mathcal{UDS}} \rightsquigarrow_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by state transition $t'$.

(d) **The process has started $\mathcal{CLDSA}$, not yet completed it, and has already received a marker from the incoming channel.** Let us consider the case in which $t$ is constructed from the transition rule that describes 4 st case of Receipt of Message in $M_{\mathcal{CLDSA}}$.

base-state((p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M$ | $MMS$) $BC$)

control((prog$[P]$ : started) (done$[Q, P, N]$ : true) $CC$)

$\Rightarrow$

base-state((p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $MMS$) $BC$)

control((prog$[P]$ : started) (done$[Q, P, N]$ : true) $CC$)

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state$[p]$ : $ps1$) (c-state$[q, p, n]$ : $m \mid mms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog$[p]$ : started) (done$[q, p, n]$ : true) $cc$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.
Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM(b-state($s_{\mathcal{CLDSA}}$)) = delM((p-state$[p]$ : $ps1$) (c-state$[q, p, n]$ : $m \mid mms$) $bc$)) = (p-state$[p]$ : $ps1$) (c-state$[q, p, n]$ : delChan($m \mid mms$)) delM($bc$) = (p-state$[p]$ : $ps1$) (c-state$[q, p, n]$ : $m \mid$ delChan($mms$)) delM($bc$) from definition of function delM.
$s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition t, then $s'_{\mathcal{CLDSA}}$ is base-state((p-state$[p]$ : $ps2$) (c-state$[q, p, n]$ : $mms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog$[p]$ : started) (done$[q, p, n]$ : true) $cc$).
Let us choose the state transition $t'$ is constructed from the transition rule that describes Receipt of Message in $M_{\mathcal{UDS}}$.

(p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M \mid CS$)

$\Rightarrow$

(p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $CS$)

Let $s'_{\mathcal{UDS}}$ is (p-state$[p]$ : $ps2$) (c-state$[q, p, n]$ : delChan($mms$)) delM($bc$). Then $s_{\mathcal{UDS}}$ can goes to $s'_{\mathcal{UDS}}$ by state transition $t'$.

We have:
$s'_{\mathcal{UDS}}$ is (p-state$[p]$ : $ps2$) (c-state$[q, p, n]$ : delChan($mms$)) delM($bc$) (*) and,

delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state$[p]$ : $ps2$) (c-state$[q, p, n]$ : $mms$) $bc$))= (p-state$[p]$ : $ps1$) (c-state$[q, p, n]$ : delChan($mms$)) delM($bc$) from definition of function delM(**),

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$ and $s_{\mathcal{UDS}} \rightsquigarrow_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by state transition $t'$.

**The $\mathcal{CLDSA}$part: state transition $t$ is constructed from the transition rule in the $\mathcal{CLDSA}$ part**

Since, the $\mathcal{CLDSA}$ part is independent from the $\mathcal{UDS}$ concerned, can be constructed regardless of any $\mathcal{UDS}$s, and does not change the base-state meta configuration component of a state of $CL(M_{\mathcal{UDS}})$. Our proof show that we can choose $s'_{\mathcal{UDS}}$ the same as $s_{\mathcal{UDS}}$ then $s_{\mathcal{UDS}}$ goes to $s'_{\mathcal{UDS}}$ by zero step and $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$. This is shown in Fig. 7.3. The following prove will consider transition rules in $\mathcal{CLDSA}$ part.

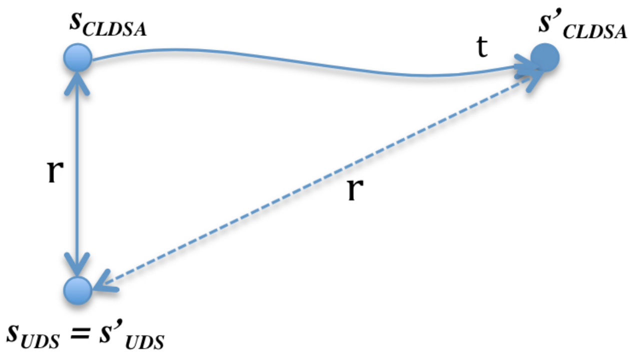


Figure 7.3: $s'_{\mathcal{UDS}}$ is the same as $s_{\mathcal{UDS}}$ when state transition $t$ in the $\mathcal{CLDSA}$ part.

- **Record of Process State** is split into two subcases:

1. **The process globally initiates $\mathcal{CLDSA}$.** This case is further split into three subcases:

   (a) The $\mathcal{UDS}$ only consists of the process.

   Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Record of Process State in $M_{\mathcal{CLDSA}}$.

   base-state((p-state[$P$] : $PS$))
   start-state(empBConfig)
   snapshot(empBConfig)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 0) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$))
   start-state((p-state[$P$] : $PS$))
   snapshot((p-state[$P$] : $PS$))
   finish-state((p-state[$P$] : $PS$))
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) $CC$)

   It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$)) start-state(empConfig) snapshot(empConfig) finish-state(empConfig) control((prog[$p$] : notYet) (cnt : 1) (#ms[$p$] : 0) $cc$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

   Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM(p-state[$p$] : $ps$) = (p-state[$p$] : $ps$) from definition of function delM.

$s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, then $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$)) start-state((p-state[$p$] : $ps$)) snapshot((p-state[$p$] : $ps$)) finish-state((p-state[$p$] : $ps$)) control((prog[$p$] : completed) (cnt : 0) (#ms[$p$] : 0) $cc$) .

Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$) (*).

We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM(p-state[$p$] : $ps$) = (p-state[$p$] : $ps$) (**).

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$

(b) The system consists of more than one process, and the process does not have any incoming channels.

Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Record of Process State in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS$) $BC$)
start-state(empBConfig)
snapshot(empBConfig)
finish-state(empBConfig)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : 0) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state((p-state[$P$] : $PS$) $BC$)
snapshot((p-state[$P$] : $PS$))
control((prog[$P$] : completed) (cnt : sd($NzN$,1)) (#ms[$P$] : 0) $CC$)
if $NzN > 1$

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$) $bc$) start-state(empConfig) snapshot(empConfig) finish-state($fc$) control((prog[$p$] : notYet) (cnt : $nzn'$) (#ms[$p$] : 0) $cc$) such that $nzn' > 1$ in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[$p$] : $ps$) $bc$) = (p-state[$p$] : $ps$) delM($bc$) from definition of function delM

$s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, then $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) bcast($bc, p$, marker)) start-state((p-state[$p$] : $ps$)) snapshot((p-state[$p$] : $ps$)) finish-state($fc$) control((prog[$p$] : completed) (cnt : 0) (#ms[$p$] : 0) $cc$).

Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$) delM($bc$) (*).

We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$] : $ps$) **bcast**($bc, p$, marker)) = (p-state[$p$] : $ps$) delM(**bcast**($bc, p$, marker)) from definition of function delM.

Function **bcast**($bc, p$, marker)) to puts markers in all the outgoing channels from a process p, then delM(**bcast**($bc, p$, marker))) = delM($bc$).

Then $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) delM($bc$) (**).

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) delM($bc$).

Therefore, $\mathbf{r}$($s'_{\mathcal{UDS}}$, $s'_{\mathcal{CLDSA}}$)

(c) The system consists of more than one process, and the process has one or more incoming channels.

Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Record of Process State in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS$) $BC$)
start-state(empBConfig)
snapshot(empBConfig)
finish-state(empBConfig)
control((prog[$P$] : notYet) (#ms[$P$] : $NzN'$) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state((p-state[$P$] : $PS$) $BC$)
snapshot((p-state[$P$] : $PS$) inchans($BC$, $P$))
control((prog[$P$] : started) (#ms[$P$] : $NzN'$) $CC$)

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$) $bc$) start-state(empConfig) snapshot(empConfig) finish-state($fc$) control((prog[$p$] : notYet) ( #ms[$p$] : $nzn$) $cc$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

Because of $\mathbf{r}$($s_{\mathcal{UDS}}$, $s_{\mathcal{CLDSA}}$), $s_{\mathcal{UDS}}$ is delM((p-state[$p$] : $ps$) $bc$) = (p-state[$p$] : $ps$) delM($bc$) from definition of function delM.

Since $s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) bcast($bc, p$, marker)) start-state((p-state[$p$] : $ps$) $bc$) snapshot((p-state[$p$] : $ps$) inchans($bc, p$)) finish-state($fc$) control((prog[$p$] : started) (cnt : 0) (#ms[$p$] : 0) $cc$).

Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[p] : ps) delM(bc) (*).

Function **bcast**($bc, p$, marker)) to put markers in all the outgoing channels of the process $p$, delM(**bcast**($bc, p$, marker))) = delM($bc$) .

Then we have delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) delM($bc$) (**).

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) delM($bc$).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}},\ s'_{\mathcal{CLDSA}})$

2. The process does not globally initiates $\mathcal{CLDSA}$. This case is further split into three subcases:

   (a) The process does not have any incoming channels, and there are no processes except for the process that has not completed $\mathcal{CLDSA}$.

   Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Record of Process State in $M_{\mathcal{CLDSA}}$.

   base-state((p-state[$P$] : $PS$) $BC$)
   start-state($SC$)
   snapshot($SSC$)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 0) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$))
   start-state($SC$)
   snapshot((p-state[$P$] : $PS$) $SSC$)
   finish-state((p-state[$P$] : $PS$) $BC$)
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) $CC$)
   if ($SC \neq$ empBConfig) .

   It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$) $bc$) start-state($sc$) snapshot($ssc$) finish-state(empConfig) control((prog[p] : notYet) (cnt : 1) ( #ms[$p$] : 0) $cc$), such that ($sc$ =/= empConfig) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

   Because of $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[$p$] : $ps$) $bc$) = (p-state[$p$] : $ps$) delM($bc$) from definition of function delM.

   Since $s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) $bc$) start-state($sc$) snapshot((p-state[$p$] : $ps$) $ssc$) finish-state((p-state[$p$] : $ps$) $fc$) control((prog[$p$] : complated) (cnt : 0) (#ms[$p$] : 0) $cc$) .

   Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$)

delM($bc$) (*).

We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$] : $ps$) $bc$) = (p-state[$p$] : $ps$) delM($bc$) from definition of function delM(**).

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) delM($bc$).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}},\ s'_{\mathcal{CLDSA}})$

(b) The process does not have any incoming channels, and there are some other processes that have not completed $\mathcal{CLDSA}$.

Let us consider the case in which $t$ is constructed from the transition rule that describes 5 st case of Record of Process State in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS$) $BC$)
start-state($SC$)
snapshot($SSC$)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : 0) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state($SC$)
snapshot($SSC$)
control((prog[$P$] : completed) (cnt : sd($NzN$,1)) (#ms[$P$] : 0) $CC$)
if ($SC \neq$ empBConfig) $\wedge$ ($NzN > 1$)

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog[$p$] : notYet) (cnt : $nzn$) ( #ms[$p$] : 0) $cc$), such that ($nzn$ > 1) and ($sc$ =/= empConfig) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

Because of $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[$p$] : $ps$) $bc$) = (p-state[$p$] : $ps$) delM($bc$) from definition of function delM.

$s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, then $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) **bcast**($bc, p$, marker)) start-state($sc$) snapshot((p-state[$p$] : $ps$) $ssc$) finish-state($fc$) control((prog[$p$] : completed) (cnt : sd($nzn$,1)) $cc$) .

Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$) delM($bc$)(*).

We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$] : $ps$) **bcast**($bc, p$, marker)) = (p-state[$p$] : $ps$) delM(**bcast**($bc, p$, marker)) from definition of function del-Mand delM(**bcast**($bc, p$, marker))) = delM($bc$).

Then $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) delM($bc$) (**).

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) delM($bc$).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$

(c) The process has some incoming channels.

Let us consider the case in which $t$ is constructed from the transition rule that describes 6 st case of Record of Process State in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS$) $BC$)
start-state($SC$)
snapshot($SSC$)
control((prog[$P$] : notYet) (#ms[$P$] : $NzN'$) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) bcast($BC$, $P$, marker))
start-state($SC$)
snapshot((p-state[$P$] : $PS$) inchans($BC$, $P$) $SSC$)
control((prog[$P$] : started) (#ms[$P$] : $NzN'$) $CC$)
if ($SC \neq$ empBConfig)

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog[$p$] : notYet) ( #ms[$p$] : $nzn'$) $cc$) such that $sc$ =/= empConfig in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[$p$] : $ps$) $bc$) = (p-state[$p$] : $ps$) delM($bc$) from definition of function delM.

Since $s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) **bcast**($bc, p$, marker)) start-state($sc$) snapshot((p-state[$p$] : $ps$) inchans($bc, p$) $ssc$) finish-state($fc$) control((prog[$p$] : started) (#ms[$p$] : $nzn'$) $cc$) .

Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$) delM($bc$) (*).

