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Utilization of multi-dimensional source correlation in multi-dimensional single parity check codes

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Abstract This paper proposes a joint source-channel coding (JSCC) technique that well utilizes multi-dimensional (MD) source correlation using MD single parity check codes (MD-SPCCs). The source is assumed to be described by the coupling of multiple first-order binary Markov processes. The knowledge about the source correlation is utilized in the channel decoding process where each component decoder utilizes a single dimension correlation of the MD source. To enhance performance and reduce the error floor, a rate-1 recursive systematic convolutional code (RSCC) is serially concatenated to the MD-SPCC via a random interleaver. Two decoding techniques are proposed for each component decoder, and the selection of the decoding technique depends on the strength of the source correlation, which may further enhance the performance of the proposed JSCC technique. Simulation results reveal that a significant performance gain can be achieved by exploiting the MD source correlation with the proposed JSCC technique compared with the case in which the source correlation is not utilized; more significant gains can be achieved with stronger source correlation, and with a larger dimensionality source correlation as well.

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1 Introduction

Shannon's separation theorem [18] states that if source entropy is less than the channel capacity, source and channel codes can be independently designed; if the designed code is optimal both in terms of source and channel coding independently, arbitrarily small error probability can be achieved. However, the theorem assumes infinite codeword lengths and hence, in theory, communication systems designed based on the separation theorem require infinite latency. In practical communication systems, however, the latency requirement does not allow the design of source and channel coding processes based on the separation theorem as the computational complexity of the systems is finite and the systems are not free of latency constraints. Furthermore, although existing source coding techniques for specific applications such as voice, image, and/or video communications are quite efficient, redundancy of the source still remains at the output of the source encoders. The shortcomings described above have motivated the need for joint optimization of source and channel coding (JSCC), particularly in iterative source-channel decoding approach [1, 12–14, 16] and also, in utilizing source redundancy with channel coding [3, 6–8, 10, 11, 17, 19, 22–26]. The JSCC techniques utilizing source redundancy with channel coding may be classified into two categories, depending on the characteristics of the source: one utilizes non-uniform source distribution knowledge [3, 19, 24, 26]; the other utilizes the time-domain source correlation knowledge, for example, described by hidden Markov model [6,7,17] or Markov process [10,11,22,23,25] to describe the source behavior.

Most of the existing works described above only assume the source to be one-dimensional (1D) correlated and in [10,11], the source was assumed to be expressed by a 2D coupled Markov model. In this paper, we extend the work [10,11] to consider sources with multi-dimensional (MD) correlation. The source is modeled by a coupling of multiple first-order binary Markov processes [5]. In many wireless applications, such as transmission of multimedia contents (e.g. 3D video), the pixels/symbols within the source are correlated in various dimensions. Sources with higher dimensional correlation can provide additional information and this information can be utilized to further improve the error correction capability of a channel code. To the best of the authors' knowledge, the utilization of MD source correlation has never been considered in any previous publications.

The main challenge of this work would be on the complexity issue. References [10, 11] use turbo block codes composed of two Bose, Chaudhuri, Hocquenghem (BCH) codes as the component codes, and a modified version of the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm is performed at the receiver to better exploit the source correlation. Hence, the larger the parity length of the BCH codes, the heavier the computational complexity required to decode the codes. It is therefore not practical to employ this BCH-based JSCC system for the exploitation of higher dimensional source correlation due to the high computational complexity required. This paper replaces the parallel concatenated BCH code by an MD single parity check code (MD-SPCC), which is composed of an MD parallel concatenation of memory-1 SPCCs, and hence the decoding complexity is significantly reduced compared with the technique presented in [10,11]. Besides the challenge in terms of complexity, this work needs to address the negative effect of the correlation among the extrinsic log-likelihood ratios (LLRs) which can cause degradation in the system performance [10]. This paper proposes an alternative decoding technique that based on LLR modification technique in order to avoid the performance degradation. Moreover, a switching algorithm between the modified BCJR algorithm and the LLR modification algorithm is proposed to improve further the performance.

The rest of the paper is organized as follows. In Section 2, we provide the information theoretic property of sources generated from coupled MD Markov processes. The system design of the utilization of MD source correlation in the framework of MD-SPCC is explained in Section 3. Decoding techniques used to utilize the source correlation in the SPC decoding are presented in Section 4. Simulation results of the proposed JSCC technique are presented and discussed in Section 5. Finally, concluding remarks are provided in Section 6.

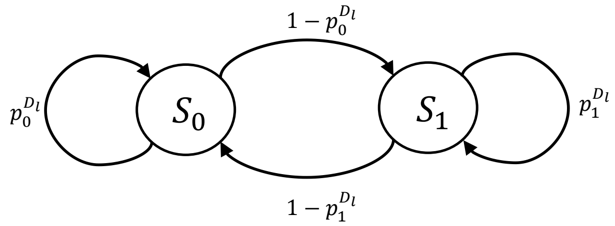

Fig. 1 Two-state Markov process. State S_i emits binary output i, $i \in \{0,1\}$

2 Multi-dimensional Markov sources

In this paper, sources with M-dimensional correlation are characterized by the coupling of M first-order binary state-emitting Markov processes. Figure 1 describes the source behavior for the l-th dimension (D_l), where $l = 1,2,..,M$. S_0 and S_1 are the states that emit binary source information 0 and 1, respectively. $p_0^{D_l}$ and $p_1^{D_l}$ are the transition probabilities from S_0 to S_0 and from S_1 to S_1, respectively. There are M different 1D sequences that can be formed from a source having M-dimensional source correlation because of the source coupling. Each of the 1D sequences corresponds to the different dimensions of the source correlation. Figure 2 illustrates an example of a source with 3D correlation coupled by three different 1D sequences.

The transition probabilities of a source in lth dimension, can be represented in matrix form as

$$\mathbf{A}^{D_l} = [a_{i',i}^{D_l}] = \begin{bmatrix} p_0^{D_l} & 1-p_0^{D_l} \\ 1-p_1^{D_l} & p_1^{D_l} \end{bmatrix} \quad i', i \in \{0,1\}, \quad (1)$$

where i' and i are the previous and current binary value emitted by a Markov source, respectively.

