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Expansion of Game Refinement Theory into Continuous Movement Games with Consideration on Functional Brain Measurement

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Abstract

Game theory has expanded far and beyond its original contexts into all manner of subjects. There are still many unsettled questions at both ends of the spectrum of applied, normative uses, and theoretical, descriptive inquiry. A wide range of scientific inquiry explores the domain of games for various purposes from that of pure play to serious business, and all of the many combinations thereof. Among these, game refinement theory is a child of the computer chess problem, and a close relative of artificial intelligence for games.

Survey work of game theory, game refinement theory, the game progress model, and functional brain imaging for gamers during gaming is briefly undertaken throughout the relevant sections. The introductory chapter presents considerations on the study of recreational games and strategic interplay, and some of the problems facing game refinement theory. Namely, there is a lack of experimentation to test the theory that information accelerates in the mind of players and observers as the game progresses.

In Chapter 2, game theory as the game player's paradigm is discussed. Some of the tools which have been adopted for use in the study of recreational gaming are examined, and game refinement theory is explained in the framework of its theoretical relatives.

In Chapter 3, game refinement theory as the game maker's paradigm is discussed. Past studies of the state of AI for board games are mentioned and updated. The game refinement model and prior works are broken down, and game refinement values for various games and game types are compared. Also, recent work in the search for reasonable quantities to relate the model of discrete board game measures to that of continuous movement games is presented. The results of sub-studies and experiments relating discrete elements of non-discrete games in the board game format are considered. A bridging principle is now sought for guidance to help prove the acceleration of information in the brain during games. Game information dynamic theory makes a bold claim that information flow is governed by physical laws of motion. Without denying or supporting this claim, it is explored briefly with consideration on the principle bridging information and hydrodynamics.

Preliminary work in functional brain measurement of gamers during gaming is presented in Chapter 4 with potential for becoming a useful component of verification for game refinement theory. The intersection of fNIRS brain measurement and games is an expanding field with excellent potential for game scientists. Prior studies have been carried out within the established protocols and frameworks of cognitive neuroscience. The well-developed model of games as a vehicle of experimentation for neuroscientists is being established, and games are recommended for those engaged in brain studies, as with brain activity measurement for those in the research of AI and games.

Lastly, results and implications from observations in continuous movement games are discussed along with the fitness of the model. Rule changes in most of the observed games show game refinement values for continuous movement games gravitating towards that seen in early work in board games. Theorists have been careful not to venture why the phenomenon of a game refinement window appears, noting a dearth of knowledge of the physics of information in the mind. Game information dynamics proposes that information might have measurable physical properties. Considering both these thoughts, the next experiments using functional brain imaging are outlined with the intention of capturing evidence of game information accelerating at game's end in the brains of gamers.

Keywords: *game theory, game refinement theory, sports, brain imaging,*

fNIRS

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 The Role of Games

Play is an artifact of society that precedes culture and even transcends species. It is supposed that the necessary condition of being able to form rules logically places the existence of games before human civilization itself [39]. Evidence of games, sometimes quite elaborate, have been found among the most ancient of civilizations, and pretty much anywhere humans go [8]. Not only the hunting game, combatives and the sometimes deadly stadium games, but also the more benign leisure sports, dice and board games have developed steadily with mankind [107] [109]. Many games and variations have been enjoyed by elite and common peoples throughout civilization. Games have a strong civilizing influence, and in turn, games change to meet the sophistication of the players and watchers.

There are several established frameworks for evaluating the decisions and the information in games. For the purposes of this study they are referred to collectively as the game sciences; among them are game theory and game refinement theory. Game theory concerns foremost with the optimization of individual player strategy. Game refinement theory was developed after several decades of game theory to consider the optimization not of individual play, but rather to consider the optimization of games. Game refinement theory shows that the evolution of board games is related to the complexity and delivery of game information, and also proposes that information accelerates at game's end. What each of these admittedly quite different approaches have in common is their aim of a better understanding of the decision-making process under varying states of uncertainty. Original game theory suffered from a distrust or misunderstanding of experimental verification, while its pioneers continued to develop more and more games and models. Although there may have been some small dissatisfaction with it, for over 40 years expected utility theory provided the basic proof for one of game theory's necessary assumptions. A player is assumed to be rational, which in one sense means that if presented games with identical payoffs, the player should make the same choice each time. What was found in Kahneman and Tversky's experiment in 1979 [54], however, was that players' choices are highly dependent upon context in which they are presented. This discussion of rationality leads to the normative branch, which has derived many new discoveries from experimentation with games, perhaps especially since [54].

Claude Shannon is often credited as the father of the information age, but he also began the age of computer gaming with a 1950 paper outlining a sample evaluation function for chess. What ensued was said to be the world's longest running computer experiment, concluding (more or less) in 1997 with the triumph of Deep Blue over Gary Kasparov. Now with the benefit of time passed, the so-named Computer Chess Challenge appears like a crucible for the richness of scientific and computing resources and brilliant minds that worked on, and were developed by it. Because of its development with the computer chess problem, recreational game science was almost synonymous with AI in the latter half of the 20th century. Having begun from the perspective of the board game, game refinement theory pursues a line of inquiry into the study of game information comprising all games. AI for games is best considered along with game refinement theory in the game maker's paradigm. Both are concerned with the final product of enjoyability for players, and both require the dissection and accounting of whole games from the point of view of the game designer. While the pioneers of computer gaming and game refinement theory have not been averse to experimentation, it can be seen that game refinement theory requires a different kind of experimentation in order to verify its claims and expand into stronger, more general applications.

In the centuries and millennia of chess' history, several main and many minor variants developed across wide swathes Asia and Europe. Only a few out of many remain [69]. As it was shown in Iida et al. [42] the surviving variants had undergone long processes of sophistication of game rules and customs. They were optimized for entertainment which made the depth of look-ahead more critical to the game outcome [16]. Like tic-tac-toe, games lacking sufficient complexity quickly lose their appeal. Of course the opposite problem also exists, and experienced players have noted that excessively complex games also cease to be interesting at some point. This concept was further explored with incongruity theory applied to games by Lankveld, et al. [59]. The process called game sophistication [16], according to the metric called game refinement, offers chess as a model for well-balanced search space complexity and depth of look-ahead optimized for entertainment.

1.2 Problem Statement

Trying to select from among the trove of possible metrics of a game is a durable problem for sports statisticians. Those making evaluations might be coaches, players, parents, or maybe a fan trying to decide whether or not to spend part of her paycheck on a ticket to a game. Regardless of being high level professionals or local recreational enthusiasts, often pride or money are at stake–even small amounts of which have proven to be worthy of strictest competition. It is essential to have good data and methods in order analyze and be able to make good decisions based on those measures. To take the sport of hockey for example, there are dozens if not hundreds of different actions which can be measured, and used for evaluation. Such actions could be the number of occurrences of attempting a goal, or of blocking an attempt. Those measures can then be correlated with the number of wins and losses among a number of games to test their "winning-ness" or "losing-ness." Linear regression models are used to evaluate those measures [15] to see whether they are strongly or weakly related to winning or losing, or not related at all. While this in itself cannot show that more occurrences of some measure *causes* more wins or losses, it seems easy enough to find the correlation of the winningness or losingness of an action, e.g. blocking or taking shots, to another knowable quantity such as goals scored or games won. It should be obvious that this can be a useful technique for helping to evaluate player or team behavior. In the case of game refinement theory, a function of two knowns (breadth and depth of game information) produces a third metric related to something a bit more elusive, a measure for game entertainment. Discovering the fitness or lack of fitness for this third metric called game refinement or sometimes GR, sometimes simply R, is not as easy. The game refinement measure has been in use for nearly 15 years, primarily in the realm of board games. Assuming that the new measure is fit one, **Can the logistic model of game uncertainty be developed for board games be applied more generally?** In order to answer this question, appropriate measures for the breadth and depth of game for new game types must be decided and evaluated for efficacy.

The game information dynamic model [47] offers a bridge between physics and information, based on correspondences of physical flows of, e.g. water or photon particles, and flows of data particles comprising information. It has been stated by researchers of game refinement theory that there is a deficit of understanding how physics of information operate, especially in the difficult-to-observe inner space of the brain. Functional brain imaging and theoretical advancement such as the game information dynamic model are proposed for pointing the way to a better game refinement theory. **Are there any intersections of the theoretical world of all game sciences (including game theory, game refinement theory and others) with the applied world of functional brain imaging that could indicate a promising goal for these studies? One that leads to a bridge between information and mind?**

Recently, functional brain activity measurement of players during gaming (henceforth BAMING) has been showing gains in several fields. Since the beginning of the new millennium, findings from BAMING have led to better models of cognition, deeper understanding of social behavior, more and better maps of human brain connectivity and function, and improved methods and analytics. A few mentionable works include brain measurements during currency auctions [28], reciprocity in the context of a prisoner's dilemma [71] [86], the ability of players to perform under stress [52], and various recreational gaming functional brain studies, like as in Mathiak et al. [62], Matsuda and Hiraki [63], and Saito et al. [89]. There are many implications for these advancements in brain-to-machine interfaces, affective gaming, neural connectivity studies, and research on the disabled, to name a few [38] [64] [79] [101] [99] [106]. Each one urges caution in making the correct conclusions about results from fNIRS, while recognizing the potential pitfalls for this very promising but nascent technology. One of the most basic goals of these and most brain studies has been identifying regions of interest (ROI) by measuring neural activation with reference to various cognitive tasks. Advances in the field of functional brain imaging now permit the transition to more dynamical assessments than merely mapping the activated regions [25] [79] [83]. In one sense, the present research proposes to continue on that same path. In another sense, prior studies have tended to focus on finding some effects of video gaming, or achieving greater understanding of the psychology of players through the study of games [63] [70] [89], while the present work proposes the study of play and game information in order to achieve a greater understanding of games.

Grether et al. contains a call for functional brain experiments in economic games [28] which also extends well to recreational games as reasonable proxies for general gaming behavior. Functional brain measurement of human players can provide valuable evidentiary support for game refinement theory. A long-discussed point of study is that of critical positions in games [55]. In chess for example, any turn in a game which is apt to determine the game outcome is generally referred to as a critical position. More specifically, on a game tree, the critical position is that node which leads to the inevitability of an outcome, such as a win, a loss, or a draw. Critical positions are, as suggested, a moment of crises. In the mind of the game player the critical position is the moment when that crises manifests. Critical positions hold significance in many areas in human behavior. It can be expected that in the mind of the player, the arrival of a critical position marks a moment of change from a player's focus on evaluating and deciding moves to something else, namely resignation or victory.

A pre-requisite to being able to investigate the happenings in the mind during critical positions will be the ability to time those positions in the game player vis-a-vis the game tree, steps, or temporal time of the game. Game refinement theory hypothesizes that the branching factor, the depth (length) of game (perhaps the most basic measure of the quantity of game information) and any changes to the velocity with which that information flows are directly related to the entertainment value of the game. It is believed that wins and losses in recreational games correlate with the powerful emotional signals found in association with monetary gains and losses [10]. Particularly for this study's purpose, the acceleration of information near game's end culminates when the certainty of game outcome reaches some reasonable level.

Merely reaching that "reasonable" level of information of game outcome belies the nature of the endgame somewhat, given that this passage almost always occurs in the presence of a steep sweeping curve (as in Figure 1.1, Logistic Model of Game Progress, and Figure 4.4 Uncertainty of Game Outcome). The convex polynomial curve in Figure 1.1 is regarded as the acceleration of information, increasing rapidly near game's end. This is discussed in greater detail in Sections 3.1.1 and 3.2. about the model. Previous discussions raise a serious question relevant to these propositions; **How do the physics of information operate, e.g. in the brains of gamers?** This remains an open question, and a question motivating this avenue of inquiry.

To date, there have been no studies done presenting any direct physical evidence of a causal relationship between information acceleration at

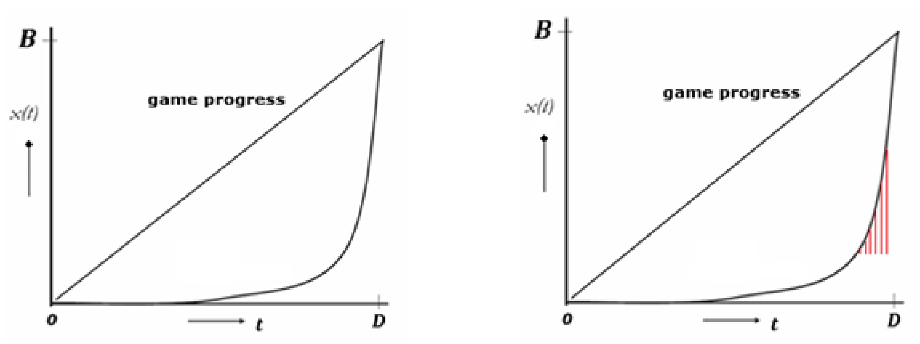

Figure 1.1: (Left)The Logistic Model of Game Progress
(Right) An area of interest under the curve
[45]

endgame and rising excitement or entertainment in players or observers. This study contains preliminary theoretical and experimental work, and identifies a way towards validating one of the principle claims of game refinement theory, that information accelerates in the mind near game's end.

Chapter 2

Game Theory–The Player's Paradigm

Game theory proffers a complex of utility matrices in the aim to evaluate optimal strategies for individual players, while game refinement proposes measures of informational complexity of games for extrapolating information about entertainment optimization. The original is argued to be a logical, mathematical expression, a tautology by some, or a messy interpretation of normative terms by others. The context of game refinement remains almost exclusively in the realm of zero-sum (or nearly zero-sum) recreational game constructs. Yet, game refinement theory is undeniably restrained by the subjectivities in its core objective to identify or measure the key ingredients of human enjoyability in games. Game refinement theory is distinct from original game theory in its fundamental approach. Nonetheless there are some important cross-overs between the two. Since game refinement theory comes about more or less directly from game theory, it will be useful to start with a brief review of the first in order to identify those commonalities, or at the very least to identify what game refinement theory is not. Also, many of the tools developed under game theory are now the tools used in game refinement theory, therefore a closer examination of them is taken here.

2.1 A Brief Connecting History of Game Theory and Recreational Game Studies

Renowned mathematician John von Neumann's genius was already well known in American academia by the 1930's when he was a colleague of Albert Einstein at Princeton University. It would be difficult to overstate von Neumann's influence. Von Neumann's ground-breaking work in fluid dynamics, set theory, linear programming, geometry, lattice networks, computing, nuclear weapons, rocketry, military and political strategy continues to touch or threaten every life on Earth. Together with economist Oskar Morgenstern, he published the formative Theory of Games and Economic Behavior in 1944. Scientific approaches to gaming had already been attempted years before, for example by Ernst Zermelo, Emile Borel and von Neumann himself. Combining Morgenstern's insight on economic philosophy and von Neumann's innovations in mathematics, they vastly expanded the applicability of games in a wide range of studies. Game theory continues as a dynamic and still expanding field of study today. Serious consideration of game theory was sporadic among economists of the day however, prolonging its further development [73].

The following analogy of Charles Darwin and the theory of evolution neatly summarizes von Neumann's contribution to game theory: Darwin did not invent the concept; he defined the mechanism called "natural selection" which made the theory of evolution work. In the same way, while von Neumann and Morgenstern did not invent the scientific study of games or strategy, their introduction of cardinal utility functions revolutionized the study of strategic interaction in games. They discovered a way to compute utility functions that made game theory workable. Also, by the virtue of identifying players' utility in games as the same sense of utility used in the study of economy, an important connection to economic philosophy was established. As it is defined for use in games, utility is the degree and quantity of enjoyment, or winning-ness, an agent derives from a gaming position. A utility function is a matrix of player preferences, i.e. a player's preference for one position compared to others.

