| | |
|---|---|
| Title | Fading Correlations for Wireless Cooperative Communications: Diversity and Coding Gains |
| Author(s) | Qian, Shen; He, Jiguang; Juntti, Markku; Matsumoto, Tad |
| Citation | IEEE Access, 5: 8001-8016 |
| Issue Date | 2017-04-28 |
| Type | Journal Article |
| Text version | author |
| URL | http://hdl.handle.net/10119/14211 |
| Rights | This is the author's version of the work. Copyright (C) 2017 IEEE. IEEE Access, 5, 2017, pp.8001-8016. DOI:10.1109/ACCESS.2017.2699785. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. |
| Description | |

# Fading Correlations for Wireless Cooperative Communications: Diversity and Coding Gains

Shen Qian* †, Jiguang He†, Markku Juntti† and Tad Matsumoto* †
*School of Information Science, Japan Advanced Institute of Science and Technology (JAIST)
1-1 Asahidai, Nomi, Ishikawa, 923-1292 Japan Email: {shen.qian, matumoto}@jaist.ac.jp
†Centre for Wireless Communications, FI-90014 University of Oulu, Finland
Email: {jiguang.he, markku.juntti}@oulu.fi

***Abstract*—We theoretically analyze outage probabilities of the lossy-forward (LF), decode-and-forward (DF) and adaptive decode-and-forward (ADF) relaying techniques, with the aim of characterizing the impact of the spatial and temporal correlations of the fading variations. First, the exact outage probability expressions are analytically obtained for LF, DF, and ADF relaying assuming each link suffers from statistically independent Rayleigh fading. Then, approximated, yet accurate closed-form expressions of the outage probabilities for the relaying schemes are derived. Based on the expressions, diversity and coding gains are found. The mathematical methodology for the derivation of the explicit outage probability is further extended to the case where the account is taken of the spatial and temporal correlations of the fading variations. The diversity gains with LF, DF, and ADF are then derived in the presence of the correlations. It is found out that the three techniques can achieve full diversity, as long as the fading variations are not fully correlated. We then investigate the optimal relay location and the optimal power allocation for the three relaying techniques. It is shown that the optimal solutions can be obtained under the framework of convex optimization. It is reveal that in correlated fading, the relay should move close to the destination or allocate more transmit power to the relay for achieving lower outage probabilities, compared to the case in independent fading.**

***Keywords*—Cooperation techniques, relay channels, fading variation, outage probability, diversity and coding gains, spatial and temporal correlations, source-channel separation theorem.***

## I. Introduction

A promising cooperative technique called lossy-forward (LF) relaying [1] has gained a lot of attention recently, since it improves throughput efficiency and reduces the outage probability, compared to the conventional decode-and-forward (DF) relaying [2]. Unlike the conventional DF strategy [3], [4], the decoded information sequence at the relay (R) is interleaved, re-encoded, and transmitted to destination (D), even though errors may be detected in the information sequence after decoding at R. Iterative processing between two decoders at the destination, one for decoding the signal received via the source (S)-D link and the other for that via the R-D link, with log-likelihood ratio (LLR) exchange via a LLR modification function improves decoding performance [5]. The LF technique can be viewed as a distributed joint source-channel coding system with side information [6]–[9]. It has been found that LF can achieve turbo-cliff-like bit error rate (BER) performance over additive white Gaussian noise (AWGN) channels [10].

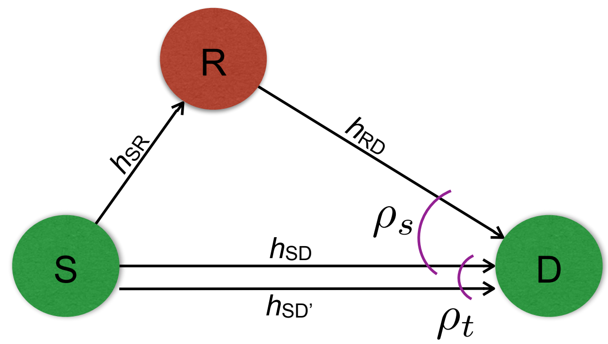



Fig. 1. The schematic scenario of one relay aided communication system.

In [11], the initial idea for LF is provided by utilizing Slepian-Wolf type cooperation for wireless communications. The coding algorithms are proposed for fading relay channel in [12], [13] with the purpose of achieving the turbo processing gain. The key concept of the coding technique for LF is introduced in [1], where it assumes that the relay does not need to necessarily recover the information sent from S perfectly. In [14], a three-node LF relaying over Rayleigh fading channels is studied by identifying the relationship between the DF protocol and Slepian-Wolf coding [15]. However, a drawback of [1], [14] is that the admissible rate region is determined by the Slepian-Wolf theorem which does not perfectly match the problem setup, since only the information of S needs to be recovered at the destination. Zhou *et al.* [2] eliminate the aforementioned drawback by utilizing the theorem of the source coding with side information in the network information theory. Based on [2], the technique is further extended to multiple access relay channel (MARC) [16], where a pair of correlated sources are transmitted to a destination with the aid

This research was supported in part by the European Unions FP7 project, ICT-619555 Links-on-the-fly Technology for Robust, Efficient and Smart Communication in Unpredictable Environments (RESCUE), in part by Academy of Finland project, No. 268209 the network compression based wireless cooperative communication systems (NETCOBRA), in part by Nokia Foundation and HPY Foundation, and also in part by the JAIST Doctoral Research Fellow program.

of one relay. Furthermore, He *et al.* [17] and Qian *et al.* [18] apply the LF technique to multi-source multi-relay systems.

Quite recently, a two-relay LF (TLF) transmission system is proposed and a power allocation scheme for minimizing the outage probability of the TLF system is presented in [20]. The technique is applied to wireless sensor networks (WSNs), where a simple, yet efficient, power allocation scheme for an arbitrary number of sensors is derived [21]. Brulatout *et al.* present a medium access protocol in [22] specifically for the LF-based network design framework. Kosek-Szott *et al.* [23] propose a centralized medium access protocol specifically designed to operate with LF relaying for coordinated wireless local area network (WLAN). It is shown that the centralized approach provides considerable gains compared to the distributed approach. The major contributions under the LF relaying framework are summarized in Table I.

Even though the superiority in LF over Rayleigh fading channels in terms of outage probability is shown in [2], [16], [18], all the evaluations related to the LF technique are based on the area integral over the admissible rate region [2], [16], [20], [21], which needs to be evaluated numerically with respect to the probability density functions (pdfs) of instantaneous signal-to-noise power ratio (SNR). Therefore, the diversity and coding gains [24] cannot be identified separately from the numerically obtained outage probability and thereby it is impossible to derive insightful expressions indicating the performance gains due to diversity and coding in fading channels.

Furthermore, in practice, it is often the case that the fading conditions experienced by different links are correlated due to the insufficient separations in the space or time domains between the nodes or transmissions, respectively [25]–[27]. In fact, the channel capacity depends heavily on the fading correlation [28], [29]. Therefore, examining the performance of diversity techniques in correlated fading conditions is a long lasting problem with great importance. Cheng *et al.* [14] has analyzed the performance of LF relaying in correlated fading channels. However, in [14], the S-R link is modeled by a binary symmetric channel (BSC) model with fixed crossover probability, which is not realistic in practical applications.

This paper aims at filling the gap between numerically obtained results and the explicit expressions indicating the diversity and coding gains, and investigating the impact of fading correlations on the system performance. We use adaptive decode-and-forward (ADF) relaying [30] as a counterpart of LF relaying since they require the same transmit phases. The LF, DF, and ADF relaying strategies specify the second phase operation performed by the relay node, upon the information part obtained after decoding at R.

For fading correlations, we consider the following two cases: (1) fading variations in S-D and R-D links are spatially correlated for LF, DF, and ADF; (2) fading variations on the two slots in S-D link are temporally correlated for ADF. The outage probability is derived by assuming the S-R link is also suffering from fading variation and hence the bit error probability of the information part after decoding at R is also a random variable. Block fading assumption is used throughout the paper, and, hence the relationship between the S-R link error probability and the S-R link instantaneous SNR can be expressed by using the theorem of lossy source-channel separation.

The discussions are conducted from four aspects: 1) outage probability derivations for LF, DF, and ADF relaying in independent and correlated fading; 2) diversity and coding gains analyses based on approximated, yet accurate closed-form outage expressions; 3) optimal relay location identification by taking into account the geometric gains; 4) optimal power allocation between S and R, under the total transmit power constraint.

The main contributions of this paper are summarized as follows:

* The exact outage probabilities for LF, DF, and ADF relaying over independent and correlated Rayleigh fading channels are derived. Approximated, yet accurate closed-form outage expressions for each relaying techniques are then obtained.
* Explicit diversity order and coding gains are derived for LF, DF, and ADF over the links suffering from statistically independent fading. It is proven that all the three relaying techniques can achieve full diversity, and the coding gain with LF is larger than that of DF, and smaller than that of ADF.
* Diversity orders of LF, DF, and ADF relaying over the links suffering from correlated fading are obtained. It is found that, full diversity order can be achieved unless fading is fully correlated.
* Optimal relay locations for achieving the lowest outage probability are investigated in independent and correlated fading. It is shown that with high fading correlation values, the relay should move closer to the destination for obtaining better outage performance. However, for specific relaying technique, the optimal relay location keeps the same (e.g., at midpoint for LF), regardless the value of the S-D link average SNRs.
* Optimal power allocations between S and R for achieving the lowest outage probability are also analyzed for LF, DF, and ADF relaying in independent and correlated fading. It is found that the higher the channel correlation, the more power should be allocated to R commonly to the relaying techniques considered.

The outline of the rest of the paper is as follows. Section II presents the system and channel models, and the assumptions used throughout this paper. Section III derives the outage probabilities of DF, LF, and ADF relaying in statistically independent fading. The diversity and coding gains are also discussed in Section III. Section IV derives the outage probabilities in correlated fading. Optimal relay locations are investigated in Section V. Optimal power allocation analysis is provided in SectionVI. Section VII concludes the paper.

## II. System and Channels Models

The wireless relay network model used in this paper is shown in Fig. 1. A source S communicates with a destination D with help of a relay R. Each terminal is equipped with single antenna. Transmission phases are orthogonal and no

TABLE I. **SUMMARY OF MAJOR CONTRIBUTIONS ON LF RELAYING.**

| Year | Authors | Contributions |
|---|---|---|
| 2005 | Hu and Li [11] | Proposed Slepian-Wolf cooperation, which exploits distributed source coding technologies in wireless cooperative communications. |
| 2007 | Woldegebreal and Karl [19] | Considered a network-coding-based MARC in the presence of non-ideal source-relay links, and analyzed the outage performance and coverage. |
| 2007 | Sneessens *et al.* [13] | Derived a decoding algorithm which enables the use of turbo-coded DF relaying by taking into account the probability of error between the source and the relay. |
| 2012 | Anwar and Matsumoto [1] | Proposed a iterative decoding technique, accumulator-assisted distributed turbo code, where the correlation knowledge between the source and the relay is estimated and exploited. |
| 2013 | Cheng *et al.* [14] | Proposed a scheme for exploiting the source-relay correlation in joint decoding process at the destination, based on the Slepian-Wolf theorem. |
| 2014 | Zhou *et al.* [2] | Derived the exact outage probabilities by utilizing the theorems of lossy source-channel separation and source coding with side information. |
| 2015 | Wolf *et al.* [20] | Proposed an optimal power allocation strategy for a two-relay system based on convex optimization to minimize outage probability. |
| 2015 | Lu *et al.* [16] | Derived the outage probability for orthogonal MARC for correlated source transmission where erroneous source information estimates at the relay are forwarded. |
| 2016 | Wolf *et al.* [21] | Proposed asymptotically optimal power allocation scheme for WSNs with correlated data from an arbitrary amount of sensors. |
| 2016 | Brulatout *et al.* [22] | Proposed a medium access control (MAC) layer protocol for LF relaying and introduced testbed implementation based on USRP and GNU radio frameworks. |

multiple access channel is involved. The cooperation protocols considered in this paper are LF, DF, and ADF relaying.

## A. Relaying

In LF relaying, S broadcasts the coded information sequence to D and R at the first time slot. The information sequence, obtained as the result of decoding at R, is interleaved, re-encoded and transmitted to D at the second time slot, even if decoder detects errors. The bit level correlation between the information sequences transmitted from S and R is estimated and utilized in an iterative joint decoding process at D to retrieve the original message of S. In this paper, we assumes that the correlation is known to the receiver. For more details about the correlation estimation at D, readers may refer to [1], [5].

In the conventional DF relaying, S broadcasts the coded information sequence to D and R at the first time slot. R tries to fully recover the received information sequence. If it is successfully recovered, the information sequence is forwarded to D at the second time slot. R keeps silent if error is detected after decoding at R.

In ADF relaying, S broadcasts the coded information sequence to D and R at the first time slot. If the transmitted information is successfully recovered at R, the recovered information sequence is interleaved, re-encoded and transmitted to D at the second time slot. Then, D performs joint decoding by exchanging LLRs between the two decoders, one corresponds to the S-D link, and the other to the R-D link. If R detects error in the information part after decoding, R notifies S of the information recovery failure via a feedback link, and S interleaves the information sequence, re-encodes, and retransmits the information sequence to D again. D performs joint decoding by exchanging LLRs between two decoders, one for the sequence transmitted at the first time slot and the other for that at the second time slot.

