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Measuring Sophistication of Sports Games: The First Result from Baseball

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Abstract: This paper demonstrates a game informatical analysis of sports to examine if its rules are reasonably established for maintaining the attractiveness. It focuses mainly on the baseball and game refinement measure is employed for the assessment. The baseball leagues from several countries that conducted from 2000 to 2015 are analyzed. The results obtained show that game refinement values of baseball are around 0.065. These values are slightly lower than other sophisticated sports such as soccer and basketball. Under the assumption that game refinement measure indicates a comfortable degree of game outcome uncertainty or a good balance between skill and chance, a tentative conclusion is that baseball has slightly more skill-based weight than soccer and basketball. It implies that baseball may be sufficiently sophisticated to attract people but may be of slightly less popularity than soccer and basketball. Possible enhancements are discussed.

1. Introduction

In the zero-sum games, the summation of position scoring values during a game is not reduced. One gains a value while another one losses the same value. Studying a model of decision making in this situation originates in game theory [8] [10]. The idea of the existence of mixed-strategy equilibrium in two-person zero-sum games is its main concept. In many fields such as economics and computer science, the concept of game theory has been widely applied [3]. However, game is not only about winning or losing. Instead, entertainment is one of the major aspects that should be concerned. Thus, studying the attractiveness and sophistication of games has been issued, for which game refinement theory is used. While game theory concerns a player's winning strategy, in contrast, game refinement theory concerns entertainment and player's satisfaction in games. Game refinement theory was originally proposed in the domain of board games [5].

The idea of game refinement theory that was cultivated in the domain of board games can be extended to other type of games such as sports games. Popular sports such as soccer and basketball have a historical background. In order to obtain the popularity, its rules have been changed throughout its history. Interestingly, the experimental results from the previous works [5] [13], in which game refinement measures were applied to various sophisticated board games and popular sports, have shown the same or similar values of game refinement measures, i.e., somewhere between 0.07 and 0.08. In term of entertainment and engagement for the spectators and players, it relates to the game refinement measure, which is supported by the previous works [9] [14] [16]. However,

each sports has unique rules, especially the rule for deciding the winner of a game. For example, deciding the winner of popular sports such as soccer and basketball is based on the score of each team at the end of time. In contrast, other popular sports like tennis and table tennis, a game ends when players scores reach some point which is set in advance. This kind of decision separates sports types into two major groups: time limit sports and score limit sports.

Baseball is a competitive sports between two teams. Each game consists of nine innings that each team plays as an offense and defense side, respectively. In order to earn scores, the players on the offense team must run through four bases around the field, while the players on the defense team try to defend the base. Baseball has been quite popular especially in Japan. However, nowadays, it seems that baseball is less attractive than it was. The reasons behind its decreased popularity are doubtful. After being admitted to the Olympics as a medal sport beginning with the 1992 Games, baseball was dropped from the 2012 Summer Olympic Games at the 2005 International Olympic Committee (IOC) meeting. It remained part of the 2008 Games. The elimination of baseball, along with softball, from the 2012 Olympic program enabled the IOC to consider adding two different sports, but none received the votes required for inclusion [1] [7].

This paper therefore aims to quantify entertainment impact of baseball. Game refinement measure is applied to inspect the trend of baseball. Moreover, forecasting the trend of baseball is performed if its rules remain unchanged. In addition, the difference between several regions is observed. The leagues that have been conducted in each country are chosen as a representative for each region. Moreover, the difference between these leagues is considered. The classification of leagues represents each type of leagues. For this study, the data from leagues in each year from 2000 to 2015 were collected. These data include the leagues from

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several countries such as USA, Korea and Japan. Moreover, the classification of leagues such as Major, Triple-A, Double-A are gathered.

The structure of this paper is as follows. Section 2 introduces the basic idea of game refinement theory with its recent development in the domain of sports games. Application of game refinement theory to baseball is shown. In Section 4, the results from the previous section are discussed. Moreover, we mention about the related works. Finally, the concluding remarks are given in Section 5.

2. Game Refinement Theory and Its Application to Baseball

In this section, the basic idea of game refinement theory and its measure are explained. Then, the previous works that apply game refinement theory to various domains are described. Next, a model of game refinement theory that applies to measure baseball is presented.