Game variables are either parametric or non-parametric. Parametric variables tend to be static or at least linear, but non-parametric ones require a player to anticipate (and model) the actions of another player's response. One of the important developments from game theory is the way this difference between knowns and unknowns is treated. The idea of cardinal utility functions also was not new, and had already had a long and controversial past in other economic studies [108]. Whether von Neumann's utility functions are truly cardinal, or merely complications on ordinal utility functions endures as another subject for argument. Von Neumann and Morgenstern re-introduced the concept of the cardinal utility function as a method for forecasting an agent's probability of choosing any particular move in the game scenario. By cardinal utility, estimates of players' preferences about the choices in a game as the inputs are used to devise an index of utility for a player's options, which form the values of an individual agent's cardinal utility function. They are cardinal because beyond just the degree preference of ordinal utility, cardinal utility functions also enumerate a quantity, the amount of preference for outcomes, as derivatives of possible future outcomes, e.g. new positions and probabilities for winning or losing in a game. Beyond the complete subjectivity of trying to measure individual preferences, even an approximately objective scale for the preferences of any one individual gaming agent will likely differ from agent one agent to another. Perhaps most importantly, while utility functions can sometimes offer accurate measures of player preference, they cannot be counted on to accurately predict an agent's likelihood to choose once expectations for competing agents' preferences and choices come into consideration. Nonetheless the usefulness of utility functions as an indicator of preference, if not always as a predictor of actual choice, is well accepted.

In game theory the basic assumptions are that players are rational, self-interested and maximizing agents. As simple or self-evident as those assumptions may seem, even these are not without equivocation. The concept of rationality is often considered to be a uniquely human quality, yet games as defined in game theory are also used to describe the choices made by non-human agents such as other animals, corporations, herds or computers. There are any number of definitions for just what, exactly, rationality entails. By most definitions, rationality requires a will to act with intention toward some goal, a desired result. This definition can still work for game theory with a minimum of stretching of the notion of "intention," as long as autonomous action is present. That is to say that if an amoeba, for example, moves to engulf a smaller object, then can we fairly assume it has a will to eat it? It is simply a different standard from the moralist's requirements for rationality.

2.1.1 Some Game Theory Games

The zero-sum game is a formalization of the economics of a pie-cutting problem. In the case of a fixed number of parties dividing a finite good, one party's gain is another party's loss. There exists an argument that there are no instances of the zero-sum game in human interaction. Being as we are at very least complicated social creatures embodying many externalities in the form of competing utilities, preferences and objectives, truly zero-sum games may be possible only in theory. Assume then for example, two very hungry agents who have never met each other, who in fact cannot even see each other, and will never meet again, with an opportunity to divide a small pie. (Assume also that neither will use the knife against the other!) In a case like the pie-cutting example above, there is understood to be a close approximation of the zero-sum dynamic. Each of these examples has in-common that they can be evaluated as one-shot games between anonymous, self-interested and self-maximizing agents in a zero-sum situation. In a mathematical sense, recreational games are zero-sum, but in the psychological sense recreational game players are expected to receive the benefit of social enjoyment and healthful mental or physical exercise even when they lose. Then there are many situations which are not zero-sum; those games with net payoffs of more than or less than zero. Almost any trade is sum-positive–the parties to a trade enjoy the division of what becomes for them a larger pie, due to the surplus of utility. In the cases of pollution, crime, punishment and much of politics, parties experience the division of negative goods which is sum-negative, i.e. a choice of the lesser evils. Game theory triumphs of recent years include the modelling for complicated auctions and bids, lotteries, traffic flows and the division of rights to public utility services.

Games can be as simple as a situation with two interdependent decision makers, each with one decision to make, and each with two options, seeking to maximize his own benefit or minimize his own harm. Each player's choice has an effect on the other player(s). Such a one-shot puzzle can be represented with a four-square decision graph called a payoff matrix, as in the prisoner's dilemma shown in Figure 2.1. The prisoner's dilemma is a member of this simplest, one-shot game. What defines the prisoner's dilemma is not the variables or the roles assigned to the players but the payoff structure. The agents playing "Column" or "Row" can be two companies competing in a market, office mates engaging in some subterfuge, drivers on the road preparing to negotiate an intersection or what have you. The form of the prisoner's dilemma just happens to lend itself to many moral decision problems, otherwise known as social dilemmas, because the payoff structure provides some conflict for the agents making the decision. In the original example, two players who would normally be cooperative with each other find themselves in the following predicament: a pair of suspects in a crime, "Row and Column" have both promised to play "Co-operate." If they both honor their promise, they will each receive a utility of say, "2" (i.e., less punishment). However, they choose simultaneously or without the information of the other's move, so there is a substantial risk in the case that one honors their promise, and the other does not. Additionally, there is substantial incentive NOT to honor the promise (despite that doing so is a violation of the agreement, and substantially decreases the other's utility). If the other player chooses Co-operate, one's utility increases, and in either case, each individual player's payoff, regardless of what the other player chooses, is better if they do not honor their promise. Mathematically, the only rational choice for each player is to choose "Defect" which is the equilibrium position. (Of course each player would be better off, and they would also be better off as a group if both players choose to honor their promise to each other, but the information of the other's decision is not known until afterwards.)

The prisoner's dilemma strikes many people as ironic the first time they see it, because the mathematically correct payoff structure gets the players less utility than what they could get by simply cooperating. A real world example of a prisoner's dilemma can be seen in the interactions of players in oligopolic competition, a notoriously unstable structure of interdependence and confidence, with many unilateral and bilateral defections. The United States' airlines industry provides several samples for study, such as [9] [60] and [81] to name a few.

Among the first instances of the political use of game theory was by its founder. The United States government and the U.S. Air Force heavy RAND Corporation strongly supported game theory research for the use of developing a nuclear war policy. Incidentally, von Neumann was not only a lead developer of today's chief of all weapons of mass destruction, but also a formative American military policy maker regarding the use of that power. Von Neumann applied his concepts of games to the very real world of the Cold War arms race, and was the one to coin the phrase "mutually assured destruction" (MAD). Von Neumann advocated for the annihilation of Kyoto as a member of Manhattan Project's target selection committee and later for a nuclear attack on the USSR [29]. To summarize Don Ross, MAD was poorly modeled on a one-shot prisoner's dilemma game, which would be a highly simplified representation of the complex reality of World War Two and the Cold War. In the case of the MAD paradox, noting the executives of the U.S.A. and U.S.S.R. for the players, assign "Don't Shoot" for the Co-operate decision, "Shoot" for the Defect decision. Some reasonable values of consequences for four basic MAD outcomes can be gathered, based on the following assumptions:

1. That the executives' motives, and those of the nations they represent, are the same.

2. That it was already well understood by executive decision makers by the time of the invention of MAD in the early 1950's (when multiple nuclear players were producing multiple delivery systems) that "limited nuclear strikes" were always an unlikely scenario.

3. That one likely response to a nuclear attack by any nuclear-armed player is to issue their best immediate, all-out, total nuclear counter-attack.

4. That any nuclear attack and counter-attack would have a relatively high possibility to result in profound world-wide catastrophic destruction, especially to that of both attacker and counter-attacker nations.

There are arguments against each of these points, but after all they are assumptions. The fourth assumption might be mitigated by defensive actions such as anti-missile systems, or effective underground bomb shelters. However, since extensive environmental damages to vital air, water and crop resources are also likely, the fourth assumption is not an unreasonable one.

The result is the MAD deterrent of von Neumann's invention, as it was supported by then-Defense Secretary Robert McNamara. A payoff schedule for nuclear war between the two Cold War powers like the one in Figure 2.1, shows an important difference from the prisoner's dilemma. In the prisoner's dilemma, defection is the dominant strategy. In MAD, defection results in mutually assured destruction, the opposite of a prisoner's dilemma strategy. The incentive to defect from the nuclear detente either bilaterally

Social dilemma payoff structure		Column	
		Defect	Co-operate
Row	Defect	1,1	3,-3
	Co-operate	-3,3	2,2

The "MAD" mutual confidence game		U.S.S.R.	
		Shoot	Don't Shoot
U.S.A.	Shoot	-∞,-∞	-∞,-5x
	Don't Shoot	-5x,-∞	x,x

Figure 2.1: (Left) A prisoner's dilemma/social dilemma type matrix (Right) Payoff matrix for a Cold War nuclear attack

or unilaterally is prohibitive, and provides the equilibrium position of Don't Shoot as the only choice with positive utility for either player. The theoretical outcome (as well as the actual outcome thus far in history) presents as the equilibrium position of a solvable game. (Bearing in mind players do not always necessarily choose their optimal moves.) Meanwhile, the extreme costs of MAD in terms of military expenditures for the player nations of U.S.A. and U.S.S.R. alone, and the fighting of several proxy wars (Iran, Greece, Korea, Indochina and Vietnam, Cuba, Afghanistan, etc.) have led to some devastating consequences for both players and non-players.

There are alternative valuations which allow for the presentation of Cold War nuclear policy in a Prisoner's Dilemma (e.g. replacing "arm" or "don't arm" to provide credible and balanced threats). Once the large scale production of nuclear armaments had become a real possibility soon after WWII, there was little question of whether the capable parties would produce and maintain them. The real questions beyond whether or how to use the bombs, if ever, were about what scales their system deployments should be taken–primarily questions of funding and administration.

As with many theories, game theory is occasionally cited for justification for the actions and agendas of various interests and individuals. "A wise cynic might suggest that the operations researchers on both sides were playing a cunning strategy in a game over funding...cooperating with one another in order to convince their politicians to allocate more resources to weapons." Except in presentation, it is not likely that MAD, at least not as a prisoner's dilemma, was ever a serious national war policy. In a prisoner's dilemma, the only mathematically correct decision would be to try to strike first, using the fullest available catastrophic force [88]. MAD is an extreme counter-example to the proposal that games are a trivial concern, and an example how costly some games can be.

2.1.2 Experimentation Early in Game Theory

During the first three decades of game theory, theorists worked on expansion, producing cross-over studies in economic behavior, war strategy, diplomacy, evolutionary biology and more. During that time, few experiments were conducted to verify the theory, or for that matter in economics generally [87]. Eventually though it became clear that individuals choose in ways counter to that predicted in games too frequently to be considered anomalies. This apparent lapse of self-interest in games has been called "altruism," such as in the case of making a move that is costly to oneself for the purpose of reciprocating goodwill or punishing the ill-will of other players. Nash's equilibrium [72] received attention from statisticians like McKelvey and Palfrey [65] to address probabilistic variability, offering the quantal response model. Recent decades have produced experiments leading to many substantial refinements in game theory, utility theory, and the assumption of rationality [3] [6] [10]. The 2000's are a heyday for game theory experimentation. As it turns out, one-shot games are not actually very common, with repeated games being more the rule, whether in the marketplace, diplomacy, or on the chess board. This simple fact alone affects games in a number of most significant ways. In addition to the raft of refinements, a few serious refutations such as that of Francesco Guala [30] have also been made.

Since the beginning of serious experimental inquiry into applied games, game theory has been making remarkable insights on such phenomena as reciprocity, punishment, reputation, sensitivity to initial conditions and a taste for fairness. The heterogeneity of subjective individual, social and cultural traits has surprised game theorists at times, at times contradicting well-accepted theories such as expected utility and Nash equilibrium as predictors of actual behavior [6],[33] [54]. These discoveries have necessitated key refinements in order to fit the theory to the reality. Human players are extremely sensitive to the gaming environment and rules, and display a wide range of divergent values and attitudes [21]. Players often gamble, make mistakes, or choose “wrong”ly for their apparent objectives [6] [13] [54] [84]. Also, preferences are unstable over time, and depend heavily on prior experiences [21] [34]. Brain measurement experiments of players engaged in social dilemma and other economic games have been well underway for over a decade [10] [86].

In his ground-shifting Behavioral Game Theory: experiments in strategic interaction, Colin Camerer [13] summarizes the defense of some game theorists to game theory refuters “If people don’t play the way theory says, their behavior has not proved the mathematics wrong,” he says, “any more than finding that cashiers sometimes give the wrong change disproves arithmetic.” However, this is only true in the strictest of descriptive terms. If it is not already obvious, the inaccuracy of this analogy is that arithmetic is not a study in how cashiers behave. Normative game theory is, however, a study of behavior in games, often human behavior, and an application of the concept of rationality. Only in so far as game theorists are not trying to predict, engineer, or model any behavior or economic activity is theirs a purely mathematical concern [30] [31].

Quoting from Camerer, “It is important to distinguish games from game theory” [13]. Game refinement theory tends towards the former category, but recreational game sciences like game AI and game refinement theory are another *kind* of game theory. In game refinement theory, theorists want to discern the meaning of the discrete game information comprising the building blocks, nuts and bolts that hold up all games. Game refinement theory seeks an answer to the question it has raised; How does the complexity of a game’s information affect players’ or observers’ involvement in the game? Prior research seems to show a correlation between the two. It is not enough for us to say this is true because it is mathematically sound. Prior research shows that some board games share a window of refinement values. Similarly, it would not be enough to say that this is sensible, even if it seems to be so. Firm numbers are required, and sound logic must be in place to produce a result of any lasting value. This study proposes that, like game theory, game refinement theory stands to benefit from a new experimental approach.

2.1.3 Preferences, Utility Curves, Trees

Cardinal utility functions describe fully known information, such as that in a complete game tree. They also provide the mathematical basis for opponent modeling used in recreational gaming and artificial intelligence (AI). When the whole tree, and utility index are not known, or are impractical, heuristic searches employ evaluation functions, which are faithful representations of the utility functions. In turn, knowledge of the opponent's evaluation functions form the heart of opponent modelling [11] [24] [40] [41]. A player who correctly estimates an opponent's proclivities puts him/her/itself at a distinct advantage in any game. Opponent modelling works well in computer games or recreational games, where the rules are tightly constrained, well defined and relatively simple. Trying to identify and measure players' preferences is considerably more difficult where the wider spectrum of human relations are concerned. (For example the potentially endless loop of calculations beginning "I think that you think, that I think that you think, that...") As noted by Ross [88] people should understand that "game theory isn't useful for modelling every possible empirical circumstance that comes along."

The extensive form game tree is a powerful analytical tool for mapping the multitude of possible decisions in a game. The game tree in Figure 2.2 is similar to the extensive form game tree developed for game theory, only the payoffs are positions, which lead to new position potentials in the next ply. It starts by showing what happens if player X makes one of three typical moves, and what O can do in response. Each play and each position can be evaluated mathematically with an evaluation function. Despite its simple appearance, the game tree actually contains quite a bit of information, and it expands geometrically, as shown above. It shows the state-space complexity for a finite number of players for the length of the game, including its outcome, and path delineated to that conclusion. A complete tree contains the information of each player's knowledge of each other's past, present, and possible future moves. So for a solved game, a complete tree can be shown, down to the end moves, for each possible game. This opens the possibility for reverse-induction analysis. Prior knowledge of the game, the values of the various positions, especially those likely to lead to a win, a loss, or a draw will determine the valuations (preferences) of moves, and the knowledge of arrival of a critical position in the game.