The value of a current source U depends only on its immediate previous values in all the M dimensions. The previous value of U in the l-th dimension is denoted as U_{D_l}, where $l = 1,2,...,M$. Based on the coupled Markov chain (CMC) model in [5], the transition probability of the source U given the previous values in all the M dimensions, $\Pr(U|U_{D_1}, U_{D_2}, ..., U_{D_M})$ can be represented in a matrix form as

$$\mathbf{B} = \left[b_{i'_1, i'_2, ..., i'_M, i} \right], \quad (2)$$

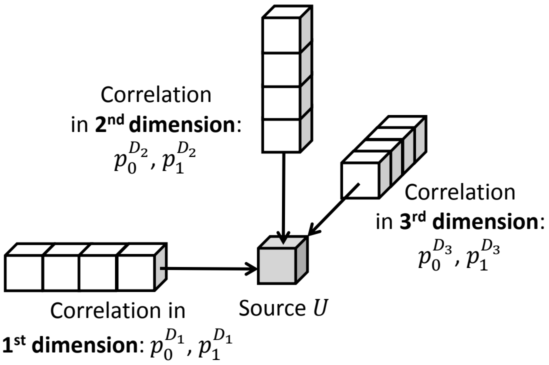

Fig. 2 A source with 3D correlation

where

$$\begin{aligned} & b_{i'_1,i'_2,...,i'_M,i} \\ & = \Pr(U = i|U_{D_1} = i'_1, U_{D_2} = i'_2, ..., U_{D_M} = i'_M) \\ & = \frac{\prod_{l=1}^{M} a^{D_l}_{i'_l,i}}{\sum_{f=0}^{1}\prod_{l=1}^{M} a^{D_l}_{i'_l,f}}, \quad i'_1, i'_2, ..., i'_M, i \in \{0,1\}. \end{aligned} \tag{3}$$

The entropy rate of U given $U_{D_1}, U_{D_2}, ..., U_{D_M}$ can be derived as

$$\begin{aligned} & H(U|U_{D_1}, U_{D_2}, ..., U_{D_M}) \\ & = H(U_{D_1}, U_{D_2}, ..., U_{D_M}|U) + H(U) \\ & \qquad - H(U_{D_1}, U_{D_2}, ..., U_{D_M}), \end{aligned} \tag{4}$$

where U_{D_1}, U_{D_2}, ..., and U_{D_M} are assumed to be statistically independent to each other given U. Thus, (4) can be simplified to

$$\begin{aligned} & H(U|U_{D_1}, U_{D_2}, ..., U_{D_M}) \\ & = H(U_{D_1}|U) + H(U_{D_2}|U) + ... + H(U_{D_M}|U) \\ & \quad + H(U) - H(U_{D_1}, U_{D_2}, ..., U_{D_M}) \\ & = H_{D_1}(\mathcal{S}) + H_{D_2}(\mathcal{S}) + ... + H_{D_M}(\mathcal{S}) \\ & \quad + H(U) - H(U_{D_1}, U_{D_2}, ..., U_{D_M}) \\ & = \sum_{l=1}^{M} H_{D_l}(\mathcal{S}) + H(U) - H(U_{D_1}, U_{D_2}, ..., U_{D_M}), \end{aligned} \tag{5}$$

where the values of $H(U)$ and $H(U_{D_1}, U_{D_2}, ..., U_{D_M})$ can be calculated empirically by measurements. $H_{D_l}(\mathcal{S})$ is the 1D entropy rate of the Markov source in the l-th dimension which can be calculated by [4]:

$$H_{D_l}(\mathcal{S}) = -\sum_{i',i\in\{0,1\}} \mu^{D_l}_{i'} a^{D_l}_{i',i} \log_2 a^{D_l}_{i',i}, \tag{6}$$

where $\mathcal{S}$ is the stochastic process of source U and $\mu^{D_l}_{i'}$ is the stationary state distribution. In the case of symmetric Markov sources where $p_0^{D_l} = p_1^{D_l} = p^{D_l}$, then $\mu^{D_l}_{i'} = 0.5$; If the Markov source is symmetric in all M dimensions, then the entropy rate $H(U) = 1$ bit.

Table 1 Entropy rate of 1D, 2D, 3D and 4D source correlation with various p values

p	H (bit)			
	1D	2D	3D	4D
0.7	0.88	0.78	0.71	0.69
0.8	0.72	0.54	0.47	0.45
0.9	0.47	0.26	0.24	0.23

The entropy rates for the sources with 1D, 2D, 3D and 4D source correlation are summarized in Table 1. For simplicity we assume the Markov sources are symmetric in all dimensions and the transition probabilities in all dimensions have identical value, e.g. for 4D source correlation case, $p^{D_1} = p^{D_2} = p^{D_3} = p^{D_4} = p$. It is found in Table 1 the entropy rate becomes lower as the dimensionality and the strength of the source correlation increases. The lowest entropy rate is 0.23 for sources with 4D source correlation and $p = 0.9$, whereas the highest entropy rate is 0.88, in the case of 1D source with $p = 0.7$. The difference in entropy rate resulted by the increase in the dimensionality, which acquired by observing the source from multiple dimensions, however, becomes smaller as the total dimensionality becomes larger. Hence, it can be expected that the relative difference in the entropy rates becomes even smaller for sources with 5D and larger dimensionality.

3 System model

3.1 Transmitter

In this work, we extend the system model in [11] by replacing the turbo BCH code with the MD SPCC to avoid the complexity explosion in the decoding process, and moreover, multiple (not necessary 2) component codes are considered. The proposed JSCC system utilizing M-dimensional source correlation by an M-dimensional SPCC at the transmitter side is illustrated in Fig. 3. It should be emphasized that the dimensionality of the SPCC should not necessarily be equal to the number of dimensions of the source correlation. More specifically, the proposed system can work well with any dimensional SPCC as long as the number of dimensions of the SPCC is not less than M. Throughout this section, the SPCC dimensionality is assumed to be equal to M.

