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Author(s)	Qian, Shen; Cheng, Meng; Matsumoto, Tad
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# Outage based Power Allocation for a Lossy-Forwarding Relaying System

Shen Qian

School of Information Science  
Japan Advanced Institute of  
Science and Technology (JAIST)  
1-1 Asahidai, Nomi, Ishikawa,  
Japan 923-1292,  
Centre for Wireless Communication  
University of Oulu  
FI-90014 Finland  
Email: shen.qian@jaist.ac.jp

Meng Cheng

Huawei Technologies Co. Ltd.,  
NO. 2222, Xin Jinqiao road, Pudong district,  
Shanghai, China 201206  
Email: chengmeng@jaist.ac.jp

Tad Matsumoto

School of Information Science  
Japan Advanced Institute of  
Science and Technology (JAIST)  
1-1 Asahidai, Nomi, Ishikawa,  
Japan 923-1292,  
Centre for Wireless Communication  
University of Oulu  
FI-90014 Finland  
Email: tadashi.matsumoto@ee.oulu.fi

**Abstract**—In this paper, an optimal power allocation scheme for a simple lossy-forwarding relaying system is provided. Here, we extend our previous work of optimal power allocation of Slepian-Wolf relay system with constant source-relay error probability, to such that block Rayleigh fading is assumed for all transmission channel and lossy transmission is assumed in source-relay channel. Since data sequences transmitted from source and relay nodes are sent from one original source, they are correlated. At destination node, by exploiting the correlation knowledge between the two data sequences in joint decoding, the performance of the whole system can be significantly enhanced. This is because the network can be seen as a distributed Turbo code. A closed-form expression of the outage probability is derived at high signal-to-noisy ratios regime. It is shown that the outage curves calculated from the closed-form expression are sufficiently close to that obtained by the numerical calculation. Then, the optimum power allocation to the nodes for the system can be formulated as a convex optimization problem. Specifically, we minimize the outage probability while assuming the total transmit power is fixed, and also to minimize the total power under an given outage requirement. It is found that the system performance with the proposed optimum power allocation scheme outperform that with equal power allocation.

## I. INTRODUCTION

Next generation wireless communication networks are expected to accommodate massive mobile nodes, e.g., Internet of Things (IoT), Machine-to-Machine (M2M) communications. With the explosion of the number of network nodes, higher transmit power efficiency is demanded. Therefore, researchers are always seeking for new ways of node cooperation techniques which enable further improvement of the transmission energy- and spectrum-efficiency.

In [1], a novel lossy-forwarding relay (LosFoR) system allowing source-relay (S-R) channel errors is proposed. The information sequence, obtained as the results of relay decoding, is interleaved, re-encoded and transmitted to the destination, even it includes errors occurring in the S-R transmission (lossy-forwarding). Since the information sequences received at the destination (D) transmitted from the source (S) and the relay (R) are from same original source, they are correlated. At

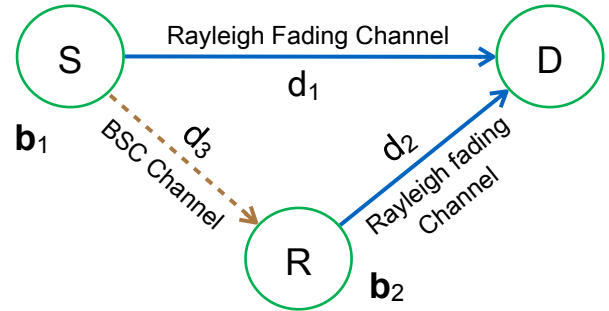


Fig. 1. Single relay transmission system.

the destination, the S-R link error probability is estimated, and used as a correlation knowledge between the information sent from source and relay. Under this mechanism, iterative processing utilizing log-likelihood ratio (LLR) function [2] reduce the decoding error, which achieves significant gain over the conventional decode-and-forward (DF) strategy [3]. This is because interleaving process at R assumed in this paper changes the network, as a whole, can be seen as a distributed turbo code.

The theoretical outage bound of the LosFoR system is derived over block Rayleigh fading channels [4]. The outage probability is expressed by a double integral of the probability density function (PDF) with respect to the admissible Slepian-Wolf rate region [5]. Furthermore, the power allocation is analysed in [6]. In [4] and [6], the S-R channel is assumed to be represented by a static binary symmetric channel (BSC) channel model, which a constant bit error probability after decoding at the relay as a parameter represents the S-R correlation. By manually adjusting the value of the parameter, the impact of the correlation on the whole system performance can be observed. Although the effects of the S-R link variation is not considered, it is quite natural that the S-R link also suffers from the fading variation.

