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Description	

CEO Problem based Analysis of D2D Cooperative User Pairing

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***Abstract*—We interpret the device-to-device (D2D) cooperative relaying framework as an instance of a two-node binary chief executive officer (CEO) problem. Noise-corrupted versions of a binary sequence are transmitted by two nodes to a single destination node over orthogonal multiple access channel. A lower bound of the bit error probability (BEP) is obtained by minimizing a distortion function subject to constraints on inequalities based on the source-channel separation theorem. We derive the rate-distortion function for a binary multiterminal source coding problem. The distortion function is then established by evaluating the relationship between two problems for majority and optimal decision criteria. Our proposed encoding/decoding algorithms using concatenated convolutional codes and joint decoding scheme are used to verify the lower bound of the BEP. The results are interpreted as a criterion to select cooperating D2D user pairs.**

I. Introduction

Cooperative relaying is an enabler for improving the reliability of connectivity. In a multiuser system with several communicating devices, the routing protocol must form pairs of cooperative users to aid each other to obtain reliable communication via relaying. In other words, the device-to-device (D2D) link can be used to enable cooperative transmission to a destination node. In the case that the relay node is not able to reliably decode the transmitted message of the source node, the information theory tells us to use compress-and-forward (CF) or estimate-and-forward (EF) relaying with Wyner-Ziv type compression. If the quality of the source-to-relay (SR) link is good enough, decode-and-forward (DF) type relaying is the optimal cooperation strategy.

For practical reasons like processing simplicity, the DF relaying is often preferred over CF. If successful decoding is not possible, the forwarding can be dropped, which is the conventional DF policy. Another option is to forward the decoded frame with some erroneous bits, a method sometimes called *lossy forwarding* (LF). This methodology can be seen as one suboptimal realization of EF relaying strategy. In the D2D cooperative setting, a simple scheme would be to share the bits locally with uncoded connectivity, because that would avoid the processing complexity and latency imposed by the encoding, decoding and the interleaving. In many delay critical application, the imposed latency cut can be crucial. However, the uncoded communication would make the data sharing even more prone to errors also resulting in the erroneous data forwarding. Therefore, we are interested in analyzing the performance of such LF schemes.

In this paper, we interpret the D2D cooperative relaying as an instance of the *chief executive officer* (CEO) problem, where a CEO aims at reproducing a common source which cannot be directly observed [1]. In the D2D setup, the source and the relay are modeled as the correlated sources and the destination decoder corresponds to the CEO. We focus on the binary case, where a binary source is transmitted by cooperative relaying to a destination node. Our goal of this work is to theoretically provide a lower bound of the bit error probability (BEP) and to compare it to the performance of the practical encoding/decoding algorithms. Subsequently, the error probability can then be used as a criterion to form cooperating D2D user pairs. We derive a theoretical lower bound of the BEP for a two-node communication network which estimates a single binary source over noisy orthogonal multiple access channel (MAC), corresponding the relaying phase of the cooperative relaying. The theoretical lower bound of the BEP is equivalent to minimizing a distortion function subject to a series of inequalities based on the source-channel separation theorem for lossy source coding.

II. Problem Statement

The CEO based system model of estimating a single source through two nodes is depicted in Fig. 1. A common i.i.d source $X^n = \{x^t\}_{t=1}^n$ taking values from a binary set $\mathcal{X} = \{0, 1\}$ with equal probability is forwarded by two nodes to a single destination. In general, both of the sequences observed by the nodes contain errors. The nodes forward the erroneous sequences to the destination, which is referred as LF [2], [3], which in the relay channel is one special instance of EF relaying. The error probabilities $\Pr(x_1^t \neq x^t)$ and $\Pr(x_2^t \neq x^t)$ are denoted as p_1 and p_2, respectively, i.e., $\Pr(z_i^t = 1) = p_i$ for the binary noise sequence $Z_i^n = \{(z_i)^t\}_{t=1}^n$, $i = 1, 2$. At the nodes, the noisy versions $X_1^n = \{(x_1)^t\}_{t=1}^n$ and $X_2^n = \{(x_2)^t\}_{t=1}^n$ of X^n are separately encoded by two joint source channel (JSC) encoders to generate symbol sequences $S_1^{k_1} = \{(s_1)^t\}_{t=1}^{k_1}$ and $S_2^{k_2} = \{(s_2)^t\}_{t=1}^{k_2}$ with coding rates $r_i = n/k_i$, $i = 1, 2$. We focus in most of our discussion to the case where X_1^n is error-free data stream of interest and X_2^n is the decoded and forwarded data stream of the cooperating relay node containing possibly some bit errors. However, the theoretical framework herein is more general.