## B. Channel Model

The S-D and R-D links are considered to be spatially correlated with $\rho_s = \langle h_{\mathrm{SD}} h^*_{\mathrm{RD}} \rangle$ $(0 \leq \rho_s \leq 1)$ representing the spatial correlation between $h_{\mathrm{SD}}$ and $h_{\mathrm{RD}}$. $h_{ij}$ $(i \in \{\mathrm{S}, \mathrm{R}\}, j \in \{\mathrm{R}, \mathrm{D}\}, i \neq j)$ denotes the complex channel gain of $i$-$j$ link. The fading gains with the two transmissions over the S-D links are also correlated with $\rho_t = \langle h_{\mathrm{SD}} h^*_{\mathrm{S'D}} \rangle$ $(0 \leq \rho_t \leq 1)$ representing the temporal correlation between $h_{\mathrm{SD}}$ and $h_{\mathrm{S'D}}$, where $h_{\mathrm{S'D}}$ denotes the complex fading gain of the S-D link at the second time slot. The variation model can be represented by the first-order Gauss-Markov random process [31].

Let $k \in \{1, 2, \cdots, K\}$ represent symbol index. The received symbol $y^k_{\mathrm{SD}}$ via the S-D link and $y^k_{\mathrm{RD}}$ via the R-D link both at D, and $y^k_{\mathrm{SR}}$ via the S-R link at R are expressed as

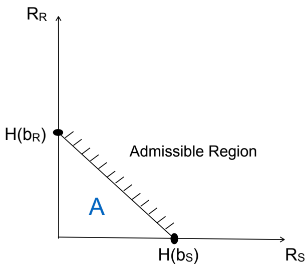
$$y^k_{\mathrm{SD}} = \sqrt{G_{\mathrm{SD}}} h_{\mathrm{SD}} x^k_{\mathrm{S}} + n^k_{\mathrm{SD}}, \tag{1}$$

$$y^k_{\mathrm{RD}} = \sqrt{G_{\mathrm{RD}}} h_{\mathrm{RD}} x^k_{\mathrm{R}} + n^k_{\mathrm{RD}}, \tag{2}$$

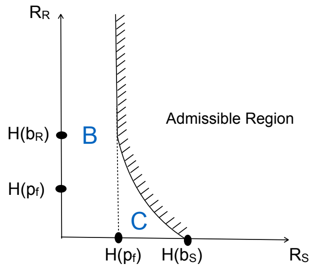
$$y^k_{\mathrm{SR}} = \sqrt{G_{\mathrm{SR}}} h_{\mathrm{SR}} x^k_{\mathrm{S}} + n^k_{\mathrm{SR}}, \tag{3}$$

respectively, where $n^k_{ij}$ $(i \in \{\mathrm{S}, \mathrm{R}\}, j \in \{\mathrm{R}, \mathrm{D}\}, i \neq j)$ denotes the zero-mean white additive Gaussian noise (AWGN) with variance of $N_0/2$ per dimension. $x^k_{\mathrm{S}}$ and $x^k_{\mathrm{R}}$ are the coded and modulated symbols transmitted from S and R with $E\{|x^k_{\mathrm{S}}|^2\} = E\{|x^k_{\mathrm{R}}|^2\} = E_s$, with $E_s$ representing the transmit symbol energy. It is assumed that the complex Gaussian link gain has zero mean and unit variance $E[|h_{ij}|^2] = 1$. $h_{ij}$ is kept constant over one block duration due to the block fading assumption. Let $G_{ij}$ represents the geometric gains of the $i$-$j$ link. The average and instantaneous SNRs can be expressed as $\overline{\gamma}_{ij} = G_{ij}(E_s/N_0)$ and $\gamma_{ij} = |h_{ij}|^2 \overline{\gamma}_{ij}$, respectively. We assume that the channel state information (CSI) is only available at the receiver sides.

Each link is assumed to suffer from frequency non-selective Rayleigh fading. The pdf of instantaneous SNR $\gamma_{ij}$ is given




(a) Rate region for S and R when $p_f = 0$.




(b) Rate region for S and R when $p_f \neq 0$.

Fig. 2. Rate region for LF relaying.

by

$$p(\gamma_{ij}) = \frac{1}{\overline{\gamma}_{ij}} \exp(-\frac{\gamma_{ij}}{\overline{\gamma}_{ij}}), (i \in \{\mathrm{S}, \mathrm{R}\}, j \in \{\mathrm{R}, \mathrm{D}\}, i \neq j), \tag{4}$$

For the sake of simplicity, the effect due to shadowing is not taken into account.

## III. Outage Analysis in Independent Fading

### A. *Outage Behavior of LF in Independent Fading*

Let $b_\mathrm{S}$ and $b_\mathrm{R}$ be the binary information sequence of S and decoding result at R, respectively. According to the theorem of source coding with side information [32], $b_\mathrm{S}$ can be reconstructed error-freely at D if the source coding rates of $b_\mathrm{S}$ and $b_\mathrm{R}$, $R_\mathrm{S}$ and $R_\mathrm{R}$, satisfy the following inequalities:

$$\begin{cases} R_\mathrm{S} & \geq H(b_\mathrm{S}|\hat{b}_\mathrm{R}), \\ R_\mathrm{R} & \geq I(b_\mathrm{R}; \hat{b}_\mathrm{R}), \end{cases} \tag{5}$$

where $\hat{b}_\mathrm{R}$ is the estimate of $b_\mathrm{R}$ at D, and $H(\cdot|\cdot)$ and $I(\cdot;\cdot)$ denote the conditional entropy and the mutual information between the arguments, respectively.

Let $p_f$ be a crossover probability of a BSC virtually modeling the S-R link. $p_f$ stays the same within one block but varies block by block. Based on the Shannon's lossy source channel separation theorem [33], [34], the relationship between $p_f$ and the instantaneous channel SNR $\gamma_\mathrm{SR}$ is given as [2]

$$p_f = H_2^{-1}\left(1 - \log_2\left(1 + \gamma_\mathrm{SR}\right)\right), \tag{6}$$

with $H_2^{-1}(\cdot)$ denoting the inverse function of the binary entropy. The minimum distortion is equivalent to $p_f$ [35] under the Hamming distortion assumption.

With $p_f = 0$ indicating perfect decoding at R, we have $H(b_\mathrm{S}|b_\mathrm{R}) = H(b_\mathrm{R}|b_\mathrm{S}) = 0$. Hence, the inadmissible rate region becomes the triangle area A as shown in Fig. 2(a). When $0 < p_f \leq 0.5$, the inadmissible region is shown in Fig. 2(b) which can be divided into two areas, B and C.

The rate region defined in (5) indicates that, although $b_\mathrm{R}$ contains error, with $0 \leq R_\mathrm{R} \leq H(b_\mathrm{R})$, it can serve as the side information for losslessly recovering $b_\mathrm{S}$. In the case $R_\mathrm{R} > H(b_\mathrm{R})$, the condition becomes to $R_\mathrm{S} \geq H(p_f)$.

If the rate pair ($R_\mathrm{S}$, $R_\mathrm{R}$) falls into the inadmissible region, the outage event occurs, and D cannot reconstruct $b_\mathrm{S}$ with an arbitrarily small error probability. Since $p_f = 0$ and $0 < p_f \leq 0.5$ are distinctive, the outage probability of LF relaying can be expressed as

$$P_\mathrm{out}^\mathrm{LF} = P_\mathrm{A} + P_\mathrm{B} + P_\mathrm{C}, \tag{7}$$

where $P_\mathrm{A}$, $P_\mathrm{B}$, and $P_\mathrm{C}$ denote the probabilities that the rate pair ($R_\mathrm{S}$, $R_\mathrm{R}$) falls into the inadmissible areas A, B, and C, respectively. Taking into account the impact of $p_f$, $P_\mathrm{A}$, $P_\mathrm{B}$, and $P_\mathrm{C}$ can be expressed as

$$P_\mathrm{A} = \Pr[p_f = 0, 0 \leq R_\mathrm{S} < 1, 0 \leq R_\mathrm{R} < H(p_f * p'_f)], \tag{8}$$

$$P_\mathrm{B} = \Pr[0 < p_f \leq 0.5, 0 \leq R_\mathrm{S} < H(p_f), R_\mathrm{R} \geq 0], \tag{9}$$

$$\begin{aligned} P_\mathrm{C} = \Pr[&0 < p_f \leq 0.5, H(p_f) \leq R_\mathrm{S} < 1, \\ &0 \leq R_\mathrm{R} < H(p_f * p'_f)], \end{aligned} \tag{10}$$

where a per-block BSC model is again used to represent the R-D link with crossover probability $p'_f$. $p_f * p'_f = (1-p_f)p'_f + (1-p'_f)p_f$ with $*$ representing convolution operation.

Based on the Shannon's lossless source channel separation theorem, the relationship between the instantaneous channel SNR $\gamma_{ij}$ and its corresponding rate $R_i$ is given by[1]

$$R_i \leq \Theta(\gamma_{ij}) = \log_2\left(1 + \gamma_{ij}\right), (i \in \{\mathrm{S}, \mathrm{R}\}, j = \mathrm{D}) \tag{11}$$

with its inverse function

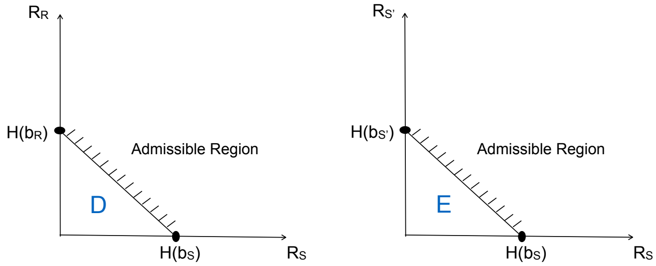
$$\gamma_{ij} \geq \Theta^{-1}(R_i) = \left(2^{R_i} - 1\right). \tag{12}$$

Solving (8), (9) and (10) based on the pdfs of the instantaneous SNRs of the corresponding channels, the outage probabilities of LF relaying over independent Rayleigh channels can be expressed as

$$\begin{aligned} P_\mathrm{out}^\mathrm{LF,\,Ind} =& \frac{1}{\overline{\gamma}_\mathrm{SD}} \exp\left(-\frac{1}{\overline{\gamma}_\mathrm{SR}}\right) \int_{\gamma_\mathrm{SD}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp\left(-\frac{\gamma_\mathrm{SD}}{\overline{\gamma}_\mathrm{SD}}\right) \\ &\left(1 - \exp\left(-\frac{\Theta^{-1}(1-\Theta(\gamma_\mathrm{SD}))}{\overline{\gamma}_\mathrm{RD}}\right)\right) d\gamma_\mathrm{SD} \\ &+ \frac{1}{\overline{\gamma}_\mathrm{SR}} \int_{\gamma_\mathrm{SR}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \left[1 - \exp\left(-\frac{\Theta^{-1}(1-\Theta(\gamma_\mathrm{SR}))}{\overline{\gamma}_\mathrm{SD}}\right)\right] \\ &\cdot \exp\left(-\frac{\gamma_\mathrm{SR}}{\overline{\gamma}_\mathrm{SR}}\right) d\gamma_\mathrm{SR} + \frac{1}{\overline{\gamma}_\mathrm{SD}\overline{\gamma}_\mathrm{SR}} \int_{\gamma_\mathrm{SR}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \\ &\int_{\gamma_\mathrm{SD}=\Theta^{-1}(1-\Theta(\gamma_\mathrm{SR}))}^{\Theta^{-1}(1)} \exp\left(-\frac{\gamma_\mathrm{SD}}{\overline{\gamma}_\mathrm{SD}}\right) \exp\left(-\frac{\gamma_\mathrm{SR}}{\overline{\gamma}_\mathrm{SR}}\right) \\ &\cdot \left(1 - \exp\left(-\frac{\xi(\gamma_\mathrm{SD}, \gamma_\mathrm{SR})}{\overline{\gamma}_\mathrm{RD}}\right)\right) d\gamma_\mathrm{SD} d\gamma_\mathrm{SR}, \end{aligned} \tag{13}$$

where $\xi(\gamma_\mathrm{SD}, \gamma_\mathrm{SR}) = H_2\{H_2^{-1}[1-\Theta(\gamma_\mathrm{SD})] * H_2^{-1}[1-\Theta(\gamma_\mathrm{SR})]\}$.

[1]Gaussian codebook is assumed. Without loss of generality, the signaling dimensionality is equal to two and the spectrum efficiency of the transmission chain, including the channel coding scheme and modulation multiplicity in each link is set to the unity. For more generic representation, readers may refer to [2].




(a) Rate region for S and R when $p_f = 0$.

(b) Rate region for twice transmit S when $p_f \neq 0$.

Fig. 3. Rate region for ADF relaying.