Since the static BSC assumption has a obviously drawback which is less realistic. Because the S-R channel variation also

follows fading distributions. Quite recently, a more practical model is proposed in [3], which considers the error probability after decoding at the relay is a random variable which changes according to the S-R variation. By imposing Shannon's lossy source-channel separation (SCS) theorem, the relationship between the S-R channel error probability and its corresponding instantaneous signal-to-noise ratio (SNR) is established. However, [3] only adopts equal power allocation for each transmitter (at both S and R), and the results are just obtained by numerical computations for multi-fold integrals. Moreover, another important performance metric – geometrical gain, which is very important for protocol designing – has not been related to power allocation analysis of the LosFoR scheme.

The major purpose of this paper is to analyse the optimum power allocation of the LosFoR system in more practical scenarios, where all channels experience fading variation. By assuming high SNR values for such a system scenario, closed-form expression of the outage probability is mathematically derived. It is shown that the power allocation for the proposed LosFoR system can be formulated as a convex optimization problem. The optimum power allocation scheme for minimizing the outage probability is then presented, where account is taken of the geometrical gain. We found that the proposed optimal power allocation scheme improve outage performance compared with the equal power allocation, when R is located close to D.

The rest of this paper is organized as follows: First of all, the system model used to analyze the outage based power allocation of the LosFoR system is described in Section II. The outage probability derivation based on the Shannon's lossy SCS theorem presented in Section III. The derivation of the closed-form expression is shown in Section IV. Convexity proof and optimal power allocation analysis are also presented in Section IV. Finally, this paper is concluded in Section V with some concluding remarks.

## II. SYSTEM MODEL

In this paper, a very simple three-node relay transmission model is considered, as shown in Fig. 1. During the first time slot, the original information sequence  $\mathbf{b}_1$  is broadcasted from the S to both R and D. As in the decode-and-forward (DF) scheme, the original information sequence is decoded and re-constructed at R, before being forwarded as  $\mathbf{b}_2$ . In contrast to the conventional DF system, our proposed technique always interleaves the re-constructed sequence at R, and then re-encodes and maps it for the second time slot transmission. It should be noted that the re-constructed bit sequence may contain some errors, due to the imperfectness transmission of S-R channel, but it is still correlated with the original one sent from S.

After receiving signals from both S and R, joint decoding process takes place at D. It is already known that by exploiting the correlation knowledge between the bit sequences sent from S and R, the minimum rate required for lossless transmission is no longer equal to the entropy of each source, but could be even lower, as stated by the Slepian-Wolf theorem. In other words, if we keep transmitting the redundancy (the correlated portion), the whole system performance will benefit from utilizing the correlation knowledge.

All the three transmission channels are assumed to suffer from independent block fading. Hence, it is reasonable to use the BSC model [2], with the crossover probability  $p_e$  as a parameter within one block. Hence,  $\mathbf{b}_2 = \mathbf{b}_1 \oplus \mathbf{e}$ , where  $\oplus$  denotes modulo-2 addition and  $\mathbf{e}$  is a binary error vector.  $\Pr(e = 1) = p_e$  is fixed within each block, and the  $p_e$  value changes block-by-block.  $p_e = 0$  indicates the perfect decoding at R, while  $p_e \neq 0$  indicates there exist S-R link errors which could not be eliminated by the decoding process at R.

In this paper, all the links are assumed to be suffering from block Rayleigh fading which the probability density function (PDF) with instantaneous SNR  $\gamma_i$  is given by

$$p(\gamma_i) = \frac{1}{\Gamma_i} \exp\left(-\frac{\gamma_i}{\Gamma_i}\right), (i = 1, 2, 3), \quad (1)$$

where  $\Gamma_i$  represent received average SNRs of the source-destination (S-D), relay-destination (R-D) and S-R channels, respectively.

With  $d_i$  ( $i=1,2,3$ ) representing the length of S-D, R-D and S-R channels respectively. With the geometrical gain of the S-D channel,  $G_1$ , being normalized to one, the geometric gains of R-D and S-R channels,  $G_2$  and  $G_3$ , can be defined as [7]

$$\begin{aligned} G_2 &= \left(\frac{d_1}{d_2}\right)^\alpha, \\ G_3 &= \left(\frac{d_1}{d_3}\right)^\alpha, \end{aligned} \quad (2)$$

where the path-loss factor  $\alpha$  is set at 3.52 [8]. For sake of simplicity, the variation due to shadowing is ignored in this paper.