We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$] : $ps$) **bcast**($bc, p$, marker)) = (p-state[$p$] : $ps$) delM($bc$) from definition of function delM(**).
From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) delM($bc$).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$

- **Receipt of Marker** is split into two subcases:

1. The process has not yet started $\mathcal{CLDSA}$. This case is further split into three subcases:

   (a) The process has only one incoming channel, and there are no processes that have not yet completed $\mathcal{CLDSA}$ except for the process.

   Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Receipt of Marker in $M_{\mathcal{CLDSA}}$.

   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
   snapshot($SSC$)
   finish-state(empBConfig)
   control((prog[$P$] : notYet) (cnt : 1) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) $SSC$)
   finish-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)

   It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state(empConfig) control((prog[$p$] : notYet) (#ms[$p$] : 1) (cnt : 1 ) (done[$q, p, n$] : false) $cc$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

   Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$)
   = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan(marker | $ms$)) delM($bc$)
   = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) from definition of function delM.

   Since $s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : $ms$) $bc$) start-state($sc$) snapshot((p-state[$p$] : $ps$) (c-state[$q, p, n$] : empChan) $ssc$) finish-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : $ms$) $bc$) control((prog[$p$] : completed) (#ms[$p$] : 0) (cnt : 0) (done[$q, p, n$] : true) $cc$)

   Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$)(*).

   We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : $ms$) $bc$)
   = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) from definition of function delM.

From (*) and (**), $s'_{\mathcal{UDS}}$ = delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}}, s'_{\mathcal{CLDSA}})$

(b) The process has only one incoming channel, and there are some other processes that have not yet completed $\mathcal{CLDSA}$.

Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Receipt of Marker in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
snapshot($SSC$)
control((prog[$P$] : notYet) (cnt : $NzN$) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) bcast($BC$, $P$, maker))
snapshot((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : empChan) $SSC$)
control((prog[$P$] : completed) (cnt : sd($NzN$,1)) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)
if $NzN > 1$

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state(empConfig) control((prog[$p$] : notYet) (#ms[$p$] : 1) (cnt : $nzn$ ) (done[$q, p, n$] : false) $cc$), where $nzn > 1$, in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$)
= (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) from definition of function delM.

Since $s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : $ms$) $bc$) start-state($sc$) snapshot((p-state[$p$] : $ps$) (c-state[$q, p, n$] : empChan) $ssc$) finish-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : $ms$) $bc$) control((prog[$p$] : completed) (#ms[$p$] : 0) (cnt : sd($nzn$,1)) (done[$q, p, n$] : true) $cc$)

Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) (*).

We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : $ms$) $bc$) = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) from definition of function delM.

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) (c-state$[q, p, n]$ : delChan($ms$)) delM($bc$).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}},\ s'_{\mathcal{CLDSA}})$

(c) The process has more than one incoming channel.

Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Receipt of Marker in $M_{\mathcal{CLDSA}}$.

base-state((p-state$[P]$ : $PS$) (c-state$[Q,\ P,\ N]$ : marker | $MMS$) $BC$ )
snapshot($SSC$)
control((prog$[P]$ : notYet) (cnt : $NzN$) (#ms$[P]$ : $NzN'$) (done$[Q,\ P,\ N]$ : false) $CC$)
$\Rightarrow$
base-state((p-state$[P]$ : $PS$) (c-state$[Q,\ P,\ N]$ : $MMS$) bcast($BC$, $P$, maker))
snapshot((p-state$[P]$ : $PS$) (c-state$[Q,\ P,\ N]$ : empChan) inchans($BC, P$) $SSC$)
control((prog$[P]$ : started) (cnt : sd($NzN$,1)) (#ms$[P]$ : sd($NzN'$,1)) (done$[Q,\ P,\ N]$ : true) $CC$)
if $NzN' > 1$
It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state$[p]$: $ps$) (c-state$[q, p, n]$ : marker | $ms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control((prog$[p]$ : notYet) (#ms[p] : $nzn'$) (cnt : $nzn$ ) (done$[q, p, n]$ : false) $cc$), such that $nzn'$ > 1, in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

Because of $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[p]: ps) (c-state[q, p, n] : marker | ms) bc)
= (p-state$[p]$ : $ps$) (c-state$[q, p, n]$ : delChan($ms$)) delM($bc$) from definition of function delM.

Since $s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, $s'_{\mathcal{CLDSA}}$ is base-state((p-state$[p]$ : $ps$) (c-state$[q, p, n]$ : $ms$) $bc$) start-state($sc$) snapshot((p-state$[p]$ : $ps$) (c-state$[q, p, n]$ : empChan) inchans($bc, p$) $ssc$) finish-state($fc$) control((prog$[p]$ : started) (#ms$[p]$ : sd($nzn'$,1)) (cnt : $nzn$) (done$[q, p, n]$ : true) $cc$)

Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (*).

We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state$[p]$ : $ps$) (c-state$[q, p, n]$ : delChan($ms$)) delM($bc$)

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state$[p]$: $ps$) (c-state$[q, p, n]$ : $ms$) $bc$)
= (p-state$[p]$ : $ps$) (c-state$[q, p, n]$ : delChan($ms$)) delM($bc$) from definition of function delM.

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}},\ s'_{\mathcal{CLDSA}})$

2. The process has already started $\mathcal{CLDSA}$. This case is further split into three subcases:

   (a) There are no incoming channels from which markers have not been received except for the incoming channel, and there are no processes that have not yet completed $\mathcal{CLDSA}$ except for the process.

   Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Receipt of Marker in $M_{\mathcal{CLDSA}}$.

   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
   finish-state(empBConfig)
   control((prog[$P$] : started) (cnt : 1) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
   $\Rightarrow$
   base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   finish-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
   control((prog[$P$] : completed) (cnt : 0) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)

   It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state(empConfig) control((prog[$p$] : started) (#ms[$p$] : 1) (cnt : 1 ) (done[$q, p, n$] : false) $cc$) in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

   Because of $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$)
   = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) from definition of function delM.

   Since $s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : $ms$) $bc$) start-state($sc$) snapshot((p-state[$p$] : $ps$) (c-state[$q, p, n$] : empChan) $ssc$) finish-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : $ms$) $bc$) control((prog[$p$] : completed) (#ms[$p$] : 0) (cnt : 0) (done[$q, p, n$] : true) $cc$) (*).

   Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$)

   We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : $ms$) $bc$)
   = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) from definition of function delM(**).

From (*) and (**), ${s'}_{\mathcal{UDS}}$= delM(b-state(${s'}_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$).

Therefore, $\mathbf{r}({s'}_{\mathcal{UDS}}, {s'}_{\mathcal{CLDSA}})$

(b) There are no incoming channels from which markers have not been received except for the incoming channel, and there are some other processes that have not yet completed $\mathcal{CLDSA}$.

Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Receipt of Marker in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
control((prog[$P$] : started) (cnt : $NzN$) (#ms[$P$] : 1) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
control((prog[$P$] : completed) (cnt : sd($NzN$,1) (#ms[$P$] : 0) (done[$Q$, $P$, $N$] : true) $CC$)
if $NzN > 1$
It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$)
control((prog[$p$] : started) (#ms[$p$] : 1) (cnt : $nzn$) (done[$q, p, n$] : false) $cc$)
such that $nzn > 1$ in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

Because of $\mathbf{r}(s_{\mathcal{UDS}}, s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$)
= (p-state[p] : ps) (c-state[q, p, n] : delChan(ms)) delM(bc) from definition of function delM.

Since $s_{\mathcal{CLDSA}}$ goes to ${s'}_{\mathcal{CLDSA}}$ by state transition $t$, ${s'}_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : $ms$) $bc$) start-state($sc$) snapshot((p-state[$p$] : $ps$) (c-state[$q, p, n$] : empChan) $ssc$) finish-state($fc$) control((prog[$p$] : completed) (#ms[$p$] : 0) (cnt : 0) (done[$q, p, n$] : true) $cc$)

Let we choose ${s'}_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that ${s'}_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) (*).

We have delM(b-state(${s'}_{\mathcal{CLDSA}}$)) = delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : $ms$) $bc$) = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) from definition of function delM

From (*) and (**), ${s'}_{\mathcal{UDS}}$= delM(b-state(${s'}_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$).

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}},\ s'_{\mathcal{CLDSA}})$

(c) There are some other incoming channels from which markers have not been received.

Let us consider the case in which $t$ is constructed from the transition rule that describes this case of Receipt of Marker in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : marker | $MMS$) $BC$ )
control((prog[$P$] : started) (cnt : $NzN$) (#ms[$P$] : $NzN'$) (done[$Q$, $P$, $N$] : false) $CC$)
$\Rightarrow$
base-state((p-state[$P$] : $PS$) (c-state[$Q$, $P$, $N$] : $MMS$) $BC$)
control((prog[$P$] : started) (cnt : $NzN$)) (#ms[$P$] : sd($NzN'$,1)) (done[$Q$, $P$, $N$] : true) $CC$)
if $NzN' > 1$

It suffices to consider $s_{\mathcal{CLDSA}}$ as base-state((p-state[$p$]: $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$) start-state($sc$) snapshot($ssc$) finish-state(empConfig) control((prog[$p$] : started) (#ms[$p$] : $nzn'$) (cnt : $nzn$ ) (done[$q, p, n$] : false) $cc$) such that $nzn' > 1$ in $S_{\mathcal{CLDSA}}$ to which the transition rule can be applied.

Because of $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$, $s_{\mathcal{UDS}}$ is delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : marker | $ms$) $bc$)
= (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) from definition of function delM.

Since $s_{\mathcal{CLDSA}}$ goes to $s'_{\mathcal{CLDSA}}$ by state transition $t$, $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : $ms$) $bc$) start-state($sc$) snapshot((p-state[$p$] : $ps$) (c-state[$q, p, n$] : empChan) $ssc$) finish-state((p-state[$p$] : $ps$) (c-state[$q, p, n$] : $ms$) $bc$) control((prog[$p$] : started) (#ms[$p$] : sd($nzn'$,1)) (cnt : $nzn$) (done[$q, p, n$] : true) $cc$)

Let we choose $s'_{\mathcal{UDS}}$ is the same $s_{\mathcal{UDS}}$, means that $s'_{\mathcal{UDS}}$ is (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) (*).

We have delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$]: $ps$) (c-state[$q, p, n$] : $ms$) $bc$)
= (p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) from definition of function delM(**).

From (*) and (**), $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) = p-state[$p$] : $ps$) (c-state[$q, p, n$] : delChan($ms$)) delM($bc$) .

Therefore, $\mathbf{r}(s'_{\mathcal{UDS}},\ s'_{\mathcal{CLDSA}})$

We have considered all transition rules in $M_{\mathcal{CLDSA}}$ and prove that for each $s_{\mathcal{CLDSA}}$, $s'_{\mathcal{CLDSA}}$ $\in S_{\mathcal{CLDSA}}$ and $s_{\mathcal{UDS}} \in S_{\mathcal{UDS}}$ such that $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$ and $s_{\mathcal{CLDSA}} \rightsquigarrow_{M_{\mathcal{CLDSA}}} s'_{\mathcal{CLDSA}}$ by one state transition of the $M_{\mathcal{CLDSA}}$, there exists $s'_{\mathcal{UDS}}$ such that $\mathbf{r}(s'_{\mathcal{UDS}},\ s'_{\mathcal{CLDSA}})$ and$s_{\mathcal{UDS}} \rightsquigarrow^*_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by zero or more. Therefor, this condition is hold.

From what have been proved above, we can see that binary relation $\mathbf{r}$ satisfies that forall state $s$ in $I_{\mathcal{CLDSA}}$, exist $s'$ in $I_{\mathcal{UDS}}$ such that $s$ is simulated by $s'$ and forall $s_1$, $s_2$ in $S_{\mathcal{CLDSA}}$ and $s_1$ moves to $s_2$ by state transition $t$, $s'_1$ in $S_{\mathcal{UDS}}$ such that $s_1$ is simulated by $s'_1$, then there exist $s'_2$ such that $s_2$ moves to $s'_2$ by zero or more state transitions and $s_2$ is simulated by $s'_2$. This mean that for any state $s$ in $S_{\mathcal{CLDSA}}$ which can be moved from any state in $I_{\mathcal{CLDSA}}$, we can find a state $s'$ in $S_{\mathcal{UDS}}$, such that $s$ is simulated by $s'$. Therefore, binary relation $\mathbf{r}$ is simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$. QED

### 7.2.2 Simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$

To prove that $\mathbf{r}$ is a simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$, we shall prove that r satisfies the following two conditions. Fig. 7.2 shows the diagrams corresponding to the two conditions.