*B. Outage Behavior of DF in Independent Fading*

In DF relaying, relay keeps silent if error is detected after decoding. When $p_f = 0$, the outage analysis for DF is the same as that for LF. When $p_f \neq 0$, it is equivalent to that of point-to-point S-D transmission. Therefore the outage probability of DF relaying can be expressed as

$$\begin{aligned} P_{\text{out}}^{\text{DF, Ind}} =& \Pr[p_f = 0, 0 \leq R_{\text{S}} < 1, 0 \leq R_{\text{R}} < H(p'_f)] \\ &+ \Pr[0 < p_f \leq 0.5, R_{\text{S}} < 1] \\ =& \frac{1}{\overline{\gamma}_{\text{SD}}} \exp\left(-\frac{1}{\overline{\gamma}_{\text{SR}}}\right) \int_{\gamma_{\text{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp\left(-\frac{\gamma_{\text{SD}}}{\overline{\gamma}_{\text{SD}}}\right) \\ & \left(1 - \exp\left(-\frac{\Theta^{-1}(1-\Theta(\gamma_{\text{SD}}))}{\overline{\gamma}_{\text{RD}}}\right)\right) d\gamma_{\text{SD}} \\ &+ \left(1 - \exp\left(-\frac{\Theta^{-1}(1)}{\overline{\gamma}_{\text{SR}}}\right)\right)\left(1 - \exp\left(-\frac{\Theta^{-1}(1)}{\overline{\gamma}_{\text{SD}}}\right)\right). \end{aligned} \tag{14}$$

*C. Outage Behavior of ADF in Independent Fading*

With ADF, S retransmits an interleaved and re-encoded version of the information, if it is notified of the decoding failure via a feedback link from R. The rate regions of ADF with $p_f = 0$ and $p_f \neq 0$ are shown in Fig. 3(a) and Fig. 3(b), respectively. Similarly to the case of LF, the outage probability is defined as the probability that the source rate pairs of S and R, ($R_{\text{S}}$, $R_{\text{R}}$) and ($R_{\text{S}}$, $R_{\text{S'}}$), fall into the inadmissible area D or E shown in Fig. 3(a) and Fig. 3(b), respectively. $R_{\text{S'}}$ is the rate of retransmitted information sequence from S. Let $P_{\text{D}}$ and $P_{\text{E}}$ denote the probabilities that ($R_{\text{S}}$, $R_{\text{R}}$) and ($R_{\text{S}}$, $R_{\text{S'}}$) fall into the inadmissible areas D and E, respectively. The outage probability of ADF relaying is then given by

$$\begin{aligned} P_{\text{out}}^{\text{ADF, Ind}} =& P_{\text{D}} + P_{\text{E}} \\ =& \Pr[p_f = 0, 0 \leq R_{\text{S}} < 1, 0 \leq R_{\text{R}} < H(p_f)] \\ &+ \Pr[0 < p_f \leq 0.5, 0 \leq R_{\text{S}} < 1, 0 \leq R_{\text{S'}} < H(p_f)] \\ =& \frac{1}{\overline{\gamma}_{\text{SD}}} \exp\left(-\frac{1}{\overline{\gamma}_{\text{SR}}}\right) \int_{\gamma_{\text{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp\left(-\frac{\gamma_{\text{SD}}}{\overline{\gamma}_{\text{SD}}}\right) \\ & \left(1 - \exp\left(-\frac{\Theta^{-1}(1-\Theta(\gamma_{\text{SD}}))}{\overline{\gamma}_{\text{RD}}}\right)\right) d\gamma_{\text{SD}} \\ &+ \frac{1}{\overline{\gamma}_{\text{S'D}}} \left(1 - \exp\left(-\frac{1}{\overline{\gamma}_{\text{SR}}}\right)\right) \int_{\gamma_{\text{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp\left(-\frac{\gamma_{\text{SD}}}{\overline{\gamma}_{\text{S'D}}}\right) \\ & \left(1 - \exp\left(-\frac{\Theta^{-1}(1-\Theta(\gamma_{\text{SD}}))}{\overline{\gamma}_{\text{SD}}}\right)\right) d\gamma_{\text{SD}}. \end{aligned} \tag{15}$$

The derivations for the explicit expressions of (13), (14), and (15) may not be possible. We use a numerical method [36] to evaluate $P_{\text{out}}^{\text{LF, Ind}}$, $P_{\text{out}}^{\text{DF, Ind}}$, and $P_{\text{out}}^{\text{ADF, Ind}}$.

*D. Approximations*

By invoking the property of exponential function $e^{-x} \approx 1 - x$ for small $x$, corresponding to high SNR regime, the outage probabilities of LF, DF, and ADF relaying, respectively, can be approximated by the closed-form expressions, as,

$$\begin{aligned} P_{\text{out}}^{\text{LF, Ind}} \approx& \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}} \left(1 - \frac{1}{\overline{\gamma}_{\text{SR}}}\right)\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}}\right) \\ &+ \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{SR}}} \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SR}}}\right), \end{aligned} \tag{16}$$

$$\begin{aligned} P_{\text{out}}^{\text{DF, Ind}} \approx& \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}} \left(1 - \frac{1}{\overline{\gamma}_{\text{SR}}}\right)\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}}\right) \\ &+ \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{SR}}}, \end{aligned} \tag{17}$$

and

$$\begin{aligned} P_{\text{out}}^{\text{ADF, Ind}} \approx& \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}} \left(1 - \frac{1}{\overline{\gamma}_{\text{SR}}}\right)\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}}\right) \\ &+ \frac{1}{\overline{\gamma}_{\text{SD}}^2\overline{\gamma}_{\text{SR}}} \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}}\right). \end{aligned} \tag{18}$$

It is found from Fig. 4 that in independent fading (i.e., $\rho_s = 0$ for LF and DF, and $\rho_s = \rho_t = 0$ for ADF) the approximated outage curves obtained from (16), (17), and (18) well match the numerically calculated outage curves from (13), (14), and (15).

*E. Diversity Order and Coding Gains*

By setting the geometric gain of each link to identical and replacing $\overline{\gamma}_{ij}$ with a generic $\overline{\gamma}$, corresponding to equilateral triangle node locations (i.e., S, R, and D are located to the vertex of an equilateral triangle), (16), (17), and (18) reduce to

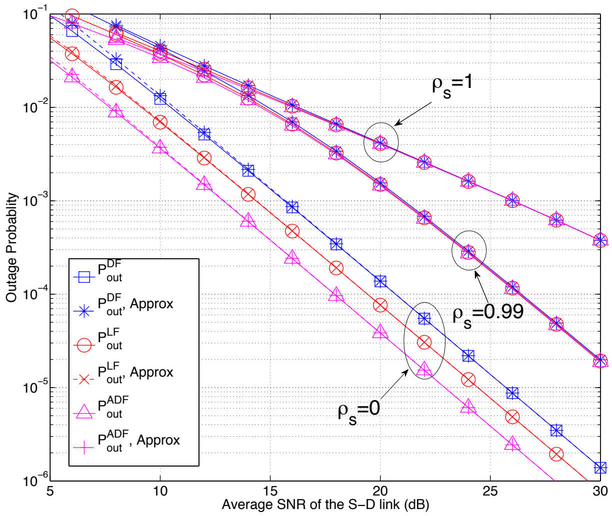



Fig. 4. Comparison between the exact and approximated outage probabilities for LF, DF, and ADF relaying over independent and spatial correlated channels, where the S-R, S-D, and R-D links have equal distances and $\rho_t = 0$.

$$P_{\text{out}}^{\text{LF, Ind}} = \frac{2\ln(4)-2}{(\overline{\gamma})^2} + \frac{8\ln(2)-2\ln(4)-4}{2(\overline{\gamma})^3} - \frac{4\ln(2)-3}{2(\overline{\gamma})^4}, \tag{19}$$

$$P_{\text{out}}^{\text{DF, Ind}} = \frac{\ln(4)}{(\overline{\gamma})^2} + \frac{4\ln(2)-2\ln(4)-2}{2(\overline{\gamma})^3} - \frac{4\ln(2)-3}{2(\overline{\gamma})^4}, \tag{20}$$

and

$$P_{\text{out}}^{\text{ADF, Ind}} = \frac{\ln(4)-1}{(\overline{\gamma})^2} + \frac{4\ln(2)-3}{2(\overline{\gamma})^3}, \tag{21}$$

respectively.

Then, in high average SNR regime, (19), (20), and (21) can further be approximated as

$$P_{\text{out}}^{\text{LF, Ind}} = \left(G_c^{\text{LF}} \cdot \overline{\gamma}\right)^{-G_d^{\text{LF}}}, \tag{22}$$

$$P_{\text{out}}^{\text{DF, Ind}} = \left(G_c^{\text{DF}} \cdot \overline{\gamma}\right)^{-G_d^{\text{DF}}}, \tag{23}$$

and

$$P_{\text{out}}^{\text{ADF, Ind}} = \left(G_c^{\text{ADF}} \cdot \overline{\gamma}\right)^{-G_d^{\text{ADF}}}, \tag{24}$$

where $G_d^{\text{LF}} = G_d^{\text{DF}} = G_d^{\text{ADF}} = 2$ are the diversity order of LF, DF, and ADF relaying. This is consistent to the decays of the cooperation protocols shown in the outage curves in Fig. 4 (i.e., those with $\rho_s = 0$). $G_c^{\text{LF}} = \frac{1}{\sqrt{2\ln(4)-2)}}$, $G_c^{\text{DF}} = \frac{1}{\sqrt{\ln(4))}}$, and $G_c^{\text{ADF}} = \frac{1}{\sqrt{\ln(4)-1)}}$ are the coding gains of LF, DF, and ADF relaying, respectively [24]. Since the coding gain expression does not include the term $E_s/N_0$, the gain appears in the form of the parallel shift of the outage curves. It is easy to find out that $G_c^{\text{ADF}} > G_c^{\text{LF}} > G_c^{\text{DF}}$, which verifies the relationship between the outage probabilities shown in Fig. 4, i.e., with the independent channels ($\rho_s = 0$, $\rho_t = 0$), $P_{\text{out}}^{\text{DF, Ind}} > P_{\text{out}}^{\text{LF, Ind}} > P_{\text{out}}^{\text{ADF, Ind}}$.

*Theorem 1:* The outage probability of LF is smaller than that of DF, i.e., $P_{\text{out}}^{\text{LF, Ind}} < P_{\text{out}}^{\text{DF, Ind}}$.

*Proof:* The difference between $P_{\text{out}}^{\text{DF, Ind}}$ in (16) and $P_{\text{out}}^{\text{LF, Ind}}$ in (17), is

$$P_{\text{out}}^{\text{LF, Ind}} - P_{\text{out}}^{\text{DF, Ind}} = \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{SR}}}\left(\ln(4) - 2 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SR}}}\right). \tag{25}$$

Let $\Delta = \left(\ln(4) - 2 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SR}}}\right)$. Since

$$\lim_{\overline{\gamma}_{\text{SR}}\to\infty} \Delta = -1.3069 < 0 \tag{26}$$

and

$$\frac{d\Delta}{d\overline{\gamma}_{\text{SR}}} = \frac{3-4\ln(2)}{\overline{\gamma}_{\text{SR}}^2} > 0, \tag{27}$$

$\Delta$ increases monotonically as $\overline{\gamma}_{\text{SR}}$ increases and limited to -1.3069. Therefore, it proves that (25) always has negative value for any given SNR, which indicates that the $P_{\text{out}}^{\text{LF, Ind}}$ is smaller than $P_{\text{out}}^{\text{DF, Ind}}$. ■

*Theorem 2:* The outage probability of LF is larger than that of ADF, i.e., $P_{\text{out}}^{\text{LF, Ind}} > P_{\text{out}}^{\text{ADF, Ind}}$.

*Proof:* The difference between $P_{\text{out}}^{\text{LF, Ind}}$ in (16) and $P_{\text{out}}^{\text{ADF, Ind}}$ in (18) is

$$\begin{aligned} P_{\text{out}}^{\text{LF, Ind}} - P_{\text{out}}^{\text{ADF, Ind}} &= \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{SR}}}\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SR}}}\right) \\ &\quad - \frac{1}{\overline{\gamma}_{\text{SD}}^2\overline{\gamma}_{\text{SR}}}\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}}\right). \end{aligned} \tag{28}$$

We assume $\frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}} \leq \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SR}}}$ with $\overline{\gamma}_{\text{SD}} > 1$[2]. Since $\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}}\right) > 0$, we have

$$\begin{aligned} &\frac{1}{\overline{\gamma}_{\text{SD}}^2\overline{\gamma}_{\text{SR}}}\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}}\right) \\ &< \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{SR}}}\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}}\right) \\ &\leq \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{SR}}}\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SR}}}\right), \end{aligned} \tag{29}$$

indicating the right hand side of (28) is greater than 0. This proves $P_{\text{out}}^{\text{LF, Ind}} > P_{\text{out}}^{\text{ADF, Ind}}$. ■

[2]Note that the approximated outage expressions (16), (17), and (18) are obtained at high SNR regime, therefore it is reasonable to give proof for $\overline{\gamma}_{\text{SD}} > 1$ and $\overline{\gamma}_{\text{SR}} > 1$.