## III. OUTAGE PROBABILITY DERIVATION

Based on Shannon's lossy SCS theorem, for S-R channel, we have  $R_3(\mathcal{D})R_{c3} \leq C(\gamma_3)$ , where  $R_3(\mathcal{D})$  is the rate function of distortion  $\mathcal{D}$  with  $\mathcal{D} = p_e$ ,  $R_{c3}$  denotes the spectrum efficiency including the channel coding scheme and the modulation multiplicity and  $C(\gamma_3)$  is the capacity with instantaneous SNR  $\gamma_3$  of S-R channel. For hamming distortion measure,  $R_3(\mathcal{D}) = 1 - H(\mathcal{D})$ .

With the relationship between  $p_e$  and  $\gamma_3$ , the rate constrains supported by the Slepian-Wolf theorem are given by [3]

$$\begin{aligned} R_1 &\geq H(\mathbf{b}_1 | \mathbf{b}_2), \\ R_2 &\geq H(\mathbf{b}_2 | \mathbf{b}_1), \\ R_1 + R_2 &\geq H(\mathbf{b}_1, \mathbf{b}_2) \\ &= 1 + H(p_e) \\ &= 1 + H\left[R^{-1}\left(\frac{C(\gamma_3)}{R_{c3}}\right)\right], \end{aligned} \quad (3)$$

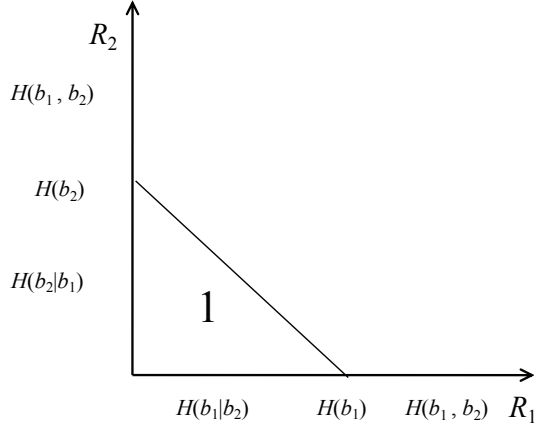
where  $R_1$  and  $R_2$  are source rate for  $\mathbf{b}_1$  and  $\mathbf{b}_2$ , respectively.  $H(\cdot|\cdot)$  and  $H(\cdot, \cdot)$  denote the conditional entropy and the joint entropy between the arguments, respectively.

Actually, the Slepian-Wolf theorem, which intends to recover two correlated sources, is not perfectly suitable for the system setup in this paper since the destination only aims to reconstruct the information sent from S. However, the difference between exact outage derived from theorem for source coding with side information and the approximation

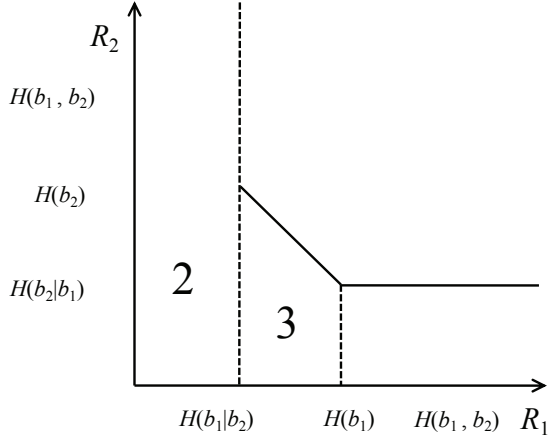
outage derived from Slepian-Wolf theorem is negligible [3]. For simplicity reason on calculation, in this paper, the outage probability is calculated based on Slepian-Wolf theorem.