**1. For each $s_{\mathcal{UDS}} \in I_{\mathcal{UDS}}$ there exists $s_{\mathcal{CLDSA}} \in I_{\mathcal{CLDSA}}$, such that $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$.**
**Prove:**

For each $s_{\mathcal{UDS}} \in I_{\mathcal{UDS}}$, from the definition of $CL$, we can find:

$s_{\mathcal{CLDSA}}$= base-state($s_{\mathcal{UDS}}$) start-state(empConfig) snapshot(empConfig)
finsh-state(empConfig) control($ctl$) where $ctl$ = InitCtlConfig($s_{\mathcal{UDS}}$).

Because of b-state($s_{\mathcal{CLDSA}}$) = $s_{\mathcal{UDS}}$, delM(b-state($s_{\mathcal{CLDSA}}$) ) = delM($s_{\mathcal{UDS}}$) = $s_{\mathcal{UDS}}$ means that $s_{\mathcal{UDS}}$= delM(b-state($s_{\mathcal{CLDSA}}$) ) then $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$.

Therefore, for each $s_{\mathcal{UDS}} \in I_{\mathcal{UDS}}$ there exists $s_{\mathcal{CLDSA}} \in I_{\mathcal{CLDSA}}$, such that $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$.

**2. For each $s_{\mathcal{UDS}}$, $s'_{\mathcal{UDS}} \in S_{\mathcal{UDS}}$ and $s_{\mathcal{CLDSA}} \in S_{\mathcal{CLDSA}}$ such that $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$ and $s_{\mathcal{UDS}} \rightsquigarrow_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by one state transition of the $M_{\mathcal{UDS}}$, there exists $s'_{\mathcal{CLDSA}}$ such that $\mathbf{r}(s'_{\mathcal{UDS}},\ s'_{\mathcal{CLDSA}})$ and $s_{\mathcal{CLDSA}} \rightsquigarrow^*_{M_{\mathcal{CLDSA}}} s'_{\mathcal{CLDSA}}$ by zero or more state transitions of the $M_{\mathcal{CLDSA}}$.**

**Prove:**

Assume that $s_{\mathcal{UDS}} \rightsquigarrow_{M_{\mathcal{UDS}}} s'_{\mathcal{UDS}}$ by state transition $t$,

To prove that $\mathbf{r}$ is simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$, it suffices to take into account the three transition rules. It means that we only need to consider state transitions, which

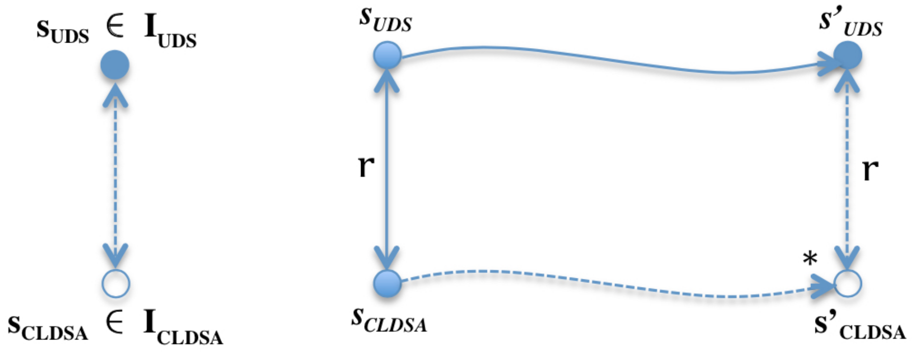


Figure 7.4: The binary relation $\mathbf{r}$ is a simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$.

are defined by the three transition rules.

We also know that for any transition rule in $M_{\mathcal{UDS}}$, there exists a transition rule in the $\mathcal{UDS}$ part in $M_{\mathcal{CLDSA}}$, which can be constructed from the transition rules of the $\mathcal{UDS}$ and changes the base-state meta configuration component of a state of $M_{\mathcal{CLDSA}}$.

More specifically in this proof, for any state transition $t$ in $M_{\mathcal{UDS}}$that moves state $s_{\mathcal{UDS}}$ to state $s'_{\mathcal{UDS}}$ in $M_{\mathcal{UDS}}$, there exists $s'_{\mathcal{CLDSA}}$ such that $\mathbf{r}(s'_{\mathcal{UDS}},\ s'_{\mathcal{CLDSA}})$ and we can find state transition $t'$ in $M_{\mathcal{CLDSA}}$such that $t'$ can moves state $s_{\mathcal{CLDSA}}$ to state $s'_{\mathcal{CLDSA}}$ in $M_{\mathcal{CLDSA}}$. This will be proven by the following. The following proof will consider all the three transition rules in $M_{\mathcal{UDS}}$. In the following proof: $p, q \in$ Pid, $ps1, ps2 \in$ PState, $n \in$ Nat, $m \in$ Msg, $cs \in$ MsgSeq, $cs' \in$ MMsgSeq, $cf \in$ Config, $bc, bc', sc, ssc, fc \in$ BConfig and $ctl, cc \in$ CtlConfig are fresh constants of those sorts.

1. **Change of Process State.**

   Let us consider the case in which $t$ is constructed from the transition rule that describes Change of Process State in $M_{\mathcal{UDS}}$.

   $$(\text{p-state}[P] : PS1) \Rightarrow (\text{p-state}[P] : PS2)$$

   It suffices to consider $s_{\mathcal{UDS}}$ as to which the transition rule can be applied:

   - $s_{\mathcal{UDS}}$= ((p-state[$p$] : $ps1$) $cf$)

   $s_{\mathcal{CLDSA}}$ is in form base-state($bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$)

   Because of $\mathbf{r}(s_{\mathcal{UDS}},\ s_{\mathcal{CLDSA}})$,

   $s_{\mathcal{UDS}}$ = delM(b-state($s_{\mathcal{CLDSA}}$)) = delM($bc$), delM($bc$) = (p-state[$p$] : $ps1$) $cf$.
   Then $bc$ = ((p-state[$p$] : $ps1$) $bc'$), where delM($bc'$) = $cf$.

$s_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps1$) $bc'$) start- state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$), where delM($bc'$) = $cf$.

Since $s_{\mathcal{UDS}}$ goes to $s'_{\mathcal{UDS}}$ by state transition $t$, $s'_{\mathcal{UDS}}$= ((p-state[$p$] : $ps2$) $cf$)

Let us choose the state transition $t'$ is constructed from the transition rule that describes Change of Process State in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS1$) $BC$)

$\Rightarrow$

base-state((p-state[$P$] : $PS2$) $BC$)

Let us choose $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps2$) $bc'$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$).

We have $s'_{\mathcal{UDS}}$ = ((p-state[$p$] : $ps2$) $cf$) (*) and,

delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$] : $ps2$) $bc'$) = (p-state[$p$] : $ps2$) delM($bc'$) = (p-state[$p$] : $ps2$) $cf$ from delM(bc') = cf.(**)

From (*) and (**), $s'_{\mathcal{UDS}}$ = delM(b-state($s'_{\mathcal{CLDSA}}$)) = (p-state[$p$] : $ps2$) $cf$.

Therefore $\mathbf{r}$($s'_{\mathcal{UDS}}$, $s'_{\mathcal{CLDSA}}$) and $s_{\mathcal{CLDSA}} \rightsquigarrow_{M_{\mathcal{CLDSA}}} s'_{\mathcal{CLDSA}}$ by state transition $t'$.

2. **Sending of Message.**

   Let us consider the case in which $t$ is constructed from the transition rule that describes Sending of Message in $M_{\mathcal{UDS}}$.

   > (p-state[$P$] : $PS1$) (c-state[$P, Q, N$] : $CS$)
   >
   > $\Rightarrow$
   >
   > (p-state[$P$] : $PS2$) (c-state[$P, Q, N$] : enq($CS, M$))

   It suffices to consider $s_{\mathcal{UDS}}$ as to which the transition rule can be applied is $s_{\mathcal{UDS}}$ is (p-state[$p$] : $ps1$) (c-state[$p, q, n$]: $cs$) $cf$.

   $s_{\mathcal{CLDSA}}$ is in form : base-state($bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$)

   Because of $\mathbf{r}$($s_{\mathcal{UDS}}$, $s_{\mathcal{CLDSA}}$), $s_{\mathcal{UDS}}$ = delM(b-state($s_{\mathcal{CLDSA}}$)) = delM($bc$).
   So delM($bc$) = (p-state[$p$] : $ps1$) (c-state[$p, q, n$] : $cs$) $cf$,

   Then $bc$ = ((p-state[$p$] : $ps1$) (c-state[$p, q, n$]: $cs'$) $bc'$), where delM($bc'$) = $cf$ and delChan($cs'$) = $cs$ from definition of function delM.

Therefore $s_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps1$) (c-state[$p, q, n$] : $cs'$) $bc'$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$), where delM($bc'$) = $cf$ and delChan($cs'$) = $cs$.

Because $s_{\mathcal{UDS}}$ goes to $s'_{\mathcal{UDS}}$ by state transition t, $s'_{\mathcal{UDS}}$= (p-state[p]: ps2) (c-state[p,q,n]: enq(cs,m))

Let us choose the state transition $t'$ is constructed from the transition rule that describes Sending of Message in $M_{\mathcal{CLDSA}}$.

base-state((p-state[$P$] : $PS1$) (c-state[$P, Q, N$] : $MMS$)

$BC$)

$\Rightarrow$

base-state((p-state[$P$] : $PS2$) (c-state[$P, Q, N$] : enq($MMS, M$))

$BC$)

Let us choose $s'_{\mathcal{CLDSA}}$ is base-state((p-state[$p$] : $ps1$) (c-state[$p, q, n$]: enq($cs', m$) $bc'$)

start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$).

We have $s'_{\mathcal{UDS}}$ = (p-state[$p$]: $ps2$) (c-state[$p, q, n$]: enq($cs, m$)) $bc$ (*) and,

delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$] : $ps2$)
(c-state[$p, q, n$] : enq($cs', m$)) $bc$)
= (p-state[$p$] : $ps2$) (c-state[$p, q, n$]: delChan(enq($cs', m$)) delM($bc'$)))

Because $m$ is message, delChan(enq($cs', m$))) = enq(delChan($cs'$), $m$), then

(p-state[$p$] : $ps2$) (c-state[$p, q, n$]: delChan(enq($cs', m$)) delM($bc'$)))
= (p-state[$p$] : $ps2$) (c-state[$p, q, n$]: enq(delChan($cs'$), $m$)) delM($bc'$)))
= (p-state[$p$] : $ps2$) (c-state[$p, q, n$]: enq($cs, m$)) $cf$)
{delM(bc') = cf and delChan(cs') = cs}.

Then $s'_{\mathcal{UDS}}$= delM(b-state($s'_{\mathcal{CLDSA}}$)) (**).

From (*) and (**), $\mathbf{r}$($s'_{\mathcal{UDS}}$, $s'_{\mathcal{CLDSA}}$) and $s_{\mathcal{CLDSA}} \rightsquigarrow_{M_{\mathcal{CLDSA}}} s'_{\mathcal{CLDSA}}$ by state transition $t'$.

3. **Receipt of Message.**

Let us consider the case in which $t$ is constructed from the transition rule that describes Receipt of Message in $M_{\mathcal{UDS}}$.

(p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M$ | $CS$)

$\Rightarrow$

(p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $CS$)

It suffices to consider an arbitrary state $s_{\mathcal{UDS}}$ in $S_{\mathcal{UDS}}$to which the rule can be applied is $s_{\mathcal{UDS}}$ = (p-state$[p]$: $ps1$) (c-state$[p, q, n]$: $m$ | $cs$) $cf$.

$s_{\mathcal{CLDSA}}$ is in form: base-state($bc$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$).

Because of $\mathbf{r}$($s_{\mathcal{UDS}}$, $s_{\mathcal{CLDSA}}$),

$s_{\mathcal{UDS}}$= delM(b-state($s_{\mathcal{CLDSA}}$)) = delM($bc$) and delM($bc$) = (p-state$[p]$: $ps1$) (c-state$[p, q, n]$: $m$ | $cs$) $cf$.

Then $bc$ = (p-state$[p]$: $ps1$) (c-state$[p, q, n]$: $m$ | $cs'$) $bc'$, where delM($bc'$) = $cf$ and delChan($cs'$) = $cs$.

Therefore, $s_{\mathcal{CLDSA}}$ is base-state((p-state$[p]$: $ps1$) (c-state$[p, q, n]$: $m$ | $cs'$) $bc'$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl$), where delM($bc'$) = $cf$ and delM($cs'$) = $cs$.

Since, $s_{\mathcal{UDS}}$ goes to $s'_{\mathcal{UDS}}$ by state transition $t$, $s'_{\mathcal{UDS}}$ = (p-state$[p]$: $ps2$) (c-state$[q, p, n]$: $cs$) $cf$

Although, there are four transition rules and $s_{\mathcal{CLDSA}}$ can apply one of them. However, No loss of generality, we can choose the following transition rule to consider.