## IV. Outage Analysis in Correlated Fading

Let the correlation of two complex fading gains be defined as $\rho = \langle h_1 h_2^* \rangle$. The joint pdf of two signal amplitudes of correlated Rayleigh fading channels is given by [37]. It is not difficult to convert the amplitude joint pdf into that of the SNRs, $\gamma_1$ and $\gamma_2$, as

$$p(\gamma_1,\gamma_2) = \frac{1}{\overline{\gamma}_1\overline{\gamma}_2(1-\rho^2)} I_0\left(\frac{2|\rho|\gamma_1\gamma_2}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}(1-|\rho|^2)}\right) \cdot \exp\left[-\frac{\frac{\gamma_1}{\overline{\gamma}_1}+\frac{\gamma_2}{\overline{\gamma}_2}}{1-|\rho|^2}\right], \tag{30}$$

where $I_0(\cdot)$ is the zero-th order modified Bessel's function of the first kind.

### A. Outage Behavior of LF in Correlated Fading

We follow the same technique as that used for analyzing the outage in independent fading. The multiple integral with the constraint (8), (9) and (10) over the joint pdf (30) leads to the outage probability expressions of LF relaying in spatially correlated S-D and R-D links, as

$$\begin{aligned} P_{\text{out}}^{\text{LF, Cor}} =& \exp\left(-\frac{1}{\overline{\gamma}_{\text{SR}}}\right)\int_{\gamma_{\text{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)}\int_{\gamma_{\text{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1-\Theta(\gamma_{\text{SD}}))} \\ & \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}(1-\rho_s^2)} I_0\left(\frac{2|\rho_s|\gamma_{\text{SD}}\gamma_{\text{RD}}}{\sqrt{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}}(1-|\rho_s|^2)}\right) \\ & \cdot\exp\left[-\frac{\frac{\gamma_{\text{SD}}}{\overline{\gamma}_{\text{RD}}}+\frac{\gamma_{\text{RD}}}{\overline{\gamma}_{\text{RD}}}}{1-|\rho_s|^2}\right] d\gamma_{\text{SD}}d\gamma_{\text{RD}}, \\ & + \int_{\gamma_{\text{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)}\int_{\gamma_{\text{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1-\Theta(\gamma_{\text{SR}}))}\int_{\gamma_{\text{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}(\infty)} \\ & \frac{1}{\overline{\gamma}_{\text{SR}}}\exp\left(-\frac{\gamma_{\text{SR}}}{\overline{\gamma}_{\text{SR}}}\right) I_0\left(\frac{2|\rho_s|\gamma_{\text{SD}}\gamma_{\text{RD}}}{\sqrt{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}}(1-|\rho_s|^2)}\right) \\ & \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}(1-\rho_s^2)}\exp\left[-\frac{\frac{\gamma_{\text{SD}}}{\overline{\gamma}_{\text{RD}}}+\frac{\gamma_{\text{RD}}}{\overline{\gamma}_{\text{RD}}}}{1-|\rho_s|^2}\right] d\gamma_{\text{SD}}d\gamma_{\text{RD}}d\gamma_{\text{SR}}, \\ & + \int_{\gamma_{\text{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)}\int_{\gamma_{\text{SD}}=\Theta^{-1}(1-\Theta(\gamma_{\text{SR}}))}^{\Theta^{-1}(1)}\int_{\gamma_{\text{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}[\xi(\gamma_{\text{SD}},\gamma_{\text{SR}})]} \\ & \frac{1}{\overline{\gamma}_{\text{SR}}}\exp\left(-\frac{\gamma_{\text{SR}}}{\overline{\gamma}_{\text{SR}}}\right) I_0\left(\frac{2|\rho_s|\gamma_{\text{SD}}\gamma_{\text{RD}}}{\sqrt{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}}(1-|\rho_s|^2)}\right) \\ & \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}(1-\rho_s^2)}\exp\left[-\frac{\frac{\gamma_{\text{SD}}}{\overline{\gamma}_{\text{RD}}}+\frac{\gamma_{\text{RD}}}{\overline{\gamma}_{\text{RD}}}}{1-|\rho_s|^2}\right] d\gamma_{\text{SD}}d\gamma_{\text{RD}}d\gamma_{\text{SR}}. \end{aligned} \tag{31}$$

### B. Outage Behavior of DF in Correlated Fading

Similarly, the outage probability expression of DF relaying in the correlated S-D and R-D link variation can be expressed as,

$$\begin{aligned} P_{\text{out}}^{\text{DF, Cor}} =& \exp\left(-\frac{1}{\overline{\gamma}_{\text{SR}}}\right)\int_{\gamma_{\text{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)}\int_{\gamma_{\text{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1-\Theta(\gamma_{\text{SD}}))} \\ & \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}(1-\rho_s^2)} I_0\left(\frac{2|\rho_s|\gamma_{\text{SD}}\gamma_{\text{RD}}}{\sqrt{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}}(1-|\rho_s|^2)}\right) \\ & \cdot\exp\left[-\frac{\frac{\gamma_{\text{SD}}}{\overline{\gamma}_{\text{RD}}}+\frac{\gamma_{\text{RD}}}{\overline{\gamma}_{\text{RD}}}}{1-|\rho_s|^2}\right] d\gamma_{\text{SD}}d\gamma_{\text{RD}}, \\ & + \left(1-\exp\left(-\frac{\Theta^{-1}(1)}{\overline{\gamma}_{\text{SR}}}\right)\right)\left(1-\exp\left(-\frac{\Theta^{-1}(1)}{\overline{\gamma}_{\text{SD}}}\right)\right). \end{aligned} \tag{32}$$

### C. Outage Behavior of ADF in Correlated Fading

When analyzing the ADF outage probability, two correlation terms, $\rho_s$ and $\rho_t$, correlations of the complex fading gains of the S-D and R-D links, and those of the two adjacent S-D transmissions, respectively, have to be taken into account. After several mathematical manipulations, we have the outage probability expression of ADF relaying over the links with correlated fading, as

$$\begin{aligned} P_{\text{out}}^{\text{ADF, Cor}} =& \exp\left(-\frac{1}{\overline{\gamma}_{\text{SR}}}\right)\int_{\gamma_{\text{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)}\int_{\gamma_{\text{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1-\Theta(\gamma_{\text{SD}}))} \\ & \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}(1-\rho_s^2)} I_0\left(\frac{2|\rho_s|\gamma_{\text{SD}}\gamma_{\text{RD}}}{\sqrt{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}}(1-|\rho_s|^2)}\right) \\ & \exp\left[-\frac{\frac{\gamma_{\text{SD}}}{\overline{\gamma}_{\text{RD}}}+\frac{\gamma_{\text{RD}}}{\overline{\gamma}_{\text{RD}}}}{1-|\rho_s|^2}\right] d\gamma_{\text{SD}}d\gamma_{\text{RD}}, \\ & + \left(1-\exp\left(\frac{1}{\overline{\gamma}_{\text{SR}}}\right)\right)\int_{\gamma_{\text{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)}\int_{\gamma_{\text{S'D}}=\Theta^{-1}(0)}^{\Theta^{-1}(1-\Theta(\gamma_{\text{SD}}))} \\ & \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{S'D}}(1-\rho_t^2)} I_0\left(\frac{2|\rho_t|\gamma_{\text{SD}}\gamma_{\text{S'D}}}{\sqrt{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{S'D}}}(1-|\rho_t|^2)}\right) \\ & \exp\left[-\frac{\frac{\gamma_{\text{SD}}}{\overline{\gamma}_{\text{SD}}}+\frac{\gamma_{\text{S'D}}}{\overline{\gamma}_{\text{S'D}}}}{1-|\rho_t|^2}\right] d\gamma_{\text{SD}}d\gamma_{\text{S'D}}. \end{aligned} \tag{33}$$

Note that, the outage expressions (31), (32), and (33) in correlated fading reduce to those over independent links (13), (14), and (15) by setting $\rho_s = 0$ and $\rho_t = 0$. In Fig. 4, we can observe that with the high spatial fading correlation, the outage performance difference among the LF, DF, and ADF relaying schemes vanishes.

The impact of spatial correlation $\rho_s$ on the outage probabilities of the LF, DF and ADF systems are demonstrated in Fig. 5. As can be seen from the figure, over the entire $\rho_s$ value region, LF exhibits superior outage performance to DF. This is because with LF, the relay system can be seen as a distributed turbo code[3]. It is also found that ADF achieves lower outage probability than LF. This is because the inadmissible rate region with ADF is made smaller by the feedback information than LF, as shown in Fig. 2 and Fig. 3. It is observed that for all the three relaying schemes, the outage probabilities increase

[3]Note that the iterative processing for LF may cause transmit delay and/or excessive power computation than DF. However, the increase in transmit delay or energy consumption is negligible, especially when the SNR is high.

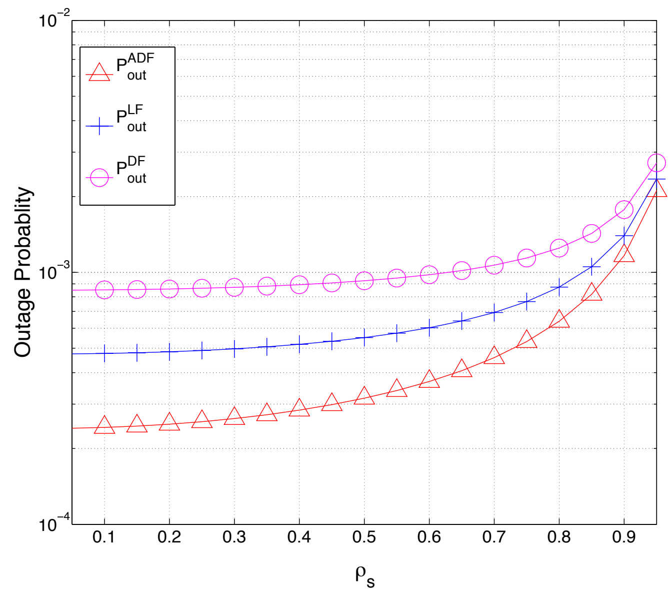



Fig. 5. Outage probability versus the spatial correlation $\rho_s$, where $\rho_t = 0$.

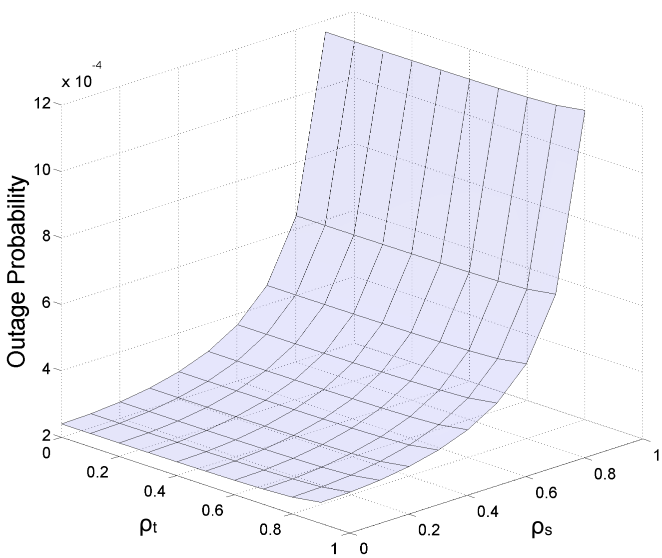



Fig. 6. Impacts of time correlation $\rho_t$ and spatial correlation $\rho_s$ on outage probability of ADF.

as the spatial correlation $\rho_s$ between the S-D and R-D links becomes larger.

Fig. 6 shows the outage performance of ADF relaying with the time correlation $\rho_t$ and spatial correlation $\rho_s$ as parameters. As can be seen from the figure, the effect of $\rho_t$ is not as significant as $\rho_s$. The larger the $\rho_t$ value, the bigger the outage probability of ADF but slightly.