Here is a partial game tree for a tic-tac-toe game (Figure 2.2). In tic-

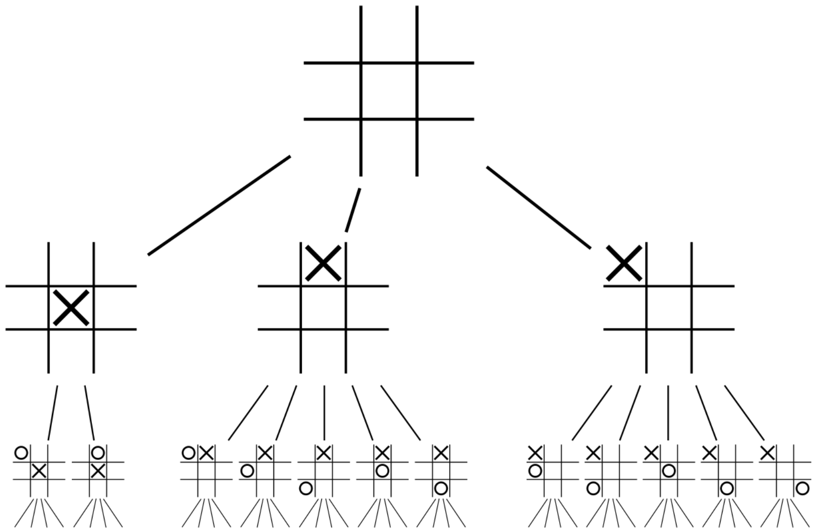

Figure 2.2: An extensive form game tree
[110]

tac-toe, players take turns, expressed by either X's or O's potential moves on each alternating ply. This one is a perfect information tree up to the end of the second move. Each layer of the tree is a "ply" and each unique position with the score/payoff for each player is a "leaf," or node, on the tree. In the example, players take turns and see their opponent's decisions, and understand the remaining moves available to each player. The search-space complexity (average number of decisions, and the number of those decisions to be made) and the form of the game tree is not always so simple. In simultaneous games, information of the other's move is unknown until after the moves of both (all) players is revealed. Hence, simultaneous games are a form of imperfect information. (Simultaneous moves can be shown in the game tree, e.g. by the removal of the other information, with a dotted line between nodes, or an oval encircling simultaneous moves.) Similarly, in games with an element of chance, such as rolling the dice in backgammon, drawing tiles from a bag of Scrabble letters, drawing cards in poker.

The latter two examples involve both chance, and imperfect information of other's letters or cards.

Game Theory provides a good framework for examining complex decisions involving competing agents by using game matrices and game trees to map the decisions of game playing agents. The quantity of information on a game tree gets out of control quickly. Consider that a complete game tree contains every iteration of every possible move and response for the duration of a game. A game for which every possible game tree can be created is a solved game. Even the most awesome computers at the time of this writing are not capable of completely graphing some games due to the phenomenal size of their game trees. For imperfect information games, a heuristic process such as abstraction can be used, although this requires making some assumptions and is not perfect [90]. Among perfect information games, chess, shogi and "go" for example, are too big to be solved. Some moves are repetitious, i.e. symmetrical moves or illegal moves, but for simplicity's sake, here are a few examples (Table 2.1). By comparison, it has been considered that there are about 10^{26} nanoseconds in cosmic history, or about 10^{80} atoms in the universe.

Table 2.1: Size of some board game trees
[35] [112]

Game	B	D	B^D nodes
Tic-tac-toe:	4	9	10^5
Checkers:	2.8	70	10^{31}
Western Chess:	35	80	10^{123}
Chinese Chess:	38	95	10^{150}
Shogi:	80	115	10^{218}
Go:	250	208	10^{360}

The use of these tools for computation and game-like consideration of problems has been a major contributor to the gains in divergent fields in recent decades, including computer hardware and programming, strategic decision-making for politics, finance, economics, and even for the study of games themselves. Game theory has provided a host of new topics and applications. Today game theory is a multifarious hybrid science benefitting from the gains in programming and (AI) which it helped to foment. It is reasonable to expect valuable new insights from the continued study of games.

Many of the discussions about the assumptions of rationality and self-interest, altruism and moral dilemmas posed in normative game theory do not appear as topics for recreational games, which occupy the remaining part of this work. Board games are objectively simple, usually two-agent, zero-sum games, or a close approximation of zero-sum games. These have a clear objective and tightly constrained number of variables, providing know-able search complexity and informational game length. Hundreds of years before von Neumann's treatise and the coining of the phrase "game theory," game masters of antiquity were cataloging their knowledge of plays and strategies in board games, such as those by Lucena in chess [12], and Ohashi or Kanju in shogi. There were many more than these, and they are also the philosophical antecedents to the computer chess problem, and to game refinement theory.

Chapter 3

Game Refinement Theory–The Maker's Paradigm

Attempting to completely separate normative and descriptive game theory constructs would be illusory. Like game theory, the computer chess challenge and game AI, game refinement theory is first of all a systematic, mathematical treatment of games very much on the cut-and-dry (descriptive) side of things. However, other reasons for pursuing game refinement are the implied human factors, art, entertainment, and emotions, which are very much definitions of rationality, hence normative. Game refinement theory attempts to provide measurability to entertainment factors, as well as interpreted game data and philosophical reference. Until now, experimentation in game refinement theory has relied primarily upon modelling, interpretation of game data, and surveys. These have produced such results as the identification of fundamental game patterns [46], the game progress curve [46], evaluation of players' winning-ness or losing-ness using normalized advantage [47], tsume shogi problem composition [51] and even more ambitious discussions such as player and observer attitudes towards fairness [45], game entertainment, evolution, and design. It has been seen that games are uniquely positioned to elicit such things as the great potential of computers, as well as the joy of victory and the agony of defeat. Game refinement theory is an endeavor that, in time, will lead to the development of a computational model with stronger, more general capabilities.

Studies of strategic decision making in the framework of games with focus on mathematical models of conflict and cooperation between players have been in progress for a long time. AI and the computer chess problem developed quickly after and along with game theory. From within this tradition game refinement theory was proposed as a measure of entertainment optimization for game creators. The logistic model of game progress was created for the domain of board games, but it may also be useful for other game types. If so, the expansion to other game types could imply a more universal meaning. The theory relies on a measure of search-space complexity to explain the evolutionary outcomes of rule changes in chess, and becomes the subject of consideration for the next few pages.

3.1 Related Works

Yannakakis reviewed the state of game AI development for game development in [111]. Player experience modeling, procedural content generation, and massive scale game data mining are identified as three flagships of present-day AI research. It is interesting to note that the game AI mentioned in [111] is of a different character from that of board games, which it is said "can only be algorithmic with respect to a certain aim (i.e. how to play a board game) in constrained board game spaces." This point of view purposely excludes board games from the field of game AI. It is of interest that related to this research, objective player experience modeling still seeks a biofeedback interface to acquire players' data, most of which are still too obtrusive, too slow, or otherwise not yet commercially viable for gameplay.

As it was stated by Yannakakis, one of the main points of divergence of computer game AI and board game research is constrained playing spaces. On this avenue, Allis published a chart in 1994 [1] comparing the strength of game AI among the original 15 Computer Game Olympiad games, and making predictions for further development, based on the complexity and perfect or imperfect nature of the games. Comparing this update with the update in van der Herik [35] and the original from Allis, there are several instances of no definitive movement up the chart 10 years or longer (Othello, backgammon, 10 x 10 draughts, bridge, go). Others have moved up one level (shogi, chess, 8 x 8 checkers). Also, to the list of solved games can now be added checkers [91] and Scrabble, which appears with an * because, like poker, it is an incomplete information game with an element of chance, but it has been solved to the degree possible. Further gains are still possible for the stochastic game aspects. Computer shogi rose to notoriety and world champion level under several computer players, including TACOS, prompting a years-long ban on shogi masters playing against computer shogi systems in 2005. According to Iida, shogi has been gaining steadily, and now on or near a World Champion level. Advances in "go" are restrained by a relatively large branching factor. According to Viennot [104] computer go is ranked as a strong 3rd or 4th dan amateur, or possibly as high as a low-level professional. Despite the optimistic predictions of many, twenty years after Allis' search for solutions, there are still many games in which humans still prevail, and much work to be done if they are to be solved. Table 3.1 shows the relative strength of game computers in some popular parlor games as of 2014. Games which have not moved up the chart since 1994 are in italics. Games which are new to the chart are below the line.

3.1.1 Early Works, and the Model of Chess

In 2003 Iida, Takeshita and Yoshimura [42] wanted a new measure to evaluate evolutionary and entertainment factors in the history of chess. They were the first to create a game-strategic-complexity measure for that purpose. As measures the informational complexity of a game, various standard metrics are available. State-space complexity (the number of possible legal moves) and decision complexity (which entails interpretation of rules and comparison of strategies and games across types) were considered too problematic. Search-space complexity (the size of the search tree, where breadth of game B is the average number of possible moves, and the depth of game D) is expressed B^D. This was considered as a reasonable measure with a good indication of a game's decision complexity, but with more general applicability from game to game. Search-space over decision complexity is also supported in van der Herik [35]. Subsequently it was reasoned that the number of plausible moves is far greater than the number of moves a player actually considers making which, in the case of chess ($B = 35$), was actually

Table 3.1: Level of AI in games 2014
[1] [35] [104]

Solved	Superhuman	World Champion	Grandmaster	Amateur
renju Connect-Four Qubic go-moku awari nine-men's-morris	chess Gipf *Othello* *backgammon*	*10x10 draughts*	*bridge* *Chinese chess*	arimaa *go*
kalah 8x8 checkers Scrabble*	Lines of Action bao	shogi	poker 11x11 hex Amazons	

found to be close to the square root of 35 on average. It is assumed that extinct games must be less entertaining than current games (at least in the opinion of current players), else they would still be in play, and therefore the following equation was proposed as a measure of game entertainment:

$$E(G) = \frac{D}{\sqrt{B}} \tag{3.1}$$

Iida et al. [42] proposed the logistic model of game uncertainty. Iida and Yoshimura [43] and Majek and Iida [61] defined the information of the game as the amount of solved uncertainty $x(t)$ where the constant n is a parameter based on the difference of skill between the two players in the game, and $x(0) = 0$ and $x(D) = B$. Note that $0 \leq t \leq D$ and $0 \leq x(t) \leq B$. The equation implies that the rate of increase in the solved information $x'(t)$ is proportional to $x(t)$ and inversely proportional to t. D is the depth of the game tree, of informatical length of game, not the temporal game length.

From this was derived the (other) original model of game refinement in [43], which forms the basis of the model used to analyze games in this paper.

$$\frac{\sqrt{B}}{D} \tag{3.2}$$

3.2 The Model

In so far as it is practical, observations in game refinement theory are drawn from championship, expert-level games. As is shown, the assumption of perfect, or near-perfect play helps to mediate the inefficiencies of less-than-perfect game play. Fundamentally, games are comprised of information. It is the spontaneous composition of that information which provides the outcome. The model of game progress as discussed in previous pages was revisited by this author, and refined in Sutiono et al. [98], and is outlined in the following paragraphs:

In games such as football or basketball, the temporal length of game is determined by elapsed time, i.e. ninety or sixty minutes, while scoring determines the outcome of winner and loser. The scoring rate is calculated by determining (a) goals scored and (b) elapsed time, or alternatively, the steps/attempts required to achieve (a). Game speed is determined by the number of successful shots (goals), where the number of attempted shots is the average number of steps required, or depth of game. For other games such as volleyball and tennis where the number of goals required to win is pre-determined in the rules of the game, the total number of points for both teams is the number of steps.

Let G and T be the average number of successful shots and the average number of shots per game, respectively. Having full information of the game progress, i.e. after its conclusion, game progress $x(t)$ will be given as a linear function of time $0 \leq t \leq T$ and $0 \leq x(t) \leq G$, as shown in Equation (1)

$$x(t) = \frac{G}{T}t \tag{3.3}$$

shown graphically as the linear function "game progress" in Figure 1.1.

Only a very boring game would progress in a linear fashion however, and most of course do not. Therefore, it is reasonable to assume a parameter n, based on the perception of game progress prior to completion. When the information of the game is known (i.e., after the end of the game) and the value of n is 1 then the game progress curve appears as a straight line. In most games, especially in competitive ones, much of the information is incomplete, the value of n cannot be assumed, and therefore game progress is a steep curve until its completion, along with B, D, $x(t)$ and t, just prior to game's end.

$$x''(T) = \frac{Gn(n-1)}{T^n} t^{n-2} \tag{3.4}$$

which is to say,

$$\frac{G}{T^2} n(n-1) \tag{3.5}$$

Suppose that, to the degree game information can be effectively processed, the greater $\frac{G}{T^2}$ becomes, and the more exciting the game becomes for players and observers. Too little game information acceleration may be easy for human observers and players to compute, and becomes boring. At some point however too much game information acceleration surpasses the entertaining range and enters frustration, and at some point beyond that could become overwhelming and incomprehensible. $\frac{G}{T^2}$ or its root square is used as a measure of game refinement or GR-value.

$$GR = \frac{\sqrt{G}}{T} \tag{3.6}$$

Furthermore, it has been observed that, of those games which have survived long periods of evolution and variation, many have similar GR values under this model (as in Table 3.2).

It is important to note that although the GR-values of many games and sports tend to gravitate toward a certain number, the findings are not prescriptive. Checkers, with its small branching factor, would seem to be one exception to the rule. Moreover, the mechanism of any possible underlying phenomenon that may exist is not certain. However, these and other observations from across game types may imply that informational complexity and its rate of flow in games is directly tied to game entertainment, and that many of those games we have analyzed tend toward a similar measure of

Table 3.2: Game refinement values of board games

Game	B	D	GR
chess	35	80	0.074
shogi	80	115	0.078
Chinese chess	38	95	0.065
go	250	208	0.076
8x8 checkers	2.8	70	0.024

that information. In cases such as volleyball [100] and badminton, the game progress model shows that recent rule changes in those sports are moving GR-values in the direction of the typical GR-value window and therefore seem to provide more evidence in favor of the theory.

It has long been understood that games continually evolve in order to maintain fairness, safety, or enjoy-ability for players and spectators. According to what is known from these and previous studies, in those games which are the surviving variants of long processes of sophistication, game information often arrives in approximately 0.07 to 0.08 GR-value. Game refinement theory hypothesizes that 0.07 to 0.08 GR-value is optimally accessible both for play and observation, although we can only speculate why this may be. Perhaps below this value, as in the case of side-out volleyball or best-of-five tennis matches, players or spectators experience a lack of excitement while awaiting the arrival of new game information. Above this value, such as in the case of side-out 3 x 15 scoring in badminton (described in section 3.7), they might find the game information moving along too fast to fully appreciate, and some of the potential enjoyment of the game is lost. Among the shortcomings of this approach are that individuals' and societies' tastes vary from group to group, and change over time. What may be an optimal GR-value today may not be optimal in the future. Since the theory has been heretofore restricted to analyzing only a narrow spectrum of all game information, it is quite limited. Implementing some testing and verification processes, such as a brain imaging study to test aspects of game refinement theory (the topic of Chapter 4) or expanding the model to include other aspects of game information, for example, could help develop the theory into a stronger, more general theory.

3.3 Continuous Movement Games

The following section is an extension of the work presented in

Nossal, N., and Iida H. (2014). Game Refinement Theory and its application to score limit games. IEEE Games and Entertainment in Media 2014, Toronto, Canada, October 22-24, 2014.

Recent studies have considered the difference between game information in board games and other type games, like those with continuous movement such as video games and sports. Game refinement theory proposes that a measurable quantity of game design exists, implying a value of entertainment, using the relationship between a game's search-space complexity and the length or depth of game. The original model was derived from studies of historical chess types, and was restricted to discrete, alternating movement board games like mah jong and go. The model offered $\frac{\sqrt{B}}{D}$ as having some relation to a game's entertainment and evolutional longevity.

Diah et al. devised experiments for a Pac-man-playing robot using the number of pellets and power pills on levels 1 and 2 of the Ms. Pac-Man game board as B, and the number of movements of the Ms. Pac-Man character on the board as D [19]. The finding of 0.045 for GR seems satisfactory as a result for the relatively simple, repetitive nature of the game. For a very different and much older type of arcade machine, Chiewvanichakorn et al. [14] investigated play and payout data for the crane, or claw type game known as UFO Catcher, and compared several different machines from Thailand and Japan. Chiewvanichakorn identified the average number of plays recorded and the frequency of payouts per cost of playing, respectively, as the measures of B and D. This is fundamentally a different kind of game because of the element of chance, and the presence of prizes meeting the basic definition of gambling. The work done analyzing the game refinement of the crane game bears many implications for other gambling games. Experiments were conducted in Japan and Thailand, and the results, based on the exchange difference of the two countries' currencies, are shown in Table 3.6.