Let us choose the state transition $t'$ is constructed from the transition rule that describes the 1st of Receipt of Message in $M_{\mathcal{CLDSA}}$.

base-state((p-state$[P]$ : $PS1$) (c-state$[Q, P, N]$ : $M$ | $MMS$) $BC$)

control((prog$[P]$ : notYet) $CC$)

$\Rightarrow$

base-state((p-state$[P]$ : $PS2$) (c-state$[Q, P, N]$ : $MMS$) $BC$)

control((prog$[P]$ : notYet) $CC$)
Let us choose $s'_{\mathcal{CLDSA}}$ is base-state((p-state$[p]$: $ps2$) (c-state$[p, q, n]$: $cs'$) $bc'$) start-state($sc$) snapshot($ssc$) finish-state($fc$) control($ctl'$).

We have $s'_{\mathcal{UDS}}$ = ((p-state[$p$] : $ps2$) (c-state[$p, q, n$]: $m \mid cs$) $cf$ (*) and,

delM(b-state($s'_{\mathcal{CLDSA}}$)) = delM((p-state[$p$]: $ps2$) (c-state[$p, q, n$]: $cs'$) $bc'$)
= (p-state[$p$]: $ps2$) (c-state[$p, q, n$]: delChan($cs'$)) delM($bc'$)
=(p-state[$p$]: $ps2$) (c-state[$q, p, n$]: $cs$) $cf$ {delM(bc') = cf and delChan(cs') = cs}.

Then $s'_{\mathcal{UDS}}$ = delM(b-state($s'_{\mathcal{CLDSA}}$)) (**) .

From (*) and (**) **r**($s'_{\mathcal{UDS}}$, $s'_{\mathcal{CLDSA}}$)and $s_{\mathcal{CLDSA}} \rightsquigarrow_{M_{\mathcal{CLDSA}}} s'_{\mathcal{CLDSA}}$ by state transition $t'$.

From what have been proved above, we can see that binary relation **r** satisfies that for all state s in $I_{\mathcal{UDS}}$, exist s' in $I_{\mathcal{CLDSA}}$ such that s is simulated by s' and for all s$_1$, s$_2$ in $S_{\mathcal{UDS}}$ and s$_1$ moves to s$_2$ by state transition $t$, s'$_1$ in $S_{\mathcal{CLDSA}}$ such that s$_1$ is simulated by s'$_1$, then there exists s'$_2$ in $S_{\mathcal{CLDSA}}$ such that s'$_1$ can moves to s'$_2$ by zero or more state transitions and s$_2$ is simulated by s'$_2$. This means that for any state s in $S_{\mathcal{UDS}}$, which can be moved from any state in $I_{\mathcal{UDS}}$, we can find a state s' in $S_{\mathcal{CLDSA}}$, such that s is simulated by s'. Therefore, binary relation r is simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$. QED

We have proven that **r** is a simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$ and it is also a simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$ to prove Theorem 2. QED

## 7.3 Chapter Summary

This chapter proposed a binary relation **r** between two state machines $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ by which for each $s1 \in S_{\mathcal{UDS}}$ and each $s2 \in S_{\mathcal{CLDSA}}$, **r**($s1, s2$) if and only if $s1$ is the same as the state obtained by deleting all markers from $s2$. To guarantee that $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$, it then proved that $M_{\mathcal{CLDSA}}$ simulates $M_{\mathcal{UDS}}$ and vice versa. Namely it proved Theorem 2 saying that **r** is a bi-simulation relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$. **r** is a bi-simulation relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ if and only if **r** is a simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$ and it is also a simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$.

To prove that **r** is a simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$, the chapter then proven that **r** satisfies the two conditions as follows: forall state $s$ in $I_{\mathcal{CLDSA}}$, exist $s'$ in $I_{\mathcal{UDS}}$ such that $s$ is simulated by $s'$ and forall $s_1$, $s_2$ in $S_{\mathcal{CLDSA}}$ and $s_1$ moves to $s_2$ by state transition $t$, $s'_1$ in $S_{\mathcal{UDS}}$ such that $s_1$ is simulated by $s'_1$, then there exist $s'_2$ such that $s_2$ moves to $s'_2$ by zero or more state transitions and $s_2$ is simulated by $s'_2$. This mean that for any state $s$ in $S_{\mathcal{CLDSA}}$ which can be moved from any state in $I_{\mathcal{CLDSA}}$, we can find a state $s'$ in $S_{\mathcal{UDS}}$, such that $s$ is simulated by $s'$.

To prove that **r** is a simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$, it then proven that **r** satisfies the two conditions as follows: for all state $s$ in $I_{\mathcal{UDS}}$, exist $s'$ in $I_{\mathcal{CLDSA}}$ such that s is simulated by s' and for all $s_1$, $s_2$ in $S_{\mathcal{UDS}}$ and $s_1$ moves to $s_2$ by state transition $t$, $s'_1$ in $S_{\mathcal{CLDSA}}$ such that $s_1$ is simulated by $s'_1$, then there exists $s'_2$ in $S_{\mathcal{CLDSA}}$ such that $s'_1$ can moves to ${s'}_2$ by zero or more state transitions and $s_2$ is simulated by ${s'}_2$. This means that for any state $s$ in $S_{\mathcal{UDS}}$, which can be moved from any state in $I_{\mathcal{UDS}}$, we can find a state $s'$ in $S_{\mathcal{CLDSA}}$, such that $s$ is simulated by $s'$. Therefore, binary relation **r** is simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$. Since **r** was proven that it is a simulation from $M_{\mathcal{CLDSA}}$ to $M_{\mathcal{UDS}}$ and also a simulation from $M_{\mathcal{UDS}}$ to $M_{\mathcal{CLDSA}}$, Theorem 2 was proven.

Theorem 2 was proven asserting that $\mathcal{CLDSA}$ does not alter the behaviors of a $\mathcal{UDS}$.

# Chapter 8

# Future Work

## 8.1 The Future Work

In recent decades, key software systems on which human beings heavily rely on are in the form of distributed systems, which consist of multiple nodes (or processes) connected with networks (or channels). Such systems should be fault tolerant because they need to run for a long time, keeping on providing services to human beings, other systems, etc. To make distributed systems fault tolerant, it is necessary to use many non-trivial distributed algorithms, such as snapshot algorithms, checkpointing algorithms and self-stabilizing algorithms. One common characteristic of these algorithms is that distributed systems are superimposed by them and may be regarded and treated as data by them. Therefore, such distributed algorithms can be called meta-distributed algorithms (MDAs) (see Fig. 8.1).

Since meta-distributed algorithms play the core part in our software-centric society and will become more and more important in our future highly advanced software-centric society, it is necessary as well as demanding to verify that such algorithms enjoy their desired properties. However, model checking that meta-distributed algorithms enjoy their desired properties has not been fully investigated because of the following reasons. It is challenging to specify meta-distributed algorithms in an existing specification language, such as PROMELA because it is necessary to specify distributed systems as computational targets or data that are dealt with by meta-distributed algorithms while an existing specification language, such as PROMELA is designed to fit to specify distributed systems

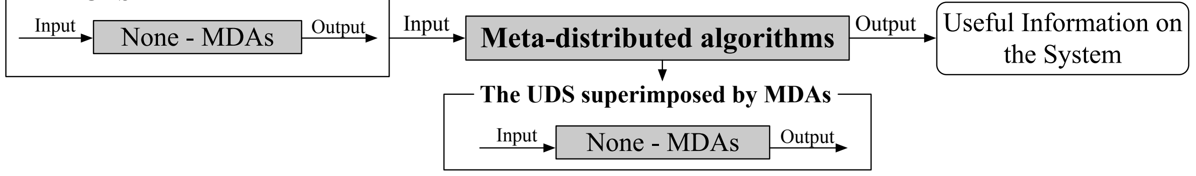

A UDS
Input
None - MDAs
Output
Input
Meta-distributed algorithms
Output
Useful Information on the System
The UDS superimposed by MDAs
Input
None - MDAs
Output

Figure 8.1: Meta-distributed algorithms.

but not computations of distributed systems (sequences of states that satisfy some conditions) as data, and to specify their desired properties in an existing temporal logic such as LTL because it is necessary to take into account computations of distributed systems. Therefore, it is also challenging to model check that meta-distributed algorithms enjoy their desired properties with an existing model checker, such as Spin [3]. It is therefore necessary to investigate how meta-distributed algorithms are formally analyzed.

Therefore, what we intend to do as the future work is to come up with how to specify meta-distributed algorithms, how to specify their desired properties, how to model check that meta-distributed algorithms enjoy their desired properties, and design and implement a model checker for meta-distributed algorithms.

It is necessary to deal with states of a distributed system, its computations and/or even a distributed system itself as data so as to specify an meta-distributed algorithm and its desired properties. From what we have conducted the current research on model checking of $\mathcal{CLDSA}$ revisited, we have learned that Maude is one promising specification language for the purpose. Maude is a specification and programming language and system based on rewriting logic. Rewriting logic is a logic designed to formally deal with concurrent systems including distributed systems, and then Maude is suited to formally deal with distributed systems. Maude is equipped with many useful functionalities. Among them are model checking facilities and meta-programming facilities. The model checking facilities mainly consists of an LTL model checker and a search command that can be used as a reachability checker. Specifying a distributed system in Maude, its states are expressed as terms. Therefore, states of a distributed system can be treated as data. Although computations of a distributed system as they are cannot be directly treated as data in Maude, the Maude search command is able to analyze the computations, checking if a given state is reachable from another given state. The Maude meta-programming functionalities can treat Maude specifications as data, and then can deal with distributed systems as data. MDAs are typically used by superimposing distributed systems, and then the Maude meta-programming facilities make it possible to describe generic specifications of meta-distributed algorithms as meta-programs that take specifications of UDSs and generate specifications of the UDSs superimposed by an meta-distributed algorithm.

To achieve the research goal, the first to do is to write a meta-program in Maude that corresponds to the function $CL$. The meta-program takes a specification of a $\mathcal{UDS}$ in Maude and generates a specification of the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ in Maude. The next to do is to implement a model checker as a meta-program in Maude that can directly deal with the more faithful formalization of the $\mathcal{DSR}$ property. Then, some more concrete meta-distributed algorithms, such as Koo-Toueg Checkpointing Algorithm, will be taken and similar case studies will be repeated, coming up with a generic way to specify meta-distributed algorithms, their desired properties and model check that such algorithms enjoy such properties.

We will first specify a $\mathcal{UDS}$ as a program in Maude. The specification of a $\mathcal{UDS}$ corresponds to state machine $M_{\mathcal{UDS}}$. The specification of a $\mathcal{UDS}$ corresponds to state machine $M_{\mathcal{UDS}}$. The system is specified as the module UDS in Maude. In which, the sorts Pid for process identifiers, PState for process states, Msg for messages, MsgQueue for

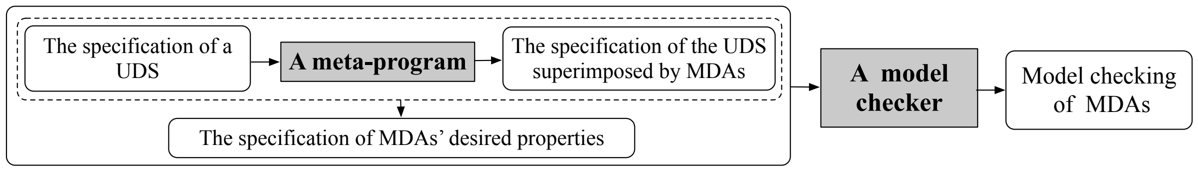

The specification of a UDS
A meta-program
The specification of the UDS superimposed by MDAs
The specification of MDAs' desired properties
A model checker
Model checking of MDAs

Figure 8.2: Meta-distributed algorithms.

queues of messages, OCom for observable components and Config for soups of observable components will be declared. Sub-sorts will also declared.

```
sorts Pid Msg PState .
sort MsgSeq .
sort OCom .
sort Config .

subsort OCom < Config .
```

Then the operator to constructor the sort are defined.

```
op empChan : -> MsgSeq [ctor] .
op _|_ : Msg MsgSeq -> MsgSeq [ctor] .

op p-state[_]:_ : Pid PState -> OCom [ctor] .
op c-state[_, _, _]:_ : Pid Pid Nat MsgSeq -> OCom [ctor] .

op empConfig : -> Config [ctor] .
op _ _ : Config Config -> Config [ctor assoc comm id: empConfig] .
```

Variables will be declared.

```
vars P P1 Q Q1 : Pid .
vars M1 M2 : Msg .
var N N1 : Nat .
var MS : MsgSeq .
var PS PS1 PS2 : PState .
var CS : MsgSeq .
vars OC OC1 : OCom .
vars CF CF1 : Config .
```