Sources with M-dimensional correlation are considered. The length of the l-th dimension of the source in bits is denoted as L_l, with $l = 1, 2, .., M$. It is also assumed that the receiver knows the transition proba-

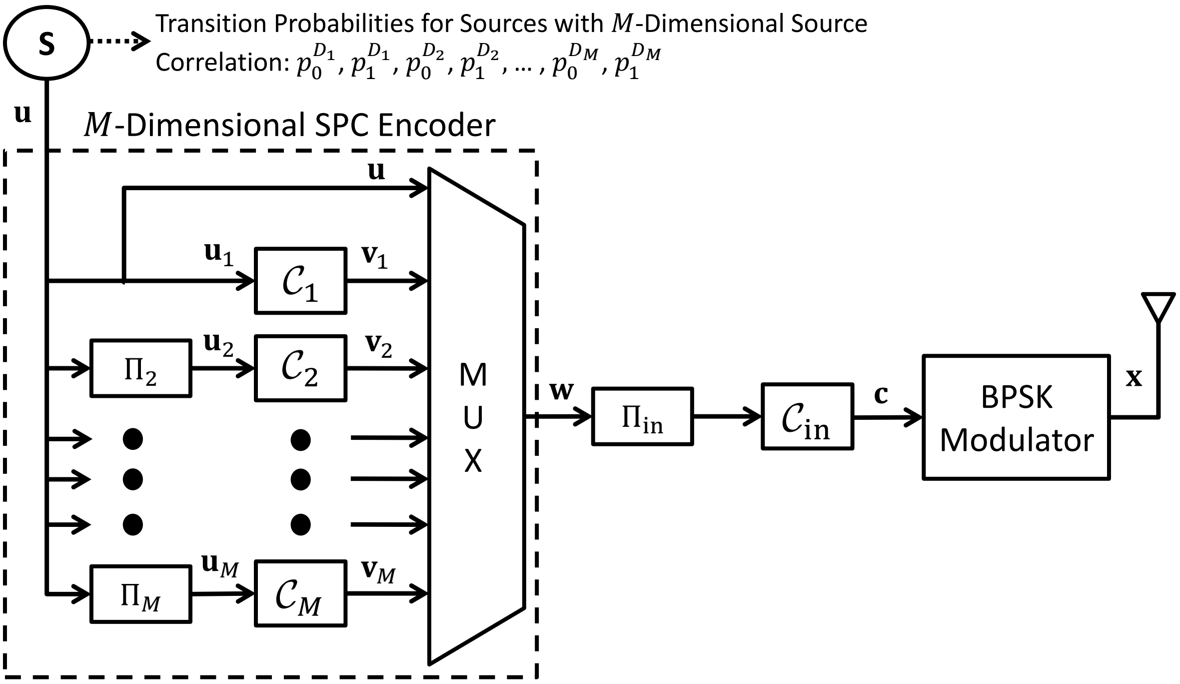

Fig. 3 Block diagram of the proposed JSCC system utilizing M-dimensional source correlation using an M-dimensional SPCC at the transmitter side.

bilities, $p_0^{D_l}$ and $p_1^{D_l}$ for all the dimensions of the source. The M-dimensional source is transformed into a 1D sequence $\mathbf{u}$ before being fed to the channel encoders for which the frame length L_T of $\mathbf{u}$ is given as

$$L_\mathrm{T} = \prod_{l=1}^{M} L_l. \tag{7}$$

In our proposed system, there are two channel codes employed. These two codes are serially concatenated via a random interleaver, Π_in separating the two codes. The outer channel code is an M-dimensional SPCC, consisting of M SPC component codes, $\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_M$ arranged in parallel structure. SPCC is one of the simplest codes, and it has very limited error correction capability. However, by combining multiple SPCCs, powerful error correction capability can be achieved [15, 20]. Every SPC component code, $\mathcal{C}_l$, adds a single parity check bit for every K_l length information bits; hence, the codeword length for $\mathcal{C}_l$ is $N_l = K_l + 1$. The SPCC with this set of parameters is denoted as SPC(N_l,K_l) code.

Π_2,Π_3,...,Π_M are block interleavers, which are used to re-arrange the long 1D source sequence $\mathbf{u}$ to a sequence following each dimension's 1D correlation property of the source correlation. For instance, $\mathbf{u}_2$ follows the sequence of the source correlation in D_2, $\mathbf{u}_3$ in D_3 and so on. The only exception is $\mathbf{u}_1$, which has the same sequence as $\mathbf{u}$ since it follows the sequence of the source correlation in D_1. Each $\mathbf{u}_l$ sequence is fed to the corresponding SPC encoder $\mathcal{C}_l$ to generate parity bits. The SPCC-coded 1D sequences having corresponding parity bits, $\mathbf{v}_1$, $\mathbf{v}_2$, ..., and $\mathbf{v}_M$, are then multiplexed with the original source sequence $\mathbf{u}$ before interleaved by Π_in.

The interleaved sequence is then encoded by an inner code, $\mathcal{C}_\mathrm{in}$, which is a rate-1 recursive systematic convolutional code (RSCC). We found that adding $\mathcal{C}_\mathrm{in}$ to the proposed system helps eliminating the high error floor problem inherently involved in MD-SPCC without sacrificing the bandwidth efficiency. Moreover, it enhances the performance of the proposed system especially for sources with strong correlation. The encoded sequence from $\mathcal{C}_\mathrm{in}$, $\mathbf{c}$, is then modulated using binary-phase shift keying (BPSK) modulator before being transmitted over an additive white Gaussian noise (AWGN) channel. The overall code rate of the proposed system is

$$R_\mathrm{c} = \frac{1}{1 + \sum_{l=1}^{M} \frac{1}{K_l}}. \tag{8}$$

3.2 Receiver

At the receiver, the received sequence $\mathbf{r}$ is demodulated to produce a channel output sequence $\mathbf{y}$ as depicted in Fig. 4. Iterative decoding process following the turbo principle is then invoked between the inner