Eq. (3) can be understood from the viewpoint of the Slepian-Wolf rate region presented in Fig. 2. According to the value of  $R_3(\mathcal{D})$ , the rate constrains can be divided into two cases for discussing: 1)  $R_3(\mathcal{D}) \geq 1$  ( $p_e = 0$ ), implying that perfect decoding is conducted at R. In this case, the outage event happens when  $R_1$  and  $R_2$  are within Part 1 shown in Fig. 2(a). The probability which  $R_1$  and  $R_2$  fall into Part 1 is denoted as  $P_1$ ; 2)  $0 \leq R_3(\mathcal{D}) < 1$  ( $0 < p_e \leq 0.5$ ), indicating that decoding at the relay is imperfect. S-R channel SNR can not support error-free transmission. Then, the outage event occurs when  $R_1$  and  $R_2$  are within Part 2 and Part 3 shown in Fig. 2(b), which the probabilities is denoted by  $P_2$  and  $P_3$ , respectively. Hence, the total outage probability of the LoFoR system can be defined by

$$P_{out} = P_1 + P_2 + P_3. \quad (4)$$



(a) Lossless transmission of the S-R channel,  $p_e = 0$ .



(b) Lossy transmission of the S-R channel,  $0 < p_e \leq 0.5$ .

Fig. 2. Slepian-Wolf rate regions of the proposed relay system.

It is found that the outage probability can be calculated by using a triple integral with respect to the joint pdf of

the instantaneous SNRs  $p(\gamma_1, \gamma_2, \gamma_3)$  [3], given the range defined in (3). Since we assume that the variation of each link instantaneous SNR is statistically independent,

$$p(\gamma_1, \gamma_2, \gamma_3) = p(\gamma_1)p(\gamma_2)p(\gamma_3), \quad (5)$$

With the assumption that Gaussian codebook is used, the relationship between the instantaneous channel SNR  $\gamma_i$  and its corresponding source rate  $R_i$  is given by

$$R_i \leq \frac{E^n}{2R_{ci}} \log_2 \left( 1 + \frac{2\gamma_i}{E^n} \right), (i = 1, 2, 3) \quad (6)$$

where  $E^n$  ( $= 2$  assumed in this paper) denotes the signalling dimensionality.

In practical systems, the channel capacity is upper bounded by the multiplicity of modulation. However, the difference between the constellation constrained capacity (CCC) and Gaussian capacity is very small in low SNR region. While in high SNR region, distortion  $\mathcal{D}$  level is very close to 0, resulting in  $p_e = 0$ . The difference between CCC and Gaussian capacity does not have any significant impact on  $\mathcal{D}$  for the entire SNR region. Therefore, it reasonable to replace CCC by Gaussian capacity for outage derivation.

By converting the rate constrains into the instantaneous SNR constrains [4], the theoretical calculation of the outage probability  $P_1$ ,  $P_2$  and  $P_3$  can then be mathematically expressed as,

$$\begin{aligned} P_1 &= \Pr [0 < R_1 < H(\mathbf{b}_1), 0 < R_2 < H(\mathbf{b}_1, \mathbf{b}_2), R_3 \geq 1] \\ &= \int_{\gamma_1=0}^1 \int_{\gamma_2=0}^{2^{1-\log_2(1+\gamma_1)}-1} \int_{\gamma_3=1}^{\infty} p(\gamma_1)p(\gamma_2)p(\gamma_3)d\gamma_1d\gamma_2d\gamma_3 \\ &= \frac{1}{\Gamma_1} \exp\left(-\frac{1}{\Gamma_3}\right) \int_1^{\gamma_1=0} \exp\left(-\frac{\gamma_1}{\Gamma_1}\right) \\ &\quad \cdot \left[1 - \exp\left(-\frac{2^{1-\log_2(1+\gamma_1)}-1}{\Gamma_2}\right)\right] d\gamma_1, \end{aligned} \quad (7)$$

$$\begin{aligned} P_2 &= \Pr [0 < R_1 < H(\mathbf{b}_1 | \mathbf{b}_2), R_2 > 0, 0 \leq R_3 < 1] \\ &= \int_{\gamma_1=0}^{2^{H(p_e)}-1} \int_{\gamma_2=0}^{\infty} \int_{\gamma_3=0}^1 p(\gamma_1)p(\gamma_2)p(\gamma_3)d\gamma_1d\gamma_2d\gamma_3 \\ &= \frac{1}{\Gamma_3} \int_{\gamma_3=0}^1 \exp\left(-\frac{\gamma_3}{\Gamma_3}\right) \\ &\quad \cdot \left[1 - \exp\left(-\frac{2^{1-\log_2(1+\gamma_3)}-1}{\Gamma_1}\right)\right] d\gamma_3, \end{aligned} \quad (8)$$