3.3.1 Considerations on the Model: Three Approaches

Sutiono showed that the model could be migrated to other game types and used G and T for the breadth and depth of game. For game refinement theory to work in the realm of continuous action games, it is necessary to find appropriate representations of the G and T values. In chess there are numerical values for board positions, piece values, and declinations of a game tree. The graphing of chess games is fairly well researched and understood as a result of long and intense periods of its development and study of play. Then video and arcade type games, while though they be of continuous action, are comprised of mercifully discrete, measurable game information which after some interpretation, it is possible to compare. For the simultaneous, continuous action sports which follow, a bit more special consideration is required. Finding faithful representations of depth and branching factors in the game is not as intuitive as for board games. To be considered using the model, continuous games must be greatly simplified, with interpretation focusing only on the most basic numerical expressions of what is actually an appalling complex of human and game factors. Even a relatively small set of discrete variables becomes unmanageable without the use of powerful computers (as in Table 2.1).

Higuchi first attempted an application of game refinement theory to a non-board game, as reported in Diah et al. [18]. Initially, two approaches were considered for applying game refinement theory to football (*Am.* soccer). The results for both approaches were very similar to that of a quite different set of games like chess, in the neighborhood of R = 0.07 to 0.08. Coincidentally, applying the game progress model by Sutiono yeilded approximately the same result for football, which is the value reported in Table 3.6. Using either of these quite different measures yielded the same GR-value. The excess of convening GR-values from different approaches to the model presents an unusual predicament of having to decide which of the approaches is right, if not all, or perhaps none of them. Anyone can use any model they want to get some pre-determined result. The question now becomes whether one of these models is an accurate representation of the same measure in the different game types.

3.3.2 Board Game Approach

In Diah et al. [18] the approach of Higuchi was first considered, which offers the idea of football as a chess-like game. Based on the board (field) size in terms of the number of players on the field plus the option to shoot as a measure of decision complexity, B was set at 11. The temporal length of football games is 90 minutes, though we chose to consider a more functional relationship to number of potential pass receivers. T was derived by the number of average passes per each shot. The average number of passes per shot in a UEFA Champions League game that year was 41.18. The result was very near that of chess and other sophisticated board games of past studies.

$$\frac{\sqrt{11}}{41.18} = 0.081 \tag{3.7}$$

3.3.3 Round-Match Approach

In the same paper, as a method to derive some meaningful correspondence of the round-match in chess to that of the singular football game, a round-match approach was considered. To do so, the football game was compartmentalized into sub-games, each of which culminates in a shot-on-goal. The average number of shots taken in a UEFA game is 22, so an average football game would have 22 sub-games. The chance of winning the sub-game is a win/lose situation for the scorer and score-ee, or a draw in the case of a save. Therefore the value of B in the sub-game would be 3. We get a GR of

$$\frac{\sqrt{3}}{22} = 0.079. \tag{3.8}$$

For the type of sports which are defined by a pre-determined G value, i.e. score-limit games, a modified round-match approach was applied. Such games include but are not limited to most of the service type games (volleyball, tennis, badminton, etc.). In this approach developed in [100] and modified slightly in this thesis (see also Section 3.7), G is considered as the total winning points in the game (match), however a work-around is needed to provide an accurate value of the length of game. For this, the total number of points played for both sides provides the number of tries T.

3.3.4 Time Limited Approach

Consider then a time-limited approach, applying Sutiono's game progress model, with $G = 2.64$ the average number of goals (using again the UEFA data from Diah, et al.) and $T = 22$ the average number of shots taken per game. The result

$$\frac{\sqrt{2.64}}{22} = 0.073 \tag{3.9}$$

was derived with this simple approach.

3.3.5 A Note on the Non-Interchangeability of Approaches

Football may be an unusual case. Each of the methods conceived for measuring game refinement of football produced similar results. Not only does the time-limited approach, but also the board game and round-match approaches each result in the football GR in a range from roughly 0.07 to 0.08. Whether this is a meaningful coincidence or not, for the time it is seen fit to apply successful attempts (i.e. goals) and shot attempts as B and D (G and T). Controlling for the particulars of divergent game types (such as drawing a logical comparison between the branching factor in chess and the number of passing options in football, for example) might hold promise when equal comparisons are available. However, even for many high-level competitions of some sports, statistics for variables such as passes completed are not available, rendering this anaysis of transferability between sports types yet more difficult. Statistics for passing in the National Basketball Association and National Hockey League are said to be coming in 2015-2016. Prior to the advent of those more sophisticated tracking and statistics however, it is possible to make some educated guesses.

Jimmy Coverdale [17] provides data of passing in the 2014 professional basketball finals. Although it is a narrowly defined sample, we can use it to estimate what might happen in a board game approach to basketball. Coverdale counted 380 passes made by the San Antonio Spurs. Using the board game application of the model as it was applied to football above, substitute five as the number of pass or shoot options to obtain G, and the quotient of passes and average shots [98] to obtain T.

$$\frac{\sqrt{5}}{380/85} = 0.500 \tag{3.10}$$

To apply a board game approach to volleyball, for example, consider that players have six pass or shoot options. Since each side may only touch the ball a maximum of three times, including the shot to the other side, then the average number of passes must be less than two. For argument, let us say 1.7. A rough average of FIVB [23] tournaments suggests that about 45 rallies occur per game in Olympic level volleyball ('set' in the new volleyball parlance), or slightly less in beach volleyball. The number of exchanges from side to side per each rally does not seem to be recorded anywhere, unfortunately. If we could assume an average of three exchanges per rally, then the number of passes per game (set) would be 382. Assuming a match length average of 4.25 games (sets), approximately 975 passes per volleyball match is possible. Now what represents the number of attempts in volleyball? Probably that number of exchanges that was assumed to be three, which can be used to divide the total number of average passes. We get

$$\frac{\sqrt{6}}{325} = 0.0075 \tag{3.11}$$

Similarly, consider what happens when the modified round-match approach of score-limit games is applied to some time-limited sports. In the case of football, let us assume some likely winning scores and total scores of an average 1.6, and 2.65 respectively. Applying the modified round-match approach used for score-limited games to football, we get

$$\frac{\sqrt{1.6}}{2.65} = 0.477 \tag{3.12}$$

In the case of hockey, take some likely winning scores and total scores of an average 3.25, and 5.5 respectively. Applying the modified round-match approach used for score-limited games to football, we get

$$\frac{\sqrt{3.25}}{5.5} = 0.328 \tag{3.13}$$

When we apply the model for round-match type games (as in volleyball or other service sports, described in sections 3.6 and 3.7) to some plausible values in other game types like football, GR values present no discernable pattern. It may not make sense to use the modified round-match approach used for score-limit sports to measure football though. Recall that the reason for using average total goals as T in the case of score-limit games is as a proxy for average shots or depth of game. A statistic for attempts in football exists, hence no proxy is required. Another way to look at this might be to notice the natural tendency of the model $\frac{\sqrt{G}}{T}$ when it is applied to some typical game scores. That is, one approach may normally produce GR values in the expected range with the high-scoring service sports, but not with low-scoring time-limited sports. While the model may yet have some relevance, it does not apply universally between game types without some consideration. Within game types, results tend to fall within the same approximate balanced range as with chess and other sophisticated board game types previously measured.

To derive game progress, the breadth and depth of most game types can with some minimal extrapolation be interpreted in terms of attempts (tries) and successful attempts (goals). Two approaches employing measures of tries and attempts are round-match and time limited approaches. As applied to continuous movement sports, these two appear to provide the most sensible measures of GR across game types. Game refinement analysis of several games by one of these two approaches, depending on each game's fundamental grouping, follows in sections 3.4 to 3.8.

3.4 Hockey

The modern game of hockey (also known as "ice hockey") originated on the many frozen streams and ponds of Canada in the latter half of the 19th century. The codification of its rules for the professional level was taking place over 100 years ago, followed with a long and ongoing process of game sophistication. Many of the recent changes in the National Hockey League (NHL) rules of hockey directly affect the sport's GR-value. As has been shown by the author of QuantHockey.com [82] the number of shots taken per professional hockey game has been remarkably stable at around 60 per game since the keeping of those records in 1967. Goals per game, and the balance between offensive and defensive systems have gone through several cycles, but historically the winningest ways favor a strong defense. The most recent high-scoring era began in the 1980's with huge goal producing forwards like Wayne Gretzky, Marcel Dionne, and Mike Bossy. Goals per game averaged in the high 7 to 8 points range. Applying the time limited game approach, a relatively stable value for game refinement throughout the history of the NHL can be seen in Table 3.3.

This dynamic presents another interesting challenge to game refinement theory. Given the endurance of the 60-shot average per hockey game, goals per game will be the more independent of these two variables. This happens to be the opposite in golf, where the number of holes G is fixed at 18, with only the number of tries D as unknown.

There has seemingly always been a great deal of discussion about scoring rates in exhibition hockey. Judging from the vast amount of commentary on the subject, the enduring opinion would seem to be that high-scoring games are more exciting and desirable than low-scoring games. Among the many commentators, rarely is it ever proposed that the tension created by frustrated scoring efforts also creates excitement, or that low-scoring games can ever be as entertaining as high-scoring games. According to Sports Illustrated "Many players told SI last month that they didn't think lack of scoring was an issue in the game today, but conceded that it was something to consider in the future." At the same time [All-Star player] Patrick Kane acknowledges "There's probably two things the fans love seeing most, goals and fights" [58].

A brief review of rule changes in the NHL since the League's creation in 1917 shows that the League has often made rulings to maintain the balance in favor of higher scoring. Defensive technology seems to rise without much extra assistance. In the mid-1990's teams increasingly began playing an improved defensive strategy called the "neutral zone trap," and as a result game scores again languished. Fans and officials were troubled, and a number of changes were proposed to change the relative size of the goal area, and offensive:neutral zones on the playing surface. Both items directly affect the balance of offense-defense in hockey. First consider the relative size of open space in the 6 x 4 foot (1.8 x 1.2 m) hockey net. The people who play hockey today are bigger than they were 100 years ago, but the size of the opening between the goalposts and crossbar have not changed. NHL players are an average of 2 inches (5 cm) taller than in 1970 [36]. The League has been reluctant to make any adjustments to the size of goal frames, however; goalkeepers' equipment has also been steadily improving and growing bigger to protect the players from injury. The NHL decided that some of these equipment modifications (e.g. longer catch gloves, bigger blocker pads, webbed jersey sleeves, and the ever-increasing leg pads) were good for stopping pucks, but not related to protection. Since the identification of the

Table 3.3: Game refinement values for NHL hockey [82]

Season	G	T	GR
1919-20	9.521	60 (est.)	0.051 (est.)
1928-29	2.918	60 (est.)	0.028 (est.)
1939-40	4.988	60 (est.)	0.037 (est.)
1949-50	5.469	60 (est.)	0.039 (est.)
1959-60	5.895	60 (est.)	0.040 (est.)
1969-70	5.809	65.518	0.037
1979-80	7.025	58.792	0.045
1989-90	7.368	60.530	0.045
1999-2000	5.492	55.950	0.042
2005-06	6.050	59.954	0.041
2006-07	5.758	59.966	0.040
2007-08	5.440	58.126	0.040
2008-09	5.695	60.404	0.040
2009-10	5.531	60.624	0.039
2010-11	5.464	60.778	0.038
2011-12	5.320	59.496	0.039
2012-13*	5.307	58.284	0.040
2013-14	5.343	60.076	0.038
2014-15	5.324	59.834	0.039

so-called “goal draught” and the “Dead Puck Era” of recent seasons, the legal size of goalie uniforms, leg pads and gloves has been reduced several times, and remains under strict scrutiny. The rules must allow fairness to offensive play but without exposing goaltenders to undue risk. The second item deals with the sizes of the two offensive zones, and the intervening neutral zone on the playing surface. The neutral zone was reduced twice, from 58 to 54 feet in 1998, and again from 54 to 50 feet in 2006. Offensive zones increased from 70 to 75 feet each. Changes to the size of the board (or field, or ice) are a factor in game refinement.

Low GR values might be interpreted as support for hockey’s reputation as a game that is inaccessible for the casual spectator. The pace is more than double that of field games like soccer, on a surface area of less than one-quarter ($1560m^2 : 7140m^2$). Casual spectators and uninitiated fans typically complain that the puck is too small and fast-moving to see. Probably the depth of look-ahead and fast changes require a higher level of involvement than other sports, but the goal scoring event is easy for everyone appreciate. The League is aware and actively attempting to raise the number of goals per game, but without harming the fundamental nature of the game it is a difficult task. Knowingly or unknowingly, they are also attempting to raise the hockey GR.

3.5 Golf

The model described in the previous sections is used to evaluate the scoring systems of golf, volleyball and tennis. There are several points unique to the golf game which have been considered without coming to a definitive conclusion using game refinement theory. In golf, as in tennis and volleyball, the length of game is determined already before play begins. According to the International Golf Federation’s History of Golf webpage, the Old Course at Saint Andrews, Scotland redesigned to an 18 hole course in 1764–this would become the recognized format for golf around the world [49]. Archeological and historical evidence indicates that golf is a game that has been played continuously for several hundred years. International standardization of such items as yardages for golf links and “par” values, expected scores for expert level play, was completed after the First World War. Golf is unique

Table 3.4: Game refinement values for professional golf

2014 Championship Leaders [80]	G	average T	GR
PGA par	18	71	0.060
Top 73	18	69.96	0.061
Rory McIlroy	18	67	0.0633
Phil Mickelson	18	67.25	0.0631
If perfect play	18	54	0.079

in that, while the number of holes in one round has traditionally been fixed at 18, rather than the player earning the most points, it is the player who accomplishes the course with the fewest number of tries or "strokes" who wins. In this case it is T and not G which will determine winner or loser. It was unclear whether this could pose a problem for evaluating with the model, but at least in one instance when we see the result of refinement values for golf, they are like those of previously studied game types.

The final scores for the top finishers were evaluated after completion of the PGA Championship of 2014 at Valhalla Golf Club [80]. The course is a par 71 and the championship consisted of four rounds, for a total score of 284. Most of the 73 leaders were slightly under par, by an average of one shot per round. G can be set at can set $G = 18$ and $T = 71$ to find the expected game refinement value of the golf game at $GR = 0.060$ assuming par. The actual values for leaders is a few shots lower, as shown in Table 3.4. It is not expected that refinement values for golf will change, even given some very remarkable jump in the training and skill of professional golfers. For example, under our model, a world record-breaking score of, say 54, produces a GR value only slightly above that of chess. PGAs have also been very careful about admitting even slight technological advancements (whether belly-putters, deep U-grooves on a club's face, or otherwise) and it is likely that changes in equipment specifications present the only possibility for significantly lower golf scores.

Equation 3.7 shows the highest plausible GR-value for a "perfect round" of golf under this model.

$$\frac{\sqrt{18}}{54} = 0.079 \tag{3.14}$$

These results appear to be consistent with the results from other games under the model. It needs pointing out that this application differs from other score-limited sports in that one round of golf is considered a game, whereas the other sports were considered in the match. To consider golf in the match context using the model as applied above, the result is substantially lower:

$$\frac{\sqrt{72}}{216} = 0.039 \tag{3.15}$$

for completely perfect play, or

$$\frac{\sqrt{72}}{284} = 0.030 \tag{3.16}$$

for par.