### D. Approximations

The Bessel function of the first kind can be expressed with its series expansion by Frobenius method [38, 9.1.10], as:

$$I_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(m+1)} \left(\frac{x}{2}\right)^{2m}. \quad (34)$$

Then, (31), (32), and (33) can be, respectively, approximated as

$$\begin{aligned} P_{\text{out}}^{\text{LF, Cor}} \approx & \left(1 - \frac{1}{\overline{\gamma}_{\text{SR}}}\right)\left(\frac{\ln(4)-1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}(1-\rho_s^2)} + \frac{2\ln(2)-3}{2\overline{\gamma}_{\text{SD}}^2\overline{\gamma}_{\text{RD}}(1-\rho_s^2)} \right.\\ & \left. + \frac{\ln(4)-\frac{3}{2}}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}^2(1-\rho_s^2)}\right) + \frac{2\ln(4)-3}{2\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{SR}}^2} + \frac{2\ln(4)-3}{2\overline{\gamma}_{\text{SD}}^2\overline{\gamma}_{\text{SR}}} \\ & + \frac{\ln(4)-1}{2\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{SR}}} + \frac{4\ln(16)-11}{4\overline{\gamma}_{\text{SD}}^2\overline{\gamma}_{\text{SR}}^2}, \end{aligned} \quad (35)$$

$$\begin{aligned} P_{\text{out}}^{\text{DF, Cor}} \approx & \left(1 - \frac{1}{\overline{\gamma}_{\text{SR}}}\right)\left(\frac{\ln(4)-1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}(1-\rho_s^2)} + \frac{2\ln(2)-3}{2\overline{\gamma}_{\text{SD}}^2\overline{\gamma}_{\text{RD}}(1-\rho_s^2)} \right.\\ & \left. + \frac{\ln(4)-\frac{3}{2}}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}^2(1-\rho_s^2)}\right) + \frac{1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{SR}}}, \end{aligned} \quad (36)$$

and

$$\begin{aligned} P_{\text{out}}^{\text{ADF, Cor}} \approx & \left(1 - \frac{1}{\overline{\gamma}_{\text{SR}}}\right)\left(\frac{\ln(4)-1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}(1-\rho_s^2)} + \frac{2\ln(2)-3}{2\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{S'D}}\overline{\gamma}_{\text{RD}}(1-\rho_s^2)} \right.\\ & \left. + \frac{\ln(4)-\frac{3}{2}}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}^2(1-\rho_s^2)}\right) + \frac{\ln(4)-1}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}\overline{\gamma}_{\text{SR}}(1-\rho_t^2)} \\ & + \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{S'D}}\overline{\gamma}_{\text{RD}}\overline{\gamma}_{\text{SR}}(1-\rho_t^2)} + \frac{\ln(4)-\frac{3}{2}}{\overline{\gamma}_{\text{SD}}\overline{\gamma}_{\text{RD}}^2\overline{\gamma}_{\text{SR}}(1-\rho_t^2)}. \end{aligned} \quad (37)$$

The accuracy of the approximations is demonstrated in Fig. 7 in next section.

### E. Diversity Order

The diversity order $d$ is defined as [39]

$$d = -\lim_{\overline{\gamma}\to\infty} \frac{\log(P_{\text{out}})}{\log(\overline{\gamma})}. \quad (38)$$

According to (38), the diversity order of LF, DF, ADF over correlated channels can be expressed as

$$\begin{aligned} d_{\text{LF}} &= -\lim_{\overline{\gamma}\to\infty} \frac{\ln(P_{\text{out}}^{\text{LF, Cor}})}{\ln(\overline{\gamma})} \\ &= -\lim_{\overline{\gamma}\to\infty} \left\{ \ln\left[\frac{5\ln(4)-7}{2\overline{\gamma}^3} + \frac{4\ln(16)-11}{4\overline{\gamma}^4} + \left(1 - \frac{1}{\overline{\gamma}}\right)\right.\right. \\ &\quad \left.\left. \cdot \left(\frac{\ln(4)-1}{\overline{\gamma}^2(1-\rho_s^2)} + \frac{2\ln(2)-2\ln(4)-6}{2\overline{\gamma}^3(1-\rho_s^2)}\right)\right] / \ln(\overline{\gamma}) \right\} = 2, \\ &\quad \rho_s \neq 1, \end{aligned} \quad (39)$$

$$d_{\rm DF} = -\lim_{\overline{\gamma}\to\infty} \frac{\ln(P_{\rm out}^{\text{DF, Cor}})}{\ln(\overline{\gamma})}$$
$$= -\lim_{\overline{\gamma}\to\infty} \Bigg\{ \ln \Bigg[ \frac{2\ln(4)-3}{\overline{\gamma}^3} + \frac{\ln(4)-1}{2\overline{\gamma}^2} + \frac{4\ln(16)-11}{4\overline{\gamma}^4} + \left(1-\frac{1}{\overline{\gamma}}\right)\left(\frac{\ln(4)-1}{\overline{\gamma}^2(1-\rho_s^2)} + \frac{2\ln(2)+2\ln(4)-6}{2\overline{\gamma}^3(1-\rho_s^2)}\right)\Bigg] / \ln(\overline{\gamma})\Bigg\} = 2, \rho_s \neq 1, \tag{40}$$

and

$$d_{\rm ADF} = -\lim_{\overline{\gamma}\to\infty} \frac{\ln(P_{\rm out}^{\text{ADF, Cor}})}{\ln(\overline{\gamma})}$$
$$= -\lim_{\overline{\gamma}\to\infty} \Bigg\{ \ln \Bigg[ \frac{\ln(4)-1}{\overline{\gamma}^3(1-\rho_t^2)} + \frac{4\ln(2)+2\ln(4)-6}{2\overline{\gamma}^4(1-\rho_t^2)} + \left(1-\frac{1}{\overline{\gamma}}\right)\left(\frac{\ln(4)-1}{\overline{\gamma}^2(1-\rho_s^2)} + \frac{2\ln(2)-3}{2\overline{\gamma}^3(1-\rho_s^2)} + \frac{\ln(4)-\frac{3}{2}}{\overline{\gamma}^3(1-\rho_s^2)}\right)\Bigg] / \ln(\overline{\gamma})\Bigg\} = 2, \rho_s \neq 1, \rho_t \neq 1, \tag{41}$$

where $\overline{\gamma}_{\rm SD}$, $\overline{\gamma}_{\rm RD}$, and $\overline{\gamma}_{\rm SR}$ are represented by a generic symbol $\overline{\gamma}$ under the equilateral triangle nodes location assumption. It is shown that the full diversity gain can be achieved by LF, DF, and ADF relaying as long as $\rho_s \neq 1$ for LF and DF, and $\rho_s \neq 1$ and $\rho_t \neq 1$ for ADF. When $\rho_s = 1$ and $\rho_t = 1$, i.e., in fully correlated fading, the diversity order reduces to one.

## V. Optimal Relay Locations for Minimizing the Outage Probability

In this section, we investigate the optimal relay locations which minimize the outage probability of the LF, DF, and ADF relaying schemes. The outage expressions can be rewritten so that they are functions of position of relay node by taking the geometric gain into consideration. It is shown that, the relay location optimization can be formulated as a convex optimization problem.

### A. Optimal Relay Locations in Independent Fading

Let $d_{\rm SD}$, $d_{\rm RD}$, and $d_{\rm SR}$ denote the distances between S and D, R and D, and S and R respectively. With $G_{\rm SD}$ being normalized to unity, $G_{\rm SR}$ and $G_{\rm RD}$ can be defined as $G_{\rm SR} = \left(\frac{d_{\rm SD}}{d_{\rm SR}}\right)^\alpha$ and $G_{\rm RD} = \left(\frac{d_{\rm SD}}{d_{\rm RD}}\right)^\alpha$ [40], respectively, where $\alpha$ is the path loss exponent. Then, the average SNRs of the S-R and R-D links can be given as

$$\overline{\gamma}_{\rm SR} = \overline{\gamma}_{\rm SD} + 10\text{lg}(G_{\rm SR}) \text{ (dB)}, \tag{42}$$
$$\overline{\gamma}_{\rm RD} = \overline{\gamma}_{\rm SD} + 10\text{lg}(G_{\rm RD}) \text{ (dB)}. \tag{43}$$

With the normalized S-D link length $d_{\rm SD} = 1$, substituting (42) and (43) into (16), (17), and (18), yields the outage probability expressions of LF, DF, and ADF relaying over the links suffering from independent fading with respect to the position of R as

$$P_{\rm out}^{\text{LF, Ind}} = \frac{1}{\overline{\gamma}_{\rm SD}\left(\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{1-d})^\alpha\right)}\left(1-\frac{1}{\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{d})^\alpha}\right) \cdot \left(\ln(4)-1+\frac{4\ln(2)-3}{2\overline{\gamma}_{\rm SD}}\right) + \frac{1}{\overline{\gamma}_{\rm SD}\left(\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{d})^\alpha\right)} \cdot \left(\ln(4)-1+\frac{4\ln(2)-3}{2\left(\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{d})^\alpha\right)}\right), \tag{44}$$

$$P_{\rm out}^{\text{DF, Ind}} = \frac{1}{\overline{\gamma}_{\rm SD}\left(\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{1-d})^\alpha\right)}\left(1-\frac{1}{\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{d})^\alpha}\right) \cdot \left(\ln(4)-1+\frac{4\ln(2)-3}{2\overline{\gamma}_{\rm SD}}\right) + \frac{1}{\overline{\gamma}_{\rm SD}\left(\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{d})^\alpha\right)}, \tag{45}$$

and

$$P_{\rm out}^{\text{ADF, Ind}} = \frac{1}{\overline{\gamma}_{\rm SD}\left(\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{1-d})^\alpha\right)}\left(1-\frac{1}{\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{d})^\alpha}\right) \cdot \left(\ln(4)-1+\frac{4\ln(2)-3}{2\overline{\gamma}_{\rm SD}}\right) + \frac{1}{\overline{\gamma}_{\rm SD}\left(\overline{\gamma}_{\rm SD}^2+10\,\text{lg}(\frac{1}{d})^\alpha\right)} \cdot \left(\ln(4)-1+\frac{4\ln(2)-3}{2\overline{\gamma}_{\rm SD}^2}\right), \tag{46}$$

under the assumption that R moves along the line between S and D, with which $d_{\rm SR} = d$ and $d_{\rm RD} = 1-d$.

The general optimization problem with regard to $d$ can be formulated as

$$d^* = \arg\min_d \quad P_{\rm out}(d) \qquad \text{subject to:} \quad d \in (0,1). \tag{47}$$

*Proposition 1:* Outage probability expressions of LF, DF, and ADF relaying in (44), (45), and (46), respectively, are convex with respect to $d \in$ (0, 1).

*Proof:* See Appendix A. ■

Taking the first-order derivative of $P_{\rm out}^{\text{LF, Ind}}$, $P_{\rm out}^{\text{DF, Ind}}$, and $P_{\rm out}^{\text{ADF, Ind}}$, respectively, in (44), (45), and (46) with respect to $d$ and setting the derivative equal to zero, we have

$$\frac{\partial P_{\rm out}^{\text{LF, Ind}}}{\partial d} = \frac{\partial \frac{1}{\overline{\gamma}_{\rm SD}\left(\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{1-d})^\alpha\right)}\left(1-\frac{1}{\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{d})^\alpha}\right)}{\partial d} \left(\ln(4)-1+\frac{4\ln(2)-3}{2\overline{\gamma}_{\rm SD}}\right) + \frac{\partial \frac{1}{\overline{\gamma}_{\rm SD}\left(\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{d})^\alpha\right)}\left(\ln(4)-1+\frac{4\ln(2)-3}{2\left(\overline{\gamma}_{\rm SD}+10\,\text{lg}(\frac{1}{d})^\alpha\right)}\right)}{\partial d} = 0, \tag{48}$$

TABLE II. Optimal Relay Location $d^*$

| $\overline{\gamma}_{\mathrm{SD}}$(dB) | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|
| $P_{\mathrm{out}}^{\mathrm{DF}}$, $\rho_s = 0$ | 0.3321 | 0.3322 | 0.3323 | 0.3324 | 0.3328 |
| $P_{\mathrm{out}}^{\mathrm{LF}}$, $\rho_s = 0$ | 0.5017 | 0.5018 | 0.5019 | 0.5020 | 0.5020 |
| $P_{\mathrm{out}}^{\mathrm{ADF}}$, $\rho_s = 0$ | 0.9916 | 0.9937 | 0.9944 | 0.9957 | 0.9978 |
| $P_{\mathrm{out}}^{\mathrm{DF}}$, $\rho_s = 0.99$ | 0.6101 | 0.6147 | 0.6253 | 0.6177 | 0.6182 |
| $P_{\mathrm{out}}^{\mathrm{LF}}$, $\rho_s = 0.99$ | 0.7819 | 0.7822 | 0.7829 | 0.7839 | 0.7841 |
| $P_{\mathrm{out}}^{\mathrm{ADF}}$, $\rho_s = 0.99$ | 0.9991 | 0.9993 | 0.9995 | 0.9998 | 0.9999 |

$$\frac{\partial P_{\mathrm{out}}^{\mathrm{DF,\,Ind}}}{\partial d} = \frac{\partial \frac{1}{\overline{\gamma}_{\mathrm{SD}}\left(\overline{\gamma}_{\mathrm{SD}}+10\lg(\frac{1}{1-d})^{\alpha}\right)}\left(1-\frac{1}{\overline{\gamma}_{\mathrm{SD}}+10\lg(\frac{1}{d})^{\alpha}}\right)}{\partial d} \left(\ln(4)-1+\frac{4\ln(2)-3}{2\overline{\gamma}_{\mathrm{SD}}}\right) + \frac{\partial \frac{1}{\overline{\gamma}_{\mathrm{SD}}\left(\overline{\gamma}_{\mathrm{SD}}+10\lg(\frac{1}{d})^{\alpha}\right)}}{\partial d} = 0, \quad (49)$$

and

$$\frac{\partial P_{\mathrm{out}}^{\mathrm{ADF,\,Ind}}}{\partial d} = \frac{\partial \frac{1}{\overline{\gamma}_{\mathrm{SD}}\left(\overline{\gamma}_{\mathrm{SD}}+10\lg(\frac{1}{1-d})^{\alpha}\right)}\left(1-\frac{1}{\overline{\gamma}_{\mathrm{SD}}+10\lg(\frac{1}{d})^{\alpha}}\right)}{\partial d} \left(\ln(4)-1+\frac{4\ln(2)-3}{2\overline{\gamma}_{\mathrm{SD}}}\right) + \frac{\partial \frac{1}{\overline{\gamma}_{\mathrm{SD}}\left(\overline{\gamma}_{\mathrm{SD}}^2+10\lg(\frac{1}{d})^{\alpha}\right)}\left(\ln(4)-1+\frac{4\ln(2)-3}{2\overline{\gamma}_{\mathrm{SD}}^2}\right)}{\partial d} = 0, \quad (50)$$

It may be excessively complex to derive the explicit expression for $d^*$ from (48), (49), and (50). Hence, the Newton-Raphson method [41, Chapter 9] is used to numerically calculate the solution of $d^*$.

