Using Conspiracy Numbers for Improving Move Selection in Minimax Game-Tree Search

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Abstract: In a two-person perfect-information game, Conspiracy Number Search (CNS) was invented as a possible search algorithm but did not find much success. However, we believe that the conspiracy number, which is the core of CNS, has not been used to its full potential. In this paper, we propose a novel way to utilize the conspiracy number in the minimax framework. Instead of using conspiracy numbers separately, we combine them together. An example way of combining conspiracy numbers with the evaluation value is suggested. Empirical results obtained for the game of Othello show the potential of the proposed method.

1 INTRODUCTION

In a two-person perfect-information game, it is common to employ a minimax-based procedure to estimate the current game situation as well as decide the next move based on that information. A minimax-based search produces a search tree, using an evaluation function to estimate the value of leaf nodes and use those value to determine the value of the root node(Shannon, 1950)(Turing et al., 1953). For most games, the size of the tree renders the search inefficient. As a result, many algorithms such as alpha-beta pruning (Knuth and Moore, 1975) and SSS* (Stockman, 1979) have been invented to solve this problem. Most algorithms focus on selectively choosing appearing paths to search while ignoring a large portion of the tree (for example, singular extensions (Anantharaman et al., 1990) and null move (Beal, 1990)(Goetsch and Campbell, 1990)). However, the effectiveness of these algorithms also depends on the quality of the evaluation function, because no matter how fast the algorithm is, it cannot search the whole tree. It needs to stop at some points and use static evaluation values. As a result, a weak evaluation function may remove the branch with the best move and bias the search toward non-optimal moves.

Conspiracy Number Search (CNS) (McAllester, 1988) was invented as a game-independent best-first search which expands the game tree non-uniformly to establish a stable value for the root node. The algorithm was based on the concept of conspiracy numbers which, in a sense, show how unlikely the root value would change to a certain value. Conspiracy numbers were also used in alpha-beta-conspiracy (McAllester and Yuret, 2002); theirs usage were different and less computationally intensive. However, both algorithms were not very successful (Schaeffer, 1990), and conspiracy numbers did not receive much attention after that. Recently, the conspiracy number was investigated again, but for another usage (Khalid et al., 2015). In our opinion, the failure of these previous methods occurred because their methods used conspiracy numbers separately. A single conspiracy number would not have much information about the game situation, so it is not beneficial to decide based on it. In this paper, we propose a new method of combining the conspiracy numbers to evaluate the game situation. It is shown that the proposed method has a potential to improve the evaluation accuracy.

The structure of this paper is as follows. Section 2 presents related works in this domain and the basic idea of conspiracy numbers is described in section 3. Then we propose our new method in section 4, and experiments which we performed and their results are shown together with some discussion in section 5. Finally, concluding remarks are given in section 6.

2 RELATED WORKS

In this section, we describe in more details about conspiracy numbers and their application in previous

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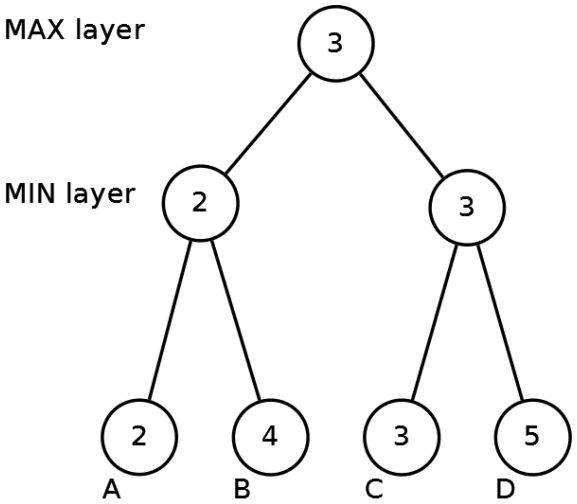
methods.

2.1 Conspiracy Number

In (McAllester, 1988), conspiracy numbers are the measurement of the difficulty of changing the minimax root value of a given tree. The conspiracy number of a particular value v in a given tree is the minimum number of leaf nodes that have to change their values (called **conspirators**) so that the value of the root node will change to v.



Root value	1	2	3	4	5	6
Conspiracy number	2	1	0	1	1	2

Figure 1: A simple minimax tree. The table shows conspiracy numbers for several values. $CN(3) = 0$ because 3 is the value of the root node, so we do not need to change anything. To change the root's value to 2, at least node C or D needs to change to 2. To change to 1, node A or B and node C or D need to change to 1.