The operators and equations for the functions on the state of a $\mathcal{UDS}$, such as chans, msg, enq, # and = are defined.

```
op chans : Config -> Config .
op msg : OCom  -> MsgSeq .
op enq : MsgSeq Msg ->  MsgSeq .
op # : Config Pid -> Nat .

eq chans(empConfig) = empConfig .
eq chans((p-state[P]: PS) CF) = chans(CF) .
eq chans((c-state[P, Q, N]: CS) CF) = (c-state[P, Q, N] : CS) chans(CF) .

eq msg(c-state[P, Q, N] : CS) = CS .
eq msg(p-state[P]: PS) = empChan .

eq enq(empChan, M2) = M2 | empChan .
eq enq(M1 | MS, M2) = M1 | enq(MS, M2) .

eq #(empConfig, P) = 0 .
eq #((p-state[P1]: PS) CF, P) = if P == P1 then 1 + #(CF, P)
 else #(CF, P) fi .
eq #((c-state[P1, Q1, N1]: CS) CF, P) = #(CF, P) .
op # : Config Pid Pid Nat -> Nat .

eq #(empConfig, P, Q, N) = 0 .
eq #((p-state[P1] : PS) CF, P, Q, N) =  #(CF, P, Q, N) .
eq #((c-state[P1, Q1, N1]: CS) CF, P, Q, N) = if (P == P1)
and (Q == Q1) and (N == N1) then 1 + #(CF, P, Q, N) else #(CF, P, Q, N) fi .
```

The three transition rules will be written.

```
 rl [chgStt] :
(p-state[P]: PS1)
=>
(p-state[P]: PS2) .

rl[sndMsg] :
(p-state[P]: PS1) (c-state[P,Q,N]: CS)
 =>
(p-state[P]: PS2) (c-state[P,Q,N]: enq(CS,M)) .

rl[recMsg] :
(p-state[P]: PS1) (c-state[Q,P,N]: M | CS)
 =>
(p-state[P]: PS2) (c-state[Q,P,N]: CS) .
```

Next, the meta-program corresponding to the function $CL$ will be written, in which the specification of a $\mathcal{UDS}$ is treated as input data of the meta-program. Taking look at the meta-programming facilities in Maude, META-LEVEL module, in which terms and modules can be meta-reprsented, can be imported to use to generate the specification of the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ from the specification of a $\mathcal{UDS}$. The meta-program corresponding to the function $CL$ will be written in the module CLDSA in Maude that import the function module META-LEVEL, which is imported to write meta-programs in Maude. Module CLDSA define the meta-program corresponding to the function $CL$ as the function CL that takes a module, namely module UDS, as an its argument and return another module.

```
mod CLDSA is
pr META-LEVEL .

op CL : Module -> Module .
```

The function CL will call three other functions CLstate, CLinit and CLtrans corresponding to $CL_{State}$, $CL_{Init}$ and $CL_{Trans}$, respectively.

```
op CLstate : Module -> Module .
op CLinit : Module -> Module .
op CLtrans : ReluSet -> ReluSet .
```

Function CLstate use other functions to takes a part of the specifications of $M_{\mathcal{UDS}}$ and return a part of the specification of $M_{\mathcal{CLDSA}}$.

```
op modImportList : ImportList -> ImportList .
op modSortSet :  SortSet -> SortSet .
op modSubsortDeclSet : SubsortDeclSet -> SubsortDeclSet .
op modOpDeclSet : OpDeclSet -> OpDeclSet .
op modEquationSet : EquationSet -> EquationSet .
```

Function CLtrans use other functions to takes the specification of the transition rules of $M_{\mathcal{UDS}}$ and return the specification of the transition rules of $M_{\mathcal{CLDSA}}$.

```
op getReluSet : Module -> ReluSet .
op modReluSet : ReluSet -> ReluSet .
op addReluSet : ReluSet ReluSet -> Module .
```

Beside, the more faithful formalization of the $\mathcal{DSR}$ property will be specified based on the two specifications. After that, we will design and implement a model checker as a meta-program in Maude. To implement the model checker, metalevel-operations in

META-LEVEL module such as metaRewrite and metaSearch, which can be used to handle a specification, can be used to analyze computations of distributed systems. Continuously, we will conduct some similar studies on other meta-distributed algorithms such as Dijkstra-Scholten algorithm, Chandy-Misra-Haas algorithm and Koo-Toueg coordinated checkpointing to come up with a generic way to specify meta-distributed algorithms, their desired properties and model check that these algorithms enjoy such properties. The process to model check that meta-distributed algorithms enjoy desired properties is depicted in Fig. 8.2.

The concept "meta-distributed algorithm" is brand-new. One possible important achievement of the future work is to clarify meta-distributed algorithms and identify a class of meta-distributed algorithms and the main achievements of the research are model checking techniques and a model checker suited for the class of meta-distributed algorithms that cannot be reasonably well tackled by any existing model checkers.

## 8.2 Chapter Summary

This chapter mentioned the future work. As what is mentioned, the future work is to come up with how to specify meta-distributed algorithms, how to specify their desired properties, how to model check that meta-distributed algorithms enjoy their desired properties, and design and implement a model checker for meta-distributed algorithms. In detail, the chapter gave the concept of meta-distributed algorithms, then discussed the problems to resolve.

The chapter then discussed the procedures and the contents of the future work. The plan were also mentioned by which the first to do is to write a meta-program in Maude that corresponds to the function $CL$, The next to do is to implement a model checker as a meta-program in Maude that can directly deal with the more faithful formalization of the DSR property and similar case studies will be repeated, coming up with a generic way to specify meta-distributed algorithms, their desired properties and model check that such algorithms enjoy such properties.

At the end of the chapter, the desired achievements of the future work were discussed by which the main desired achievements of the research are model checking techniques and a model checker suited for the class of meta-distributed algorithms that cannot be reasonably well tackled by any existing model checkers.

# Chapter 9

# Conclusion

## The Contribution of the Research

Carefully taking into account an informal description of the $\mathcal{DSR}$ property, we have given a new formal definition of the property, which more faithfully express the informal description of the $\mathcal{DSR}$ property.

Since the definition of the $\mathcal{DSR}$ property in the existing study involves only one state machine $M_{\mathcal{CLDSA}}$ and our definition involves two state machines $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$, our definition is seemingly different from the one used in the existing study. It is checked that $\mathcal{CLDSA}$ terminates in $M_{\mathcal{CLDSA}}$ and then the snapshot and the final state are reachable from the start state and the snapshot, respectively, in $M_{\mathcal{UDS}}$ in our definition, while it is checked that the snapshot and the final state are reachable from the start state and the snapshot, respectively, in $M_{\mathcal{CLDSA}}$ in their definition. However, we have conducted an analysis on the similarities between our definition and the existing definition. We recognized that the new definition is equivalent to the existing definition. We have proved Theorem 1 saying that our formalization of the $\mathcal{DSR}$ property is equivalent to the existing definition for each $M_{\mathcal{UDS}}$. The theorem guarantees the validity of the model checking approach used in the existing study. Since the existing definition have been model checked with Maude and the new definition is not straightforward to be directly model checked. The theorem also guarantees that we can use the model checking approach used in the existing study to model check the new definition. To prove Theorem 1, we have proven Proposition 1 and Lemma 1. It is required to prove two other lemmas, Lemma 2 and Lemma 3, to prove Lemma 1. The proof of Theorem 1 follows from Proposition 1 and Lemma 1.

Moreover, we have proposed a binary relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ and then proved that $M_{\mathcal{CLDSA}}$ simulates $M_{\mathcal{UDS}}$ and vice versa, namely we have proved another theorem saying that the binary relation is a bi-simulation relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$, to guarantee that $\mathcal{CLDSA}$ does not alter the behaviors of the $\mathcal{UDS}$.

The contributions of our research are:

- Completely giving the more faithful formal definition of the $\mathcal{DSR}$ property, which more faithfully express the informal description of the $\mathcal{DSR}$ property.

- Proving a theorem saying that our formalization of the $\mathcal{DSR}$ property is equivalent to the existing definition for each $M_{\mathcal{UDS}}$ to guarantees that the validity of the model checking approach used in the existing study and the existing model checking approach can be used to model check the new definition.
- Proposing a binary relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ and then proving another theorem asserting that $M_{\mathcal{CLDSA}}$ simulates $M_{\mathcal{UDS}}$ and vice versa to guarantee that $\mathcal{CLDSA}$ does not alter the behaviors of the $\mathcal{UDS}$.

## What I Have Done on the Research

On the research, what I have done are as follows:

1. Learning necessary basic technical knowledge, such as underlying distributed systems, state machine, model checking and Maude specification and programming language.
2. Learning $\mathcal{CLDSA}$ and the $\mathcal{DSR}$ property.
3. Investigating how to specify $\mathcal{CLDSA}$ and how to model check that the algorithm enjoy the $\mathcal{DSR}$ property in the existing study.
4. Investigating how to formalize a $\mathcal{UDS}$ and the $\mathcal{UDS}$ superimposed by $\mathcal{CLDSA}$ as state machines $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$.
5. Studying on completely giving the more faithful formal definition of the $\mathcal{DSR}$ property.
6. Proving a theorem saying that the new definition of the $\mathcal{DSR}$ property is equivalent to the existing definition for each $M_{\mathcal{UDS}}$.
7. Proposing a binary relation between $M_{\mathcal{UDS}}$ and $M_{\mathcal{CLDSA}}$ and then proving another theorem asserting that $M_{\mathcal{CLDSA}}$ simulates $M_{\mathcal{UDS}}$ and vice versa.

## Summary of the Future Work

In the future, we want to come up with how to specify meta-distributed algorithms, how to specify their desired properties, how to model check that meta-distributed algorithms enjoy their desired properties, and design and implement a model checker for meta-distributed algorithms. We want to clarify meta-distributed algorithms and identify a class of meta-distributed algorithms. The main expected achievements of the future work is model checking techniques and a model checker suited for the class of meta-distributed algorithms that cannot be reasonably well tackled by any existing model checkers.