Both streams are transmitted to the destination over orthogonal MAC (i.e., using time-division multiple- access (TDMA) or some other scheduled multiple-access scheme), as

$$Y_i^{k_i} = h_i \cdot S_i^{k_i} + W_i^{k_i}, i = 1, 2, \qquad (1)$$

where h_i and $W_i^{k_i} = \{w^t\}_{t=1}^{k_i}$ represent the channel gain and the additive white Gaussian noise (AWGN) sequence at the destination, respectively. The destination performs JSC decoding to form estimates $\hat{X}_i^n$ of the sequences X_i^n, $i = 1, 2$. We define the expected Hamming distortion measures $E[\frac{1}{n}\sum_{t=1}^n d(x_i^t, \hat{x}_i^t)]$ to evaluate the error probability $\Pr(x_i^t \neq \hat{x}_i^t)$ with

$$d(x_i^t, \hat{x}_i^t) = \begin{cases} 1, & \text{if} \quad x_i^t \neq \hat{x}_i^t, \\ 0, & \text{if} \quad x_i^t = \hat{x}_i^t. \end{cases} \qquad (2)$$

Finally, the destination reconstructs the source information X^n of which the estimate is denoted as $\hat{X}^n$ based on a decision rule from $\hat{X}_1^n$ and $\hat{X}_2^n$. Therefore, the distortion measure $E[\frac{1}{n}\sum_{t=1}^n d(x^t, \hat{x}^t)]$ can be formulated as a function of $E[\frac{1}{n}\sum_{t=1}^n d(x_i^t, \hat{x}_i^t)]$, $i = 1, 2$, as $D = f(D_1, D_2)$, where function $f(\cdot)$ which is detailed in section IV calculates distortion D based on D_1 and D_2, and

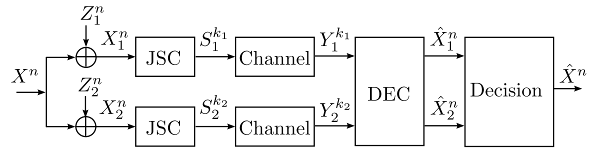

Fig. 1. The abstract system model of estimating a single source through two independent nodes.

$$E[\frac{1}{n}\sum_{t=1}^n d(x_i^t, \hat{x}_i^t)] \leq D_i + \epsilon, \quad i = 1, 2, \qquad (3)$$

$$E[\frac{1}{n}\sum_{t=1}^n d(x^t, \hat{x}^t)] \leq D + \epsilon, \qquad (4)$$

with ϵ representing an arbitrarily small positive number.

According to the source-channel separation theorem for lossy source coding [4], [5], [6], [2], distortion $D = f(D_1, D_2)$ can be achieved if the following inequalities hold:

$$\left.\begin{array}{l} R_1(D_1) \cdot r_1 \leq C(\gamma_1) \\ R_2(D_2) \cdot r_2 \leq C(\gamma_2) \end{array}\right\}, \qquad (5)$$

where $R_i(D_i)$ is the rate-distortion function for the source coding and $C(\gamma)$ is the Shannon capacity[1] with the argument γ denoting the signal-to-noise ratio (SNR) of the channel. Our goal is to derive the theoretical lower bound of the BEP for the system shown in Fig. 1. It is equivalent to the minimal expected Hamming distortion D can be achieved if the conditions shown in (5) are satisfied, as

$$\min_{D_1, D_2} \quad D = f(D_1, D_2) \qquad (6)$$
$$\text{s.t.} \quad (5).$$

To achieve this goal by solving (6), we turn to derive the rate-distortion function $R_i(D_i)$ for the problem shown in Fig. 1 and to establish the function $D = f(D_1, D_2)$ for the decision rule used at the destination.

III. Rate-Distortion Region Analysis

In network information theory, the source coding of the communication system shown in Fig. 1 is modeled by the binary CEO problem. The abstract model of the binary CEO problem is illustrated in Fig. 2. In order to derive the rate-distortion function $R_i(D_i)$, we first reduce the binary CEO problem to a binary multiterminal source coding problem. An outer bound of the rate-distortion

[1]For one dimensional signal, $C(\gamma) = \frac{1}{2}\log_2(1 + 2\gamma)$, and for two dimensional signal, $C(\gamma) = \log_2(1 + \gamma)$ [7].