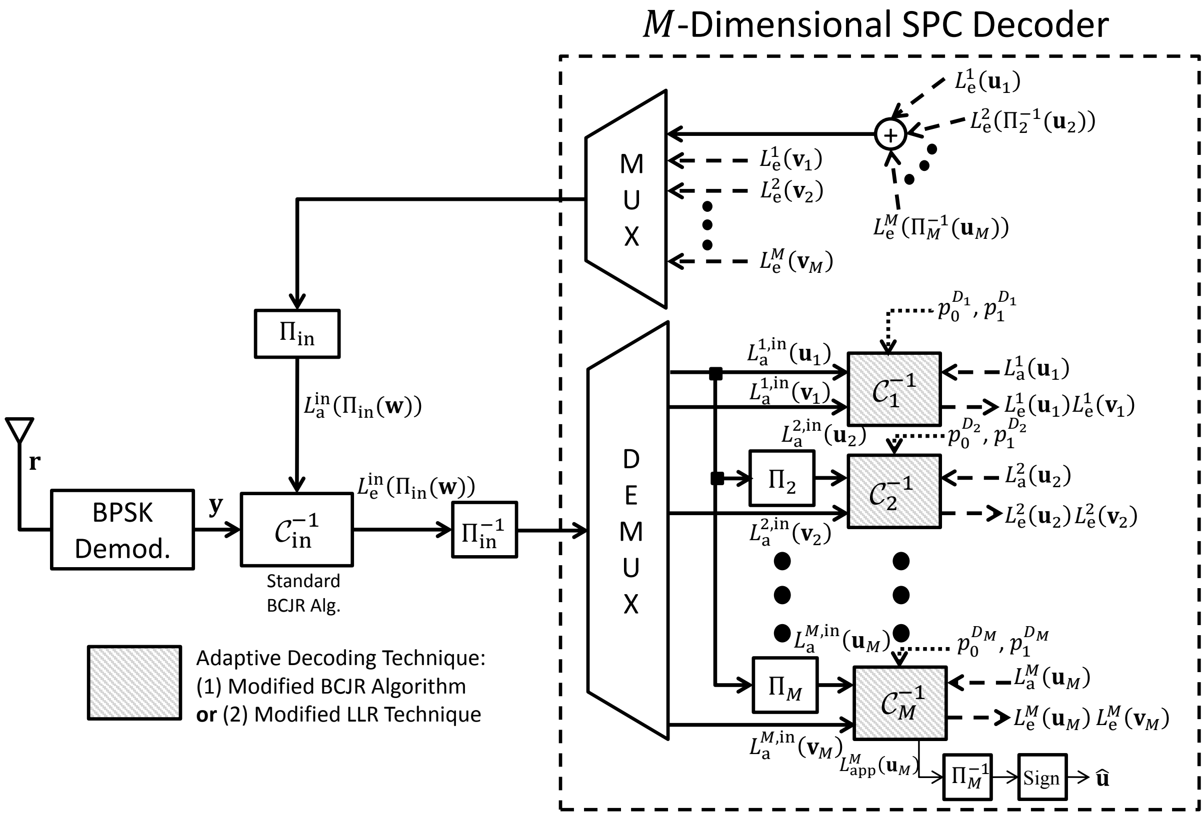

Fig. 4 Block diagram of the proposed JSCC system utilizing M-dimensional source correlation using an M-dimensional SPCC at the receiver side.

decoder, $\mathcal{C}_{\text{in}}^{-1}$ and the M SPC component decoders, $\mathcal{C}_l^{-1}$, $l = 1, 2, ..., M$. There are two inputs to $\mathcal{C}_{\text{in}}^{-1}$, one is the input from the channel observation and the other input is the *a priori* LLR L_{a}^{in} fed back from the M SPC component decoders. During the first iteration, the *a priori* LLR input L_{a}^{in} has zero value and $\mathcal{C}_{\text{in}}^{-1}$ performs decoding only with the input from the channel using the standard BCJR algorithm [2] to produce the extrinsic LLR output, denoted as L_{e}^{in}. Each SPC component decoder has two inputs from $\mathcal{C}_{\text{in}}^{-1}$, the *a priori* LLR corresponding to its information bits, and another *a priori* LLR corresponding to its parity bits. In addition, there is also *a priori* LLR input from the other SPC component decoders calculated as

$$L_{\text{a}}^{l}(\mathbf{u}_l) = \sum_{j=1, j\neq l}^{M} L_{\text{e}}^{j}(\mathbf{u}_l), \tag{9}$$

for SPC decoder $\mathcal{C}_l^{-1}$.

The source memory structure (given in transition probabilities) is assumed to be known to the corresponding SPC component decoders and the knowledge of the memory structure is utilized in the channel decoding process. In order for the SPC decoders to utilize the source memory structure, the SPC component decoders adopt either modified version of the BCJR algorithm [11, 22], to be explained in Section 4.1, or the LLR modification technique [25] together with simplified maximum *a posteriori* (MAP) algorithm [9], to be presented in Section 4.2. The modified BCJR algorithm can achieve better performance than the LLR modification technique, however, in our previous research work [10], it was found out that employing modified BCJR algorithm in multiple component decoders may result in performance degradation due to the correlation between the extrinsic LLRs, especially for sources having strong correlation. It should be noticed that the LLR correlation is caused by the insufficient randomness of the interleaver, which is different from the source correlation. This performance degradation can be avoided by using the second decoding technique that based on LLR modification technique. In order to fully exploit the performance advantage of using the modified BCJR algorithm while avoiding the performance degradation effect, an algorithm for the selection between the two decoding techniques is proposed in Section 4.3. The proposed selection algorithm, however, is an empirical technique, and optimal scheduling of decoder activation with the specified algorithms is still left as a future study.

When the source correlation is not utilized in the decoding of the component codes, simplified MAP algorithm is used instead of the standard BCJR algorithm because it can offer almost equivalent performance but require significantly lower decoding complexity than the standard BCJR algorithm. After the decoding process for all M SPC component decoders are completed, the extrinsic LLRs output from $\mathcal{C}_1^{-1}$ to $\mathcal{C}_M^{-1}$ corresponding to the bits in the information sequence $\mathbf{u}$ are summed up and then, multiplexed with the extrinsic LLRs corresponding to the parity bits, $\mathbf{v}_1$, $\mathbf{v}_2$,..., $\mathbf{v}_M$. The reconstructed sequence obtained by combining the information and parity parts is then interleaved using Π_{in} before they are fed back to $\mathcal{C}_{\text{in}}^{-1}$. The LLR exchange is repeated for a number of iterations and after the final iteration, the *a posteriori* LLRs from the last component decoder, L_{app}^M, are de-interleaved by Π_M^{-1} and hard-decision is then made to obtain the estimated information bits sequence $\hat{\mathbf{u}}$.

4 Decoding techniques

4.1 Modified BCJR algorithm

In [11,22], the standard BCJR algorithm [2] is modified in order to take into account the temporal correlation of the source during the decoding process of a convolutional code and BCH code, respectively. The modified BCJR algorithm [11, 22] can be employed for the decoding of an SPCC. Since SPCC has two states in the trellis diagram, there are four states when it is combined with a two-state Markov source in the trellis diagram. However, this is not the case when a BCH code is used because the number of states in the trellis diagram is larger than two and it depends on the parity length of the code.