# Appendix A

# Specification of $\mathcal{CLDSA}$ in Maude

```
***
*** Specification of the Chandy-Lamport Distributed Snapshot
Algorithm in Maude
***

fmod PID is
  pr NAT .
  sort Pid .
  op p : Nat -> Pid [ctor] .
endfm

fmod TOKEN is
  pr NAT .
  sort Token .
  op t : Nat -> Token [ctor] .
endfm

fmod MARKER is
  sort Marker .
  op marker : -> Marker [ctor] .
endfm

fmod MESSAGE is
  pr TOKEN .
  pr MARKER .
  sort Msg .
  subsorts Token Marker < Msg .
endfm

fmod CHANNEL is
```

```
  pr MESSAGE .
  sorts EmpChan NeChan Chan .
  subsorts EmpChan NeChan < Chan .
  op empChan : -> EmpChan [ctor] .
  op _|_ : Msg Chan -> NeChan [ctor] .
  op put : Chan Msg -> NeChan .
  op delMC : Chan -> Chan .

  vars M1 M2 : Msg .
  var C : Chan .
  var T : Token .

  eq put(empChan,M2) = M2 | empChan .
  eq put(M1 | C,M2) = M1 | put(C,M2) .

endfm

fmod PROCESS-STATE is
  pr BOOL .
  pr TOKEN .
  sort PState .
  subsort Token < PState .
  op noToken : -> PState [ctor] .
  op __ : PState PState -> PState [ctor assoc comm
  id: noToken] .

  var T : Token .
  eq T T = T .
endfm

fmod PROGRESS is
  sort Prog .
  ops notYet started completed : -> Prog [ctor] .
endfm

fmod OBSERVABLE-COMPONENT is
  pr PID .
  pr CHANNEL .
  pr PROCESS-STATE .
  pr PROGRESS .
  sort OCom .

  *** p-state[p] is the state of process p.
  op (p-state[_] :_) : Pid PState -> OCom [ctor] .
  *** c-state[p,q,n] is the n th channel from p to q.
  op (c-state[_,_,_] :_) : Pid Pid Nat Chan -> OCom [ctor] .
  *** When cnt becomes 0, the snapshot has been taken.
  op (cnt :_) : Nat -> OCom [ctor] .
  *** the number of markers not yet received by a process.
  op (#ms[_] :_) : Pid Nat -> OCom [ctor] .
  *** indicating whether a marker has been received from
  *** a channel from p to q.
  op (done[_,_,_] :_) : Pid Pid Nat Bool -> OCom [ctor] .
  *** indicating that a process has not yet started,
  *** has started,
  *** or completed the algorithm.
  op (prog[_] :_) : Pid Prog -> OCom [ctor] .
  *** indicating whether messages are consumed.
  op (consume :_) : Bool -> OCom [ctor] .
endfm

fmod CONFIGURATIONS is
  pr OBSERVABLE-COMPONENT .
  sort Config .
  subsort OCom < Config .
  op empConfig : -> Config [ctor] .
  op _ _ : Config Config -> Config [ctor assoc comm id:
  empConfig] .

  var OC : OCom .
  eq OC OC = OC .

  var CF : Config .
  vars P' P Q : Pid .
  var PS : PState .
  var C : Chan .
  var M : Msg .
  var N : Nat .

  op bcast : Config Pid Marker -> Config .
  op inchans : Config Pid -> Config .
  op delM : Config -> Config .

  --- Function bcast to puts markers in all the outgoing channels
  from a process P;
  eq bcast(empConfig,P,M) = empConfig .
  eq bcast((c-state[P,Q,N] : C) CF,P,M)
  = (c-state[P,Q,N] : put(C,M)) bcast(CF,P,M) .
  eq bcast(OC CF,P,M) = OC bcast(CF,P,M) [owise] .

  --- Function inchans to initialize the states of all incoming
  channels of a process as empty channel;
  eq inchans(empConfig,P) = empConfig .
  eq inchans((c-state[Q,P,N] : C) CF,P)
   = (c-state[Q,P,N] : empChan) inchans(CF,P) .
  eq inchans(OC CF,P) = inchans(CF,P) [owise] .

endfm

fmod META-CONFIGURATION-COMPONENT is
  pr CONFIGURATIONS .
  sort MCComp .

  ops base-state start-state finish-state
      snapshot control : Config -> MCComp .
endfm

fmod META-CONFIGURATION is
  pr META-CONFIGURATION-COMPONENT .
  sort MConfig .
  subsort MCComp < MConfig .
  op __ : MConfig MConfig -> MConfig [assoc comm] .

  var MOC : MCComp .
  eq MOC MOC = MOC .
endfm

fmod INIT-META-CONFIG is
  pr META-CONFIGURATION .

***(

Let us consider the following system:

- There are two processes p(0), p(1).

- There is one token t(0) in the system.

- The state of each process only depends on the tokens
```

```
owned by the process. So, the state of each process can
be expressed as \empty, {t(0)}. Initially, p(0) has the token
t(0), and p(1) does not have any tokens.

- There are two channels: p(0) --> p(1), p(1) --> p(0).
Initially each channel is empty.

- Each process repeatedly does the following:

i. If the process has a token, then it puts the token in
the outgoing channel. Accordingly its state changes.

ii. If the incoming channel to the process is not empty,
then the process gets the token from it. Accordingly
its state changes.

Let imc00 be the initial state of the system.

)
  op imc00 : -> MConfig .
  eq imc00
  = base-state((p-state[p(0)]: t(0)) (p-state[p(1)]: noToken)
  (c-state[p(0),p(1),0]: empChan) (c-state[p(1),p(0),0]: empChan))
  start-state(empConfig)
  finish-state(empConfig)
  snapshot(empConfig)
  control((cnt : 2) (#ms[p(0)]: 1) (#ms[p(1)]: 1)
  (done[p(0),p(1),0]: false) (done[p(1),p(0),0]: false)
  (prog[p(0)]: notYet) (prog[p(1)]: notYet)
  (consume : false)) .
---
***(

Let us consider the following system:

- There are two processes p(0), p(1).

- There is one token t(0) in the system.

- The state of each process only depends on the tokens
owned by the process. So, the state of each process can
be expressed as \empty, {t(0)}. Initially, p(0) has the token
t(0), and p(1) does not have any tokens.
```

```
- There are two channels: p(0) --> p(1), p(1) --> p(0).
Initially each channel is empty.

- Each process repeatedly does the following:

i. The process may consume a token owned by
the process.

ii. If the process has a token, then it puts the token in
the outgoing channel. Accordingly its state changes.

iii. If the incoming channel to the process is not empty,
then the process gets the token from it. Accordingly its
state changes.

Let imc01 be the initial state of the system.

)
  op imc01 : -> MConfig .
  eq imc01
  = base-state((p-state[p(0)]: t(0))
  (p-state[p(1)]: noToken)
  (c-state[p(0),p(1),0]: empChan)
  (c-state[p(1),p(0),0]: empChan))
  start-state(empConfig)
  finish-state(empConfig)
  snapshot(empConfig)
  control((cnt : 2) (#ms[p(0)]: 1) (#ms[p(1)]: 1)
  (done[p(0),p(1),0]: false) (done[p(1),p(0),0]: false)
  (prog[p(0)]: notYet) (prog[p(1)]: notYet)
  (consume : true)) .
---
***
  op imc02 : -> MConfig .
  eq imc02
  = base-state((p-state[p(0)]: t(0))
  (p-state[p(1)]: noToken)
 (p-state[p(2)]: t(2))
 (c-state[p(0),p(1),0]: empChan)
  (c-state[p(0),p(1),1]: (t(1) | empChan)))
  start-state(empConfig)
  finish-state(empConfig)
```

```
   snapshot(empConfig)
   control((cnt : 3) (#ms[p(0)]: 0)
   (#ms[p(1)]: 2) (#ms[p(2)]: 0)
   (done[p(0),p(1),0]: false)
   (done[p(0),p(1),1]: false)
   (prog[p(0)]: notYet) (prog[p(1)]: notYet)
    (prog[p(2)]: notYet)
   (consume : true)) .
---
***
   op imc03 : -> MConfig .
   eq imc03
      = base-state((p-state[p(0)]: (t(0) t(1)))
           (p-state[p(1)]: noToken)
           (c-state[p(0),p(1),0]: empChan)
           (c-state[p(0),p(1),1]: empChan)
           (c-state[p(1),p(0),0]: empChan))
         start-state(empConfig)
         finish-state(empConfig)
         snapshot(empConfig)
         control((cnt : 2) (#ms[p(0)]: 1) (#ms[p(1)]: 2)
           (done[p(0),p(1),0]: false) (done[p(0),p(1),1]: false)
           (done[p(1),p(0),0]: false)
           (prog[p(0)]: notYet) (prog[p(1)]: notYet)
           (consume : true)) .
---
***
   op imc04 : -> MConfig .
   eq imc04
      = base-state((p-state[p(0)]: (t(0) t(1)))
          (p-state[p(1)]: noToken)
           (p-state[p(2)]: noToken)
           (c-state[p(0),p(1),0]: empChan)
           (c-state[p(0),p(2),0]: empChan)
           (c-state[p(1),p(0),0]: empChan)
           (c-state[p(1),p(2),0]: empChan))
         start-state(empConfig)
         finish-state(empConfig)
         snapshot(empConfig)
           control((cnt : 3) (#ms[p(0)]: 1)
           (#ms[p(1)]: 1) (#ms[p(2)]: 2)
           (done[p(0),p(1),0]: false)
           (done[p(0),p(2),0]: false)
```

```
            (done[p(1),p(0),0]: false)
            (done[p(1),p(2),0]: false)
            (prog[p(0)]: notYet) (prog[p(1)]: notYet)
            (prog[p(2)]: notYet)
            (consume : false)) .
---
***
    op imc05 : -> MConfig .
    eq imc05
      = base-state((p-state[p(0)]: (t(0) t(1)))
            (p-state[p(1)]: noToken)
            (p-state[p(2)]: noToken)
            (c-state[p(0),p(1),0]: empChan)
            (c-state[p(0),p(2),0]: empChan)
            (c-state[p(1),p(0),0]: empChan)
            (c-state[p(1),p(2),0]: empChan))
        start-state(empConfig)
        finish-state(empConfig)
        snapshot(empConfig)
        control((cnt : 3) (#ms[p(0)]: 1)
            (#ms[p(1)]: 1) (#ms[p(2)]: 2)
            (done[p(0),p(1),0]: false)
            (done[p(0),p(2),0]: false)
            (done[p(1),p(0),0]: false)
            (done[p(1),p(2),0]: false)
            (prog[p(0)]: notYet) (prog[p(1)]: notYet)
            (prog[p(2)]: notYet)
            (consume : true)) .
---
***
    op imc06 : -> MConfig .
    eq imc06
      = base-state((p-state[p(0)]: (t(0) t(1)))
           (p-state[p(1)]: noToken)
            (p-state[p(2)]: noToken)
            (c-state[p(0),p(1),0]: empChan)
            (c-state[p(0),p(1),1]: empChan)
            (c-state[p(1),p(2),0]: empChan)
            (c-state[p(2),p(0),0]: empChan))
        start-state(empConfig)
        finish-state(empConfig)
        snapshot(empConfig)
        control((cnt : 3) (#ms[p(0)]: 1)
            (#ms[p(1)]: 2) (#ms[p(2)]: 1)
            (done[p(0),p(1),0]: false)
            (done[p(0),p(1),1]: false)
            (done[p(1),p(2),0]: false)
            (done[p(2),p(0),0]: false)
            (prog[p(0)]: notYet) (prog[p(1)]: notYet)
            (prog[p(2)]: notYet)
            (consume : true)) .
  ---
  ***
    op imc07 : -> MConfig .
    eq imc07
      = base-state((p-state[p(0)]: (t(0) t(1) t(2)))
            (p-state[p(1)]: noToken)
            (p-state[p(2)]: noToken)
            (c-state[p(0),p(1),0]: empChan)
            (c-state[p(0),p(1),1]: empChan)
            (c-state[p(1),p(2),0]: empChan)
            (c-state[p(2),p(0),0]: empChan))
        start-state(empConfig)
        finish-state(empConfig)
        snapshot(empConfig)
        control((cnt : 3) (#ms[p(0)]: 1)
        (#ms[p(1)]: 2) (#ms[p(2)]: 1)
            (done[p(0),p(1),0]: false)
            (done[p(0),p(1),1]: false)
            (done[p(1),p(2),0]: false)
            (done[p(2),p(0),0]: false)
            (prog[p(0)]: notYet) (prog[p(1)]: notYet)
            (prog[p(2)]: notYet)
            (consume : true)) .
  ---
  ***
    op imc08 : -> MConfig .
    eq imc08
      = base-state((p-state[p(0)]: (t(0) t(1)))
            (p-state[p(1)]: noToken)
            (p-state[p(2)]: noToken)
            (c-state[p(0),p(1),0]: empChan)
            (c-state[p(0),p(1),1]: empChan)
            (c-state[p(0),p(2),0]: empChan)
            (c-state[p(1),p(2),0]: empChan)
            (c-state[p(2),p(0),0]: empChan))
```

```
        start-state(empConfig)
        finish-state(empConfig)
        snapshot(empConfig)
        control((cnt : 3) (#ms[p(0)]: 1)
          (#ms[p(1)]: 2) (#ms[p(2)]: 2)
          (done[p(0),p(1),0]: false)
          (done[p(0),p(1),1]: false)
          (done[p(0),p(2),0]: false)
          (done[p(1),p(2),0]: false)
          (done[p(2),p(0),0]: false)
          (prog[p(0)]: notYet) (prog[p(1)]: notYet)
          (prog[p(2)]: notYet)
          (consume : true)) .
---
***
  op imc09 : -> MConfig .
  eq imc09
     = base-state((p-state[p(0)]: noToken)
          (p-state[p(1)]: noToken) (p-state[p(2)]: noToken)
          (p-state[p(3)]: t(0)) (p-state[p(4)]: noToken)
          (c-state[p(0),p(1),0]: empChan)
          (c-state[p(1),p(2),0]: empChan)
          (c-state[p(2),p(3),0]: empChan)
          (c-state[p(2),p(4),0]: empChan)
          (c-state[p(3),p(0),0]: empChan)
          (c-state[p(3),p(0),1]: empChan)
          (c-state[p(3),p(2),0]: empChan)
          (c-state[p(4),p(3),0]: empChan))
        start-state(empConfig)
        finish-state(empConfig)
        snapshot(empConfig)
        control((cnt : 5) (#ms[p(0)]: 2) (#ms[p(1)]: 1)
          (#ms[p(2)]: 2) (#ms[p(3)]: 2) (#ms[p(4)]: 1)
          (done[p(0),p(1),0]: false)
          (done[p(1),p(2),0]: false)
          (done[p(2),p(3),0]: false)
          (done[p(2),p(4),0]: false)
          (done[p(3),p(0),0]: false)
          (done[p(3),p(0),1]: false)
          (done[p(3),p(2),0]: false)