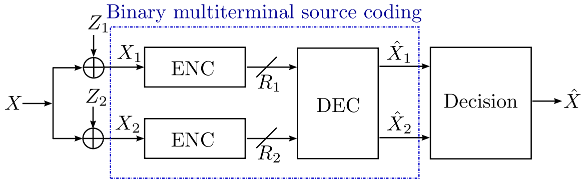

Fig. 2. The abstract model of the binary CEO problem with two independent nodes.

region which is determined by the rate-distortion function $R_i(D_i)$ is then derived for the binary multiterminal source coding problem through the converse proof, as in the Gaussian case [8].

Since X_1^n and X_2^n originate from the same source X^n, the random variable pair $[(x_1)^t, (x_2)^t]$ follows joint probability distribution $\Pr(x_1, x_2)$ given by

$$\Pr_{X_1,X_2}(x_1, x_2) = \begin{cases} \frac{1}{2} \cdot \rho & \text{if } x_1 \neq x_2, \\ \frac{1}{2} \cdot (1-\rho) & \text{otherwise,} \end{cases} \tag{7}$$

where $\rho = \Pr(x_1 \neq x_2)$ is the correlation parameter between the sources X_1 and X_2, i.e., X_2 can be seen as the output of a binary symmetric channel (BSC) with the crossover probability ρ where X_1 is the input, which is also directly the relay channel interpretation. Two encoders separately encode the data sequences X_1^n and X_2^n at rates R_1 and R_2 as

$$\varphi_1 : \mathcal{X}^n \rightarrow \mathcal{M}_1 = \{1, 2, \cdots, 2^{nR_1}\},$$
$$\varphi_2 : \mathcal{X}^n \rightarrow \mathcal{M}_2 = \{1, 2, \cdots, 2^{nR_2}\}.$$

The encoder output sequences $U_1 = \varphi_1(X_1^n)$ and $U_2 = \varphi_2(X_2^n)$ are transmitted to a common receiver. It jointly decodes the received samples to construct the estimates $(\hat{X}_1^n, \hat{X}_2^n)$ of the source pair (X_1^n, X_2^n) denoted as $(\hat{X}_1^n, \hat{X}_2^n) = \psi(\varphi_1(X_1^n), \varphi_2(X_2^n))$. In the relaying context, these transmissions typically take place during different time intervals.

For given distortion values $D_1 \in [0, \frac{1}{2}]$ and $D_2 \in [0, \frac{1}{2}]$, the rate-distortion region $\mathcal{R}(D_1, D_2)$ is defined as

$$\mathcal{R}(D_1, D_2) = \{(R_1, R_2) : (R_1, R_2) \text{ is admissible such that } E\frac{1}{n}\sum_{t=1}^{n} d(X_i^t, \hat{X}_i^t) \leq D_i + \epsilon, i = 1, 2\}.$$

We provide an outer bound $\mathcal{R}^{\text{o}}(D_1, D_2)$ of the rate-distortion region $\mathcal{R}(D_1, D_2)$.

Theorem 1 ([9]): The outer bound is the intersection of three rate-distortion regions, as

$$\mathcal{R}^{\text{o}}(D_1, D_2) = \mathcal{R}_1^{\text{o}}(D_1) \bigcap \mathcal{R}_2^{\text{o}}(D_2) \bigcap \mathcal{R}_{12}^{\text{o}}(D_1, D_2),$$

where

$$\mathcal{R}_1^{\text{o}}(D_1) = \{(R_1, R_2) : \forall R_2 \leq 1 \quad R_1 \geq H_b(\rho * H_b^{-1}(1 - R_2)) - H_b(D_1)\}, \tag{8}$$

with $H_b(a) = -a \cdot \log_2(a) - (1-a) \cdot \log_2(1-a)$ and $H_b^{-1}(a)$ representing the binary entropy function and its inverse function, respectively. It should be emphasized here that $H_b^{-1}(a)$ only takes values from the interval $[0, \frac{1}{2}]$ since distortion is assumed to be within this range. The operator $*$ calculates the binary convolution of the two variables, i.e., $a * b = a(1-b) + b(1-a)$.

$$\mathcal{R}_2^{\text{o}}(D_2) = \{(R_1, R_2) : \forall R_1 \leq 1 \quad R_2 \geq H_b(\rho * H_b^{-1}(1 - R_1)) - H_b(D_2)\}, \tag{9}$$

$$\mathcal{R}_{12}^{\text{o}}(D_1, D_2) = \{(R_1, R_2) : R_1 + R_2 \geq 1 + H_b(\rho) - H_b(D_1) - H_b(D_2)\}. \tag{10}$$

Since regions $\mathcal{R}_1^{\text{o}}(D_1)$, $\mathcal{R}_2^{\text{o}}(D_2)$ and $\mathcal{R}_{12}^{\text{o}}(D_1, D_2)$ are the outside of the regions of $\mathcal{R}(D_1, D_2)$, the following theorem holds

Theorem 2 ([9]): $\mathcal{R}(D_1, D_2) \subseteq \mathcal{R}^{\text{o}}(D_1, D_2)$.