2.1 Basic idea of game refinement theory

A general model of game refinement theory was proposed based on the concept of game information progress [13]. It bridges the difference between sports games and board games. We first describe a general model of game progress in order to derive a game refinement measure. Then, we apply this idea to various games while identifying reasonable game progress models of given games, and compare them using game refinement measures.

The game progress is twofold [2]. One is game progress model that is speed or scoring rate with a focus on scoring, while another one is game information progress with a focus on the game outcome. Game information progress presents the degree of certainty of a game's result in time or in steps. Having full information of the game progress, i.e. after its conclusion, game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq t_k$ and $0 \leq x(t) \leq x(t_k)$, as shown in Equation (1).

$$x(t) = \frac{x(t_k)}{t_k}t \quad (1)$$

However, the game information progress given by Equation (1) is unknown during the in-game period. The presence of uncertainty during the game, often until the final moments of a game, reasonably renders game progress as exponential. Hence, a realistic model of game information progress is given by Equation (2).

$$x(t) = x(t_k)\left(\frac{t}{t_k}\right)^n \quad (2)$$

Here n stands for a constant parameter which is given based on the perspective of an observer of the game considered. Only a very boring game would progress in a linear function however, and most of course do not. Therefore, it is reasonable to assume a parameter n , based on the perception of game progress prior to completion. If the information of the game is completely known (i.e., after the end of the game) and the value of n is 1, the game progress curve appears as a straight line. In most games, especially in competitive ones, much of the information is incomplete, the value of n cannot be assumed, and therefore game progress is a steep curve until its completion, along with $x(t_k)$, t_k , $x(t)$ and t , just

prior to game's end.

Then acceleration of game information progress is obtained by deriving Equation (2) twice. Solving it at $t = t_k$, we have Equation (3).

$$x''(t_k) = \frac{x(t_k)}{(t_k)^n} (t_k)^{n-2} n(n-1) = \frac{x(t_k)}{(t_k)^2} n(n-1) \quad (3)$$

It is assumed in the current model that game information progress in any type of game is encoded and transported in our brains. We do not yet know about the physics of information in the brain, but it is likely that the acceleration of information progress is subject to the forces and laws of physics. Too little game information acceleration may be easy for human observers and players to compute, and becomes boring. In contrast, too much game information acceleration surpasses the entertaining range and will be frustration, and at some points beyond that could become overwhelming and incomprehensible. Therefore, we expect that the larger the value $\frac{x(t_k)}{(t_k)^2}$ is, the more the game becomes exciting, due in part to the uncertainty of game outcome. Thus, we use its root square, $\frac{\sqrt{x(t_k)}}{t_k}$, as a game refinement measure for the game under consideration. We call it GR value for short as shown in Equation (4).

$$GR = \frac{\sqrt{x(t_k)}}{t_k} \quad (4)$$

2.2 Basic models of game progress in sports

To obtain a measure of game refinement for a game under consideration, we need to figure out an appropriate game progress model. It usually can be done with a focus on the scoring which is essential for deciding the winner. In the previous works [14] [16], in score limit sports domains such as volleyball, badminton and table tennis, the game refinement measure GR was calculated by $GR = \frac{\sqrt{W}}{T}$ where W and T stand for the average winner's scores and the average total scores of entire game, respectively. The values W and T correspond to $x(t_k)$ and t_k in Equation (1). We show the results from [9] in Table 1.

Table 1: Measurement for score limit sports

Sports	Version	W	T	GR
Badminton	Old scoring system	30.07	45.15	0.121
	New scoring system	46.34	79.34	0.086
Table Tennis	Pre-2000	57.87	101.53	0.075
	Post-2000	54.86	96.47	0.077

We consider another type of sports such as soccer and basketball. In these sports, there are no score limit but the game is regulated by a time limit. For this type of sport, a game progress model is constructed with a focus on the number of goals called G and the number of attacks or shot attempts called T . We then obtain $GR = \frac{\sqrt{G}}{T}$. This approach was applied in time limit sports such as football and basketball and the results are shown in Table 2 [13].