## B. Optimal Relay Locations in Correlated Fading

Similarly to the independent fading case, the outage probability expressions of LF, DF and ADF relaying in correlated fading with respect to the position of R can be expressed as (51), (52), and (53).

The optimization problem for the correlated fading channels' case can also be formulated by (47) for the LF, DF, and ADF relaying techniques.

*Proposition 2:* Outage probability expressions of LF, DF, and ADF relaying, respectively, given by (51), (52), and (53) respectively, are convex with respect to $d \in (0, 1)$.

*Proof:* The convexity of (51), (52), and (53) can be proven by taking second-order partial derivative of $P_{\mathrm{out}}^{\mathrm{LF,\,Ind}}$, $P_{\mathrm{out}}^{\mathrm{DF,\,Ind}}$, and $P_{\mathrm{out}}^{\mathrm{ADF,\,Ind}}$ in (51), (52), and (53) with respect to $d$, and showing that the derivative results are positive in the range $d \in (0, 1)$[4]. ■

Take the first-order derivative of $P_{\mathrm{out}}^{\mathrm{LF,\,Cor}}$, $P_{\mathrm{out}}^{\mathrm{DF,\,Cor}}$, and $P_{\mathrm{out}}^{\mathrm{ADF,\,Cor}}$, respectively, in (51), (52), and (53) with respect to $d$ and set the derivative result equal to zero, $\frac{\partial P_{\mathrm{out}}^{\mathrm{LF,\,Cor}}}{\partial d} = 0$, $\frac{\partial P_{\mathrm{out}}^{\mathrm{DF,\,Cor}}}{\partial d} = 0$, and $\frac{\partial P_{\mathrm{out}}^{\mathrm{ADF,\,Cor}}}{\partial d} = 0$. As in the independent fading case, the optimal relay location $d^*$ for each relaying schemes over correlated fading channels can also be obtained by utilizing the Newton-Raphson method [41, Chapter 9].

[4]The details of the proof are straightforward but lengthy, and therefore are omitted here for brevity.

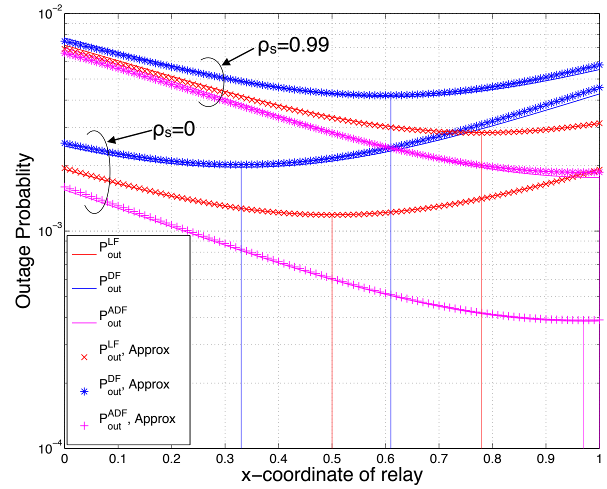



Fig. 7. Outage probabilities versus relay locations for different relaying schemes over independent and correlated channels, where $\overline{\gamma}_{\mathrm{SD}} = 15$ (dB) and $\rho_t = 0$.

The optimal relay location $d^*$ for different relaying schemes over spatially independent ($\rho_s = 0$) as well as spatially correlated ($\rho_s = 0.99$) channels are shown in Table II[5]. Obviously, with the correlated S-D and R-D links, the optimal relay positions for achieving the smallest outage probability shift closer to D than the independent fading case. However, for specific relay scheme the optimal relay positions stay the same (e.g., for LF in spatially independent fading ($\rho_s = 0$) the optimal relay position is at the midpoint, $d^* = 0.5$) regardless of the average SNR values. The deviations of $d^*$ are due to the numerical calculation errors

Fig. 7 depicts the impact of the relay location on outage performances of LF, DF and ADF relaying in spatially independent ($\rho_s = 0$) and correlated ($\rho_s = 0.99$) cases. The relay is assumed to move along the line between S ($x = 0$) and D ($x = 1$). Both the numerically calculated outage probabilities and approximated outage probabilities are plotted. It is found from the figure that the approximated and numerically calculated outage probability curves are consistent with each other. This observation indicates the accuracy of the approximations. It is found from Fig. 7 that in statistically independent fading the optimal relay locations for LF, DF, and ADF that achieve the lowest probabilities are $d = 0.5$, $d < 0.5$ and, $d > 0.5$, respectively. On the contrary, in correlated fading, the optimal relay location for all the relaying schemes are closer to D. This is because the fading correlation reduces the coding gain achieved by the transmission over the S-D and R-D links.

[5]We only focus on the spatial correlation $\rho_s$ which have impact on outage of DF, LF, and ADF relaying.

$$P_{\text{out}}^{\text{LF, Cor}} = \left(1 - \frac{1}{\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)}\right)\left(\frac{\ln(4)-1}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)(1-\rho_s^2)} + \frac{2\ln(2)-3}{2\overline{\gamma}_{\text{SD}}^2\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)(1-\rho_s^2)}\right.$$
$$\left. + \frac{\ln(4)-\frac{3}{2}}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)^2(1-\rho_s^2)}\right)$$
$$+ \frac{2\ln(4)-3}{2\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)^2} + \frac{2\ln(4)-3}{2\overline{\gamma}_{\text{SD}}^2\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)} + \frac{\ln(4)-1}{2\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)} + \frac{4\ln(16)-11}{4\overline{\gamma}_{\text{SD}}^2\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)^2}, \tag{51}$$

$$P_{\text{out}}^{\text{DF, Cor}} = \left(1 - \frac{1}{\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)}\right)\left(\frac{\ln(4)-1}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)(1-\rho_s^2)} + \frac{2\ln(2)-3}{2\overline{\gamma}_{\text{SD}}^2\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)(1-\rho_s^2)}\right.$$
$$\left. + \frac{\ln(4)-\frac{3}{2}}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)^2(1-\rho_s^2)}\right) + \frac{1}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)}, \tag{52}$$

$$P_{\text{out}}^{\text{ADF, Cor}} = \left(1 - \frac{1}{\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)}\right)\left(\frac{\ln(4)-1}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)(1-\rho_s^2)} + \frac{2\ln(2)-3}{2\overline{\gamma}_{\text{SD}}^2\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)(1-\rho_s^2)}\right.$$
$$\left. + \frac{\ln(4)-\frac{3}{2}}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)^2(1-\rho_s^2)}\right) + \frac{\ln(4)-1}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)(1-\rho_t^2)}$$
$$+ \frac{4\ln(2)-3}{2\overline{\gamma}_{\text{SD}}^2\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)(1-\rho_t^2)} + \frac{\ln(4)-\frac{3}{2}}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{1-d})^{\alpha}\right)^2\left(\overline{\gamma}_{\text{SD}} + 10\lg(\frac{1}{d})^{\alpha}\right)(1-\rho_t^2)}. \tag{53}$$

The impact of the time correlation $\rho_t$ of the complex fading gains between the two transmission slots over the S-D link as well as the spatial correlation $\rho_s$ of the complex fading gains between the S-D and R-D links on optimal relay location $d^*$ with LF, DF, ADF relaying is depicted in Fig. 8. With LF and DF, the value of $d^*$ increases as $\rho_s$ increases, which indicate that the optimal relay locations shift closer to D as the spatial correlation increases. Obviously, $\rho_t$ has no impact on $d^*$ with LF and DF relaying. With ADF system, interestingly, it is found that $d^*$ does not change regardless of the $\rho_s$ value, when $\rho_t$ is small. As the time correlation $\rho_t$ becomes large, $d^*$ with ADF exhibits the same tendency as that with LF and DF. In general, the optimal $d^*$ with LF is larger than that with DF and smaller than that with ADF.

## VI. Optimal Power Allocation for Minimizing the Outage Probability

In this section, our aim is to minimize the outage probabilities for the LF, DF, and ADF relaying techniques by adjusting powers allocated to S and R. Let the powers allocated to S and R be denoted as $E_T k$ and $E_T(1-k)$, respectively, where $E_T$ represents the total transmit power and $k$ ($0 \leq k \leq 1$) is the power allocation ratio. With the noise variance of each link being normalized to the unity, the geometric gain times transmit power is equivalent to their corresponding average SNR. Then the average SNRs of each link can be expressed as functions of $k$ as $\overline{\gamma}_{\text{SD}} = E_T k G_{\text{SD}}$, $\overline{\gamma}_{\text{RD}} = E_T(1-k)G_{\text{RD}}$, and $\overline{\gamma}_{\text{SR}} = E_T k G_{\text{SR}}$.

### A. Optimal Power Allocation in Independent Fading

The outage expressions of LF, DF, and ADF in independent fading with respect to $k$ can be written as

$$P_{\text{out}}^{\text{LF, Ind}} = \frac{1}{E_T^2 k(1-k)G_{\text{SD}}G_{\text{RD}}}\left(1 - \frac{1}{E_T k G_{\text{SR}}}\right) \cdot \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SD}}}\right) + \frac{1}{E_T^2 k^2 G_{\text{SD}}G_{\text{SR}}}\left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SR}}}\right), \tag{54}$$

$$P_{\text{out}}^{\text{DF, Ind}} = \frac{1}{E_T^2 k(1-k)G_{\text{SD}}G_{\text{RD}}}\left(1 - \frac{1}{E_T k G_{\text{SR}}}\right) \cdot \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SD}}}\right) + \frac{1}{E_T^2 k^2 G_{\text{SD}}G_{\text{SR}}}, \tag{55}$$

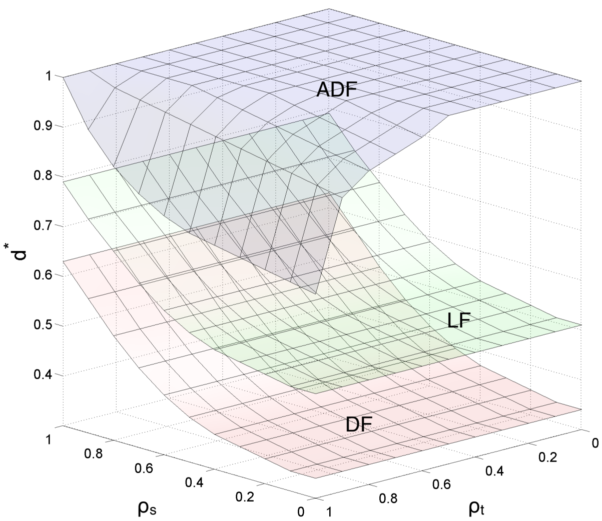



Fig. 8. Impact of time correlation $\rho_t$ and spatial correlation $\rho_s$ on optimal relay location $d^*$, where $\overline{\gamma}_{\text{SD}} = 15$ (dB).

and

$$P_{\text{out}}^{\text{ADF, Ind}} = \frac{1}{E_T^2 k(1-k) G_{\text{SD}} G_{\text{RD}}} \left(1 - \frac{1}{E_T k G_{\text{SR}}}\right) \cdot \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SD}}}\right) + \frac{1}{E_T^3 k^3 G_{\text{SR}} G_{\text{SD}}} \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SD}}}\right), \tag{56}$$

respectively.

The optimization problem with regards to $k$ can be formulated as

$$\begin{aligned} k^* = \arg\min_k \quad & P_{\text{out}}(k) \\ \text{subject to:} \quad & k \in (0,1). \end{aligned} \tag{57}$$

*Proposition 3:* Outage probability expressions of LF, DF, and ADF relaying, respectively, in (54), (55), and (56) are convex with respect to $k \in (0,\,1)$.