This identifies two problematic questions for this use of the game progress model for golf. Badminton, table tennis, volleyball, and tennis are the same in that the match consists of games, or games and sets which are normally completed on the same day. Therefore, if the model as used above is a valid one, should not golf matches which are commonly played over four rounds not on the same day to be considered to be one game? Or, are the individual rounds of golf one game? It would seem, and the results above would seem to support, that golf matches of this kind are composed of separate events. Golf, like chess, is a game which has clearly been optimized over many centuries of sophistication, and there is little denying its entertainment, at least for players. If one can ignore that 4 rounds' scores are cumulative in the match, then perfect or nearly perfect play in golf supports the use of this model with a GR value in or very near the window previously observed for other game types, but if not, there are still other explanations which could account for the low GR. If golf, like in hockey, is a niche sport which may be optimal for players but not for observers, then the lower GR value is probably accurate.

3.6 Tennis

According to the 2015 International Tennis Federation Handbook [50] standard scoring of a tennis game proceeds as follows: (0) also "love;"(15) First Point; (30) Second Point; (40) Third Point (with a tie at 40 being known as a “deuce;” and Fourth or Game Point, except in the case of a deuce, in which case this point becomes known as Advantage. This ensures that games must be won by a margin of at least two points (and in the case of a deuce, by two consecutive points). There is some ambivalence, however, since the rules of tennis are open to several alternative scoring methods. Under the “No-Ad” or “tiebreak” scoring alternative to the advantage rule, a game proceeds to a decisive game point directly from a deuce. Similarly, a set of at least six games must be won by a margin of two games under Advantage Set rules, with no limitation to the number of games required. Alternatively in a tiebreak set, winners are determined within a maximum of 13 games. Matches too can be comprised of best-of-three or best-of five (providing a two-game lead), or any of several alternative Short Sets (first to win four, provided a two-game lead) and tiebreak match systems. Let us consider some of the major differences between the top level tennis events by applying a round-match approach of the model.

The Wimbledon, U.S. Open, French Open, and Australian Open are the best known annual tennis events. These tournaments employ best-of-five set matches for men’s singles and doubles, and best-of-three set matches for women’s and mixed tournaments. All four tournaments now employ the tiebreak rules, except in the fifth (final) set, which only the U.S. Open allows to be decided in a tiebreaker. Masters 1000 includes: Indian Wells, Miami, Monte-Carlo, Madrid, Rome, Canada, Cincinnati, Shanghai, and Paris Masters, all of which are played in the best-of-three, tiebreak format. For the purpose of comparison to the other formats we only need consider only the data of Masters 1000 Men’s. The intended research requires complete point score totals for the matches, which are not published on ITF. While IBM SlamTracker tracks point totals per player, their dataset is not available for independent research. Observations are taken from data of recent years’ four major world tournaments “the Majors,” and Masters 1000 tournaments as they were found on a popular gambling website [102]. The data consists of total points for the winning side, and total match points for every Majors (also “Grand Slam”) tournament match from 2004 to 2014, 5080 matches; and Masters 1000 Men’s tournaments from 2006 to 2014, 4536 matches.

Best-of-five matches seem to appear only in the domain of men’s Major tournaments. Most competitions are decided in best-of-three, including mixed doubles, women’s, and all Masters 1000 events. Of the many proximal or distal reasons which could be imagined for having a best-of-five for men's competition in the Majors, perhaps tradition or the test of endurance might receive the most mention. The topic seems to be under somewhat vigorous discussion in the tennis community. Many good arguments exist on both sides of course, so the discussion here will be confined to the mathematical realm. However, it is interesting that two typical arguments, one of each for and one against the 5-set match, are of a mathematical nature. One states that the best-of-five is more interesting than a best-of-three contest, because best-of-three format (say, Player A vs. Player B) can produce only four distinct three-set outcomes: ABA, ABB, BAA and BAB. A five-set match, on the other hand, has 12 possible set outcomes, creating a host of ways momentum can twist and turn over the course of several hours [94]. Opposite this, it is opined that best-of-five-set matches have little effect on outcome in men's tennis [67], with Miller citing that only about 2.5 to 3.5 percent of game outcomes change between the third and fifth sets.

From a game refinement standpoint, it can be seen that the GR-values of best-of-three competitions more closely conform to the GR-values found in other games under the model than do best-of-fives. This is clearly the greatest difference in the measures of game refinement among the tournaments. It can also be noticed that, although it is true that there is a gender difference to consider between best-of-three and best-of-five contests in the Majors, the GR-value for men's best-of-three contests in the Masters 1000 are quite nearly the same as those for women's in the Majors. For the eight years of observation from Masters 1000 matches, we found a GR-value of 0.064, while those for the three-set Majors ranged from 0.062 to 0.064. For this reason we believe that the difference in GR-values between best-of-three and best-of-five-set competition is not attributable to gender.

Except for the separation between three-set and five-set matches, the next greatest divider of GR-values among tennis championships is probably the tiebreak final sets used only in the U.S. Open. The purpose of the advantage point and advantage set rules was to assure a certain margin for victory by requiring consistency and endurance from competitors. Because of this fact, the length of tennis matches played under advantage rules are unknowable and potentially interminable. In the most extreme example, the Isner-Mahut match at Wimbledon in 2010 played out over 11 hours, including eight hours tied in the final set. One could suppose that uncontrollability of game length was tolerated or preferable in the game context

Table 3.5: Game refinement values for professional tennis tournaments

Association	Rules	Tournament	Surface	Avg.G	Avg.T	GR
ATP (Men)	5 tiebreak sets	U.S. Open	Hard	105.4	193.3	0.053
	No T/B final	Australian Open	Hard	119.8	218.9	0.05
	No T/B final	French Open	Clay	119.5	217.9	0.05
	No T/B final	Wimbledon	Grass	122.8	226.6	0.049
WTA(Women)	3 tiebreak sets	U.S. Open	Hard	77.9	138	0.064
	No T/B final	Australian Open	Hard	77.53	139.4	0.063
	No T/B final	French Open	Clay	77.4	139.4	0.063
	No T/B final	Wimbledon	Grass	77.96	140.8	0.062
Masters(Men)	3 tiebreak sets	(all)	(all)	82.37	141.8	0.064

of tennis' past, though apparently that is no longer so. No-ad (tiebreak) alternative scoring eliminates this tradition, and much of the unpredictability of match length. As can be seen in Table 3.5, among the Majors the U.S. Open has the shortest and lowest-scoring matches, and the highest game refinement value, though the difference is slight.

3.7 Other Score Limited Games

Observations on table tennis and badminton are taken from the author's recently presented paper [75]. In the case of volleyball, observations are borrowed from Takeuchi, et al. [100], which uses game refinement theory to study the effects of several changes made to the game of volleyball.

Volleyball was switched from a side-out scoring system to the rally point system in 2000. Prior to the institution of the rally point system, only the side serving could win a point. It can be safely assumed that the primary impetus for this change was to make volleyball game length more predictable for the purpose of television programming. Side-out games were won by the first team reaching 15 points with a minimum 2-point advantage. It was predicted that the use of rally point scoring would significantly speed up game progress, so along with faster games, higher scoring was introduced. The points required for a win were initially doubled to 30, and the 30-point rally system was employed from 2001 to 2007, then backed down to 25 points in 2008. Experimentation with the volleyball scoring system provided three distinct opportunities for applying the model. Although Takeuchi created reasonable estimates based on average winning rates to compare rally point with side-out rules, his estimate failed to take into account that volleyball is a round-match type game consisting of best-of-five contests. Assuming an average length of match of, say, 4.25 sets (games) per match shows that the rally point system raises the game refinement value over side-out from 0.036 to 0.055.

In badminton a best-of-three, 15-point side-out scoring system (3 x 11 for women's singles) was employed until 2005 [75]. As in pre-2000 volleyball, only the server can score the point. If the service side loses the rally, no point is awarded, and the service passes to the other side. Under the new rally point system in badminton, the side which wins the rally gets the point, regardless of service. Game length is 21 points, with a minimum 2-point advantage. Average winning scores G and total scores T in men's singles under both systems were compared. Game refinement for the two variants differs significantly, indicating the change of game progress as affected by the change in scoring. The result shows that the changes in badminton service and scoring has brought the GR-value of badminton nearer to that of chess and the model considered previously in other games and game types.

Table tennis already used a rally point system prior to 2001 when the game was switched from a 21-point win to an 11-point system. Several equipment changes to both paddle and ball were then instituted, and in 2008 a doubles event was added. First, the game refinement value of table tennis prior to 2001 was already in the window of 0.07 to 0.08. Second, the net effect of changes to equipment, scoring, and match length on game refinement was nil. Also, the newly introduced team event is significantly lower than for the other events. While the introduction of the doubles event allows more players to join the tournament, the significantly lower GR value foretells some adjustments to come to the doubles' format.

3.8 Results

The histories of several games were reviewed, and the most recent rule changes and subjects of discussion were considered for the purpose of evaluating them as gaming systems under the model of game progress. The reason for doing so was to find a transferrable game information metric from board type to other type games. Several approaches were evaluated to approximate the most accurate and reasonable value of the game search tree for use with the model. For the arcade games a board game approach identifies a number of choices and movements on the game board, although the crane type game UFO Catcher also incorporated an economic element for comparison of two countries' versions of the same game. More generally, goals and tries were applied as branching factor and depth of game for both time limited and score limited games, though with a significant difference in their interpretation. As one-shot games, time limited games were evaluated in the time limited approach as in the case of football [18]. Score limit games were evaluated with the round-match approach since scores accumulate over the course of several rounds. The evaluation of golf was less conclusive, due to the ambiguity of scoring for rounds or matches. Preliminary work with hockey seems to support the use of this metric, at least according to the dominant narrative regarding a "goal drought" in recent seasons. While the number of shots per NHL hockey game holds relatively stable, and the percentage of successful shots only slightly less stable over time, it can be seen that raising the GR value of hockey entails a significant increase of goals (or some remarkable new advancement leading to increased shot accuracy). To increase the number of goals per game without completely upsetting the long and cherished sophistication process requires further changes to the offensive balance such as stricter rule enforcement (more or longer penalties), increasing offensive zones and opportunities to shoot, available shooting area (i.e. by limiting goaltender pads' size), or larger goal frames. Most of these have already been done at least to some degree, though the League acts conservatively, incrementally most of the time. These results do not imply that it is necessary or preferable to impose any changes, but just that it can be seen that doing so would likely raise the GR of hockey closer to that of other "sophisticated" games. FIDE (chess), UEFA (football), and pretty much all of the other governing boards of games and sports exhibit the same cautious approach as the NHL with regards to the sophistication processes in their hands.

A pattern of service-type games eschewing their traditional scoring systems seems to have been established with the switch in volleyball from side-out scoring to the rally system in 2000, and badminton and table tennis following up with major changes soon after. The primary reason for the changes to scoring systems has been to make those score-limited sports easier to televise, and easier for casual spectators to follow. Knowingly or unknowingly, the governing boards of many sports have also been actively involved in controlling the breadth and length of their respective games. Prior to 1974 the U.S. Open and French Open were played on grass courts, which have been known for their fast, sometimes slippery surfaces, and given to a particular type of game play. The last remaining Major to continue the tradition of grass courts, the Wimbledon has acted to harden the underlayment and slow the upper surface. Perhaps more significantly, in terms of game refinement theory, from 1980 tiebreak sets were introduced to the Majors except in the final set, which only the U.S. Open allows by tiebreak. Tennis is among the older, and more conservatively controlled sports, but we see these increments moving tennis closer toward a 0.07 GR-value. The GR-value of volleyball rose from 0.036 to 0.055, attributable to the switch to rally-point scoring. Badminton's GR-value fell from 0.121 to 0.086 in response to service and scoring changes, and table tennis, which started with

Table 3.6: Summary of GR values of various games

Approach	Game	G	T	GR
board game	Ms. Pac-Man [19]	244	345	0.045
hybrid*	UFO Catcher Thailand [14]	0.367*	10.65*	0.057
hybrid*	UFO Catcher Japan [14]	0.967*	13.30*	0.074
time limit	basketball [98]	36	82	0.073
time limit	football (soccer) [18]	2.64	22	0.073
time limit	hockey	5.34	60.08	0.039
round match	tennis	82.37	140.8	0.062
round match	golf	18	67	0.063
round match	volleyball [100]			
	– (old) 5 x 15 side-out system	63.25	223.2	0.036
	– (new) 5 x 25 rally point system	106.25	187	0.055
round match	badminton [74]			
	– (old) side-out 3 x 15 system	30.07	45.15	0.121
	– (new) rally point 3 x 21 system	46.37	79.34	0.086
round match	table tennis [74]	54.86	96.47	0.077

*UFO Catcher contains stochiastic variables not found in the other games. Number of plays P for G were evaluated with playing cost per win cT as T

GR around 0.075, did not deviate significantly from that value as a result of reduced game scores balanced with game speed changes due to equipment modifications.

While these findings are not prescriptive, observations from other sports and across game types imply that informational complexity and its rate of flow in games is directly tied to game entertainment, and that sophisticated games will tend toward a similar measure of that information. Games continually evolve in order to maintain fairness, safety, or enjoy-ability for players and spectators. In the case of score-limited sports, evaluation with the game progress model seems to prove that many of the changes in rules, particularly scoring systems, result in game refinement values nearer to those found previously in other-type games.

3.9 Game Information Dynamics

The following section is an abridged version of the work contained in

Iida, H., Nakagawa, T., and Nossal, N. (2012). Certainty of patient survival with reference to game information dynamic model, Open Journal of Preventative Medicine 2-4, 490-498.

In other previous works [47] [46], the notion of a physical "information flow" was explored. The game information dynamic model suggested correspondences between physical and the theoretical world of information. Game information dynamics takes for it basis of comparison the boundary-layer theory of Schlicting [92], and proposes approximately the following argument:

- Data has at least some mass.
- Information is comprised of data.
- Therefore, information has mass.

The theory then draws correspondences between physical fluid in the boundary-layer theory, and information in the mind. The theory is subject to an equivocating "if" information in fact has mass, or whether information is strictly formless–a concept only, completely separate from the physical universe, of which data is merely a representation.

"In the present study, information of game outcome represents the data which is certainty of the game outcome, and so the present information might correspond to Rauterbergs sixth interpretation. It may be evident that information before and after the reception of a message is not the same. There are two possible dynamical views of information (incorporating force or energy); firstly, information flows, secondly, information is entropy. The gap between the two viewpoints is not small in practice: the former considers that information is tractable within physics, but the latter views that information is beyond physics, even though there is some relation to it. If information flows, it may be natural to consider that motion of information particles having mass is governed by the basic equations for fluid mechanics.

The dependent variables in fluid mechanics are velocity, pressure, temperature and density, all of which depend on the position and time. These are considered to be related to information in games for example. We consider that information is produced as the result of motion of information particles. Solso shows that motion of visualized fluid particles is detected by the eye through light, having enormous high speed ($3x10^{10}cm/s$), and is almost instantaneously mapped on the retina of the eye. We consider that during this process, through fluid particles are transformed into information particles. The eye and brain work together in collecting light and in processing the information particles.

"The eye collects light and passes it to the neural network in the brain, which processes it and then redirects the eye to scan elsewhere. Visual signals from the physical dimensions enter the eye as light and are normally recorded on the retina as two-dimensional images. The brain interprets two-dimensional visual images as having three-dimensions by use of contextual cues and knowledge of the world as gained through a lifetime of experience. Electromagnetic signals or photons due to light carrying the information on the motion of visualized fluid particles are converted to electrochemical signals and passed along the visual cortex for further processing in other parts of the cerebral cortex. It is, therefore, considered that flow in physical world can be faithfully transformed to that in eye and brain (referred to informatical world here after). If a flow phenomenon due to the motion of fluid particles in physical world can model an information phenomenon, the latter must be caused by the same motion of information particles in informatical world. Thus, in this study it is essential to find correspondences between a flow and information phenomena."