          (done[p(4),p(3),0]: false)
          (prog[p(0)]: notYet) (prog[p(1)]: notYet)
          (prog[p(2)]: notYet)
```

```
         (prog[p(3)]: notYet) (prog[p(4)]: notYet)
         (consume : true)) .
---
***
  op imc10 : -> MConfig .
  eq imc10
     = base-state((p-state[p(0)]: (t(0) t(1)))
         (p-state[p(1)]: noToken) (p-state[p(2)]: noToken)
         (p-state[p(3)]: noToken) (p-state[p(4)]: noToken)
         (c-state[p(0),p(1),0]: empChan)
         (c-state[p(1),p(2),0]: empChan)
         (c-state[p(2),p(3),0]: empChan)
         (c-state[p(2),p(4),0]: empChan)
         (c-state[p(3),p(0),0]: empChan)
         (c-state[p(3),p(0),1]: empChan)
         (c-state[p(3),p(2),0]: empChan)
         (c-state[p(4),p(3),0]: empChan))
       start-state(empConfig)
       finish-state(empConfig)
       snapshot(empConfig)
       control((cnt : 5) (#ms[p(0)]: 2) (#ms[p(1)]: 1)
         (#ms[p(2)]: 2) (#ms[p(3)]: 2) (#ms[p(4)]: 1)
         (done[p(0),p(1),0]: false)
         (done[p(1),p(2),0]: false)
         (done[p(2),p(3),0]: false)
         (done[p(2),p(4),0]: false)
         (done[p(3),p(0),0]: false)
         (done[p(3),p(0),1]: false)
         (done[p(3),p(2),0]: false)
         (done[p(4),p(3),0]: false)
         (prog[p(0)]: notYet) (prog[p(1)]: notYet)
         (prog[p(2)]: notYet)
         (prog[p(3)]: notYet) (prog[p(4)]: notYet)
         (consume : true)) .
endfm

mod CHANDY-LAMPORT is
  pr META-CONFIGURATION .
  vars BC CC SC SSC : Config .
  vars P' P Q : Pid .
  var T : Token .
  var PS : PState .
  var N : Nat .
```

```
vars C C' : Chan .
vars NzN NzN' : NzNat .

****** Consumption of Tokens ******
***
*** When a distributed snapshot has been taken,
*** we intentionally
*** stop the base computation because we want
*** not to make the
*** size of the reachable state space too large.
***
*** Process P only changes its state.
rl [chgStt] :
  base-state((p-state[P] : (T PS)) BC)
  finish-state(empConfig)
  control((consume : true) CC)
  =>
  base-state((p-state[P] : PS) BC)
  finish-state(empConfig)
  control((consume : true) CC) .


****** Sending of Tokens ******
*** Process P sends a token to process Q.
rl [sndTkn] :
  base-state((p-state[P] : (T PS))
  (c-state[P,Q,N] : C) BC)
  finish-state(empConfig)
  =>
  base-state((p-state[P] : PS)
  (c-state[P,Q,N] : put(C,T)) BC)
  finish-state(empConfig) .


****** Receipt of Tokens ******
*** Process P receives a token along an incoming channel.
*** case-1: The process has not yet started the algorithm.
*** Note: No need (done[Q,P,N] : false) on both sides.
rl [recTkn&notYet&~done] :
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : T | C) BC)
  finish-state(empConfig)
  control((prog[P] : notYet) CC)
  =>
  base-state((p-state[P] : (T PS))
  (c-state[Q,P,N] : C) BC)
  finish-state(empConfig)
  control((prog[P] : notYet) CC) .

*** case-2: The process has completed the algorithm.
*** Note: No need (done[Q,P,N] : true) on both sides.
rl [recTkn&completed&done] :
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : T | C) BC)
  finish-state(empConfig)
  control((prog[P] : completed) CC)
  =>
  base-state((p-state[P] : (T PS))
  (c-state[Q,P,N] : C) BC)
  finish-state(empConfig)
  control((prog[P] : completed) CC) .

*** case-3: The process has started the algorithm,
*** namely that it has already recorded its state,
*** not yet completed it, and has not yet received a marker
*** from the incoming channel.
rl [recTkn&started&~done] :
  base-state((p-state[P] : PS) (c-state[Q,P,N] : T | C) BC)
  snapshot((c-state[Q,P,N] : C') SSC)
  finish-state(empConfig)
  control((prog[P] : started) (done[Q,P,N] : false) CC)
  =>
  base-state((p-state[P] : (T PS)) (c-state[Q,P,N] : C) BC)
  snapshot((c-state[Q,P,N] : put(C',T)) SSC)
  finish-state(empConfig)
  control((prog[P] : started) (done[Q,P,N] : false) CC) .

*** case-4: The process has started the algorithm,
*** not yet completed it, and has already received a marker
*** from the incoming channel.
rl [recTkn&started&done] :
  base-state((p-state[P] : PS) (c-state[Q,P,N] : T | C) BC)
  finish-state(empConfig)
  control((prog[P] : started) (done[Q,P,N] : true) CC)
  =>
  base-state((p-state[P] : (T PS)) (c-state[Q,P,N] : C) BC)
  finish-state(empConfig)
  control((prog[P] : started) (done[Q,P,N] : true) CC) .


****** Record of Process States ******
*** Process P starts taking the distributed snapshot.
*** case-1: The process globally initiates the algorithm,
*** namely the first process that records its state in the
*** system.
*** case-2: The process does not, namely that there exists
*** another process
*** that has globally initiated the algorithm.
*** case-1: is further split into three sub-cases:
*** case-1-1: The underlying system only consists of
*** the process.
*** Note: finish-state should be added.
rl [start&cnt=1&#ms=0] :
  base-state((p-state[P] : PS))
  start-state(empConfig)
  snapshot(empConfig)
  finish-state(empConfig)
  control((cnt : 1) (prog[P] : notYet)
  (#ms[P] : 0) CC)
  =>
  base-state((p-state[P] : PS))
  start-state((p-state[P] : PS))
  snapshot((p-state[P] : PS))
  finish-state((p-state[P] : PS))
  control((cnt : 0) (prog[P] : completed)
  (#ms[P] : 0) CC) .

*** case-1-2: The system consists of more than one process,
*** and the process does not have any incoming channels.
crl [start&cnt>1&#ms=0] :
  base-state((p-state[P] : PS) BC)
  start-state(empConfig)
  snapshot(empConfig)
  control((cnt : NzN') (prog[P] : notYet)
  (#ms[P] : 0) CC)
  =>
  base-state((p-state[P] : PS) bcast(BC,P,marker))
  start-state((p-state[P] : PS) BC)
  snapshot((p-state[P] : PS))
```

```
    control((cnt : sd(NzN',1)) (prog[P] : completed)
   (#ms[P] : 0) CC)
  if NzN' > 1 .

  *** case-1-3: The system consists of more than one process,
  *** and the process has one or more incoming channels.
  rl [start&cnt>1&#ms>0] :
    base-state((p-state[P] : PS) BC)
    start-state(empConfig)
    snapshot(empConfig)
    control((prog[P] : notYet) (#ms[P] : NzN') CC)
   =>
    base-state((p-state[P] : PS) bcast(BC,P,marker))
    start-state((p-state[P] : PS) BC)
    snapshot((p-state[P] : PS) inchans(BC,P))
    control((prog[P] : started) (#ms[P] : NzN') CC) .

  *** case-2: The process does not, namely that there exists
  *** another process
  *** that has globally initiated the algorithm.
  *** case-2: is further split into three sub-cases:
  *** case-2-1: The process does not have any incoming
  *** channels,
  *** and there are no processes except for the process
  *** that have not completed the algorithm.
  *** Note: finish-state should be added.
  crl [record&cnt=1&#ms=0] :
    base-state((p-state[P] : PS))
    start-state(SC)
    snapshot(SSC)
    finish-state(empConfig)
    control((cnt : 1) (prog[P] : notYet) (#ms[P ] : 0) CC)
   =>
    base-state((p-state[P] : PS))
    start-state(SC)
    snapshot((p-state[P] : PS) SSC)
    finish-state((p-state[P] : PS))
    control((cnt : 0) (prog[P] : completed) (#ms[P] : 0) CC)
  if (SC =/= empConfig) .

  *** case-2-2: The process does not have any incoming channels,
  *** and there are some other processes that have not completed
  the algorithm.
```

```
crl [record&cnt>1&#ms=0] :
  base-state((p-state[P] : PS) BC)
  start-state(SC)
  snapshot(SSC)
  control((cnt : NzN') (prog[P] : notYet)
  (#ms[P] : 0) CC)
  =>
  base-state((p-state[P] : PS) bcast(BC, P, marker))
  start-state(SC)
  snapshot((p-state[P] : PS) SSC)
  control((cnt : sd(NzN',1))
  (prog[P] : completed)
  (#ms[P] : 0) CC)
if (NzN' > 1) /\ (SC =/= empConfig) .

*** case-2-3: The process has some incoming channels.
crl [record&cnt>1&#ms>0] :
  base-state((p-state[P] : PS) BC)
  start-state(SC)
  snapshot(SSC)
  control((prog[P] : notYet) (#ms[P] : NzN') CC)
  =>
  base-state((p-state[P] : PS) bcast(BC,P,marker))
  start-state(SC)
  snapshot((p-state[P] : PS) inchans(BC,P) SSC)
  control((prog[P] : started) (#ms[P] : NzN') CC)
if (SC =/= empConfig) .


****** Receipt of Markers ******
*** Process P receives a marker along an incoming channel.
*** case-1: The process has not yet started the algorithm.
*** case-2: The process has already started the algorithm.
*** case-1 is further split into three sub-cases:
*** case-1-1: The process has only one incoming channel,
*** and there are no processes that have not yet completed
*** the algorithm
*** except for the process, which implies that the process
*** does not have any outgoing channels.
*** Note: finish-state should be added.
rl [recMkr&notYet&#ms=1&cnt=1] :
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : marker | C) BC)
  snapshot(SSC)
  finish-state(empConfig)
  control((prog[P] : notYet)
  (#ms[P] : 1) (cnt : 1)
  (done[Q,P,N] : false) CC)
  =>
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : C) BC)
  snapshot((p-state[P] : PS)
  (c-state[Q,P,N] : empChan) SSC)
  finish-state((p-state[P] : PS)
  (c-state[Q,P,N] : C) BC)
  control((prog[P] : completed)
  (#ms[P] : 0) (cnt : 0)
  (done[Q,P,N] : true) CC) .

*** case-1-2: The process has only one incoming channel,
*** and there are some other processes that have
*** not yet completed the algorithm.
crl [recMkr&notYet&#ms=1&cnt>1] :
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : marker | C) BC)
  snapshot(SSC)
  control((prog[P] : notYet) (#ms[P] : 1)
  (cnt : NzN)
  (done[Q,P,N] : false) CC)
  =>
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : C)
  bcast(BC,P,marker))
  snapshot((p-state[P] : PS )
  (c-state[Q,P,N] : empChan) SSC)
  control((prog[P] : completed)
  (#ms[P] : 0) (cnt : sd(NzN,1))
  (done[Q,P,N] : true) CC)
if NzN > 1 .

*** case-1-3: The process has more than one incoming channel.
crl [recMkr&notYet&#ms>1&cnt>1] :
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : marker | C) BC)
  snapshot(SSC)
  control((prog[P] : notYet)
  (#ms[P] : NzN') (cnt : NzN)
  (done[Q,P,N] : false) CC)
  =>
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : C)
  bcast(BC,P,marker))
  snapshot((p-state[P] : PS )
  (c-state[Q,P,N] : empChan)
  inchans(BC,P) SSC)
  control((prog[P] : started)
  (#ms[P] : sd(NzN', 1)) (cnt : NzN)
  (done[Q,P,N] : true) CC)
if NzN' > 1 .

*** case-2: The process has already started the algorithm.
*** case-2 is further split into three sub-cases:
*** case-2-1: There are no incoming channels
*** from which markers have not been
*** received except for the incoming channel,
*** and there are no processes
*** that have not yet completed the algorithm
*** except for the process.
rl [recMkr&started&#ms=1&cnt=1] :
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : marker | C) BC)
  finish-state(empConfig)
  control((prog[P] : started)
  (#ms[P] : 1) (cnt : 1)
  (done[Q,P,N] : false) CC)
  =>
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : C) BC)
  finish-state((p-state[P] : PS)
  (c-state[Q,P,N] : C) BC)
  control((prog[P] : completed)
  (#ms[P] : 0) (cnt : 0)
  (done[Q,P,N] : true) CC) .

*** case-2-2: There are no incoming channels
*** from which markers have not been
*** received except for the incoming channel,
*** and there are some other processes
*** that have not yet completed the algorithm.
*** Note: finish-state should not be added.
crl [recMkr&started&#ms=1&cnt>1] :
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : marker | C) BC)
  control((prog[P] : started)
  (#ms[P] : 1) (cnt : NzN)
  (done[Q,P,N] : false) CC)
  =>
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : C) BC)
  control((prog[P] : completed)
  (#ms[P] : 0) (cnt : sd(NzN,1))
  (done[Q,P,N] : true) CC)
if NzN > 1 .

*** case-2-3: There are some other incoming channels
*** from which markers have not
*** been received.
*** Note: finish-state should not be added.
crl [recMkr&started&#ms>1&cnt>1] :
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : marker | C) BC)
  control((prog[P] : started)
  (#ms[P] : NzN') (cnt : NzN)
  (done[Q,P,N] : false) CC)
  =>
  base-state((p-state[P] : PS)
  (c-state[Q,P,N] : C) BC)
  control((prog[P] : started)
  (#ms[P] : sd(NzN',1)) (cnt : NzN)
  (done[Q,P,N] : true) CC)
if NzN' > 1 .
endm

mod EXPERIMENT is
pr CHANDY-LAMPORT .
pr INIT-META-CONFIG .
vars SC FC SSC : Config .
vars MC : MConfig .
endm
```