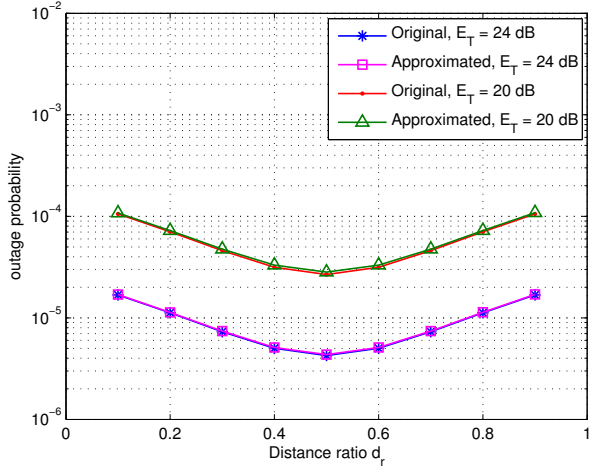


Fig. 3. Accuracy of the approximated closed-form outage expression.

$$\begin{aligned}
P_3 &= \Pr[H(\mathbf{b}_1 | \mathbf{b}_2) < R_1 < H(\mathbf{b}_1), \\
&\quad 0 < R_2 < H(\mathbf{b}_1, \mathbf{b}_2) - R_1, 0 \leq R_3 < 1] \\
&= \int_{\gamma_1=2^{H(p_e)}-1}^1 \int_{\gamma_2=0}^{2^{1+H(p_e)}-\log_2(1+\gamma_1)-1} \\
&\quad \int_{\gamma_3=0}^1 p(\gamma_1)p(\gamma_2)p(\gamma_3)d\gamma_1d\gamma_2d\gamma_3, \\
&= \frac{1}{\Gamma_1} \frac{1}{\Gamma_3} \int_{\gamma_1=2^{1-\log_2(1+\gamma_3)}-1}^1 \int_{\gamma_3=0}^1 \exp\left(-\frac{\gamma_1}{\Gamma_1}\right) \exp\left(-\frac{\gamma_3}{\Gamma_3}\right) \\
&\quad \cdot \left[1 - \exp\left(-\frac{2^{1-\log_2(1+\gamma_3)}-\log_2(1+\gamma_1)-1}{\Gamma_2}\right)\right] d\gamma_1d\gamma_3. \tag{9}
\end{aligned}$$

The values of  $P_1$ ,  $P_2$  and  $P_3$  can be calculated by numerical method [9], with accurate enough numerical calculation error control.

#### IV. OPTIMAL POWER ALLOCATION

The goal of this section is to minimize the outage probability obtained at previous section, while the total transmit power  $E_T$  is fixed and the noise variance  $\sigma_n^2$  of each channel is normalized to unity. The transmit power, which hence equivalent to their corresponding average SNR, allocated to S and R are denoted as  $kE_T$  and  $(1-k)E_T$ , respectively, where  $k$  ( $0 < k < 1$ ) is power allocation ratio. However, the specific power allocation strategies are out of the scope of this paper.

While keeping the ratio  $k$  constant and bring the average SNRs  $\Gamma_1$ ,  $\Gamma_2$  and  $\Gamma_3$  to infinity, by invoking the property of exponential function  $e^{-x} \approx 1-x$  for small  $x$ , at the high SNR regime, the the outage probability (4) can be approximated to a closed-form expression, as,

$$\begin{aligned}
P_{out} &\approx \frac{0.5}{E_T^2 k^2 G_1^2} + \frac{0.5}{E_T^2 k^2 G_3^2} + \frac{1.3863}{E_T^2 k^2 G_1 G_3} \\
&\quad + \frac{0.3863}{E_T^2 k(1-k)G_1 G_2}. \tag{10}
\end{aligned}$$

To identify the accuracy of the approximation, outage probability curves obtained by using the approximated expression (10), and by numerical calculating the (7)-(9) are presented in Fig. 3. The relay node is assumed to be allocated in the line between the source and the destination, and the distance ratio is defined by  $d_r = d_2/d_1$ . Good matching is illustrated between the curves of approximation and numerical calculation, which indicates sufficient accuracy of the approximation. Then, the optimal power allocation problem is formulated by a standard optimization problem, which can be proven to be convex.