"Imagine that the assumed flow is visualized with neutral buoyant particles, for example. Motion of the visualized particles is detected by the eye almost instantaneously through the light having enormous high speed ($3x10^{10}cm/s$) and is mapped on our retina, so that during these processes, motion of fluid particles is transformed into information particles. This is why motion of the fluid particles is intact in the physical world, but only the reflected light, or electromagnetic wave consisting of photons can reach the retina. The photons are then converted into electrochemical particles and are passed along the visual cortex for further processing in parts of the cerebral cortex. The photons and/or electrochemical particles can be considered to be information particles. It is, therefore, natural to expect that flow in the physical world is faithfully transformed to that in the informatical world. During this transformation, flow solution in the physical world changes into the information in the informatical world" [47].

From Newton's second law ($F = ma$) game refinement theory asks, what are the correspondences of force, mass and acceleration in the light speed world of information? There is already a significant take-away. However slight, if information has any mass at all, then it can be shown to flow not only metaphorically, but in the same manner as fluid, possibly even governed by the very same laws. The ramifications are far-reaching, and unfortunately far beyond the scope here today.

Figure 3.1: Correspondences of physical world and informatical world in the game information dynamic model
[47]

Physical World (flow)	Informatical World (game)
u: flow velocity	I : current uncertainty of game outcome
U_0 : plate velocity	I_0 : initial uncertainty of game outcome
y: vertical distance	L: current game length
δ: gap between two walls	L_0 : total game length

Chapter 4

Towards Verification with BAMING

This chapter is an updated and abridged version of work previously published in

1. Nossal, N. (2015). Brain Activity Measurement in Gaming: BAMING, International Review of Management and Business Research 4-2-1, 373-386.

2. Nossal N., Tsuchiyama, N., Hidaka, S., and Iida, H. (2012). fNIRS survey of brain function at the moment of winning. IPSJ Symposium, 6, 179–182.

In light of the preceding chapters, it has been considered that some additional laboratory experimentation might be necessary or useful for expanding upon and affirming the basic concepts of game refinement theory. What follows is survey work in the science of brain imaging of gamers during gaming (BAMING), with some preliminary work in the use of BAMING to describe the correlation of activation in the gamers brain regions of interest with the arrival of the critical position in a game. Critical positions were discussed in the context of original game theory for their relevance to the tools of game sciences, and identified as an interesting point of study bearing some relation on one of the bases of game refinement theory: the acceleration of game information in the end game.

Neuroanatomy is a mature field where research has advanced to the finest details of the most microscopic levels of function and connectivity. The science of functional brain activity measurement is a sub-field still in its infancy where researchers are focused mainly on identifying the locations of neural activation under various cognitive stimulation. Advances are progressing and accumulating at a steady pace. University of Washington researchers Rao and Stucco [4] posted an experimental result online purporting to show the first instance of the use of a human brain-to-brain interface. Though some peers were understandably critical of their method of publication, Rao and Stucco at least showcased an interesting development with major implications for therapeutic use, using electroencephalography (EEG) and a transcranial magnetic stimulation device. See also Grau et al. [27] for more about advances in brain to brain interfaces.

4.1 Brain Imaging

Technologies like fMRI, positron emission tomography (PET) and x-ray computed tomography (CT or CAT scan) provide excellent spatial resolution and the ability to show oxygenation or metabolic processes in action, even deep within the basal structures of the brain, however the sensitivity and massive configuration of the hardware renders the subject immobile. So while those are superior qualities for medical purposes and research, fMRI, PET and CT present special challenges for research on people during normal activities. What fNIRS loses in terms of depth of measurement and spatial resolution, it makes up for with superior usability, mobility and safety to subjects and operators. At the time of this writing, scientists wanting to image the functional brain during any activity requiring more than facial, ocular or the most minimal of head movements need EEG or fNIRS. Only fNIRS can image cortical blood-oxygen level dependent (BOLD) signal changes of mobile participants. Marco Ferrari and Valentina Quaresima [22] have noted in their review of the history of fNIRS for brain measurement that “Hitachi has introduced two battery operated wearable/wireless systems suitable for performing fNIRS measurements on adult [prefrontal cortex] PFC; i.e. a 22-channel in 2009 (WOT) and a 2-channel in 2011 (HOT 121B). Both instruments are currently only available in Japan.” The

Figure 4.1: Full mobility 22-channel fNIRS array
(courtesy Fujinami Laboratory, JAIST School of Knowledge Science)

Hitachi WOT-220R wireless fNIRS headset is shown in Figure 4.1. With temporal resolution of 5 Hz and analytical capability sufficient for producing cortical activation maps, measurements can be interpreted graphically, or with reference to an average brain (Figure 5.2).

Frans Jobsis discovered that near infrared light could penetrate skin, bone and most tissues of the human body except hemoglobin, which absorbs some. Neurons, despite their large energy requirement, do not store oxygen and glucose. During activation neurons receive these from the blood. Neural activation causes increased flow of oxygenated blood to the active areas as well as increasing the flow of deoxygenated blood from the active areas, i.e. neuro-vascular coupling.

The development of fNIRS and functional magnetic resonance imaging (fMRI) for brain imaging are tied to the discovery of the BOLD signal in 1990. In that year, Seiji Ogawa and colleagues [77] discovered that minute fluctuations in blood oxygen levels could be used to effectively measure, among other things, the functioning of the brain during MRI. Being able to detect the BOLD signal also forms the basis of fNIRS. Villringer and Chance [105] recognized that "optical measurements could be performed in walking people or under other natural conditions that are not easily accessible by

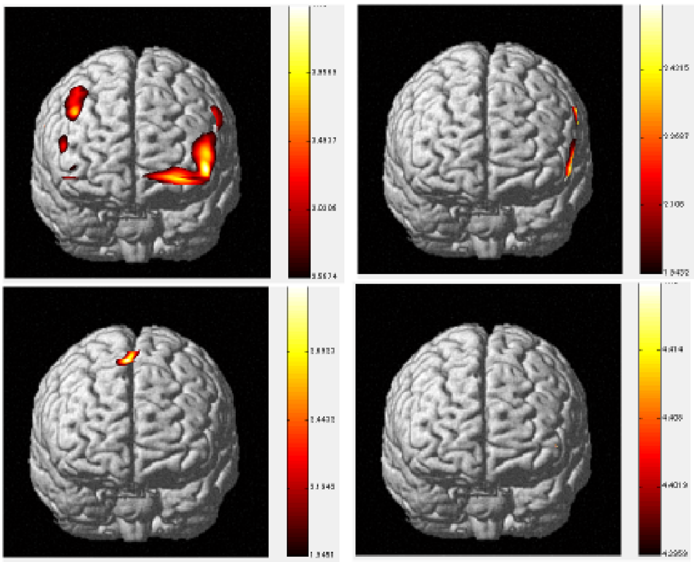

Figure 4.2: Images of fNIRS data on average brain
(courtesy Hideo Shinagawa, Institute of Social and Economic Research, Osaka University)

other functional methods." Mobility, ease of use, and accessibility are prime factors both for cognitive experimentation, and for user interfaces.

4.2 Functional Brain Measurements of Gamers During Gaming (BAMING)

BAMING (Figure 4.3) is the intersection of the physical world of brain measurement of gamers, and the informatical world of games and information. Development is growing along all axes, and the middle line (BAMING) could be of particular interest for the development of game refinement theory. The neuroscience side, with exciting new projects like the ambitious attempt to map the entirety of neural interconnectivity, the Human Connectome, is itself a cross-study of applied theoretical graph theory and neuro-anatomy [32] [96] [95]. On the more theoretical side of BAMING, Games-with-a-purpose, where players' inputs to an ostensibly recreational game are used for inputting to some other purpose of the game designer [2] and serious gamers learn vital skills, e.g. in game simulations [7]. The building of brain-machine interfaces (BMIs) is an exciting new area of discovery, identified by Matthews et al. [64] as best served by fNIRS. At the same time most fNIRS researchers recognize that fNIRS is not yet a mainstream BMI technology, and has yet to be fully exploited [64] [20]. Prior experiments in BAMING centered on economic gamers and recreational gamers in other contexts appear on parallel axes between the more theoretical line of this BAMING project, and the more applied science of brain activity measurement. The seemingly unanimous opinion of researchers in fNIRS and in gaming is that there is that there is a need for more experiments in both fields. We recognize this to be an opportunity for rapid progress in the field of fNIRS for theoretical gaming.

4.3 Survey of BAMING Experiments

Studies of players of recreational games with fNIRS began in the middle of the previous decade. Matsuda and Hiraki [63] reported 21 five-minute long trials of 13 right-handed 7-14 year olds playing either one or both of two different video game types–a fighting type, and a puzzle type. They found decreasing oxyHb during the game in children, but had also reported

A research map of BAMING related fields. The y axis Brain Measurement of Gamers is part of the physical world, while the x axis Game Sciences, starting with expected utility theory and game theory is theoretical. In between are some of the applications derived from or directly related to the two, including those currently under construction (dashed line) such as the Brain Connectome, affective gaming, strong AI, and general gaming.

Figure 4.3: BAMING

the same in adults in their previous study referenced in the same work. Nagamitsu et al. [70], responding to fears of the general public that video gaming could be detrimental to the human brain, performed nine trials on six children and six adults (three male, three female per group) during a hand-held video game. While Nagamitsu does not report any necessarily detrimental effects, the result adds to the body of evidence we seek to build for understanding the substrates of neural connectivity and interaction during games. They reported a possible age-dependent difference in the Hb oxygenation of the dorsolateral PFC (positive for adults, negative for children) which in hindsight also suggests a top-down modulation for the adult players but not for children.

In 2008, Audrey Girouard led a group of computer scientists and biomedical engineers from Tufts University to produce a ground breaking result in brain activity measurement of humans during gaming [25]. They sought to prove, using a two-channel fNIRS device and the NASA Task Load Index, that researchers could determine whether a subject was at rest, playing an easy version of Pac-Man, or a difficult version of Pac-Man. They reported that indeed they could determine with 94 percent accuracy whether a player was resting or playing, and 61 percent accuracy whether the player was playing the difficult version or the easy version. While this experiment was not the first brain measurement during gaming experiment ever, the attempt to measure differences in stress intensity of a player's experience using brain oxygen level dependent signals was largely successful. In some regards, it might seem like a very modest advance–using both user-reported and brain measurement data, with a wide margin of error it could be determined whether a person was playing the easiest, or the most difficult setting on Pac-Man. Another way to look at this result however could be amazing: Using fNIRS technology, the Tufts group could, to an extent, measure how the player was feeling or thinking during a 30 second window, which signifies the earnest beginnings of measuring human mental experience. Using the improved fNIRS and other technology, such as EEG or heart/respiratory rates, with confirmation by self-reporting, it could be very interesting, and useful to see how far that margin of error can be mitigated, and to see more and finer distinctions in functional brain measurement during gaming activity. Refinements on the method could be used to help find the measurable similarities believed to exist among players, as the result of various game dynamics such as information acceleration at the time of experiencing

Entertaining games have a great deal of uncertainty until the final moments of the game. Steadily progressing uncertainty of game outcome, like the roughly 45° curves, and games which progress quickly to certainty of game outcome (concave curves) are less entertaining.

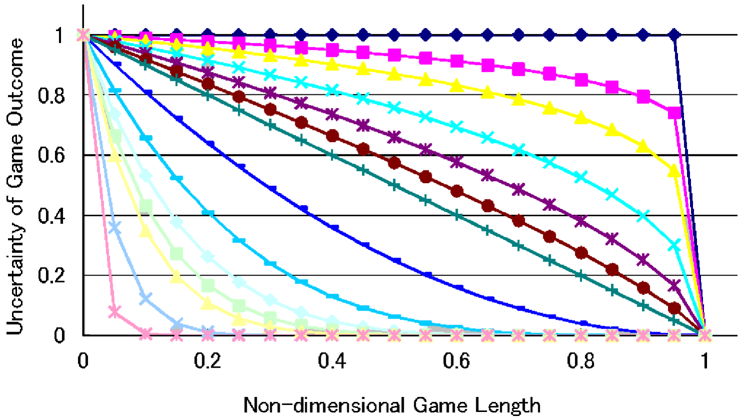

Figure 4.4: Uncertainty of game outcome

[46](by permission)

a critical position.

Since the Girouard experiments, several more studies have been published in a similar direction. Hoshi et al. [38] published results suggesting the possibility of recognizing emotional states, i.e. pleasant or unpleasant, using fNIRS. The work of identifying brain regions related to emotional responses using fNIRS is in its very beginnings, and seems to be gaining interest [68] [20] [56]. More recently Ono et al. [79] conducted four trials each of 26 adults while playing a dance game Dance Revolution, measured in the left side frontopolar cortex (FPC) and left middle temporal gyrus (MTG). Ono reported a remarkable separation of game (dance) performance in relationship to frontopolar oxyHb and sustained oxyHb in the middle temporal gyrus, with the lower performing players recording higher FPC, and lesser MTG activation, and high performers having decreased "suppressed" FPC and more persistent MTG activation, as evidenced by oxyHb.

It is necessary for operators of brain imaging platforms to develop "x-ray type knowledge" [85] of all the brain's structures, similar to that of neurosurgeon. To highlight the relationship of the outer surfaces with the underlying cortical structures, the cranium is removed from the supraorbital process and along the zygomatic arch and down the zygomatic process. The coronal suture, superior temporal line, and pterion are replaced as landmarks.

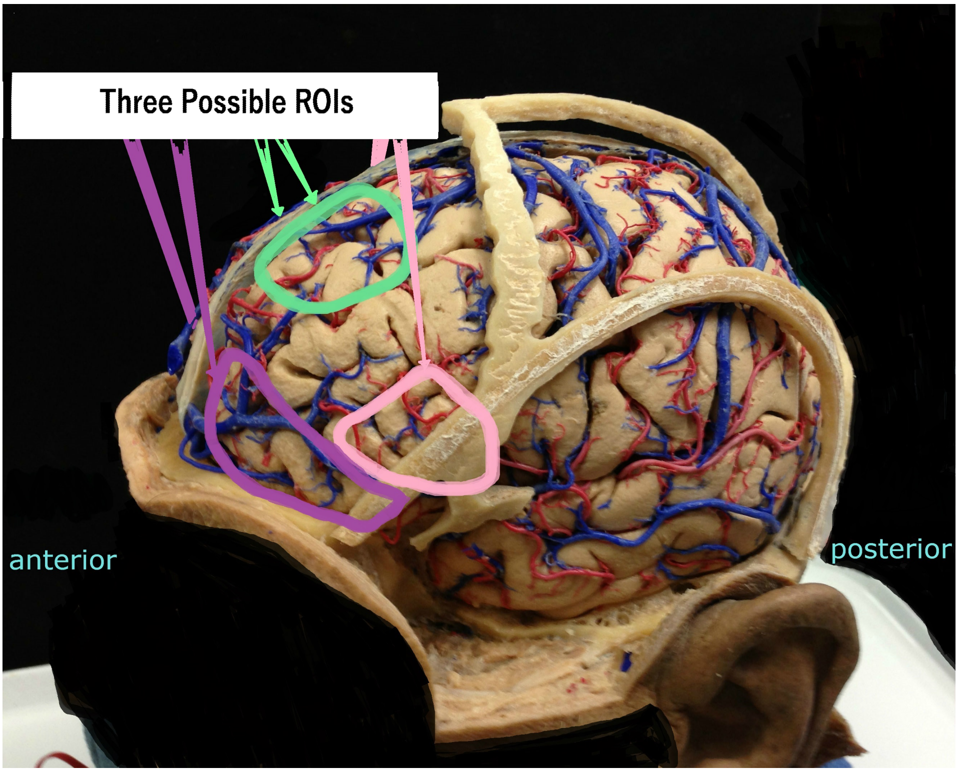

Figure 4.5: A visual for the alignment of the mobile wireless fNIRS headset on living subjects over regions of interest (ROI).
(photo by Nathan Nossal, courtesy Dr. Albert L. Rhoton Neuro-Microanatomy Laboratory)

Nossal et al. [74] attempted a BAMING experiment designed to measure, control and analyze fNIRS data for a player engaged in gaming. The experiment identified the objective for that observation; a critical position in gaming, as well as several design and implementation challenges. We refer to the recognition of players' arrival at a critical position as a Wakatta Moment, because of the Japanese word meaning "understood" or "got it." The Hitachi WOT-220R wearable infrared laser topography headset and associated hardware and software were employed.