Let T denote a node with minimax value m. The conspiracy number $CN(T,v)$ can be defined recursively as follows:

- A a leaf node, because there is no more search, the only conspirator is the node itself. However, if it is also a terminal node (an ending position), its value cannot be changed. So:

$$CN(T,v) = \begin{cases} 0 & \text{if } v = m \\ 1 & \text{if } v \neq m \\ \infty & \text{if terminal node} \end{cases} \quad (1)$$

- At a max node, in order to increase its value, at least *one* of its children needs to increase its value. Meanwhile, to decrease its value, *all* of its children need to decrease their values. Therefore, we have two cases:

$$CN\uparrow(T,v) = \begin{cases} 0 & \text{if } v \leq m \\ \min\limits_{\text{all children } T_i} CN\uparrow(T_i,v) & \text{if } v > m \end{cases} \quad (2)$$

$$CN\downarrow(T,v) = \begin{cases} \sum\limits_{\text{all children } T_i} CN\downarrow(T_i,v) & \text{if } v < m \\ 0 & \text{if } v \geq m \end{cases} \quad (3)$$

- At a min node, a similar scheme is applied:

$$CN\uparrow(T,v) = \begin{cases} 0 & \text{if } v \leq m \\ \sum\limits_{\text{all children } T_i} CN\uparrow(T_i,v) & \text{if } v > m \end{cases} \quad (4)$$

$$CN\downarrow(T,v) = \begin{cases} \min\limits_{\text{all children } T_i} CN\downarrow(T_i,v) & \text{if } v < m \\ 0 & \text{if } v \geq m \end{cases} \quad (5)$$

2.2 Conspiracy Number Search

Conspiracy number search was described in (McAllester, 1988). It is a selective search that explores the game tree non-uniformly to determine the value of the root node. It maintains a range of possible values and keeps expanding the tree until a certain degree of confidence is reached. The confidence is measured by the width of a possible values' range W and a minimum value for conspiracy numbers T. The purpose of the search is to raise the conspiracy numbers of unlikely values to greater than T in order to reduce the range of possible values to below W. At each turn, CNS tries to disprove either the highest or lowest possible value, which has the highest conspiracy numbers, by expanding one of its conspirators. Then, it recalculates conspiracy numbers and repeats the process until the desired confidence is obtained.

Using the example in Figure 1, we will demonstrate how CNS works. Let W be 1, which means that the search will only stop when there is only 1 value in the possible range. Let T be 1, which means that any value which has more than 1 conspirators is considered unlikely. The range of possible values in our example would be $2-5$, which is larger than our desired range W. To reduce it, CNS choose to raise the conspiracy number of either 2 or 5. If it selects 2, whose conspirators is either node C or node D, CNS will expand both node C and D. Figure 2 shows a possible result after expanding C.

CNS had been analyzed in several papers such as (Elkan, 1989), (Schaeffer, 1990), (VanderMeulen, 1990), (Klingbeil and Schaeffer, 1990) and (Lister

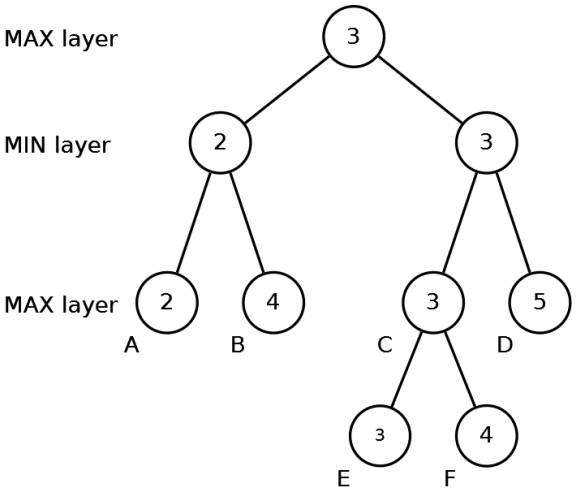


Figure 2: The tree in Figure 1 after expanding node C. Now, to change value of node C to 2, both node E and F need to change to 2. So the conspirators of 2 are node D or node E and F. Next, node D will be selected to expand.

and Schaeffer, 1994), which showed some drawbacks and proposed some improvements to CNS, but CNS did not enjoy much success or attention afterward.

2.3 Other Directions

Alpha-Beta Conspiracy search (ABC), another algorithm based on the conspiracy number, was introduced in (McAllester and Yuret, 2002). ABC uses 2 values of conspiracy numbers to guide the search; hence, it is less computationally expensive than CNS. However, to our knowledge, not much analysis or application has been done with ABC.