4.2 LLR modification

We must use a block interleaver to preserve the source correlation property; however, this contradicts the turbo principle. In fact, we have observed the cases where performance is degraded by iteration, especially when the correlation is large. In order to avoid this problem, an alternative decoding technique is suggested based on the LLR modification technique [25]. It updates the value of the *a priori* LLR corresponding to the information sequence by considering the source correlation property before it is fed to the respective SPC decoder. The *a priori* LLR $L_{\text{a}}^l(\mathbf{u}_l(t))$ to be fed to $\mathcal{C}_l^{-1}$ at a time index t can be updated as [25]

$$L_{\text{a,mod}}^l(\mathbf{u}_l(t)) = (1-\alpha)L_{\text{a}}^l(\mathbf{u}_l(t)) + \alpha \cdot \ln\left[\frac{(1-p_0^{D_l})P(\mathbf{u}_l(t-1)=0) + p_1^{D_l}P(\mathbf{u}_l(t-1)=1)}{p_0^{D_l}P(\mathbf{u}_l(t-1)=0) + (1-p_1^{D_l})P(\mathbf{u}_l(t-1)=1)}\right] \quad (10)$$

where α is a constant that specifies the weighting factor for the correction term (the second term of (10)) over the original term (the first term of (10)) and the value range of α is from 0 to 1. The optimal value of α can be determined empirically. For $t \geq 2$, $P(\mathbf{u}_l(t-1)=1)$ and $P(\mathbf{u}_l(t-1)=0)$ can be determined as

$$P(\mathbf{u}_l(t-1)=1) = \frac{e^{L_{\text{a}}^l(\mathbf{u}_l(t-1))}}{1+e^{L_{\text{a}}^l(\mathbf{u}_l(t-1))}}, \quad (11)$$

and

$$P(\mathbf{u}_l(t-1)=0) = \frac{1}{1+e^{L_{\text{a}}^l(\mathbf{u}_l(t-1))}}, \quad (12)$$

respectively, and for $t=1$,

$$P(\mathbf{u}_l(0)=0) = 1 - P(\mathbf{u}_l(0)=1) = \mu_0^{D_l}. \quad (13)$$

Similarly, the *a priori* LLR, $L_{\text{a}}^{l,\text{in}}(\mathbf{u}_l(t))$ is also modified in the same way as $L_{\text{a}}^l(\mathbf{u}_l(t))$ was modified. In this case, we do not need to use the BCJR algorithm based on the memory-extended trellis diagram algorithm because the source statistics have been utilized when modifying the *a priori* LLRs. Simplified MAP algorithm [9] is employed for the decoding of SPCCs and the modified *a priori* LLRs, according to (10) are input to the simplified MAP algorithm. The extrinsic LLR for every K_l information bits sequence and the corresponding parity bit output from decoder $\mathcal{C}_l^{-1}$ can be calculated as [9]

$$L_{\text{e}}^l(\mathbf{u}_l(q)) = 2\cdot\text{arctanh}\left(\left(\prod_{j=1,j\neq q}^{K_l}\tanh\left(\frac{L_{\text{a,mod}}^{l,\text{in}}(\mathbf{u}_l(j)) + L_{\text{a,mod}}^l(\mathbf{u}_l(j))}{2}\right)\right)\tanh\left(\frac{L_{\text{a}}^{l,\text{in}}(v_l)}{2}\right)\right), \quad (14)$$

and

$$L_{\text{e}}^l(v_l) = 2\cdot\text{arctanh}\left(\left(\prod_{j=1}^{K_l}\tanh\left(\frac{L_{\text{a,mod}}^{l,\text{in}}(\mathbf{u}_l(j)) + L_{\text{a,mod}}^l(\mathbf{u}_l(j))}{2}\right)\right)\right), \quad (15)$$

respectively, where $q = 1, 2, ..., K_l$ is the index indicating the bit position in the K_l length information bits sequence $\mathbf{u}_l$ and v_l is the corresponding parity bit.

4.3 Selection of decoding techniques

Ideally, the use of the modified BCJR algorithm should result in optimal performance, however, when the source has strong correlation, performance improvement from the utilization of higher dimensional source correlation cannot be achieved if the modified BCJR algorithm is used by all the SPC component decoders. This problem is primarily due to the correlation among the extrinsic LLRs resulted from using the modified BCJR algorithm for more than one component decoder. A negative effect of the correlated extrinsic LLRs becomes more significant when utilizing strong source correlation. It is well known that highly correlated extrinsic LLRs are not desirable for turbo decoding. Therefore, the LLR modification technique is better suited for reducing the negative impact of the correlated extrinsic LLRs and thus, helps to enhance the performance. In this section, we propose a method to select the decoding technique for each SPC component decoder, so that performance enhancement can be well achieved. However, as stated before, it is still an empirical method and no mathematical proof of the optimality is provided.

The general idea is to fully exploit the performance advantage of using the modified BCJR algorithm while no performance degradation is imposed. The selection algorithm is described below:

- Step-1: Identify and sort $\mathcal{C}_l^{-1}$ corresponding to the strongest to the weakest correlation of $\mathbf{u}_l$. $\mathcal{C}_l^{-1}$ that corresponds to the strongest, the second, and so on until the Mth strongest source correlation are denoted as $\mathcal{C}_{\mathrm{S}:1}^{-1}, \mathcal{C}_{\mathrm{S}:2}^{-1}$, ..., and $\mathcal{C}_{\mathrm{S}:M}^{-1}$, respectively
- Step-2: Employ the modified BCJR algorithm to $\mathcal{C}_{\mathrm{S}:1}^{-1}$
- Step-3: Check if the average value of the transition probabilities for 2D source correlation $\bar{p}^{\mathrm{2D}}$ exceeds or not exceeds a pre-determined threshold value p_{T}^{2D}. If $\bar{p}^{\mathrm{2D}} \leq p_{\mathrm{T}}^{2D}$, then modified BCJR algorithm is employed by $\mathcal{C}_{\mathrm{S}:2}^{-1}$. On the other hand, if $\bar{p}^{\mathrm{2D}} > p_{\mathrm{T}}^{2D}$, then the LLR modification technique is employed at $\mathcal{C}_{\mathrm{S}:2}^{-1}$. The threshold p_{T}^{2D} is determined empirically beforehand by simulations
- Step-4: Repeat Step-3 for other component decoders, $\mathcal{C}_{\mathrm{S}:3}^{-1}$ until $\mathcal{C}_{\mathrm{S}:M}^{-1}$ for 3D to MD source correlation, respectively

Once the LLR modification technique is selected at a particular dimension of source correlation, the LLR modification technique is commonly used for the rest of the decoders, and the modified BCJR algorithm is no longer used for higher dimension sources. As stated before, this scheduling technique is rather empirical and optimality is not guaranteed. Nevertheless, as shown in the next section, it can achieve excellent performance.