#### A. Proof of Convexity

Here we give the proof of the convexity of the approximated outage probability expression. Eq. (10) is comprised of four terms. If each of the four terms can be proven to be convex, (10) is also convex because it is a sum of the convex terms. The Hessian matrix of the first term  $\frac{0.5}{E_T^2 k^2 G_1^2}$  can be calculated as

$$\mathbf{H} \left[ \frac{0.5}{E_T^2 k^2 G_1^2} \right] = \frac{1}{E_T^2 G_1^2 k^2} \begin{bmatrix} \frac{3}{k^2} & \frac{2}{E_T k} \\ \frac{2}{E_T k} & \frac{3}{E_T^2} \end{bmatrix}. \tag{11}$$

The eigenvalues of (11)

$$\lambda_{1,2} = \frac{1}{2E_T^2 G_1^2 k^2} \left( \frac{3}{k^2} + \frac{3}{E_T^2} \pm \sqrt{\left(\frac{3}{k^2} + \frac{3}{E_T^2}\right)^2 - \frac{20}{E_T^2 k^2}} \right) \tag{12}$$

are clearly non-negative. Therefore, the Hessian matrix of  $\frac{0.5}{E_T^2 k^2 G_1^2}$  is positive semi-definite and hence its convexity has been proven.

The Hessian matrix of the second term  $\frac{0.5}{E_T^2 k^2 G_3^2}$  can be calculated as

$$\mathbf{H} \left[ \frac{0.5}{E_T^2 k^2 G_3^2} \right] = \frac{1}{E_T^2 G_3^2 k^2} \begin{bmatrix} \frac{3}{k^2} & \frac{2}{E_T k} \\ \frac{2}{E_T k} & \frac{3}{E_T^2} \end{bmatrix}. \tag{13}$$

The eigenvalues of (13)

$$\lambda_{1,2} = \frac{1}{2E_T^2 G_3^2 k^2} \left( \frac{3}{k^2} + \frac{3}{E_T^2} \pm \sqrt{\left(\frac{3}{k^2} + \frac{3}{E_T^2}\right)^2 - \frac{20}{E_T^2 k^2}} \right) \tag{14}$$

are clearly non-negative. Therefore, the Hessian matrix of  $\frac{0.5}{E_T^2 k^2 G_3^2}$  is also positive semi-definite and hence its convexity has been proven.

Similarly, the Hessian matrix of the third term  $\frac{1.3863}{E_T^2 k^2 G_3 G_1}$  can be calculated as

$$\mathbf{H} \left[ \frac{1.3863}{E_T^2 k^2 G_3 G_1} \right] = \frac{1}{E_T^2 G_3 G_1 k^2} \begin{bmatrix} \frac{8.3178}{k^2} & \frac{5.5452}{E_T k} \\ \frac{5.5452}{E_T k} & \frac{8.3178}{E_T^2} \end{bmatrix}. \tag{15}$$

The eigenvalues of (15)

$$\begin{aligned}
\lambda_{1,2} &= \\
&\frac{1}{2} \left( \frac{8.3178}{k^2} + \frac{8.3178}{E_T^2} \pm \sqrt{\left(\frac{8.3178}{k^2} + \frac{8.3178}{E_T^2}\right)^2 - \frac{153.7468}{E_T^2 k^2}} \right) \tag{16}
\end{aligned}$$

TABLE I. OPTIMAL POWER RATIO  $k$ 

$d_r$	optimal $k$ ( $E_T = 16$ dB)
0.1	0.500
0.2	0.502
0.3	0.520
0.4	0.574
0.5	0.666
0.6	0.774
0.7	0.871
0.8	0.943
0.9	0.985

are clearly non-negative. Therefore, the Hessian matrix of  $\frac{1.3863}{E_T^2 k^2 G_3 G_1}$  is positive semi-definite and hence its convexity has been proven.

The Hessian matrix of the fourth term  $\frac{0.3863}{E_T^2 k(1-k)G_1 G_2}$  can be calculated as

$$\mathbf{H} \begin{bmatrix} 0.7726 \\ \frac{1}{E_T^2 k(1-k)G_1 G_2} \end{bmatrix} = \frac{1}{k^3(1-k)^3 E_T^4 G_1 G_2} \begin{bmatrix} E_T^2(3k^2 - 3k + 1) & E_T k(2k^2 - 3k + 1) \\ -E_T k(2k^2 - 3k + 1) & 3k^2(k^2 - 2k + 1) \end{bmatrix}. \quad (17)$$