2.3 Game refinement theory on baseball

Baseball does not have a time limitation like soccer or basketball, nor does have a score limitation like volleyball. Baseball

Table 2: Measurement for time limit sports

Sports	G	T	GR
Soccer	36.38	82.01	0.073
Basketball	2.64	22	0.073

consists of 9 innings per one game. The players from 2 teams play as an offensive and defensive side in each inning. In addition, the winner team is decided by the score at the end of last inning. Thus, the number of innings is a limitation for each player to earn the score. Therefore, the baseball can be considered as a time limit sport even if it does not have a real time limitation in game.

In sports games such as soccer and basketball, the scoring rate is calculated by two factors: (1) goal, i.e., total score and (2) time or steps to achieve the goal. For example, in basketball the total score is given by the average number of successful shoots, whereas the steps to achieve the goal is estimated by the average number of shoots attempted [13]. Then the game speed of basketball is given by

$$\frac{\text{average_number_of_successful_shoots}}{\text{average_number_of_shoots}}$$

By considering as a time limit sports domain, the game refinement model for time limit domain can be applied to the baseball. However, each team has only one chance, which is playing as the offensive side, in each inning to gain the score. The others sport in time limit domain can gain the score for the entire game. This is a major difference between baseball and other time limit sports. In order to get the score in baseball, the batter, who swings a bat, must hit the ball, which pitcher throws, and reaches the four bases. Thus, the number of hits in each game is considered as a game progress. On the other hand, the average score in each game is directly proportional to the number of hits. Then the game speed of baseball is given by

$$\frac{\text{average_number_of_scores}}{\text{average_number_of_hits}}$$

Let S and H be the average number of scores and hits in each game, respectively. The game progress model of baseball is given by Equation (5).

$$x(t) = \frac{S}{H}t \quad (5)$$

Following the same procedure to obtain Equation (4), the game refinement measure of baseball is derived from Equation (6).

$$GR = \frac{\sqrt{S}}{H} \quad (6)$$

3. Analysis of Baseball

Game refinement measure is employed for the game informational analysis of baseball. All data in this study were collected from the baseball leagues that conducted from 2000 to 2015 [19]. We focus on three categories: years, leagues and regions.

3.1 Trends for the years 2000 - 2015

The baseball leagues are analyzed to obtain the average GR values for the years 2000-2015. The results are summarised in Table 3, which shows that baseball's GR values for the period are located at somewhere between 0.063 and 0.066. The results

Table 3: Baseball: Scores, Hits and GR values, 2000 - 2015

Years	Number of games	Scores	Hits	GR
2000	35374	176960	1190171	0.066
2001	34920	164468	1178324	0.064
2002	35240	164313	1187817	0.064
2003	35524	164126	1190869	0.064
2004	34612	169478	1168346	0.065
2005	36539	181316	1243020	0.065
2006	39896	185088	1337652	0.064
2007	42132	203348	1419498	0.065
2008	42342	205503	1430130	0.065
2009	41176	195358	1382997	0.064
2010	42120	200161	1416249	0.064
2011	41339	197598	1390301	0.065
2012	41574	193862	1397980	0.064
2013	41309	187857	1383344	0.063
2014	41128	190937	1385524	0.063
2015	41501	185823	1394460	0.063

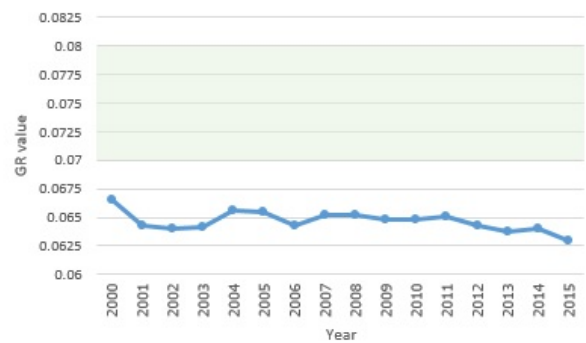


Fig. 1: Baseball's GR values for the years 2000-2015

are depicted in Figure 1. Figure 1 shows that the highest value comes from the year 2000 while the lowest one comes from the last 3 years: 2013-2015. To be more precise, GR values reduce consecutively during the years 2000 - 2015.

