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JAPAN ADVANCED INSTITUTE OF SCIENCE AND TECHNOLOGY

An Analysis of Scrabble from the Viewpoint of Gamified Learning

by

Suwanviwatana Kananat

A thesis submitted in partial fulfillment for the degree of Master of Information Science

Written under the direction of
Professor Hiroyuki Iida
School of Information Science

March 2018

Declaration of Authorship

I, SUWANVIWATANA KANANAT, declare that this thesis titled, 'Scrabble and Its Educational Use' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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"An investment in knowledge pays the best interest."

Benjamin Franklin

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Abstract

Professor Hiroyuki Iida
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by Suwanviwatana Kananat

'Gamification' is a newly defined terminology which refers to the application of game-design elements and game principles in non-game contexts, to improve users experience and their engagement. It has been used in various fields, including education. SCRABBLE, a game involving utilization of an English alphabet is our primary consideration. If learning English vocabulary is regarded as the action interested, then SCRABBLE is one of the gamified direction.

Game refinement theory, proposed by Iida *et al.* is known as active research which focuses on explaining game entertainment and sophistication with a mathematical model. Apparently, its measure indicates the rate of change in information progress. While insufficiency leads to tedious or boredom, an extreme value leads to frustrating experience. In this study, quantifying attractiveness and educational benefit in SCRABBLE, an English word anagram game is our primary concern. Notably, it has many unique characteristics.

Despite the fact that most have a singularity, SCRABBLE has dual properties of board game and scoring game, entertaining and educative. Game refinement theory indicates that SCRABBLE is an enjoyable game in which sufficient vocabulary knowledge is required to enjoy the game. This fact leads to unbalanced player distribution between native and non-native speakers. Besides, the result reflects the theory that an inconsistency between legacy models is discovered. Therefore, a 'mass-in-mind' or a shift of perceived challenge is introduced to explain.

Possible enhancements, which focuses on entertaining and educative experience are suggested and discussed theoretically to improve SCRABBLE. The proposed methodology is expected to apply to descendant works as well.

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Contents

List of Figures

List of Tables

Abbreviations

AI	**A**rtificial **I**ntelligence
DS	**D**ictionary **S**ize
GR	**G**ame **R**efinement
GRCM	**G**ame **R**efinement **C**onsidering the **M**ass
LC	**L**earning **C**oefficient
LV	Player Strength
MAPE	**M**ean **A**bsolute **P**ercentage **E**rror
MPE	**M**ean **P**ercentage **E**rror
OCTWL	**O**fficial **T**ournament and **C**lub **W**ord **L**ist
PE	**P**ercentage **E**rror
SOWPODS	Collins Scrabble Words
WESPA	**W**orld **E**nglish-Language **S**crabble **P**layers' **A**ssociation

Physical Constants

Lower limit of refined GR	=	0.070
Upper limit of refined GR	=	0.080
Lower limit of refined $GRCM$	=	0.00245
Upper limit of refined $GRCM$	=	0.00320

Symbols

a	actual intuition on a player
b	effective branching factor
B	whole branching factor
B'	perceived branching factor
C	complexity
D	game length
D'	potential-to-swing count
F	impact from a certain source
G	successful shoot count
GR	game refinement measure
LC	learning coefficient
m	mass
p	selection possibility
S	swing count
T	shoot count
$x(t)$	information progress
z	individual score
Z	total score

For/Dedicated to/To my…

Chapter 1

Introduction

Previously, games were recognized as entertainment medium which has a particular target. However, games have gained continuously spacious attention and became more accessible to most generations. Also, techniques and principles in games are brought into non-game contexts, which is called 'gamification' [3]. Gamification has been applied to various fields: work [4], education [5, 6], marketing [7], health [8], business [9], which is proven to improve user's engagement successfully.

Language is a basis of a communication system between human [10], and one of the fundamental parts of social development [11]. In the globalization era when an additional language is the source of opportunities [12], it is undeniable that linguistics has become increasingly significant. Also, learning language potentially improves the human brain functionality by various means [13].

The living of people is subjected to change as the technology develops. The advanced technology was introduced and became irreplaceable infrastructure in most organizations [14], including science, engineering, and education. Previously, the development of knowledge can be carried out only at an institution or by a textbook [15]. Later, an introduction of smart devices and the advancement of consistent connectivity brought the distant study to be doable at one's convenience [16, 17].