Let  $u(k) = 3k^3(k^2 - 2k + 1) + E_T^2(3k^2 - 2k + 1)$  and  $v(k) = 3k^2 E_T^2(3k^3 - 3k + 1)(k^2 - 2k + 1)$ , the eigenvalues of (17) can be calculated as

$$\lambda_{1,2} = \frac{u(k) \pm \sqrt{u(k)^2 - 4v(k)}}{2}. \quad (18)$$

For  $0 < k < 1$ , the value of the quadratic polynomials  $k^2 - 2k + 1$  and  $3k^2 - 2k + 1$  in  $u(k)$ , and the quadratic polynomials  $3k^3 - 3k + 1$  and  $k^2 - 2k + 1$  in  $v(k)$  are large than 0. This indicates the non-negativity of  $u(k)$  and  $v(k)$ . Hence, the Hessian matrix of  $\frac{0.3863}{E_T^2 k(1-k)G_1 G_2}$  is proven to be positive semi-definite and hence its convexity has been proven. Therefore, the outage probability expression (10) can be proven is a convex function.

### B. Optimal Power Allocation: Total Power Fixed

In this subsection, we minimize the outage probability while the total power  $E_T$  is fixed. The convex problem can be formulated as

$$\begin{aligned} & \text{minimize} && P_{out}(k, E_T) \\ & \text{subject to} && k - 1 < 0 \\ & && -k < 0. \end{aligned} \quad (19)$$

The solution to the optimization problem can be obtained by using a convex optimization tool, the optimal values of  $k$  can be obtained as shown in TABLE I. Obviously, the larger the  $d_r$  value, the more transmit power should be allocated to S.

Theoretical results for the outage performance performance with and without optimal power allocation are shown in Fig. 4, where the total transmit power  $E_T$  is set at 16 dB. First of all, it can be obviously seen that with equal power allocation, the outage curve is symmetric to the mid-point between S and D, which yields the lowest outage probability. This founding is different from the conventional DF case, of which R should

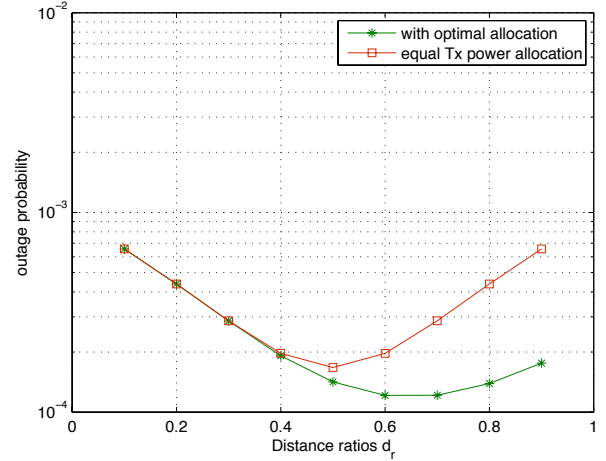


Fig. 4. Outage probabilities with and without optimal power allocation, total transmit power fixed.

TABLE II. OPTIMAL POWER RATIO  $k$  WITH FIXED  $P_{out}$  ( $d_r=0.6$ )

required $P_{out}$	required $E_T$ (equal power)	required $E_T$ (optimal power)	Gain
0.001	12.45 dB	11.35 dB ( $k=0.774$ )	1.1 dB
0.0001	17.45 dB	16.40 dB ( $k=0.774$ )	1.05 dB
0.00001	20.50 dB	21.42 dB ( $k=0.774$ )	1.08 dB

be located at a point closer to S in order to achieve the best performance [3]. The optimal power allocation is that the lowest outage probability can be achieved at the point where the contributions of both S-R and R-D channels to outage are balanced. In the other word, the proposed technique enables the transmitter to find proper relays for cooperation in a broader area. It should be noted that, by selecting the optimal power allocation ratio  $k$ , the system can much reduce the outage probability compared with the case of the equal power allocation when the relay is allocated near to the destination, while they achieve almost the same outage when relay is near to the source.

### C. Optimal Power Allocation: Outage Probability Requirement Fixed

In this sub-section, we investigate how much total transmit power can be saved through optimal power allocation, given a fixed outage probability requirement. The problem can be formulated as

$$\begin{aligned} & \text{minimize} && E_T + 0k \\ & \text{subject to} && P_{out}(k, E_T) - T_{out} \leq 0 \\ & && k - 1 < 0 \\ & && -k < 0 \\ & && -E_T < 0, \end{aligned} \quad (20)$$

where  $T_{out}$  is required outage probability. The convexity of (20) is proven in sub-section IV-A.