# Appendix B

# Model Checking of $\mathcal{CLDSA}$ in Maude

```
***
*** Model Checking of the Distributed Snapshot Reachability
*** Property
***

*** Experiment for imc00 ***

*** states: 164
search in EXPERIMENT : imc00 =>* MC such that false .

*** Solution 40 (state 163)
*** states: 164  rewrites: 1341 in 18ms cpu
*** (6159ms real) (70817 rewrites/second)
search in EXPERIMENT :
  imc00 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 40 (state 163)
*** states: 164  rewrites: 1578 in 25ms cpu
*** (5896ms real) (61566 rewrites/second)
search in EXPERIMENT :
  imc00 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (consume : false))
     =>
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (consume : false)) .

*** Solution 40 (state 163)
*** states: 164  rewrites: 1575 in 26ms cpu
*** (5871ms real) (60057 rewrites/second)
search in EXPERIMENT :
  imc00 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (consume : false))
      =>
      base-state(FC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (consume : false)) .

*** End of Experiment for imc00 ***


*** Experiment for imc01 ***

*** states: 239
search in EXPERIMENT : imc01 =>* MC such that false .

*** Solution 55 (state 238)
*** states: 239  rewrites: 1968 in 27ms cpu
*** (2169ms real) (70386 rewrites/second)
search in EXPERIMENT :
  imc01 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 55 (state 238)
*** states: 239  rewrites: 2396 in 38ms cpu
*** (2153ms real) (62003 rewrites/second)
search in EXPERIMENT :
  imc01 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
                (consume : true))
        =>
        base-state(SSC) finish-state(empConfig)
        control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
                (consume : true)) .

*** Solution 55 (state 238)
*** states: 239  rewrites: 2360 in 38ms cpu
*** (2324ms real) (61615 rewrites/second)
search in EXPERIMENT :
  imc01 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
        control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
                (consume : true))
        =>
        base-state(FC) finish-state(empConfig)
        control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
                (consume : true)) .

*** End of Experiment for imc01 ***


*** Experiment for imc02 ***

*** states: 8451  rewrites: 101397 in 706ms cpu
*** (707ms real) (143426 rewrites/second)
search in EXPERIMENT : imc02 =>* MC such that false .

*** Solution 874 (state 8450)
*** states: 8451  rewrites: 109842 in 974ms cpu
*** (59634ms real) (112729 rewrites/second)
search in EXPERIMENT :
  imc02 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 874 (state 8450)
*** states: 8451  rewrites: 139329 in 1552ms cpu
*** (44910ms real) (89716 rewrites/second)
search in EXPERIMENT :
  imc02 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
      =>
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

*** Solution 874 (state 8450)
*** states: 8451  rewrites: 121493 in 1255ms cpu
*** (40742ms real) (96790 rewrites/second)
search in EXPERIMENT :
  imc02 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
      =>
      base-state(FC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

*** End of Experiment for imc02 ***


*** Experiment for imc03 ***

*** states: 60695
search in EXPERIMENT : imc03 =>* MC such that false .

*** Solution 9315 (state 60694)
*** states: 60695  rewrites: 553158 in 8221ms cpu
*** (462784ms real) (67285 rewrites/second)
search in EXPERIMENT :
  imc03 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 9315 (state 60694)
*** states: 60695  rewrites: 1814286 in 24295ms cpu
*** (409544ms real) (74674 rewrites/second)
search in EXPERIMENT :
  imc03 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (consume : true))
      =>
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (consume : true)) .

*** Solution 9315 (state 60694)
*** states: 60695  rewrites: 1725059 in 23364ms cpu
*** (612072ms real) (73833 rewrites/second)
search in EXPERIMENT :
  imc03 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (consume : true))
      =>
      base-state(FC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (consume : true)) .

*** End of Experiment for imc03 ***


*** Experiment for imc04 ***

*** states: 269508
search in EXPERIMENT : imc04 =>* MC such that false .

*** Solution 20851 (state 269506)
*** states: 269507  rewrites: 3825380 in 43031ms cpu
*** (2074578ms real) (88898 rewrites/second)
search in EXPERIMENT :
  imc04 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig .
```

```
*** Solution 20851 (state 269506)
*** states: 269507  rewrites: 7333415 in 102291ms cpu
*** (2741392ms real) (71691 rewrites/second)
search in EXPERIMENT :
  imc04 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : false))
      =>
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : false)) .

*** Solution 20851 (state 269506)
*** states: 269507  rewrites: 6935913 in 99523ms cpu
*** (2715449ms real) (69690 rewrites/second)
search in EXPERIMENT :
  imc04 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : false))
      =>
      base-state(FC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : false)) .

*** End of Experiment for imc04 ***


*** Experiment for imc05 ***

*** states: 471295
search in EXPERIMENT : imc05 =>* MC such that false .

*** Solution 33344 (state 471293)
*** states: 471294  rewrites: 6874881 in 86366ms cpu
*** (5815273ms real) (79601 rewrites/second)
search in EXPERIMENT :
  imc05 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 33344 (state 471293)
*** states: 471294  rewrites: 14393284 in 225842ms cpu
*** (3378231ms real) (63731 rewrites/second)
search in EXPERIMENT :
  imc05 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
      =>
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

*** Solution 33344 (state 471293)
*** states: 471294  rewrites: 12802410 in 214543ms cpu
*** (3572482ms real) (59672 rewrites/second)
search in EXPERIMENT :
  imc05 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
      =>
      base-state(FC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

*** End of Experiment for imc05 ***


*** Experiment for imc06 ***

*** states: 810938 rewrites: 11021149 in
*** 5357391096ms cpu
*** (434192ms real) (2 rewrites/second)
*** search in EXPERIMENT : imc06 =>* MC such that false .
*** Solution 81740 (state 810937)
*** states: 810938  rewrites: 11832087 in
*** 5357391621ms cpu
*** (1726642ms real) (2 rewrites/second)
search in EXPERIMENT :
  imc06 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 81740 (state 810937)
*** states: 810938  rewrites: 31965889 in
*** 5357390769ms cpu
*** (2014779ms real) (5 rewrites/second)
search in EXPERIMENT :
  imc06 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
      =>
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

*** Solution 81740 (state 810937)
*** states: 810938  rewrites: 31087639 in
*** 5357394898ms cpu
*** (1914408ms real) (5 rewrites/second)
search in EXPERIMENT :
  imc06 =>* start-state(SC) finish-state(FC)
  snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
      =>
      base-state(FC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

*** End of Experiment for imc06 ***
*** Experiment for imc07 ***

*** maude.intelDarwin(708,0xac6062c0) malloc:
*** mmap(size=2097152) failed (error code=12)
*** error: can't allocate region
*** set a breakpoint in malloc_error_break to debug
*** terminate called throwing an exceptionAbort trap: 6
----
search in EXPERIMENT : imc07 =>* MC such that false .

*** Solution 137140 (state 6854875)
*** states: 6854876  rewrites: 105922148 in 5213061ms cpu
*** (27530117ms real) (20318 rewrites/second)
----
*** maude.intelDarwin(737,0xac6062c0) malloc:
*** mmap(size=2097152) failed (error code=12)
*** error: can't allocate region
*** set a breakpoint in malloc_error_break to debug
*** terminate called throwing an exceptionAbort trap: 6
----
search in EXPERIMENT :
  imc07 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 122682 (state 6400916)
*** states: 6400917  rewrites: 440565484 in 38431146ms cpu
*** (54612651ms real) (11463 rewrites/second)
----
*** maude.intelDarwin(12040,0xac1522c0) malloc:
*** mmap(size=2097152) failed (error code=12)
*** error: can't allocate region
*** set a breakpoint in malloc_error_break to debug
*** terminate called throwing an exceptionAbort trap: 6
----
search in EXPERIMENT :
  imc07 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
     =>
```

```
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

*** Solution 125725 (state 6485240)
*** states: 6485241  rewrites: 373514223 in 23539706ms cpu
*** (39141309ms real) (15867 rewrites/second)
---
*** maude.intelDarwin(205,0xac6062c0) malloc:
*** mmap(size=2097152) failed (error code=12)
*** error: can't allocate region
*** set a breakpoint in malloc_error_break to debug
*** terminate called throwing an exceptionAbort trap: 6
---
search in EXPERIMENT :
  imc07 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
      =>
      base-state(FC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

***(imc07  SSC  FC
*** Solution 123827 (state 6448377)
*** states: 6448378  rewrites: 368379995 in 32136576ms
*** cpu (46709597ms real) (11462 rewrites/second)
***maude.intelDarwin(310,0xac1522c0) malloc:
*** mmap(size=262144) failed (error code=12)
*** error: can't allocate region
*** set a breakpoint in malloc_error_break to debug
terminate called throwing an exceptionAbort trap: 6
)

*** End of Experiment for imc07 ***


*** Experiment for imc08 ***

*** states: 3587681  rewrites: 51560637 in 1914834ms cpu
*** (1915236ms real) (26926 rewrites/second)
```

```
---
search in EXPERIMENT : imc08 =>* MC such that false .

*** Solution 190434 (state 3587680)
*** states: 3587681  rewrites: 55148210 in 2047317ms cpu
*** (46673794ms real) (26936 rewrites/second)
---
search in EXPERIMENT :
  imc08 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 190434 (state 3587680)
*** states: 3587681  rewrites: 118469042 in 5181498ms cpu
*** (49712756ms real) (22863 rewrites/second)
---
search in EXPERIMENT :
  imc08 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
      =>
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

*** Solution 190434 (state 3587680)
*** states: 3587681  rewrites: 116431923 in 5081011ms cpu
*** (48872905ms real) (22915 rewrites/second)
---
search in EXPERIMENT :
  imc08 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true))
      =>
      base-state(FC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (consume : true)) .

*** End of Experiment for imc08 ***
*** Experiment for imc09 ***

*** states: 579896  rewrites: 24480376 in 192794ms cpu
*** (192858ms real) (126976 rewrites/second)
search in EXPERIMENT : imc09 =>* MC such that false .

*** Solution 2380 (state 579886)
*** states: 579887  rewrites: 25059911 in 194375ms cpu
*** (500287ms real) (128925 rewrites/second)
search in EXPERIMENT :
  imc09 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 2380 (state 579886)
*** states: 579887  rewrites: 25128901 in 236470ms cpu
*** (562877ms real) (106266 rewrites/second)
search in EXPERIMENT :
  imc09 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (prog[p(3)]: notYet)
              (prog[p(4)]: notYet) (consume : true))
      =>
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (prog[p(3)]: notYet)
              (prog[p(4)]: notYet) (consume : true)) .

*** Solution 2380 (state 579886)
*** states: 579887  rewrites: 25128496 in 267381ms cpu
*** (647594ms real) (93980 rewrites/second)
search in EXPERIMENT :
  imc09 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (prog[p(3)]: notYet)
              (prog[p(4)]: notYet) (consume : true))
      =>
      base-state(FC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
            (prog[p(2)]: notYet) (prog[p(3)]: notYet)
            (prog[p(4)]: notYet) (consume : true)) .

*** End of Experiment for imc09 ***


*** Experiment for imc10 ***

*** maude.intelDarwin(295,0xac6062c0) malloc:
*** mmap(size=262144) failed (error code=12)
*** error: can't allocate region
*** set a breakpoint in malloc_error_break to debug
*** terminate called throwing an exceptionAbort trap: 6
----
search in EXPERIMENT : imc10 =>* MC such that false .

***
search in EXPERIMENT :
  imc10 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig .

*** Solution 779 (state 5662572)
*** states: 5662573  rewrites: 254245940 in 14463958ms cpu
*** (14540092ms real) (17577 rewrites/second)
search in EXPERIMENT :
  imc10 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (prog[p(3)]: notYet)
              (prog[p(4)]: notYet) (consume : true))
     =>
      base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
              (prog[p(2)]: notYet) (prog[p(3)]: notYet)
              (prog[p(4)]: notYet) (consume : true)) .

***
search in EXPERIMENT :
  imc10 =>* start-state(SC) finish-state(FC) snapshot(SSC) MC
  such that FC =/= empConfig
   /\ base-state(SSC) finish-state(empConfig)
      control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
          (prog[p(2)]: notYet) (prog[p(3)]: notYet)
          (prog[p(4)]: notYet) (consume : true))
    =>
    base-state(FC) finish-state(empConfig)
    control((prog[p(0)]: notYet) (prog[p(1)]: notYet)
          (prog[p(2)]: notYet) (prog[p(3)]: notYet)
          (prog[p(4)]: notYet) (consume : true)) .

*** End of Experiment for imc10 ***
```

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