Remark 1: Because we are interested in reconstructing X_1, the set-up is equivalent to the Wyner-Ziv compression problem. It is found that the rate-distortion region of the Wyner-Ziv problem lies inside of $\mathcal{R}_1^{\text{o}}(D_1)$ in this case, i.e., $\mathcal{R}_1^{\text{o}}(D_1)$ is not tight.

In summary, the rate-distortion function $R_i(D_i)$ is given by

$$\begin{cases} R_1(D_1) \geq H_b(\rho * H_b^{-1}(1 - R_2)) - H_b(D_1), \\ R_2(D_2) \geq H_b(\rho * H_b^{-1}(1 - R_1)) - H_b(D_1), \\ \sum_{i=1}^{2} R_i(D_i) \geq 1 + H_b(\rho) - \sum_{i=1}^{2} H_b(D_i). \end{cases} \tag{11}$$

IV. BEP Lower Bound

As stated in Section II, distortion D is a function of distortions D_i, $i = 1, 2$. Function $f(D_1, D_2)$ is obtained by evaluating the relationship between the binary CEO and the binary multiterminal source coding problems in terms of distortions, where the model of the relationship is shown in Fig. 3. $\hat{X}$ is obtained based on the decision rule from the outputs of two parallel BSCs with crossover probabilities $p_1 * D_1$, $p_2 * D_2$ and input X. The distortion D largely depends on the decision rule used by the destination. Here we only consider two decision rules. One is the weighted majority decision and the other the optimal decision.

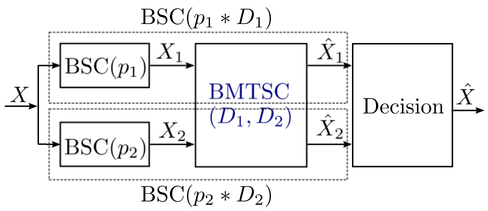

Fig. 3. The distortion model of the binary CEO problem. BMTSC represents binary multiterminal source coding.

Distortion D is obtained by evaluating the probability of an error event. Let $\theta_1 = p_1 * D_1$ and $\theta_2 = p_2 * D_2$. Without loss of generality, we assume that $\theta_1 \leq \theta_2$. Hence, the error event is composed of two independent events: node 1 makes a wrong decision and node 2 makes correct decision or both node 1 and node 2 make erroneous decisions. Therefore, the distortion D in this case is approximated by $D \cong \theta_1 \cdot (1-\theta_2) + \theta_1 \cdot \theta_2 = \theta_1$.

Since the block length is assumed to be infinite and the code is random, an optimal lower bound of the distortion D is determined by utilizing the rate-distortion function for the binary source [7], as

$$1 - H_b(\tilde{d}) = I(X;\hat{X}) \tag{12}$$

$$\leq 1 + H_b(\theta_1 * \theta_2) - H_b(\theta_1) - H_b(\theta_2). \tag{13}$$

Thus, it is obvious from (13) that for $0 \leq \tilde{d} \leq \frac{1}{2}$, $\tilde{d} \geq H_b^{-1}[H_b(\theta_1) + H_b(\theta_2) - H_b(\theta_1 * \theta_2)]$. Therefore, the distortion D is the minimum value of $\tilde{d}$, as

$$D = H_b^{-1}[H_b(\theta_1) + H_b(\theta_2) - H_b(\theta_1 * \theta_2)]. \tag{14}$$

It should be emphasized here that the optimal decision acts as a universal lower bound of the BEP. However, in the design of practical encoding/decoding algorithms, we do not consider this decision rule.

In summary, the distortion level D of the two decision rules described above is given as

$$D = \begin{cases} \min\{\theta_1, \theta_2\}, & \text{majority decision,} \\ H_b^{-1}[H_b(\theta_1) + H_b(\theta_2) - H_b(\theta_1 * \theta_2)], & \text{optimal.} \end{cases} \tag{15}$$

The minimum distortion value $D^\star$ can be shown to be [9]

$$D^\star = \begin{cases} \min\{\theta_1^\star, \theta_2^\star\}, & \text{majority decision,} \\ H_b^{-1}[H_b(\theta_1^\star) + H_b(\theta_2^\star) - H_b(\theta_1^\star * \theta_2^\star)], & \text{optimal,} \end{cases} \tag{16}$$

where $\theta_1^\star$ and $\theta_2^\star$ are $p_1 * D_1^\star$ and $p_2 * D_2^\star$, respectively.