Table 2 Simulation parameters

Parameter	Value
Outer Code	4D MD-SPCC
SPCC Type	SPC(8,7) Code
Inner Code	Rate-1 RSC$(3,2)_8$ Code
Frame Length	$28 \times 28 \times 28 \times 28$ bits
Π_2, Π_3, Π_4	Blk. Int. $28 \times 28 \times 28 \times 28$ bits
Π_{in}	Random Int. 965,888 bits length
Code Rate, R_{c}	0.64
No. of Iterations	25
α	0.2

5 Numerical results

5.1 BER performance evaluation

A series of simulations was carried out to evaluate the performance of the proposed JSCC system utilizing 1D, 2D, 3D, and 4D source correlation using a 4D MD-SPCC as the outer code and a rate-1 RSC$(3,2)_8$ code as the inner code. The parameters that we used in the simulations are summarized in Table 2. The threshold values, p_{T}^{2D}, p_{T}^{3D} and p_{T}^{4D} for determining the decoding technique of $\mathcal{C}_2^{-1}$, $\mathcal{C}_3^{-1}$ and $\mathcal{C}_4^{-1}$, respectively, were determined empirically by the preliminary simulations; They were found to be $p_{\mathrm{T}}^{2D} = 1$, $p_{\mathrm{T}}^{3D} = 0.72$ and $p_{\mathrm{T}}^{4D} = 0.67$, respectively, where as noted before, the sources are symmetric having an identical transition probability value in all source correlation dimensions.

Although the same outer code is used for 1D, 2D, 3D and 4D source correlation, the decoding techniques are different for different source dimensions. For 1D source correlation, only $\mathcal{C}_1^{-1}$ uses modified BCJR algorithm, whereas the other decoders use simplified MAP algorithm, which means that the utilization of the source correlation is only performed at $\mathcal{C}_1^{-1}$. For 2D source correlation, $\mathcal{C}_1^{-1}$ employs the modified BCJR algorithm, $\mathcal{C}_2^{-1}$ employs the modified BCJR algorithm or LLR modification technique (depending on the strength of the source correlation) and the other two decoders employ simplified MAP algorithm. For 3D source correlation, the modified BCJR algorithm is employed for $\mathcal{C}_1^{-1}$, the modified BCJR algorithm or LLR modification technique is employed for $\mathcal{C}_2^{-1}$ and $\mathcal{C}_3^{-1}$, and the simplified MAP algorithm is employed at $\mathcal{C}_4^{-1}$. Finally, for

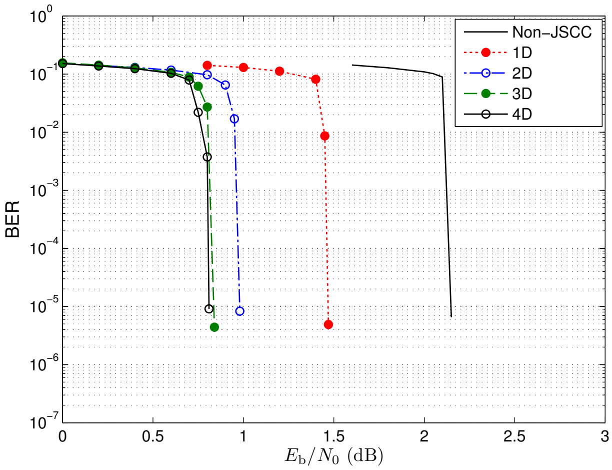

Fig. 5 Comparison of BER performance between the proposed JSCC system utilizing 1D, 2D, 3D, and 4D source correlation over the conventional non-JSCC system for identical $p = 0.7$ after 25 iterations

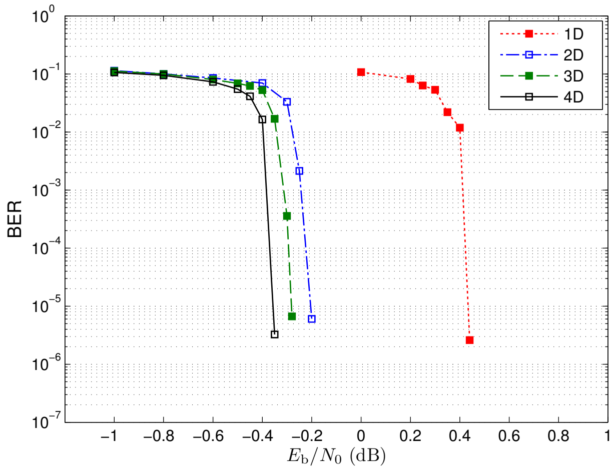

Fig. 6 BER performance of the proposed JSCC system utilizing 1D, 2D, 3D, and 4D source correlation for identical $p = 0.8$ after 25 iterations

4D source correlation, the modified BCJR algorithm is employed for $\mathcal{C}_1^{-1}$ and the modified BCJR algorithm or LLR modification technique is employed for $\mathcal{C}_2^{-1}$, $\mathcal{C}_3^{-1}$ and $\mathcal{C}_4^{-1}$.

As stated before, $p_0^{D_1} = p_1^{D_1} = p_0^{D_2} = p_1^{D_2} = p_0^{D_3} = p_1^{D_3} = p_0^{D_4} = p_1^{D_4} = p$ is also assumed in the simulations. The bit error rate (BER) performance of the proposed JSCC system utilizing 1D, 2D, 3D and 4D source correlation for sources with $p = 0.7$ are shown in Fig. 5. Significant gain achieved by using the proposed JSCC system over the conventional system that does not utilize source correlation (where all component decoders of the conventional MD-SPCC employ simplified MAP algorithm). The conventional system achieves BER 10^{-5} at $E_b/N_0 = 2.15$ dB while for the JSCC system utilizing 1D source correlation, 1.47 dB, yielding 0.68 dB gain. The gain becomes larger as higher dimensional source correlation is utilized and the gain achieved by utilizing 4D source correlation is 1.34 dB. It is worth emphasizing here that in this case, since $p < p_T^{2D}$, the modified BCJR algorithm is employed for $\mathcal{C}_2^{-1}$ to efficiently utilize the 2D source correlation. Similarly, since $p < p_T^{3D}$, the modified BCJR algorithm is employed for $\mathcal{C}_3^{-1}$ to efficiently utilize the 3D source correlation, and the modified LLR technique is employed for $\mathcal{C}_4^{-1}$ to efficiently utilize the 4D source correlation because $p > p_T^{4D}$.