Later, conspiracy numbers inspired works on proof-number search (PNS) (Allis et al., 1994) which was very successful at solving games (Schaeffer et al., 2007). PNS is applicable in AND/OR tree (Kishimoto et al., 2012), in which a node only has 3 possible values: true, false or unknown. In a game situation, these values can represent a win, a lost or undetermined, respectively. The purpose of PNS is to determine the outcome of the game as fast as possible by searching branches that have high chance of establishing result first. PNS does not require an evaluation function; it only requires the rules of the game to determine the outcome at ending positions. However, while it is capable to solve end-game positions (Seo et al., 2001), PNS cannot make decision at the opening or the middle of a game.

In (Khalid et al., 2015), another usage of conspiracy numbers is investigated. The authors suggest that the flow of conspiracy numbers can indicate critical positions which can be used to determine a change in strategy, speculative plays or early resignation.

3 IMPROVING MOVE SELECTION WITH CONSPIRACY NUMBERS

In this section, we will note some important aspects of conspiracy numbers and introduce our method.

3.1 Important Characteristics of Conspiracy Numbers

There are 2 important characteristics of conspiracy numbers. First, they increase asymmetrically with respect to the search depth. Assume that the root node is a max node, if the search depth is one, from Equation (2) and (3), it can be seen that for values higher the root value, their conspiracy numbers will be 1 (only one of its children need to change to a higher value). Meanwhile, for values lower than root value, their conspiracy numbers could be higher (all of its children with higher value than the root value need to change). If the root node is a min node, the opposite is true. Therefore, we only consider conspiracy numbers with even search depth, so that their values would be balanced for both higher values and lower values.

Another important property of conspiracy numbers is that they increase monotonically. Let m denote the root value. If $v_1 < v_2 < m$, $CN(v_1) \geq CN(v_2) > CN(m) = 0$. Also if $m < v_1 < v_2$, $0 = CN(m) < CN(v_1) \leq CN(v_2)$. Figure 3 illustrates this property.

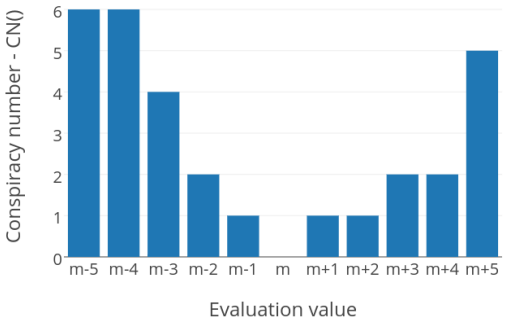


Figure 3: A simple example shows the relation between evaluation values and conspiracy numbers. m denotes the root value.

3.2 Combining Conspiracy Numbers

The above characteristics give us the intuition that Conspiracy Numbers for a game situation can be viewed as the probability distribution of evaluation value for that situation. A value with high conspiracy number indicates that it is difficult to achieve that value and vice versa. From that intuition, we

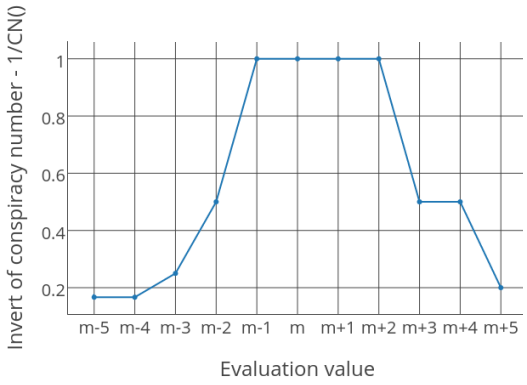


Figure 4: Evaluation values and inverted conspiracy numbers. Conspiracy numbers are the same as in Figure 3. We consider $CN(m)$ to be 1 instead of 0 for easier computation.

think that we can combine conspiracy numbers to better understand game situations. It is also a different approach toward conspiracy numbers since to our knowledge, previous methods only use conspiracy number of a single (as in CNS) or two (as in ABC search) evaluation values to determine the direction of the search.

Let v denote the current minimax value of a game situation. For any value x, $CN(x)$ is the conspiracy number of x (either $CN\uparrow(x)$ or $CN\downarrow(x)$). Then, intuitively, we choose $\frac{1}{CN(x)}$ to represent the probability of changing to x. The graph of $\frac{1}{CN(x)}$ is shown in Figure 4. To combine these numbers, we treat $CN()$ as a continuous function, and $\frac{1}{CN(x)}$ would resemble the probability distribution of the evaluation value. We then calculate a value which we call the "Conspiracy Adjusted Evaluation Value" (CAEV) as follows:

$$\frac{\int_{-\infty}^{\infty} \frac{x}{CN(x)} \mathrm{d}x}{\int_{-\infty}^{\infty} \frac{1}{CN(x)} \mathrm{d}x} \tag{6}$$

The numerator represents the expected value if we consider $\frac{1}{CN(x)}$ as the probability of having x as the evaluation value. The denominator $\int_{-\infty}^{\infty} \frac{1}{CN(x)} \mathrm{d}x$ is presented to normalize the new value. Our hypothesis is that the CAEV would be a better measure of the game position than evaluation value alone.