For the comparison, we show, in Figure 2, the trend of soccer's GR values for the years 2003 - 2013 [13]. For example, GR value in 2003 is 0.073, which is in the sophisticated zone. However, it has decreased until 2006, which takes the lowest value 0.065. Moreover, we show, in Figure 3, the trend of basketball's GR

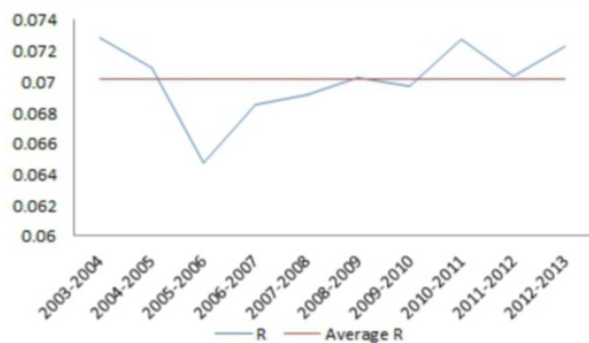


Fig. 2: Soccer's GR values for the years 2003-2013

values for the years 2003-2013 [13]. *GR* values are above the sophisticated zone, however, it has increased until 2007, which is the highest value (around 0.087), but later decreased to be in the sophisticated zone. Both popular sports: soccer and basketball,

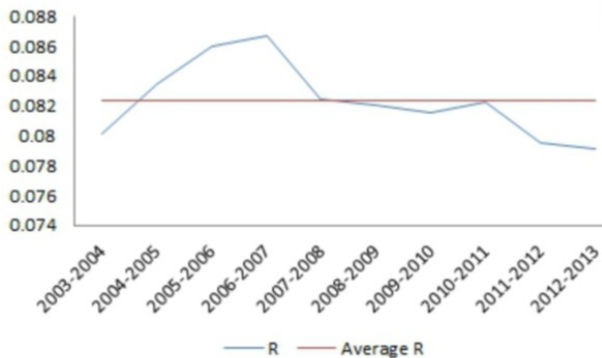


Fig. 3: Basketball's *GR* values for the years 2003-2013

are lower and higher than the sophisticated zone at some point, respectively. Baseball's *GR* value dropped in 2006, and during 2007 - 2011 *GR* values are stable at 0.065. Then, it has decreased consecutively until 2015. As *GR* values of soccer and basketball have been changed due to the minor rule changes, there is a tendency that *GR* value moves to the sophisticated zone, while it increased in soccer and it decreased in basketball, respectively. However, baseball's *GR* value did not move to the sophisticated zone yet, instead *GR* value has decreased moderately.

3.2 Leagues - performance quality

Major League Baseball (MLB) is a professional baseball organization that is the oldest of the four major professional sports leagues in the United States and Canada. A total of 30 teams now play in the American League (AL) and National League (NL), with 15 teams in each league.

Minor League Baseball is a hierarchy of professional baseball leagues in USA that compete at levels below MLB and provide opportunities for player development and a way to prepare for the major leagues. All of the minor leagues are operated as independent businesses. Most are members of the umbrella organization known as Minor League Baseball (MiLB), which operates under the Commissioner of Baseball within the scope of organized baseball. Several leagues, known as independent baseball leagues, do not have any official links to Major League Baseball.

The current minor league classification system divides leagues into one of five classes, those being Triple-A (AAA), Double-A (AA), Class A (Single-A or A), Class A Short Season, and Rookie. Furthermore, Class A is further subdivided into Class A and Class A-Advanced (often called Low-A and High-A, respectively), and Rookie is further subdivided into Rookie Advanced, Complex-based Rookie and international summer baseball. Under the rules governing the affiliated minor leagues (specifically Major League Baseball Rule 51), Class A Short Season is a separate classification from the other leagues bearing the "Class A" name, despite the similarity in name.

In this study, six baseball leagues including Major, TripleA,

DoubleA, AdvanceA, A and Rookie, are analyzed. The results are summarized in Table 4. The results are depicted in Figure 4.