*Proof:* See Appendix B. ■

Taking the first-order derivative of $P_{\text{out}}^{\text{LF, Ind}}$, $P_{\text{out}}^{\text{DF, Ind}}$, and $P_{\text{out}}^{\text{ADF, Ind}}$, respectively in (54), (55), and (56) with respect to $k$ and setting the derivative result equal to zero, we have,

$$\frac{\partial P_{\text{out}}^{\text{LF, Ind}}}{\partial d} = \frac{\partial \frac{1}{E_T^2 k(1-k) G_{\text{SD}} G_{\text{RD}}} \left(1 - \frac{1}{E_T k G_{\text{SR}}}\right) \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SD}}}\right)}{\partial k} + \frac{\partial \frac{1}{E_T^2 k^2 G_{\text{SD}} G_{\text{SR}}} \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SR}}}\right)}{\partial k} = 0, \tag{58}$$

TABLE III. Optimal power allocation ratio $k^*$

| $E_T$ (dB) | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|
| $P_{\text{out}}^{\text{DF}}$, $\rho_s = 0$ | 0.7532 | 0.7444 | 0.7417 | 0.7408 | 0.7401 |
| $P_{\text{out}}^{\text{LF}}$, $\rho_s = 0$ | 0.6989 | 0.6719 | 0.6698 | 0.6688 | 0.6685 |
| $P_{\text{out}}^{\text{ADF}}$, $\rho_s = 0$ | 0.5061 | 0.5053 | 0.5046 | 0.5039 | 0.5022 |
| $P_{\text{out}}^{\text{DF}}$, $\rho_s = 0.99$ | 0.6861 | 0.6309 | 0.5302 | 0.5298 | 0.5258 |
| $P_{\text{out}}^{\text{LF}}$, $\rho_s = 0.99$ | 0.6266 | 0.5530 | 0.5193 | 0.5131 | 0.5119 |
| $P_{\text{out}}^{\text{ADF}}$, $\rho_s = 0.99$ | 0.5019 | 0.5017 | 0.5015 | 0.5014 | 0.5013 |

$$\frac{\partial P_{\text{out}}^{\text{DF, Ind}}}{\partial d} = \frac{\partial \frac{1}{E_T^2 k(1-k) G_{\text{SD}} G_{\text{RD}}} \left(1 - \frac{1}{E_T k G_{\text{SR}}}\right) \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SD}}}\right)}{\partial k} + \frac{\partial \frac{1}{E_T^2 k^2 G_{\text{SD}} G_{\text{SR}}}}{\partial k} = 0, \tag{59}$$

and

$$\frac{\partial P_{\text{out}}^{\text{ADF, Ind}}}{\partial d} = \frac{\partial \frac{1}{E_T^2 k(1-k) G_{\text{SD}} G_{\text{RD}}} \left(1 - \frac{1}{E_T k G_{\text{SR}}}\right) \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SD}}}\right)}{\partial k} + \frac{\partial \frac{1}{E_T^3 k^3 G_{\text{SR}} G_{\text{SD}}} \left(\ln(4) - 1 + \frac{4\ln(2)-3}{2E_T k G_{\text{SD}}}\right)}{\partial k} = 0. \tag{60}$$

The optimal power allocation ratio $k^*$ can be obtained by numerically solving (58), (59), and (60).

## B. Optimal Power Allocation in Correlated Fading

The outage expressions of LF, DF, and ADF in correlated fading can be written as (61), (62), and (63), respectively. The optimization problem in correlated fading can also be formulated by (57) for LF, DF, and ADF relaying.

*Proposition 4:* Outage probability expressions of LF, DF, and ADF relaying, respectively, in (61), (62), and (63), are convex with respect to $k \in (0,\,1)$.

*Proof:* The convexity of (61), (62), and (63) can be proven by taking second-order partial derivative of $P_{\text{out}}^{\text{LF, Ind}}$, $P_{\text{out}}^{\text{DF, Ind}}$, and $P_{\text{out}}^{\text{ADF, Ind}}$ in (61), (62), and (63) with respect to $k$, and showing that the derivative results are positive in the range $k \in (0, 1)$[6]. ■

By taking the first-order derivative of $P_{\text{out}}^{\text{LF, Cor}}$, $P_{\text{out}}^{\text{DF, Cor}}$, and $P_{\text{out}}^{\text{ADF, Cor}}$, respectively, in (61), (62), and (63) with respect to $k$ and setting the derivative result equal to zero, $\frac{\partial P_{\text{out}}^{\text{LF, Cor}}}{\partial k} = 0$, $\frac{\partial P_{\text{out}}^{\text{DF, Cor}}}{\partial k} = 0$, and $\frac{\partial P_{\text{out}}^{\text{ADF, Cor}}}{\partial k} = 0$, and similarly to the independent fading case, the optimal power allocation ratio $k^*$ of each relaying schemes over correlated fading channels can be numerically obtained.

Optimal $k^*$ for the different relaying schemes are shown in Table III [7]. Obviously, the larger the total transmit power

[6] The details of the proof are straightforward but lengthy, and therefore are omitted here for brevity.

[7] We only focus on the spatial correlation $\rho_s$ which have impact on outage of DF, LF, and ADF relaying.

$$P_{\text{out}}^{\text{LF, Cor}} = \left(1 - \frac{1}{E_T k G_{\text{SD}}}\right)\left(\frac{\ln(4)-1}{E_T k G_{\text{SD}} E_T(1-k) G_{\text{RD}}(1-\rho_s^2)} + \frac{2\ln(2)-3}{2E_T k G_{\text{SD}}^2 E_T(1-k) G_{\text{RD}}(1-\rho_s^2)} + \frac{\ln(4)-\frac{3}{2}}{E_T k G_{\text{SD}} E_T(1-k) G_{\text{RD}}^2(1-\rho_s^2)}\right) + \frac{2\ln(4)-3}{2E_T k G_{\text{SD}} E_T^2 k^2 G_{\text{SR}}^2} + \frac{2\ln(4)-3}{2E_T^2 k^2 G_{\text{SD}}^2 E_T k G_{\text{SD}}} + \frac{\ln(4)-1}{2E_T k G_{\text{SD}} E_T k G_{\text{SD}}} + \frac{4\ln(16)-11}{4E_T^2 k^2 G_{\text{SD}}^2 E_T^2 k^2 G_{\text{SR}}^2}, \tag{61}$$

$$P_{\text{out}}^{\text{DF, Cor}} = \left(1 - \frac{1}{E_T k G_{\text{SD}}}\right)\left(\frac{\ln(4)-1}{E_T k G_{\text{SD}} E_T(1-k) G_{\text{RD}}(1-\rho_s^2)} + \frac{2\ln(2)-3}{2E_T k G_{\text{SD}}^2 E_T(1-k) G_{\text{RD}}(1-\rho_s^2)} + \frac{\ln(4)-\frac{3}{2}}{E_T k G_{\text{SD}} E_T(1-k) G_{\text{RD}}^2(1-\rho_s^2)}\right) + \frac{1}{E_T k G_{\text{SD}} E_T k G_{\text{SD}}}, \tag{62}$$

$$P_{\text{out}}^{\text{ADF, Cor}} = \left(1 - \frac{1}{E_T k G_{\text{SD}}}\right)\left(\frac{\ln(4)-1}{E_T k G_{\text{SD}} E_T(1-k) G_{\text{RD}}(1-\rho_s^2)} + \frac{2\ln(2)-3}{2E_T k G_{\text{SD}}^2 E_T(1-k) G_{\text{RD}}(1-\rho_s^2)} + \frac{\ln(4)-\frac{3}{2}}{E_T k G_{\text{SD}} E_T(1-k) G_{\text{RD}}^2(1-\rho_s^2)}\right) + \frac{\ln(4)-1}{E_T k G_{\text{SD}} E_T(1-k) G_{\text{RD}} E_T k G_{\text{SD}}(1-\rho_t^2)} + \frac{4\ln(2)-3}{2E_T^2 k^2 G_{\text{SD}}^2 E_T(1-k) G_{\text{RD}} E_T k G_{\text{SD}}(1-\rho_t^2)} + \frac{\ln(4)-\frac{3}{2}}{E_T k G_{\text{SD}} E_T^2 (1-k)^2 G_{\text{RD}}^2 E_T k G_{\text{SD}}(1-\rho_t^2)}. \tag{63}$$

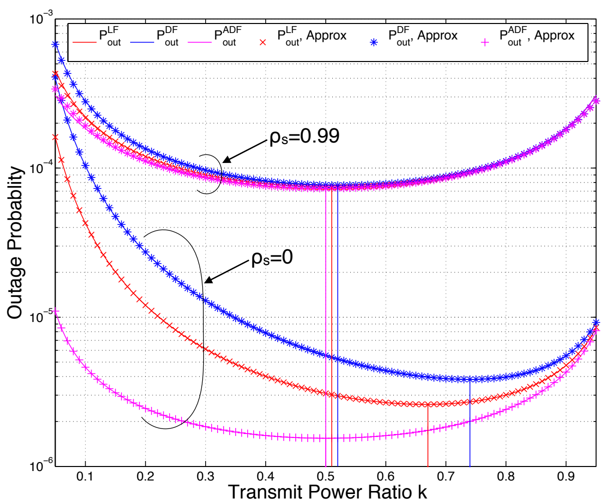



Fig. 9. Outage probabilities versus power allocations for different relying schemes over independent and correlated channels, where $E_T = 20$ (dB) and $\rho_t = 0$.

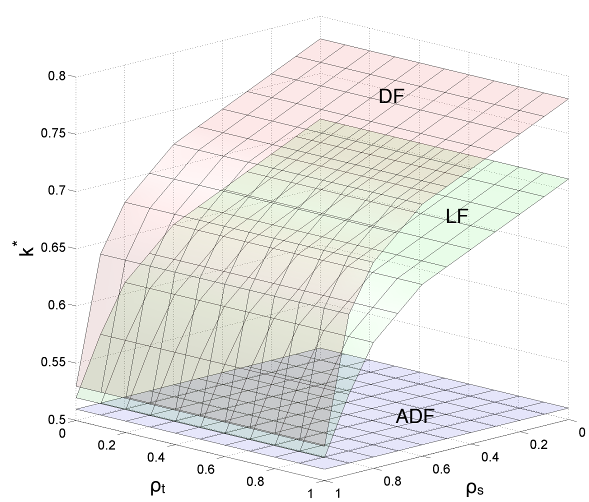



Fig. 10. Impact of time correlation $\rho_t$ and spatial correlation $\rho_s$ on optimal power allocation ratio $k^*$, where $E_T = 20$ (dB).

$E_T$, the more transmit power should be allocated to R both in the case of fading variation on the S-D and R-D links are independent and spatially correlated. It is also found that for achieving minimum outage, R needs more power with LF than with DF, and needs less power than with ADF. Moreover, with large spatial channel correlation $\rho_s$, more power should be allocated to R for all the relaying techniques.

Fig. 9 presents the impact of the power allocation ratios to S and R on outage probabilities. In numerical results, we normalize the geometric gain of each link to unity under the equilateral triangle nodes location assumption, and investigate the impact of power allocation ratio $k$ on outage performance. The total transmit power is fixed at 20 dB. Note that the approximated outage curves well match with the numerically calculated curves. It can be observed from Fig. 9 that, with high spatial correlation $\rho_s$, nearly equal power allocation ($k \approx 0.5$) yields the lowest outage probabilities for all LF, DF, and ADF relaying. On the contrary, in statistically independent fading, the equal power allocation is optimal for ADF, while DF and LF need more power for S, which is consistent with the results shown in Table III.

Fig. 10 shows the impact of $\rho_t$ and $\rho_s$ on optimal power allocation ratio $k^*$ for LF, DF, ADF relaying, where the total transmit power for S and R is set at $E_T = 20$ (dB). It can be seen from the figure that the larger the spatial correlation $\rho_s$, the smaller the optimal $k^*$ value required with DF and LF, indicating that more power needs to be allocated to R. It is also found out that, with DF, more power needs to be allocated to source node than with LF, in order to achieve the smallest outage probability. This is because DF forwards only the error-free information sequences from relay node. Note that the equal power allocation strategy is always optimal for ADF regardless of the values of $\rho_t$ and $\rho_s$.

## VII. Conclusions

We have analyzed the outage probabilities with LF, DF, and ADF relaying in statistically independent and correlated fading. The diversity and coding gains have been derived from high-SNR, yet accurate approximations in independent fading. The diversity orders for LF, DF, and ADF relaying have also been obtained in correlated fading. Our analysis demonstrates that the full diversity gain can be achieved by either of LF, DF, and ADF relaying, as long as the fading variations are not fully correlated. It is proven that coding gain with the LF strategy is larger than with DF relaying but smaller than with the ADF method, in independent fading. Furthermore, the optimal relay locations and optimal power allocations between the source and relay have been investigated. The observations suggest that compared to the independent fading case, the relay should be located closer to the destination, or more transmit power be allocated to the relay for achieving the lowest outage probabilities, in the correlated fading case. An important extension is to take into consideration of different channel models, different topologies, and dynamic network change, which is left as future work as summarized in Table IV.