It tends to be helpful for game experimental subjects to be analytically skilled, highly motivated, and playing a game which is well-understood and practiced before proceeding. In early game theory experimentation, albeit in the more traditional normative economic games, insensitivity to player motivation or incentives, and failures to present the game for maximum elicitation of the target behavior often led to specious results (i.e. without getting players to participate fully or realistically) [13]. Still, the same principles apply, with the exception that only a controlled familiarity with the puzzle, rather than practice was allowed, as will be shown. A game with the following characteristics was sought:

- one player alone can play
- has a time limit for completion
- has an incentive for winning
- is fun or interesting to play
- has never been played by the subject

A puzzle in the shape of an unfolding Kepler polyhedron (a star) was selected. The star can be inverted and reassembled in a new color, at which position the puzzle is solved. Strictly speaking, the solving of a puzzle does not make a "game," but a one-player game can be created from most any activity if a time constraint is imposed. A (rather generous) time constraint of four minutes was imposed. It would be considerably simpler to identify a moment of winning, it was thought, so the time constraint was chosen assuming that all participants would complete. To increase motivation, participants were informed that winners would receive a very modest prize. Prizes were later revealed to be packets of cookies, trinkets, or gift certificates worth about 200-300 Japanese yen (2-3 USD).

Participants were shown the puzzle, and asked if they had ever seen it before or did they know how to solve it. As long as the answer was "no," participants were told that at the start of the clock, they would receive the puzzle, which they were to disassemble, and reassemble in a new color. Eleven participants wore the headset for measurement of oxygenation/deoxygenation in their frontal cortices while they played a simple one-player puzzle game. Once the information of winning (or losing) was presented, a neural response was expected to be discernable in the oxygenation record within a few seconds of stimulus. According to spontaneous utterances of participants such as "Oh! Is that it?" as well as a post-trial survey, ten out of the eleven participants did not realize they were close to solving the puzzle. Four were even unsure of having solved it even a few seconds after completion. Data was selected from a ten second period five seconds prior and five seconds post event stimulus of a "Wakatta Moment" or, in the case of no discernable "Wakatta" moment, the moment of solving the puzzle. Channels 11 and 12 (over the longitudinal fissure and parts of the left and right superior frontal gyri) were selected for analysis. Numerical waveform data (Figure 4.6) obtained from the WOT-220 was analyzed. Linear least-squares were applied to the eleven data sets. 6 rising and 5 declining oxygenation patterns were found in the data analyzed. Average oxygenation from the eleven records is shown in the Figure.

Post-data analysis it was determined that non-uniformity of the time of stimulus compromised the ability to reliably compare records from one participant to another. This fact was determined to preclude the reaching of any significant results in the experiment, and work to assess group average oxygen change with the removal of individual means was stopped. Although conclusive fNIRS evidence of that Wakatta Moment was not immediately found in the first trial, the experience positions any like-minded group of game researchers or brain researchers to carry out the next attempt to substantiate the moment of recognition of winning/losing in the human player's brain.

Oxygenation values for 11 participants (Columns 1-11) and the mean average (Column 12) for Channel 12 of the WOT-220 Wireless fNIRS System. Five values are recorded per second.

	1	2	3	4	5	6	7	8	9	10	11	avg.
Sec. 0.2	0.770585	0.205722	1.373609	0.592175	0.085326	0.384195	0.041007	0.095437	0.187681	2.546449	−0.10512	0.561552
0.4	0.772437	0.205542	1.367625	0.599811	0.086062	0.382798	0.039478	0.061071	0.186653	2.547683	−0.11695	0.557474
0.6	0.774452	0.206103	1.359365	0.602996	0.087567	0.381153	0.038064	−0.01446	0.186323	2.549472	−0.14812	0.547538
0.8	0.77345	0.203152	1.356956	0.597136	0.09034	0.38147	0.038622	0.083333	0.183643	2.545622	−0.09985	0.559443
1	0.774409	0.197937	1.361238	0.60307	0.092089	0.382868	0.038592	−0.02626	0.183174	2.548038	−0.08099	0.552197
1.2	0.776039	0.198533	1.352201	0.609066	0.092252	0.383194	0.036602	−0.2239	0.185037	2.552421	−0.11834	0.531191
1.4	0.777785	0.2007	1.341007	0.608006	0.094039	0.381974	0.035682	0.013068	0.184447	2.554303	−0.17772	0.546663
1.6	0.776244	0.19713	1.345017	0.605772	0.097299	0.381343	0.036523	0.003752	0.183771	2.555798	−0.13679	0.549623
1.8	0.776953	0.196026	1.342415	0.599399	0.099645	0.382964	0.035871	−0.04209	0.182435	2.560756	−0.08401	0.550033
2	0.779318	0.197321	1.334835	0.599512	0.100371	0.38461	0.033358	0.018921	0.184893	2.569324	−0.0903	0.555651
2.2	0.780955	0.199205	1.328346	0.589072	0.101279	0.383031	0.032291	0.004459	0.18353	2.575029	−0.1126	0.551327
2.4	0.779159	0.193613	1.326515	0.583689	0.103049	0.382483	0.033214	−0.05082	0.182162	2.580027	−0.07694	0.548741
2.6	0.778698	0.191611	1.322752	0.582986	0.10714	0.384122	0.032565	−0.03624	0.182468	2.586874	−0.00503	0.557087
2.8	0.782087	0.19271	1.312934	0.576073	0.10988	0.386366	0.03009	−0.06061	0.182783	2.596512	−0.03444	0.552217
3	0.783779	0.194712	1.304205	0.572525	0.110682	0.385485	0.030198	−0.05576	0.180491	2.602732	−0.08419	0.547714
3.2	0.782583	0.19101	1.299118	0.570036	0.113191	0.385507	0.030366	−0.07872	0.179211	2.605649	−0.06339	0.546778
3.4	0.782578	0.191145	1.292662	0.564591	0.116885	0.387846	0.028153	−0.12625	0.177078	2.613137	−0.01556	0.546569
3.6	0.78516	0.191709	1.285297	0.563288	0.119555	0.391139	0.026608	−0.0887	0.175626	2.621137	−0.00932	0.551045
3.8	0.786343	0.189991	1.272088	0.561853	0.121012	0.3926	0.027031	−0.11195	0.17442	2.622765	−0.0486	0.544324
4	0.784538	0.186164	1.263599	0.556876	0.122282	0.393345	0.027274	−0.16084	0.171827	2.625695	−0.07656	0.535837
4.2	0.784443	0.186107	1.256256	0.55194	0.123976	0.396239	0.025655	−0.09696	0.170498	2.630556	0.024449	0.550287
4.4	0.78732	0.186771	1.248245	0.550107	0.126318	0.400484	0.024643	−0.1197	0.173809	2.63351	0.002599	0.546737
4.6	0.788783	0.185003	1.235819	0.545149	0.128491	0.40301	0.024927	−0.1667	0.175467	2.630924	−0.05589	0.535907
4.8	0.786671	0.183036	1.225305	0.531523	0.128892	0.404775	0.025682	−0.11911	0.170831	2.632835	−0.0825	0.535267
5	0.7853	0.183246	1.215451	0.52449	0.130238	0.40832	0.025021	−0.0947	0.172621	2.636718	0.016348	0.545732
5.2	0.787422	0.183714	1.206132	0.517132	0.132112	0.412692	0.023432	−0.10158	0.17549	2.632445	0.006347	0.543212
5.4	0.791365	0.180394	1.199046	0.504785	0.133717	0.415557	0.024326	−0.08803	0.17411	2.633421	−0.03721	0.539225
5.6	0.789667	0.177995	1.188546	0.502575	0.135088	0.41842	0.024406	−0.06677	0.170781	2.636796	−0.07878	0.536248
5.8	0.787601	0.177205	1.180334	0.495484	0.134151	0.421743	0.023174	−0.04808	0.168934	2.636019	−0.00255	0.543092
6	0.788463	0.177019	1.167453	0.484532	0.134481	0.426046	0.022472	−0.08085	0.169042	2.634566	−0.03466	0.535324
6.2	0.79232	0.172749	1.162536	0.483651	0.135601	0.429029	0.023228	−0.05398	0.166457	2.63784	−0.09949	0.531813
6.4	0.79234	0.171936	1.160785	0.478139	0.135372	0.432209	0.023174	−0.06086	0.162231	2.641724	−0.15638	0.525515
6.6	0.790527	0.17177	1.155303	0.467762	0.133453	0.435029	0.021499	−0.11041	0.165184	2.636914	−0.05725	0.528162
6.8	0.790156	0.169763	1.147696	0.466456	0.132377	0.439022	0.020936	−0.15636	0.16298	2.637363	−0.05498	0.523219
7	0.792337	0.166669	1.140707	0.463527	0.132714	0.442074	0.021909	−0.13279	0.156146	2.638376	−0.08232	0.52176
7.2	0.791926	0.166253	1.139316	0.455276	0.13324	0.444029	0.021514	−0.1088	0.167199	2.632198	−0.13391	0.518931
7.4	0.790085	0.165862	1.137954	0.453165	0.131866	0.446144	0.020014	−0.15286	0.162545	2.629367	−0.05988	0.520388
7.6	0.790988	0.162869	1.128345	0.456423	0.13059	0.448988	0.019298	−0.12996	0.156319	2.627775	−0.0749	0.519703
7.8	0.793489	0.159759	1.120472	0.451143	0.130555	0.452142	0.020361	−0.13672	0.162536	2.613695	−0.12424	0.513017
8	0.792445	0.159271	1.11514	0.445456	0.132113	0.453074	0.020031	−0.13433	0.171241	2.616771	−0.17621	0.508636
8.2	0.790419	0.160111	1.114551	0.444781	0.132849	0.455468	0.018347	−0.12645	0.162711	2.619394	−0.07565	0.517866
8.4	0.79103	0.158439	1.111386	0.439027	0.131577	0.457899	0.017838	−0.12081	0.161071	2.621757	−0.05835	0.51917
8.6	0.794267	0.158069	1.101283	0.433429	0.130929	0.460452	0.018392	−0.0604	0.164084	2.620634	−0.08216	0.521726
8.8	0.794895	0.157907	1.094106	0.433333	0.13043	0.461739	0.018707	−0.06742	0.16282	2.624615	−0.124	0.517013
9	0.79179	0.155963	1.090012	0.43261	0.132106	0.463316	0.017416	−0.16625	0.162983	2.629156	−0.11165	0.508859
9.2	0.790344	0.153552	1.090255	0.423772	0.132513	0.464446	0.017787	−0.11654	0.160208	2.626022	−0.06078	0.516507
9.4	0.793138	0.153143	1.084652	0.424244	0.130871	0.466976	0.019134	−0.05937	0.15994	2.628254	−0.07717	0.520347
9.6	0.796044	0.153769	1.075854	0.424712	0.131133	0.469448	0.020149	−0.11875	0.162894	2.634101	−0.1102	0.51265
9.8	0.793174	0.15256	1.07	0.414221	0.131697	0.469878	0.019447	−0.06195	0.161167	2.632519	−0.14878	0.512176
10	0.791532	0.152123	1.068188	0.416599	0.133143	0.471907	0.020023	−0.00979	0.162428	2.63223	−0.03313	0.527751

Figure 4.6: Raw waveform data captured by the WOT-220 fNIRS system

The figure below shows the mean average O^2 Mol/L (y-axis) and time in 50 increments of 0.2 seconds (x-axis); 10 seconds total.

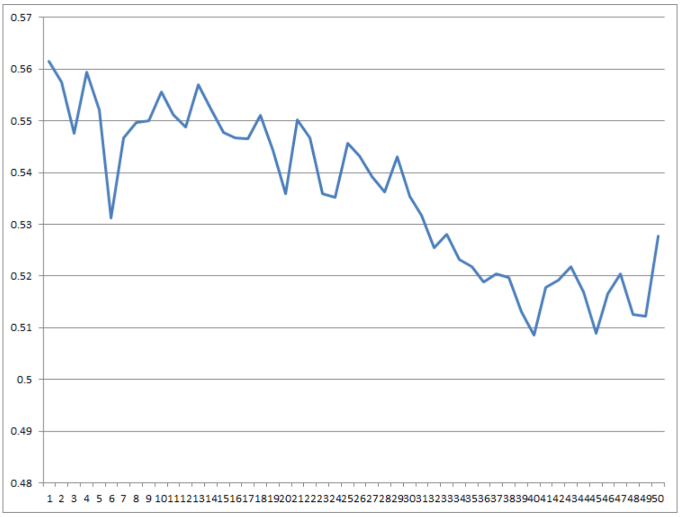

Figure 4.7: Average oxygenation during the 5 seconds pre- and post- solution of a puzzle under time constraint

4.4 BAMING and Game Refinement Theory

It has been asked why anyone should bother with a new brain imaging experiment, when we could simply ask gamers "Did you feel excited at that time?" and such questions. Indeed, the gamers should be asked this, but few participants are capable of answering precisely which part of their brain was doing the work, or which stimuli might correlate to the changes in brain activation. It is hoped that brain imaging studies will shed some light on this question, and prove or disprove what has been hypothesized about the reaction of gamers during game progress, by elucidating a statistically significant brain oxygen pattern simultaneously with the arrival of information of the game outcome.

Players must self-evaluate during the course of games. It is believed that the acceleration of information at game end has a direct effect on the state of players (or observers). Game refinement theory hypothesizes that this is also a decisive measure of a game's entertainment, as evidenced by a close game, undecided until the very last move provides more tension and excitement than a blowout victory by one clearly superior side. Furthermore, it is presumed that the marked area under the curve in the Figure 1.1 (right side) is an approximation of a final and decisive critical position of the game. This hypothesis can be strengthened by finding the information of gamers' experience, whether in terms of time, duration, or intensity. As Iida, Sutiono, Takeuchi and others have stated, there is a deficit of knowledge of the "physics of [information in] the mind." Work using fNIRS in games promises to widen that vista. A functional brain survey of the players to find the critical moment of cognition could also be useful for understanding the time and the location in the brain that players are thinking or feeling about this impending result in end games. Any experimental result identifying the area of brain activation at the arrival of the critical position would certainly hold value for both disciplines. On a game tree, the critical position is that node which leads to the inevitability of an outcome, such as a win, a loss, or a draw. In the mind of the player the critical position is the moment when that crises manifests. It can be expected that in the mind of the player, the arrival of a critical position marks a moment of change from a player's focus on evaluating and deciding moves to something else, namely resignation or victory. We expect results that will indicate a change from high-level, top-down regulatory mechanisms during play, as per Ono et al.

[79], with a return to more middling levels after most of the information of winning or losing is presented.