Table 4: Baseball: Scores, Hits and *GR* values at 6 leagues

Leagues	Number of games	Scores	Hits	<i>GR</i>
Major	77740	354978	2660699	0.062
AAA	96540	473002	3270860	0.065
AA	67210	301019	2240816	0.063
Adv A	66615	310083	2234234	0.064
A	66120	299532	2204732	0.064
Rookie	42816	215175	1433607	0.067

Baseball's *GR* value in each league is somewhere between 0.062

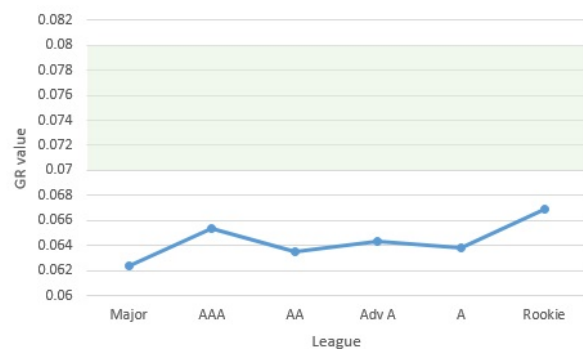


Fig. 4: Baseball's *GR* values at 6 leagues

and 0.067. It seems that *GR* value is inverse proportional to the playing performance quality.

The game refinement values of all leagues are lower than the sophisticated zone. Major league has the lowest value, whereas Rookie league has the highest value. It is likely that the main difference between Major league and Rookie league is the performance gap between batters and pitchers. In Major league, the performance of batters is lower than pitchers, whereas the performance gap in Rookie between batters and pitchers is not so significance. Then, the successful percentage of hits in Major league is lower than Rookie league. Thus, the average score in Rookie league is higher than Major league. Therefore, Rookie's *GR* value is closer to the sophisticated zone than Major league.

3.3 Regions - locality and diversity

The baseball leagues from 6 countries: United States, Australia, Holland, Italian, Japan and Korean, are analyzed. The results are summarized in Table 5. The *GR* values vary between 0.060 and 0.067. Holland takes the highest *GR* value, whereas Japan takes the lowest one. The results are depicted in Figure 5.

The game refinement measure of Japan leagues is significantly different from others. Japan baseball leagues has a different rule with others. Japan leagues use a smaller baseball, strike zone, and playing field. The Japanese baseball is wound more tightly and is harder than others. These difference directly affect the score that can be earned in each game. Thus, the game refinement of Japan is totally different from other countries.

Table 5: Baseball: Scores, Hits and GR values in 6 countries

Countries	Number of games	Scores	Hits	GR
US	550317	2619772	18549940	0.065
Australia	1612	7810	53976	0.066
Holland	2266	10813	74035	0.067
Italy	2252	10675	75054	0.065
Japan	27212	112293	915472	0.060
Korea	17287	82856	587741	0.064

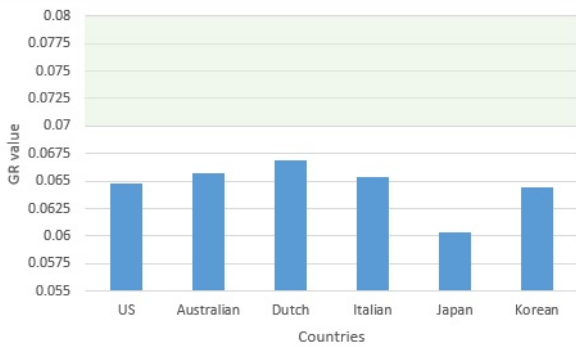


Fig. 5: Baseball's GR values in 6 countries

Table 6: Softball: Scores, Hits and GR values for the years 2010 - 2015

Years	Number of games	Scores	Hits	GR
2010	474	2106	12536	0.080
2011	454	1761	11809	0.076
2012	482	1966	12738	0.076
2013	398	1498	10471	0.074
2014	378	1438	9856	0.075
2015	398	1796	10616	0.080

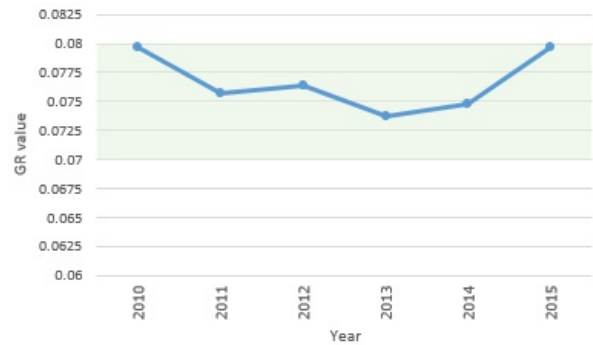


Fig. 6: Softball's GR values for the years 2010 - 2015

4. Discussion

In this section, softball is analyzed to compare with baseball. Then we discuss about the trend of baseball based on the analysis results in the previous sections. Moreover, some related works are presented.