The CAEV cannot be calculated exactly because $CN()$ is not a real continuous function. Therefore, we approximate it using the following procedure:

- First, the evaluation range is divided into small segments.
- Second, conspiracy numbers for the start and end positions of each segment are calculated.
- Then, in each segment, we consider $CN()$ to be a linear function and calculate Equation (6) for that segment.
- The summation of these values is the approximation of CAEV.

The detail of the procedure can be found in the Appendix.

4 EXPERIMENT

We conducted some experiments to assess the effectiveness of the new measure. In this section, we describe the experiments and discuss about their results.

4.1 Experimental Design

The game of Othello is chosen as our test bed. We use game transcripts from the United States 2015 National Open tournament (United States Othello Association, 2015). For our experiments, we propose a method which corporate CAEV and alpha-beta search as in Algorithm 1. Algorithm 2 shows the pseudo code for minimax procedure (which is implemented as negamax) in Algorithm 1.

Algorithm 1: Func $CN(node, cn\_depth, ab\_depth)$.

```
minimax(node, cn_depth)
for all child in node.children do
    child.eval = CAEV(child)
end for
return get_best_move(node.children)
```

Algorithm 2: Func $minimax(node, depth)$.

```
if node.is_terminal() or depth = 0 then
    node.eval ← alphabeta(node, ab_depth)
    return
end if
node.eval ← −∞
for all child in node.children do
    minimax(child, depth − 1)
    node.eval ← max(node.eval, −child.eval)
end for
```

Figure 5 illustrates the structure of our method. It consists of 2 layers. The top layer is a game tree generated by minimax algorithm; CAVE is applied to this tree. The leaf nodes of this tree are evaluated by an alpha-beta procedure. In the pseudocode, cn_depth denotes the depth of the minimax layer while ab_depth denotes the depth of the alpha-beta layer. So a player with $cn\_depth = 0$ will be identical to an alpha-beta player, while if $ab\_depth = 0$, it will be a minimax player with CAEV applied. For short, we will denote a player as $CN(x,y)$ with x being

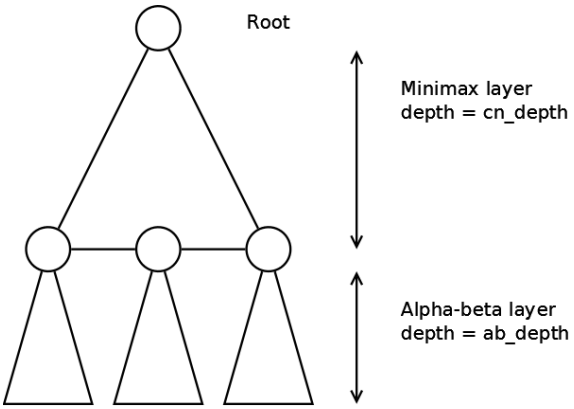


Figure 5: Structure of our proposed player which consists of a layer of minimax above a layer of alpha-beta. The CAEV will be applied to the minimax layer.

cn_depth and y being *ab_depth*. We compare 3 players: $CN(5,0)$, $CN(3,2)$ and $CN(0,5)$ to identify the influence of CAEV on a minimax-based player. Also, *2 evaluation functions* are prepared for these players to identify the effect of different evaluation functions. Details on these evaluation functions can be found in the Appendix.

Each player will play 20 games with other players. A game will start from a random position between move 5 and move 10 of a game from a set of randomly selected games in the tournament. In addition, to ensure fairness, for each selected game, each player will start first once, which means that there will be 40 matches between each player pairs. For each match, the winner gets 1 point, the loser gets 0 point, while each player will get 0.5 point in case of a draw.

4.2 Experimental Results

First, we performed matches between players with the same evaluation function. Their results are shown in Table 1 and Table 2. Then, matches between players with different evaluation functions was played and their results are shown in Table 3.

Table 1: Results of matches between players using EV1. Note that the results are symmetric.

	$CN(5,0)$	$CN(3,2)$	$CN(0,5)$
$CN(5,0)$	-	19.5 - 20.5	19 - 21
$CN(3,2)$	20.5 - 19.5	-	18 - 22
$CN(0,5)$	21 - 19	22 - 18	-

From Table 1, we cannot observe much influence of CAEV on players with EV1. Most of the matchup are seemingly equal. But for EV2, We can see that the CAEV indeed has a remarkable effect on the performance of the players. As shown in Table 2, players with higher *cn_depth* perform better.