Tables II and III show the required total transmit power with equal and optimal power allocations for outage requirements, where  $d_r$  is 0.6 and 0.7, respectively. The outage probability requirements are set as  $P_{out} = 0.00001$ , 0.0001 and 0.001, respectively. It is clearly seen that by selecting



TABLE III. OPTIMAL POWER RATIO  $k$  WITH FIXED  $P_{out}$  ( $d_r=0.7$ )

required $P_{out}$	required $E_T$ (equal power)	required $E_T$ (optimal power)	Gain
0.001	13.25 dB	11.47 dB ( $k=0.871$ )	1.88 dB
0.0001	18.30 dB	16.42 dB ( $k=0.872$ )	1.88 dB
0.00001	23.31 dB	21.44 dB ( $k=0.872$ )	1.87 dB

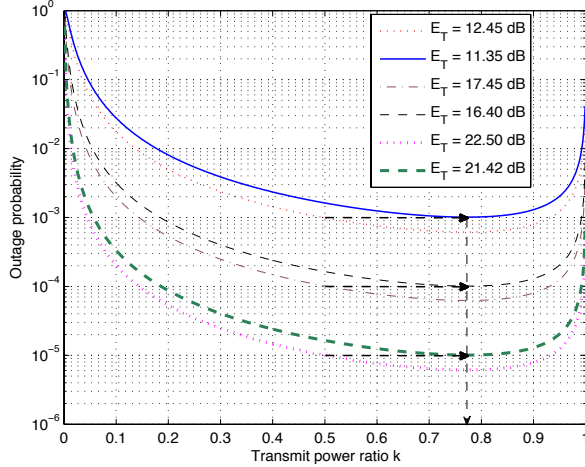


Fig. 5. Theoretical outage probabilities versus the optimal power ratio  $k$  with different total transmit powers,  $d_r = 0.6$ .

the optimal  $k$  values, roughly 1~2 dB gain can achieved by optimal power allocation compared with that with equal power allocation. It is also found that, given the relay location, the transmit power gains with the optimal power allocation over the equal power allocation are almost the same, independently of the outage requirements. As shown in Fig. 5 and Fig. 6, the optimal  $k$  values corresponding to outage requirements are exactly consistent to the the data obtained as the solution to the optimization problem.

## V. CONCLUSION

This paper has investigated the optimal power allocation for a LosFoR system based on outage probability analysis. In the proposed system model, besides the S-D and R-D channels, the S-R channel is also assumed to suffer from block Rayleigh fading and is modelled by the rate distortion function in lossy scenario. An approximated closed-form expression of outage probability for the LosFoR system has been derived, where account is taken of the geometrical gain. As shown in the results, the proposed approximation can ensure sufficient accuracy of the outage probability calculation. Based on the expression, the optimal power allocation scheme has been formulated to an convex problem for minimizing the outage probability while keeping the total transmit power constant, and for minimizing the transmit power while keeping the outage probability fixed. Compared with equal power allocation, by adjusting the power allocated to S and R, lower outage probability can be achieved. Moreover, it has been shown that when the relay is near to D, the proposed optimization scheme achieved much lower outage probability than that with the equal power allocation.

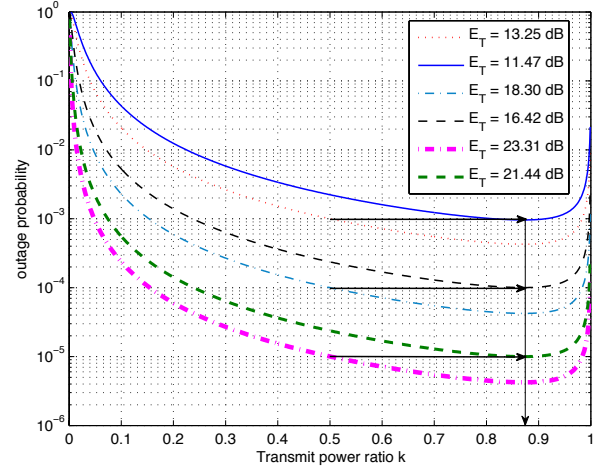


Fig. 6. Theoretical outage probabilities versus the optimal power ratio  $k$  with different total transmit powers,  $d_r = 0.7$ .

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