4.1 Softball and baseball compared

Softball is a variant of baseball played with a larger ball on a smaller field. It was invented in 1887 in Chicago as an indoor game. It was at various times called indoor baseball, mush ball, playground, softbund ball, kitten ball, and, because it was also played by women, ladies' baseball. The name softball was given to the game in 1926 [17]. Women's softball made its first Olympic appearance in 1996 and made its final Olympic appearance in the 2008 games [18]. Softball and baseball have both failed attempts to be reinstated to the Olympics for 2012 and 2016. In 2012 the heads of the International Softball and Baseball Federations announced that they were uniting to increase their chances in playing in the 2020 games.

Softball is analyzed using the data from 2010 to 2015. The results are summarized in Table 6. An important difference between softball and baseball when applying the game refinement theory is the number of innings. Baseball consists of 9 innings a game but softball consists of 7 innings. Softball's GR values fall between 0.074 and 0.080, that is the sophisticated zone. The results are depicted in Figure 6.

4.2 Baseball in the future - possible enhancements

The results from each year are slightly lower than the sophisticated zone. According to these measurements, if the rules of baseball are not changed in the future, the difference between its

game refinement measure and the sophisticated zone will increase. This forecasting is shown in Figure 1. In order to increase the game refinement value of baseball, the scores and home runs in each game must be increasing while the attempt does not increase as much as scores and home runs. According to the above result, changing some rules such as decreasing the number of innings, decreasing the field size or increasing the strike zone should be preferred to increasing the game refinement measure of baseball.

4.3 Related Works

Veer *et al.* [15] introduced a concept of measuring the excitement in sports games. They relate the excitement to the variability of the win expectancy. The larger is this variability, the higher is the excitement. The win expectancy varies more if there is a number of swings during the game, as opposed to a one-sided game. Win expectancy also changes more the closer to the end of the game a decisive event happens, or a more unexpected is the upset of a favorite team. They illustrated this concept at soccer games for which the theoretical win expectancy can be computed from a Poisson model of scoring. This approach is promising to measure the excitement with focus on an individual game under consideration. Indeed it was employed in video processing domain [6]. However, our purpose is different from this direction. Our study does not focus on an individual game, but on large number of games to assess the sophistication of the game under consideration.

Rottenberg [11] noted that the nature of sports is such that competitors must be of approximately equal ability if any are to be financially successful. In recent years, sports commentators and fans, Major League Baseball itself, and even some economists have expressed growing concern about the widening

disparities among team expenditures and the growing concentrations of postseason contenders and championships. Sanderson and Siegfried [12] compare different concepts of competitive balance, review the theoretical and empirical scholarship on competitive balance and the relationship between payrolls and performance, describe the natural forces and institutional rules and regulations that contribute to observed distributions of playing performances, and evaluate the likely impact of several popular proposals payroll and salary caps, luxury taxes, and increased revenue sharing on competitive balance. They made frequent comparisons to other sports leagues, including collegiate athletics and individual sports.

5. Concluding Remarks

Baseball has been one of the most popular sports, especially in Japan, with a very long history and widely played in many countries. However, it does not have any of changed rules that impact the sports. Thus, it is likely that the popularity of baseball has been decreasing over time. In this study, baseball's entertainment is evaluated by the game refinement theory, which concerns about attractiveness and sophistication of the considered sports. The model of time limit sports was applied to baseball, which considers the innings as the time. The baseball leagues both major and minor from many countries were chosen to apply the game refinement theory.

The range of game refinement value of baseball is around 0.065, which is lower than the sophisticated zone. Moreover, it trends to decreasing in the future. Thus, baseball will not attract some viewers. The value of game refinement is decreasing because the rules does not change for a while. In contrast, the rules of other popular sports have been changed, which lead their game refinement value to the sophisticated zone. In term of countries, baseball was the most popular sports in Japan. However, the game refinement value shows that Japan has the lowest value among other countries compared. The reason is that the rules of Japan leagues that difference, which trend to get score harder than others. In term of leagues, Rookie league has the highest game refinement value. In contrast, Major league has the lowest one. Thus, the average scores in each game of Major league is lower than Rookies league. The reason is that the performance gap between batters and pitchers. Lastly, the softball's game refinement value falls in the sophisticated zone due to the effort for changing its rules, which are a huge impact to the games, promote the scoring than baseball.