Table 2: Comparison between players with EV2.

	$CN(5,0)$	$CN(3,2)$	$CN(0,5)$
$CN(5,0)$	-	23 - 17	28 - 12
$CN(3,2)$	17 - 23	-	22 - 18
$CN(0,5)$	12 - 28	18 - 22	-

Table 3: Comparison between players with EV1 and EV2. In the column are players with EV1, while in the row are players with EV2.

		EV2		
		CN(5,0)	CN(3,2)	CN(0,5)
EV1	CN(5,0)	25 - 15	19 - 21	15 - 25
	CN(3,2)	23 - 17	27 - 13	25 - 15
	CN(0,5)	22 - 18	25.5 - 14.5	20.5 - 19.5

To find the difference between the 2 evaluation functions, we let players with EV1 compete with players with EV2. The results are shown in Table 3. Although the results look complicated, it is clear that players with EV1 performs better than players with EV2 since most players with EV1 won over players with EV2. Among them, CN(3,2) with EV1 achieves the best results (second row). However, there are still ambiguity, such as indicated by the fact that $CN(5,0)$ with EV1 lost against $CN(0,5)$ (alpha-beta player) with EV2 while both $CN(3,2)$ and $CN(0,5)$ did not, and for EV2, the player with more *cn_depth* obtains worst results, which contradict with the results in Table 2. Further investigation is needed to clarify the effects of CAEV and to know the best way to utilize it.

5 CONCLUDING REMARKS

We have presented a move selection policy which incorporates conspiracy numbers. The novel idea of our method is treating conspiracy numbers as a whole, contrasting with previous methods which treat conspiracy numbers separately for each individual evaluation value. By doing so, we can better understand a game situation and make better decisions.

A simple method using the proposed idea was suggested. The method combines conspiracy numbers into evaluation values by an integration method and using the new value to select moves. Our experiments show that the value can have huge improvement on certain evaluation functions.

To clearly understand the effects of the proposed idea, more experiments are required. For example, another evaluation function or another domain should be tested. We expect that the effect would be greater on weak evaluation functions than on strong evaluation functions. Also, we could change the function

$\frac{1}{CN(x)}$. It was chosen based on intuition, and since it is an approximation, other functions may also perform well.

The method also has some drawbacks. The most notable one is the expensive cost of building a minimax tree which is inherited from conspiracy number. If we could calculate or approximate conspiracy numbers without the need of a minimax tree, we could use the method much more freely, such as using alpha-beta or other fast game playing algorithms.

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APPENDIX

CAEV. Here is the procedure to calculate the CAEV of a node.

Algorithm 3: Func $CAEV(node)$.

$ev \leftarrow node.eval$
$sum \leftarrow 0$
for $v = ev - RANGE + STEP$; $v <= ev + RANGE$; $v+ = STEP$ **do**
  $sum+ = intgr(v - STEP, \frac{1}{CN(v-STEP)}, v, \frac{1}{CN(v)})$
end for
return sum

In the above procedure, *RANGE* is the maximum distance of the evaluation value to the current value of the root. We do not need to calculate the maximum or minimum possible evaluation values because such values would have very high conspiracy numbers, which means that their inverse will be extremely small and can be disregarded. $STEP$ is the segments' length; a smaller $STEP$ indicates a better approximation and thus a higher computational cost. In our experiments, we set *RANGE* to 1000, which is equal to the difference of 1 corner stone in our evaluation function, and $STEP$ to 50. The function *intgr(x,fx,y,fy)* will calculate the integral from x to y of a straight line which goes through (x, fx) to (y, fy).

Evaluation Function. In our experiment, we use 2 simple evaluation functions which relies on several features of the game with hand-tuned weight. The functions are *symmetric* in relation to the side to move. For the first evaluation function (EV1), the included features are the **differences** between 2 players in the number of **corners**, **mobility** (the number of moves), the number of **discs**, the number of **next-to-open-corner discs** and the number of **frontier discs** (discs next to an open cell).

Algorithm 4: Evaluation Function 1.

```
return 1000*corners + 100*mobility - 200*next_to_corners - 100*frontier_discs + 1*discs
```

The second evaluation function (EV2) is simpler than the first one. It only depends on the number of **corners**, the **mobility** and the number of **discs**.

Algorithm 5: Evaluation Function 2.

```
return 1000*corners + 100*mobility + 1*discs
```