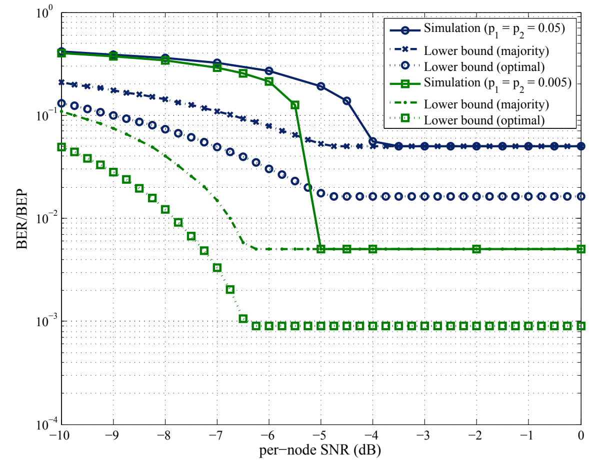

Fig. 4. Symmetric P and SNR. BPSK is used for both nodes.

V. Numerical Examples

The encoding/decoding algorithm which we proposed in [10], [11] is considered. Each node encodes its erroneous sequence by using a serially concatenated memory-1 convolutional code and an accumulator (ACC). The encoder output sequences are then modulated and transmitted to the destination over statistically independent AWGN and block Rayleigh fading channels, where the channel gain h_i is static within each block but varies independently block-by-block. At the destination, iterative decoding process is carried out between the decoders of the convolutional code and the ACC, as well as between the two decoders of the convolutional codes through the LLR updating function f_c to modify the extrinsic LLR, according to the error probabilities p_1 and p_2.

The lower bounds of the BEP for different SNR values γ_1, γ_2 are obtained through solving the convex optimization problem [9] The common parameters used in the simulations are

- Frame length: $n = 10000$ bits for AWGN channels and $n = 2048$ bits for block Rayleigh fading channels.
- The number of frames: 1000 for AWGN channels and 10000 for block Rayleigh fading channels.
- Interleavers: random.
- Encoder C_i: half-rate nonrecursive systematic convolutional code with generator polynomial $G = [03, 02]_8$, where $[\cdot]_8$ represents the argument is an octal number.
- Modulation: binary phase-shift keying (BPSK) and quadrature phase-shift keying (QPSK) with coherent detection, where channel state information is as-

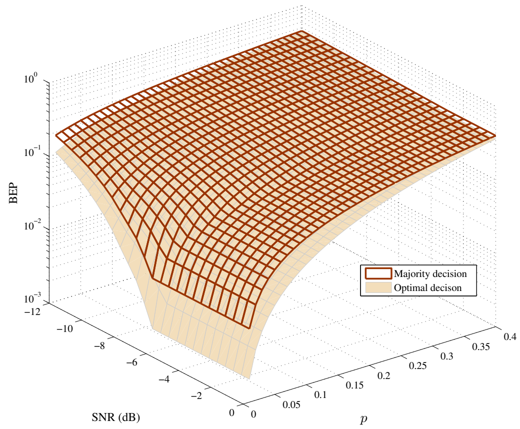

Fig. 5. The lower bound versus the error probabilities p_1, p_2 and SNR between the relay nodes and the destination.

sumed to be known to the receiver. Natural mapping is used as the mapping rule in QPSK [12, Example 18.2].

- Doping ratio P_d: 1 for BPSK and 8 for QPSK.
- Decoding algorithm for DCC_i and ACC^{-1}: log-maximum *a posteriori* (MAP).
- The number of iterations: 30 times.

Fig. 4 shows the error probability lower bounds and the BER versus SNR when p_1, p_2 and SNRs of the two nodes are set identically; this is referred as the symmetric case. It can be found that, the BER curves obtained by simulations and the theoretical lower bounds of the BEP exhibit a similar tendency. Furthermore, it is clearly found that the error floor of the BER obtained by the simulation and the lower bound of the BEP based on majority decision match exactly. The reason is that if the SNRs of two nodes are large enough, the distortion levels D_1 and D_2 are almost 0, which results in the error floor is determined by the error probabilities p_1 and p_2.

Fig. 5 shows the error probability lower bound for the cooperative relaying case as a three dimensional surface vs. SNR and the error probability parameter of the SR link. The BEP value can then be used for system level user pairing optimization.

VI. Conclusion

We examined theoretically the lower bound of the BEP for the binary CEO problem to approximate the BEP of lossy DF based relaying. The results can be used as an approximation to find D2D cooperating user pairs. Further work will focus on finding efficient algorithms for that.

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