TABLE IV. Future Work and Challenges

| Future Work | Challenges |
|---|---|
| Complex Structure (Multi-relay) | Multiple helper problem is one of the open questions in Network Information Theory. |
| Channel Models (No Rayleigh) | In many cases, joint pdf for more than one channel realization is still unknown. |

## Appendix A
## Proof of Convexity of (44), (45), and (46)

By taking second-order partial derivative of $P_{\text{out}}^{\text{LF, Ind}}$, $P_{\text{out}}^{\text{DF, Ind}}$, and $P_{\text{out}}^{\text{ADF, Ind}}$ in (44), (45), and (46) with respect to $d$, respectively, we have

$$\frac{\partial^2 P_{\text{out}}^{\text{LF, Ind}}}{\partial d^2} = \frac{9\alpha d^{\alpha}(\alpha+1)}{4\overline{\gamma}_{\text{SD}} d^{(\alpha+2)}\left(\overline{\gamma}_{\text{SD}} + 10\lg\left(\frac{1}{d^{\alpha}}\right)\right)^3} + \frac{90\alpha^2 d^{2\alpha}}{\overline{\gamma}_{\text{SD}} d^{(2\alpha+2)}\left(\overline{\gamma}_{\text{SD}} + 10\lg\left(\frac{1}{d^{\alpha}}\right)\right)^4}, \tag{64}$$

$$\frac{\partial^2 P_{\text{out}}^{\text{DF, Ind}}}{\partial d^2} = \frac{20\alpha^2 d^{2\alpha}}{\overline{\gamma}_{\text{SD}} d^{(2\alpha+2)}\left(\overline{\gamma}_{\text{SD}} + 10\lg\left(\frac{1}{d^{\alpha}}\right)\right)^3} + \frac{10\alpha^2 d^{(\alpha-1)}}{\overline{\gamma}_{\text{SD}} d^{(\alpha+1)}\left(\overline{\gamma}_{\text{SD}} + 10\lg\left(\frac{1}{d^{\alpha}}\right)\right)^2}, \tag{65}$$

and

$$\frac{\partial^2 P_{\text{out}}^{\text{ADF, Ind}}}{\partial d^2} = \frac{20\alpha^2\left(\frac{1}{\overline{\gamma}_{\text{SD}} + 10\lg\left(\frac{1}{d^{\alpha}}\right)} - 1\right)(1-d)^{2\alpha}}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg\left(\frac{1}{d^{\alpha}}\right)\right)^3 (1-d)^{(2\alpha+2)}} + \frac{\alpha^2 d^{(\alpha-1)}}{\overline{\gamma}_{\text{SD}}\left(\overline{\gamma}_{\text{SD}} + 10\lg\left(\frac{1}{d^{\alpha}}\right)\right)^2 d^{(\alpha+1)}}. \tag{66}$$

Obviously, (64), (65), and (66) are positive in the range $0 \le d \le 1$ which indicates that the objective functions are convex with respect to $0 \le d \le 1$.

## Appendix B
## Proof of Convexity of (54), (55), and (56)

Taking the second-order derivative of $P_{\text{out}}^{\text{LF, Cor}}$, $P_{\text{out}}^{\text{DF, Cor}}$, and $P_{\text{out}}^{\text{ADF, Cor}}$ in (54), (55), and (56) with respect to $k$, respectively, we have

$$\frac{\partial^2 P_{\text{out}}^{\text{LF, Cor}}}{\partial k^2} = \frac{\left(\frac{1}{E^T G_{\text{SR}} k} + 1\right)}{5E_T^3 G_{\text{SD}}^2 G_{\text{RD}} k^3 (1-k)^2} + \frac{6\left(\frac{1}{10E^T G_{\text{SR}} k} + \frac{19}{50}\right)}{E_T^2 G_{\text{SD}} G_{\text{RD}} k^4} + \frac{17}{25E_T^3 G_{\text{SD}} G_{\text{SR}}^2 k^5} + \frac{9\left(\frac{1}{E^T G_{\text{SR}} k} + 1\right)}{20E_T^3 G_{\text{SD}}^2 G_{\text{RD}} G_{\text{SR}} k^4 (1-k)}, \tag{67}$$

$$\frac{\partial^2 P_{\text{out}}^{\text{DF, Cor}}}{\partial k^2} = \frac{9\left(\frac{1}{E^T G_{\text{SR}} k} + 1\right)}{2E_T^3 G_{\text{SD}}^2 G_{\text{RD}} k^4 (1-k)} + \frac{5\left(\frac{1}{E^T G_{\text{SR}} k} + 1\right)}{E_T^3 G_{\text{SD}}^2 G_{\text{RD}} k^3 (1-k)^2} + \frac{6}{E_T^2 G_{\text{SD}} G_{\text{SR}} k^4} + \frac{5}{2E_T^4 G_{\text{SD}}^2 G_{\text{RD}} G_{\text{SR}} k^5 (1-k)}, \tag{68}$$

and

$$\frac{\partial^2 P_{\text{out}}^{\text{ADF, Cor}}}{\partial k^2} = \frac{\left(\frac{1}{E^T G_{\text{SR}} k} + 1\right)}{5E_T^3 G_{\text{SD}}^2 G_{\text{RD}} k^3 (1-k)^2} + \frac{12\left(\frac{1}{10E^T G_{\text{SR}} k} + \frac{19}{50}\right)}{E_T^3 G_{\text{SD}} G_{\text{RD}} k^5} \quad \frac{9}{10E_T^4 G_{\text{SD}}^2 G_{\text{SR}} k^6} + \frac{9\left(\frac{1}{E^T G_{\text{SR}} k} + 1\right)}{20E_T^3 G_{\text{SD}}^2 G_{\text{RD}} G_{\text{SR}} k^4 (1-k)}. \tag{69}$$

It is not difficult to find that (68), (67), and (69) are positive in the range $0 \le k \le 1$ which indicates that the objective functions are convex with respect to $0 \le k \le 1$.

## References


[1] K. Anwar and T. Matsumoto, "Accumulator-assisted distributed turbo codes for relay systems exploiting source-relay correlation," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1114–1117, July 2012.

[2] X. Zhou, M. Cheng, X. He, and T. Matsumoto, “Exact and approximated outage probability analyses for decode-and-forward relaying system allowing intra-link errors,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 7062–7071, Dec 2014.

[3] T. Cover and A. E. Gamal, “Capacity theorems for the relay channel,” *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572–584, Sep 1979.

[4] J. Laneman, D. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec 2004.

[5] J. Garcia-Frias and Y. Zhao, “Near-Shannon/Slepian-Wolf performance for unknown correlated sources over AWGN channels,” *IEEE Trans. Commun.*, vol. 53, no. 4, pp. 555–559, April 2005.

[6] A. Aaron and B. Girod, “Compression with side information using turbo codes,” in *Proc. Data Compr. Conf.*, 2002, pp. 252–261.

[7] J. Garcia-Frias, Y. Zhao, and W. Zhong, “Turbo-like codes for transmission of correlated sources over noisy channels,” *IEEE Signal Processing Mag.*, vol. 24, no. 5, pp. 58–66, Sept 2007.

[8] D. Varodayan, Y. C. Lin, and B. Girod, “Adaptive distributed source coding,” *IEEE Trans. Image Processing*, vol. 21, no. 5, pp. 2630–2640, May 2012.

[9] I. Shahid and P. Yahampath, “Distributed joint source-channel coding using unequal error protection LDPC codes,” *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3472–3482, August 2013.

[10] A. J. Aljohani, S. X. Ng, and L. Hanzo, “Distributed source coding and its applications in relaying-based transmission,” *IEEE Access*, vol. 4, pp. 1940–1970, 2016.

[11] R. Hu and J. Li, “Exploiting Slepian-Wolf codes in wireless user cooperation,” in *Proc. IEEE Works. on Sign. Proc. Adv. in Wirel. Comms.*, June 2005, pp. 275–279.

[12] B. Zhao and M. C. Valenti, “Distributed turbo coded diversity for relay channel,” *IEE Electron. Lett.*, vol. 39, no. 10, pp. 786–787, May 2003.

[13] H. H. Sneessens, J. Louveaux, and L. Vandendorpe, “Turbo-coded decode-and-forward strategy resilient to relay errors,” in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, March 2008, pp. 3213–3216.

[14] M. Cheng, K. Anwar, and T. Matsumoto, “Outage probability of a relay strategy allowing intra-link errors utilizing Slepian-Wolf theorem,” *EURASIP J. Advances Signal Processing*, vol. 2013, no. 1, p. 34, 2013.

[15] D. Slepian and J. Wolf, “Noiseless coding of correlated information sources,” *IEEE Trans. Inform. Theory*, vol. 19, no. 4, pp. 471–480, Jul 1973.

[16] P.-S. Lu, X. Zhou, and T. Matsumoto, “Outage probabilities of orthogonal multiple-access relaying techniques with imperfect source-relay links,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 2269–2280, April 2015.

[17] J. He, I. Hussain, M. Juntti, and T. Matsumotov, “End-to-end outage probability analysis for multi-source multi-relay systems,” in *Proc. IEEE Int. Conf. Commun.*, May 2016, pp. 1–6.

[18] S. Qian, J. He, M. Juntti, and T. Matsumotov, “Performance analysis for multi-source multi-relay transmission over $\kappa$-$\mu$ fading channels,” in *Proc. Annual Asilomar Conf. Signals, Syst., Comp.*, November 2016, pp. 1–6.

[19] D. H. Woldegebreal and H. Karl, “Multiple-access relay channel with network coding and non-ideal source-relay channels,” in *Proc. Int. Symp. Wireless Commun. Systems*, Oct 2007, pp. 732–736.

[20] A. Wolf, M. Matth, A. Festag, and G. Fettweis, “Outage based power allocation for a lossy forwarding two-relaying system,” in *Proc. IEEE Int. Workshop on Computer Aided Modelling and Design of Commun. Links and Networks*, Sept 2015, pp. 267–272.

[21] A. Wolf, D. C. Gonzlez, J. C. S. S. Filho, and G. Fettweis, “Asymptotically optimal power allocation for WSNs with mutually correlated sensing data,” *IEEE Commun. Lett.*, vol. 20, no. 7, pp. 1317–1320, 2016.

[22] M. Brulatout, H. Khalif, V. Conan, S. Szott, M. Natkaniec, K. Kosek-Szott, and L. Prasnal, “A cooperative MAC protocol for lossy forwarding networks,” in *Proc. Wireless Days Conf.*, March 2016, pp. 1–3.

[23] K. Kosek-Szott, M. Natkaniec, L. Prasnal, and S. Szott, “CLF-MAC: A coordinated MAC protocol supporting lossy forwarding in WLANs,” in *Proc. Int. Symp. Wireless Commun. Systems*, Sept 2016, pp. 434–438.

[24] Z. Wang and G. B. Giannakis, “A simple and general parameterization quantifying performance in fading channels,” *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1389–1398, Aug 2003.

[25] D.-S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, “Fading correlation and its effect on the capacity of multielement antenna systems,” *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 502–513, Mar 2000.

[26] C.-N. Chuah, D. N. C. Tse, J. M. Kahn, and R. A. Valenzuela, “Capacity scaling in MIMO wireless systems under correlated fading,” *IEEE Trans. Inform. Theory*, vol. 48, no. 3, pp. 637–650, Mar 2002.

[27] R. Y. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, “Spatial modulation,” *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228–2241, July 2008.

[28] H. Shin and J. H. Lee, “Capacity of multiple-antenna fading channels: spatial fading correlation, double scattering, and keyhole,” *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2636–2647, Oct 2003.

[29] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, “Capacity limits of MIMO channels,” *IEEE J. Select. Areas Commun.*, vol. 21, no. 5, pp. 684–702, June 2003.

[30] M. Souryal and B. Vojcic, “Performance of amplify-and-forward and decode-and-forward relaying in Rayleigh fading with turbo codes,” in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, vol. 4, May 2006, pp. 5–5.

[31] R. H. Etkin and D. N. C. Tse, “Degrees of freedom in some underspread MIMO fading channels,” *IEEE Trans. Inform. Theory*, vol. 52, no. 4, pp. 1576–1608, April 2006.

[32] A. E. Gamal and Y.-H. Kim, *Network Information Theory*. New York: Cambridge University, 2011.

[33] C. E. Shannon, “Coding theorems for a discrete source with a fidelity criterion,” *IRE Nat. Conv. Rec*, vol. 4, no. 142-163, p. 1, 1959.

[34] F. J. Vazquez-Araujo, O. Fresnedo, L. Castedo, and J. Garcia-Frias, “Analog joint source-channel coding over MIMO channels,” *EURASIP J. Wireless Comm. and Netw.*, vol. 2014, no. 1, p. 25, 2014.

[35] G.-C. Zhu and F. Alajaji, “Joint source-channel turbo coding for binary Markov sources,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 5, pp. 1065–1075, May 2006.

[36] W. Gander and W. Gautschi, “Adaptive quadrature-revisited,” *BIT Numer. Math., Springer*, vol. 40, pp. 84–101, 2000.

[37] W. R. B. Mischa Schwartz and S. Stein, *Communication Systems and Techniques*, 1st ed. John Wiley & Sons, 1995.

[38] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Tenth Printing ed. New York: Dover, 1964.

[39] M. Dohler and Y. Li, *Cooperative Communications: Hardware, Channel and PHY*. Wiley Publishing, 2010.

[40] D. Liang, S. X. Ng, and L. Hanzo, “Relay-induced error propagation reduction for decode-and-forward cooperative communications,” in *Proc. IEEE Global Commun. Conf.*, Dec 2010, pp. 1–5.

[41] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes: The Art of Scientific Computing*, Third ed. New York: Cambridge University Press, 2007.