In conclusion, game refinement theory can be used in order to evaluate the attractiveness or entertainment in games or sports. However, it must have an information about the game progress to measure. Baseball is one of the sports that was applied in order to measure its entertainment impact. This can help the game designers create game or change the rules to achieve the game with more sophisticated. From the previous studies, we see that the suitable game refinement measure is around 0.07 - 0.08. Baseball is slightly lower than this range. Further works may include the analysis of various data such as from other similar sports or other sports. Moreover, this model can be applied to other games such as board games.

References

- [1] BBC News (2008). "Fewer Sports for London Olympics". July 8, 2005.
- [2] P. Chetprayoon, S. Xiong and H. Iida (2015). An Approach to Quantifying Pokemon's Entertainment Impact with focus on Battle. *3rd International Conference on Applied Computing & Information Technology (ACIT2015)*.
- [3] Furth, D. (2000). Equilibrium: Some recent types of equilibrium models, *De Economist* 141, NR. 3, 1993 *Game Equilibrium Modelling*, 2, 319.
- [4] H. Iida, N. Takeshita, and J. Yoshimura. (2003). A metric for entertainment of boardgames: Its implication for evolution of chess variants. *Entertainment Computing Technologies and Applications*, 65–72.
- [5] H. Iida, K. Takahara, J. Nagashima, Y. Kajihara and T. Hashimoto. (2004). An application of game-refinement theory to Mah Jong. *Entertainment Computing-ICEC2004*, 333–338. Springer.
- [6] G. G. Lee, H. k. Kim and W. Y. Kim (2009). "Highlight generation for basketball video using probabilistic excitement," *IEEE International Conference on Multimedia and Expo*, New York, NY, pp. 318–321.
- [7] J. McCauley (August 23, 2008). "MLB Wants Baseball Back in Olympics". *Washington Times*. Associated Press.
- [8] J.Nash (1950). "Equilibrium points in n-person games" *Proceedings of the National Academy of Sciences* 36(1):48–49.
- [9] N. Nossal and H. Iida (2014). Game Refinement Theory and Its Application to Score Limit Games. *IEEE Games and Entertainment in Media 2014*.
- [10] J. Neumann (1928). Zur theorie der gesellschaftsspiele. *Mathematische Annalen*, 100(1):295–320.
- [11] S.Rottenberg (1956). The Baseball Player's Labor Market, *The Journal of Political Economy*, Vol.64, No.3, pp.242–258.
- [12] A. R. Sanderson and J. J. Siegfried (2003). Thinking about Competitive Balance, *Journal of Sports Economics* Vol. 4, No. 4, pp.255–279.
- [13] A. P. Sutiono, R. Ramadan, P. Jarukasetporn, J. Takeuchi, A. Purwarianti and H. Iida (2015). A mathematical model of game refinement and its applications to sports games, *EAI Endorsed Transactions on Creative Technologies*, Vol.2, Issue 5, 1–7.
- [14] J. Takeuchi, R. Ramadan and H. Iida. (2014). Game refinement theory and its application to Volleyball, *Research Report 2014-GI-31* (3), Information Processing Society of Japan, 1–6.
- [15] J. Vecer, T. Ichiba, M. Laudanovic (2007). "On probabilistic excitement of sports games", *Journal of Quantitative Analysis in Sports*, 3(3).
- [16] XIONG Shuo, ZUO Long, R. Chiewvanichakorn and H. Iida (2014). Quantifying Engagement of Various Games. *The 19th Game Programming Workshop 2014*, 101–106.
- [17] "softball." *The Columbia Encyclopedia*. New York: Columbia University Press, 2008.
- [18] "Olympic Preview: Softball ". *Infoplease.com*. 2000-07-01.
- [19] Baseball reference, <http://www.baseball-reference.com>, 2016.
- [20] Softball reference, <http://www.horizonleague.org>.