In the case of chess, the depth of lookahead would be a factor complicating enough to render the experiment rather difficult, if not useless. A great player might envision some 20 or more plies beyond the present board. Correlating the informatical critical position and the player's recognition of the fact would be an amazing yield, if it could be done! Instead, a game with only a few plies, where the arrival of the critical position is either a) controlled by the design or the administrator of the experimental game, or b) well-enough defined so that it is exact and unequivocal. At this early juncture in the study attempting to synthesize the uses of various sciences, it seems wise to strive for simplicity and economy.

Information of the game and its outcome is encoded somewhere in the brain. Girouard et al. [25] proved this dramatically with the successful use of fNIRS to evaluate player experience of a Pac-Man game. We know that the critical moment must come when players (or observers) greet victory and acknowledge defeat. We suppose that this moment comes just prior to the end of game after delivery of the information of the final critical position, and that the event manifests physically in the forebrain as well as autonomic structures. It is likely to also include the calming responses of the parasympathetic nervous system, as like in a post-acute stress response [53] [78]. The primary neural actors for that response, the amygdala, hypothalamus and pituitary gland are too deep in the brain to be directly measured by fNIRS, however we are also interested in the significations of that event in the areas of the outer cortex. fNIRS can detect BOLD signal changes in the outer cortical areas of the forebrain where it has been shown to be effective for locating indicators of higher cognition, and emotion [22] [97]. There are also several other well-known biological indicators for excitement such as heart and breathing rates, sphincter flexion, pupil dilation and vasodilation. Monitoring these in the game player with the event stimulus of a controlled critical position will provide the opportunity to identify these in the player experience, as well as the opportunity to categorize player BOLD signal responses among players. Successful experimentation in this direction would naturally lead to bigger and better things, such as possibly experimentation leading to more understanding of the physics of information flow inside the human brain.

The PFC presents good measurability for "emotionally-charged tasks"

[97], such as the event of winning or losing a game. The 22 channel WOT-220 is presumed to be capable of measuring any of 22 channels extending in a semi-circle, roughly from the frontal to the temporal poles. The outer channels 1-4 and 19-22 often fail to record reliable data however, possibly due to the presence of hair, or gaps caused by the curvature of the headset over players' crania. In our experience without shaving participants' heads, the reliably measurable regions of the cranial surface are over the frontopolar (FP), orbital (O), ventrolateral (VL), dorsolateral (DL), and ventral anterior (VA) PFC. The VLPFC, the DLPFC, or the OFC are known regions relating to higher planning, decisions, rewards, and in the case of the OFC and VLPFC, inhibition of surprise, fear and sensory inputs [37]. Each of these regions could hold interest for the task, and additionally the superior frontal gyrus (SFG) has been identified in Connectomic studies as a hub of connectivity for the whole brain [26]. Using the wireless headset (in Figure 4.1), an upward adjustment of 4 centimeters on the average wearer's forehead permits measurement of the SFG and much of Brodmann area 9. Channels 6 and 18 are positioned to monitor the right and left VLPFC respectively, and channels 9 and 15 can monitor the region over the OFC.

Chapter 5

Conclusions and Future Work

Earlier it was asked; **Can the logistic model of game uncertainty developed for board games be applied more generally, such as to video games, continuous movement parlor games or sports?**

In connection with this thesis, several games and sports were reviewed and analyzed under the game refinement theory model of game progress. Although the original measure, and the modified version of game refinement for continuous games and sports is rather simple, it was found to be applicable to other game types. Using the model of game progress indeed produces game refinement values for some continuous movement games that are in or near the window of game refinement of sophisticated board games from studies past. Several approaches were attempted, and tentatively it was found that the number of tries and goals as relative measures of the branching factor and depth of game could be used across several different games and game types. Extending the model of game progress in this way requires a simple approach, holding all other things equal. Still, clear and intuitive measures for G and T are not always readily available for every game, and not all of the attempts yielded the expected results.

The question encountered while trying to decide whether to score for rounds or matches in golf is a good example. The process of sophistication spanned centuries and continents, and would appear to be sufficient if not ideal for this purpose. At least to the degree that games can be fully evolved, game information in golf would seem to have little room left for change.

The number of holes per round and the ideal number of tries to achieve this have both been long-ago determined, with no major changes being expected. Assuming the perfect round score for golf validates our method in the case of golf, however departing from the method exactly as it was applied to other score limited games could portend some trouble with the model. Likewise, trying to apply the model rigidly seems to produce a value of game refinement that does not pass the sniff test in the case of golf.

The measures we seek relate to the informational complexity of games and their respective trees, and any meaning those measures can provide us for analyzing the entertainment and evolution of games. The use of defined terms "sophistication" and "refinement" have other uses in English which can be quite emotional and connotative. Although there is some slight connection intended with the originals, obviously they are not interchangeable. While assisting with the analysis of game refinement in the case study of tennis, one tennis aficionado confessed a feeling of great shock and disappointment that his beloved sport registered GR values only in the low range of 0.050 to 0.064. It is necessary to stress that the findings are only descriptive, not prescriptive. Prior observations imply that informational complexity and its rate of flow in games is directly tied to game entertainment, and that sophisticated games will tend toward a similar measure of that information. This is not the same as saying "Tennis is not a fully sophisticated game" or "Tennis is less entertaining than U.F.O. Catcher" or any such affects.

Preliminary work with hockey seems to support the use of this metric, at least according to the dominant narrative regarding a "goal draught" in recent seasons. While the number of shots per NHL hockey game holds relatively stable, and the percentage of successful shots only slightly less stable over time, it can be seen that raising the GR value of hockey entails either a significant increase of shots, or increased shot accuracy. To increase the number of goals per game without completely upsetting hockey's long and cherished history will require further changes to the offensive balance. A few solutions are known to be under consideration such as stricter rule enforcement (more or longer penalties), increasing the offensive zones and opportunities to shoot, and increasing the available shooting area, i.e. limiting goaltender pads' size or increasing the size of goal frames. With the single exception of increasing goal frames, each of the foregoing offense-favoring rule changes have already been attempted. Seen through the lens of game refinement theory, it is understood that acting to increase offense in the NHL would likely raise the measure of GR in hockey closer to that of other games.

There is some tension inherent in the proposition that offense, or at least scoring, needs to increase in order to raise entertainment in hockey. What fans seem to want (more fighting and more scoring, according to one professional) may be at odds with that which produces winning teams with statistical regularity, namely a strong defense. Similarly among boxing aficionados, it is said that an impenetrable defense (recently that of Floyd Mayweather) trumps a superior offense (Mayweathers erstwhile opponent Manuel Pacquiao). Those same fans also claim to prefer to watch more daring, offense-oriented boxing matches. At heart is the question–which is more entertaining, a high-scoring hockey team, or a hockey team with a high winning rate? Unfortunately, the model of game progress, although it can tell a value for game refinement, has no method for evaluating competing factors like this. The NHL, like most governing bodies, tends to act conservatively and incrementally in addressing its concerns. FIDE (chess), UEFA and FIFA (football), and the other sport and game leagues must use the same cautious approach as the NHL in regards to the delicate balance of managing the sophistication processes in their hands.

Games continually evolve in order to maintain fairness, safety, or enjoyability for players and spectators. In the case of score-limited sports, evaluation with the game progress model seems to prove that many of the changes in rules, particularly scoring systems, result in game refinement values nearer to those found previously in other-type games. A pattern of service-type games eschewing their traditional scoring systems seems to have been established with the switch in volleyball from side-out scoring to the rally system in 2000. Badminton and table tennis followed with major changes soon after. The primary reason for the changes to scoring systems is to make those score-limited sports easier to televise, and easier for casual spectators to follow. Whether knowingly or unknowingly, the governing boards of these sports have been actively involved in controlling the breadth and length of game of their respective games. Prior to 1974 the U.S. Open and French Open were played on grass courts, which have been known for their fast, sometimes slippery surfaces, and given to a particular type of game play. The last remaining Major to continue the tradition of grass courts, the Wimbledon has acted to harden the underlayment and slow the upper surface. Perhaps more significantly, in terms of game refinement theory, from 1980 tiebreak sets were introduced to the Majors except in the final set, which only the U.S. Open allows by tiebreak. Tennis is among the older, possibly more conservatively controlled sports, but we see these increments moving tennis closer toward a 0.07 GR-value. The GR-value of volleyball rose from 0.036 to 0.055, which this work attributes directly to the switch to rally-point scoring. Badminton's GR-value fell from 0.121 to 0.086 in response to service and scoring changes, and table tennis, which started with GR around 0.075, did not deviate significantly from that value as a result of reduced game scores balanced with game speed changes due to equipment modifications.

A prospective route toward the expansion of game refinement theory in an experimental vein was discussed. Previously it was asked; **Are there any intersections of the theoretical world of all game studies, including game theory, game refinement theory and others, with the applied world of functional brain imaging that could indicate a promising goal for these studies? One that leads to a bridge between information and mind?** fNIRS presents as a promising new technology for brain to game interfaces and also for cognitive game research. One or more new experiments could be done to replicate, verify and improve the result of Girouard et al., distinguishing the difficulty levels of play. Using mobile fNIRS along with heart rate, blood pressure, and video monitoring, it should be possible to match or exceed their 61 percent success rate for differentiating between high and low difficulty. The inclusion of cross-physiological measures and an abbreviated form of the NASA-TLX or similar questionnaire could also be used to help interpret the results.

We propose a survey of 20 Japanese, or Japanese and American game players. The experimental vehicle will be able to isolate the particular moments and parameters of game progress, and the areas of the brain, and stimuli to be measured. A well-controlled game in terms of game progress such that the progression from opening to mid-game, end-game and win or lose is optimal, and could be achieved in a number of ways, such as with a prisoner's dilemma with approval stage or a T-puzzle. Previous studies have used a timer to start and stop a continuous-play game. In order to better elicit the effects of winning or losing, a step-based event stimulus could be used to construct discrete sets of game information. A four or five-move game culminating in a decisive win or lose in the final step will be used, and the rules clearly explained to participants shortly before testing. In the well-controlled time frame, a tolerance of less than 3 seconds deviation from the norm is expected. It has been noted that 3 seconds is the approximate time required to convert neural responses to measurable oxygenation/deoxygenation patterns in the fNIRS medium. The information of winning or losing is the stimulus. Return to base should be expected within 30 seconds of the stimulus.

How do the physics of information operate, e.g. in the brains of gamers? This remains an open question, and a question which motivates this avenue of inquiry. The fields of functional brain imaging, theoretical neuroscience, and experimental cognitive research are in states of very high production. New approaches to data, new advances in optics and imaging, and theoretical advances seem to be combined for an explosive growth of knowledge now and in the future. Attempting to graph all neural connectivity [95] and the study of information flow dynamics in the brain (and a paper by that name) [83] have recently begun. There is reason to be optimistic that the advancement from brain mapping to checking on how the brain *works* and what information *is* could be on the near horizon.

The critical position on a game tree has relevance beyond chess or tennis, or any other game, crossing into numerous other fields of thought, decision-making and human endeavor. Game refinement theory comprises many attempts to interpret the relationship of information complexity in games and the human experience. The methods available are simplistic, and based on assumptions which require more and more qualification of the results. Assumptions bring noise. This is a beginning, based on a few intuitions and observations, not yet an end by far. The support derived from BAMING tends to raise more questions than it answers, but going forward into a new period of vigorous and fearless experimentation promises much for the future game refinement theory.

Appendix A

Experimental Design

This is a the outline of an experiment for finding any correlation of an informatical critical position and a cognitive critical position in a T-puzzle.

If necessary, proceed to wider survey of literature:
-More/different papers on T-Puzzle (e.g. Wajima, Abe, and Nakagawa)
-A paper on analysis (e.g. Tanaka, Katsura and Sato)

Premises:

* More than analysis, measurement, design must be correct (It's OK to get help with analysis, measurement.)

** Good design, which produces any result will prove something valuable (but, poor design which produces any result "good" or "bad" still proves nothing)

*** Is likely to collect analyze-able, statistically significant data of the associated brain activity (or alternatively, associated eye movement) around a Wakatta Moment.

Definitions:

The Wakatta Moment–This requires a clear working definition, not arbi-

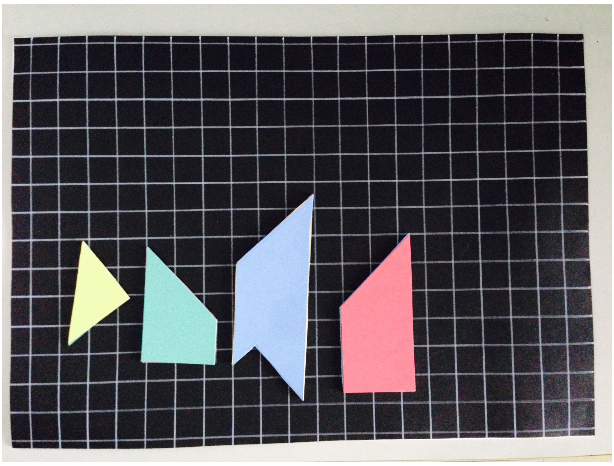

Figure A.1: T-puzzle board and pieces

trary, and reproducible. In the information paradigm, the Wakatta Moment is the arrival at a critical position. The critical position is the first node on the game tree which leads to that branch containing the solution to the T-puzzle. In the cognitive paradigm, the Wakatta Moment is the player's own understanding of having arrived at the critical position. In the case of the proposed T-puzzle, this would be e.g. key placement of the pentagonal piece, the filling or not filling of the notch, and a corresponding definitive curve, shape or value of brain/eye data. Actual values for the data of Wakatta Moment must be decided during experimentation.

Components–
6 attributes
puzzle state
time
distance
more?

Puzzle State–
1-The key positions (attributes) of the puzzle pieces (i.e. the notched pentagons relationship to trapezoids)?
2-What is the center of field?

3-What is the given stage of development/distance to solution?
4-Is an indicator of the solver/players state

I. Process:
1. Define a physical working environment for the trials:
   a. The four pieces of the T-puzzle (one-sided, or two-sided)
   b. Size and shape of workspace (table) appropriate for the task
   c. Appropriate color coding for: table, T-puzzle pieces, gloves...
   d. Divide workspace (b.) into number of regions

2. Apparatus:
   a. Black background, white grid (as in Figure, a zoned 2.5 cm 14 x 20 grid)
   b. Wide-angle plasma lens video camera and I-bar support over (a.)
   c. Durable non-reflective, bright color-coded T-puzzle(s)
   d. Tobii Glasses eye tracking system (takes 4MB discs)
   e. Hitachi WOT 220 fNIRS system
   f. EKG machine
   g. plenty of numerated mini discs for data recording

II. Test Metrics
1. Review use of Eye Tracker, EKG, fNIRS machines for the task
2. Test background, color for video monitoring and recording

3. Pilot experiment (guest-test)
→ can we measure? → can we clearly identify the (defined) Wakatta Moment?
   3. Adjust/re-design after II.2. as necessary

III. Experiment
1. Apply for permission of the experiment under school and accepted international ethical guidelines for treatment of information, health, safety, and handling of private personal data.

2. Begin advertising and interviews for subjects about 1 month prior. (Control for handedness and color vision. Observe gender, nationality, other factors?)

3. Select and schedule 15-20 subjects

4. Conduct the experiments.

1 Instructions, Pre-tests–Is the task understood? Is the subject capable and prepared for the task?→
2 Trials →
3 Identify critical position(s) →
4 Advances quickly to solution from 3

5. Analyze:
-What is the number of steps in the problem? (i.e. 1-thinking 2-assembling 3-disassembling 4-achieving a critical position 5-solving ?)
-Observe and record the working definition of the critical position or positions
-and failing positions
-and other, less critical positions

-What are the measurable cues of behavior?
(location and pupil movement from eye tracker; Δ heart rate from EKG; Δ brain O^2 from fNIRS) → Interpret: indicators of ... engagement, difficulty, others ?

6. Report

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