Similar observations are found when evaluating the BER performance for sources with $p = 0.8$ and 0.9 as shown in Fig. 6 and Fig. 7, respectively. In the same way as in the case of $p = 0.7$, better performance can be achieved by utilizing higher dimensional source correlation, and moreover, utilizing sources with larger correlation also improves the performance. The decoding

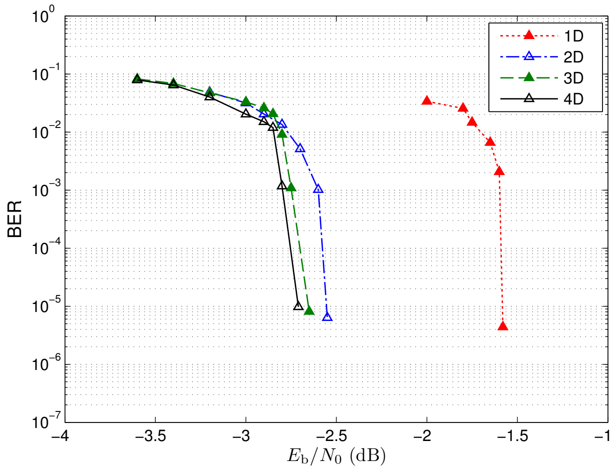

Fig. 7 BER performance of the proposed JSCC system utilizing 1D, 2D, 3D, and 4D source correlation for identical $p = 0.9$ after 25 iterations

scheduling was determined in the same way as in the case $p = 0.7$, according to the technique presented in Section 4. The benefit of using the selection decoding technique is shown Fig. 8 as the system that based on modified BCJR algorithm performs worse than the proposed system especially when utilizing higher dimensional source correlation.

The performance gains with the proposed JSCC system over the conventional system are summarized in Table 3. Among those systems evaluated, the largest improvement that can be observed over the conventional system is by utilizing 4D source correlation for sources with $p = 0.9$, and the gain is 4.86 dB, while the utilization of the 1D source correlation with $p = 0.7$ achieves the smallest gain of 0.68 dB. Thus, it is reasonable to expect that utilizing 5D or even large source cor-

Table 3 Gain in performance of the proposed JSCC system over the conventional system

p	E_b/N_0 at BER 10^{-5} (dB)				Gain (dB)			
	1D	2D	3D	4D	1D	2D	3D	4D
0.7	1.47	0.98	0.82	0.81	0.68	1.17	1.33	1.34
0.8	0.44	-0.20	-0.28	-0.35	1.71	2.35	2.43	2.50
0.9	-1.58	-2.55	-2.65	-2.71	3.73	4.70	4.80	4.86

Table 4 Gap to theoretical limit of the proposed JSCC system utilizing 1D, 2D, 3D, and 4D source correlation for $p = 0.7$, 0.8 and 0.9

p	Theoretical Limit (dB)				Gap to Limit (dB)			
	1D	2D	3D	4D	1D	2D	3D	4D
0.7	-0.07	-0.90	-1.55	-1.73	1.54	1.88	2.37	2.54
0.8	-1.41	-3.14	-3.89	-4.14	1.85	2.94	3.61	3.79
0.9	-3.91	-6.97	-7.19	-7.33	2.33	4.42	4.54	4.62

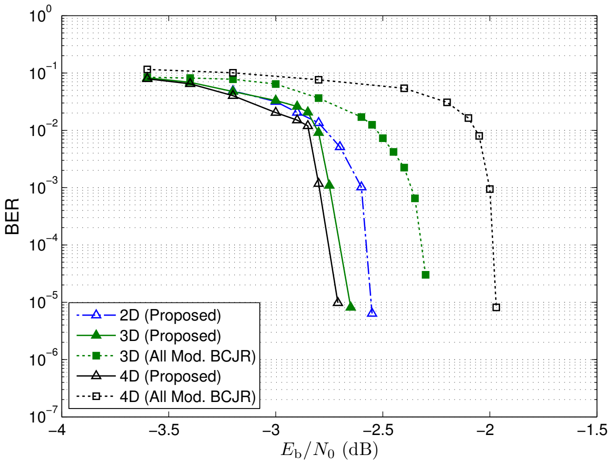

Fig. 8 Comparison in BER performance between the proposed JSCC system (with selection decoding technique) and the JSCC system based on modification BCJR algorithm (without selection decoding technique) to utilize MD source correlation with identical $p = 0.9$ after 25 iterations

relation using the proposed JSCC system can achieve further gain.

The theoretical limits were calculated using channel constraint capacity (CCC) [21] in order to evaluate the gap in E_b/N_0 between the performance of the proposed JSCC system at BER level 10^{-5} and the threshold E_b/N_0 calculated from the theoretical CCC. In this case, where sources with memory are considered, the capacity, C is subjected to the constraints of $H \cdot R_c \leq C$ where R_c is the code rate of the system and H is the entropy rate. By using the values of the entropy rate shown in Table 1, the theoretical limit was calculated for the proposed JSCC system utilizing 1D, 2D, 3D, and 4D source correlation with p as a parameter. The gap to the theoretical limit was then evaluated based on the curves shown in Fig. 5, Fig. 6 and Fig. 7. Table 4 tabulates the theoretical limit and the gap to the limit of the proposed JSCC system utilizing 1D, 2D, 3D, and 4D source correlation for $p = 0.7$, 0.8 and 0.9.

The theoretical limit for the conventional system with $R_c = 0.64$ and $H = 1$ is at 0.89 dB and hence, the gap to the theoretical limit with the conventional system is 1.26 dB. Utilizing larger dimensionality and stronger source correlation using the proposed JSCC system results in lower theoretical limit, however, the gap to the limit becomes larger. For example, utilizing 1D source correlation with $p = 0.7$ achieves the smallest gain compared to the conventional system, however the gap to the limit is 1.54 dB which is also the smallest among those systems compared. On the other hand, utilizing 4D source correlation with $p = 0.9$ achieves the largest gain compared to the conventional system, however, gap to the limit of 4.62 dB is also the largest. Hence, it is reasonable to predict that the utilization of larger than four correlation dimensions and p larger than 0.9 even increase the gap to the theoretical limit. The design of a more sophisticated JSCC system that can achieve close-limit performance without requiring high computational complexity for sources having high dimensionality and strong source correlation is left as future study.

5.2 Computational complexity evaluation

In [10, 11], it is shown that significant improvement can be achieved by exploiting 1D and 2D source correlation using turbo BCH codes. Similar to the JSCC system proposed in this paper, the modified BCJR algorithm is employed to exploit the source correlation during the decoding of a BCH code in [10, 11]. However, unlike the SPCC, number of the states in the trellis diagram for the BCH code is larger than $2 \cdot 2^1$

Table 5 The fine-tuned code parameters, the corresponding code rate and theoretical limit for the BCH-based and the proposed JSCC systems exploiting 2D source correlation for $p = 0.7$, 0.8 and 0.9

JSCC System	p	Codes ($\mathcal{C}_1$,$\mathcal{C}_2$,$\mathcal{C}_{\text{in}}$)	R_c	Limit(dB)
BCH	0.7	BCH(255, 247),BCH(15, 11),RSC(77, 40)$_8$	0.72	-0.60
Proposed	0.7	SPC(128, 127),SPC(4, 3),RSC(77, 40)$_8$	0.75	-0.48
BCH	0.8	BCH(255, 247),BCH(15, 11),RSC(777, 400)$_8$	0.72	-2.97
Proposed	0.8	SPC(128, 127),SPC(5, 4),RSC(177, 100)$_8$	0.80	-2.80
BCH	0.9	BCH(127, 120),BCH(15, 11),RSC(37, 20)$_8$	0.70	-6.84
Proposed	0.9	SPC(128, 127),SPC(7, 6),RSC(77, 40)$_8$	0.85	-6.79

Table 6 Comparison of the BCH-based and the proposed JSCC systems exploiting 2D source correlation

p	Code Rate		Gap to Limit(dB)		No. of Trellis States	
	BCH	Proposed	BCH	Proposed	BCH	Proposed
0.7	0.72	0.75	0.71	1.03	$2 \cdot 2^8 + 2 \cdot 2^4 + 2^5 = 576$	$2 \cdot 2^1 + 2 \cdot 2^1 + 2^5 = 40$
0.8	0.72	0.80	1.50	1.76	$2 \cdot 2^8 + 2 \cdot 2^4 + 2^8 = 800$	$2 \cdot 2^1 + 2 \cdot 2^1 + 2^6 = 72$
0.9	0.70	0.85	3.43	3.65	$2 \cdot 2^7 + 2 \cdot 2^4 + 2^4 = 304$	$2 \cdot 2^1 + 2 \cdot 2^1 + 2^5 = 40$

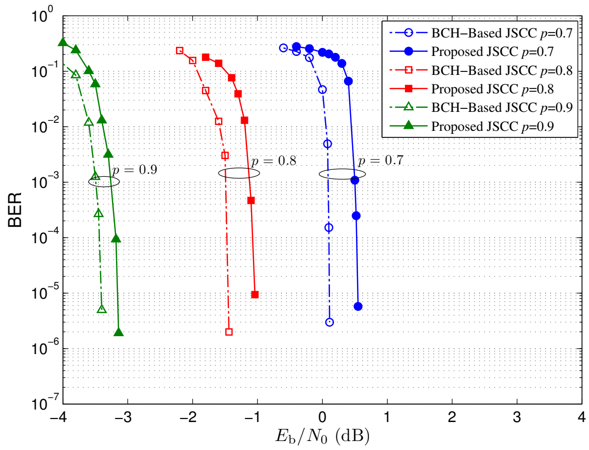

Fig. 9 BER performance of the fine-tuned BCH-based and the proposed JSCC systems exploiting 2D source correlation with p=0.7, 0.8 and 0.9 after 40 iterations

(multiply by 2 to include the Markov states) depending on the length of the parity-check part of the BCH code. For instance, there are $2 \cdot 2^7$ states in the trellis for BCH($N = 127$,$K = 120$). The complexity is determined by the number of trellis states involved during the decoding process and therefore, it is not desirable to employ a code requiring large number of the trellis states.

The code parameters for the BCH-based JSCC system are fine-tuned to yield close-limit performance as presented in [11]. Similarly to the exploitation of 2D source correlation, the code parameters for the proposed SPCC-based JSCC system (with 2D MD-SPCC used as the outer code) were fine-tuned as well. For the codes with the fine-tuned parameters, their corresponding code rates and the theoretical limit for both the proposed and BCH-based JSCC schemes are presented in Table 5.

The BER performance curves with fine-tuning of parameters for the BCH-based [11] and the proposed JSCC systems exploiting 2D source correlation are compared in Figure 9. To achieve BER 10^{-5}, the BCH-based JSCC system requires E_b/N_0 value of 0.11 dB, -1.47 dB and -3.41 dB for p=0.7, 0.8 and 0.9, respectively. The proposed JSCC system on the other hand, the required E_b/N_0 values needed to achieve BER 10^{-5} are 0.55 dB, -1.04 dB and -3.14 dB for p=0.7, 0.8 and 0.9, respectively. It should be noticed that the BCH-based JSCC system achieves slightly better performance than the proposed JSCC system in terms of turbo cliff and hence, smaller gap to the theoretical limit, as shown in Figure 9 and Table 6. However, the BCH-based JSCC system requires larger number of trellis states involved in the decoding process compared with the proposed JSCC system, as indicated in Table 6. Since the same iteration rounds were performed in both systems, it can be concluded that the proposed JSCC system, decoded with much smaller number of trellis states requires lower computational complexity, resulting in much lower latency.

6 Conclusions

In this paper, the JSCC technique proposed in [11] for the utilization of 2D source correlation has been extended to better utilize MD source correlation using MD-SPCCs. The source is characterized by the coupling of multiple first-order Markov processes. The source statistics, dimension-by-dimension of the source correlation are utilized during SPC decoding process by using either the modified BCJR algorithm or the LLR modification technique. An empirical but yet efficient method has been proposed to select the suitable decoding technique for each component decoder of the MD-SPCC. It has been shown through simulations that the utilization of higher dimensional and stronger source correlation with the proposed JSCC technique achieves larger performance gain over the conventional system that does not utilize source correlation. The proposed JSCC technique can be applied in a number of emerging wireless multimedia applications, especially for the transmission of MD multimedia contents (e.g. 3D video).

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