Game refinement theory, proposed by Iida *et al.* in 2004, is ongoing research, which focuses on the measurement of attractiveness and sophistication of a game [18]. The mathematical theory involves physics-in-mind and game outcome uncertainty [19]. Two original works, known as game progress model and board game model have been used to quantify engagement of the scoring game and the board game respectively [20]. While an entertaining aspect of the game is the concern of game refinement theory, we quantify an educational benefit by learning coefficient, a newly proposed measurement [21].

Scrabble [22], an English word anagram game is the primary test-bed of this study. An analysis of game refinement theory and learning coefficient exposes the possible enhancements and further development directions. Supposing learning English vocabulary is considered as the main action, then Scrabble is the possible gamified outcome.

1.1 Scrabble

This section explains fundamental regulations, regular observations and the brief history of Scrabble. Scrabble is a word anagram game, which up-to-four players competitively score points by placing tiles on a board [23]. Each tile bears an English alphabet and its respective mark. A formed word is required to be a valid word in a standard dictionary and adjacent either horizontally or vertically to preceding words.

The trademark of Scrabble belongs to Hasbro, Inc., a toy and board game company in the United States and Canada, and belongs to Mattel, Inc., a toy manufacturing business in other countries [22]. The game was originally published in America in 1938, then their popularity has spread widely beyond hundred nations, and over 150 million copies have been sold worldwide [24].

Despite the fact that there exist several sets of regulations in Scrabble, we primarily focus on two players setting with OCTWL dictionary. This setting can be classified as a board game, a scoring game, a zero-sum game, and an imperfect information game. Notably, the information completeness in Scrabble keep increasing as game steps, then transforms to a perfect information game in the endgame phase.

History shows that several games have been adjusted to match people's taste [25]. The complexity had been continuously decreased in Shogi, the Japanese chess [26]. Fairness has been ensured, and brand-new contents are introduced continuously in Dota2 [27, 28]. Those techniques are implemented to maintain the player's engagement and prevent their extinction. On the contrary, there is neither an explicit modification nor a visible change in Scrabble regulation throughout its history. Most of the changes were the justification of the ambiguous issues [29].

1.1.1 Regulation

Players alternately take their turn to arrange tiles on a board. The words formed are required to be a legit word in a standard dictionary and either horizontally or vertically adjacent to the prior. In the endgame, a player with the highest score becomes the winner [23].

TABLE 1.1: SCRABBLE tiles distribution

Tile	*Point*	*Quantity*	*Tile*	*Point*	*Quantity*	*Tile*	*Point*	*Quantity*
A	1	9	J	8	1	S	1	4
B	3	2	K	5	1	T	1	6
C	3	2	L	1	4	U	1	4
D	2	4	M	3	2	V	4	2
E	1	12	N	1	6	W	4	2
F	4	2	O	1	8	X	8	1
G	2	3	P	3	2	Y	4	2
H	4	2	Q	10	1	Z	10	1
I	1	9	R	1	6	Blank	0	2

In total, there are hundred total tiles with various score distribution [23] which is shown in Table 1.1. There are two unique tiles called blank, which contains zero points but be able to be assigned to any alphabet. Standard SCRABBLE is played on the fifteen by fifteen board comprising fixed hot-spot locations, which grant either a single letter or a word score multiplier. A single letter bonus takes priority over a word bonus, while bonuses from multiple hot-spot stacks multiplicatively [1]. Standard scrabble board is given in Figure 1.1.

The playing sequence is determined by a tile each player randomly draw in the beginning [23]. Supposing the blank tile is the highest priority [2], a player with a letter that is closest to the alphabet 'A' or the blank tile will begin the game. After that, the tiles are put back into the bag. Each player starts his/her turn by drawing tiles until he/she has seven tiles or the bag is empty, then choose to do the followings.

- Forming a word by placing tiles
- Exchanging one tile
- Exchanging all tiles
- Passing a turn

'Bingo', an official name of the fifty points bonus [3], is the special points given to a player who manages to utilize all seven tiles in one round [23]. This rule is examined as the well-refined rule [30]. In a competitive game, 'challenge' is the act of a player questioning the validity of a word formed by another player. A one-round penalty is given to a loser of the challenge [29].

[1] This is made clear in 1953.
[2] This is made clear in 1999.
[3] This is made clear in 1999.

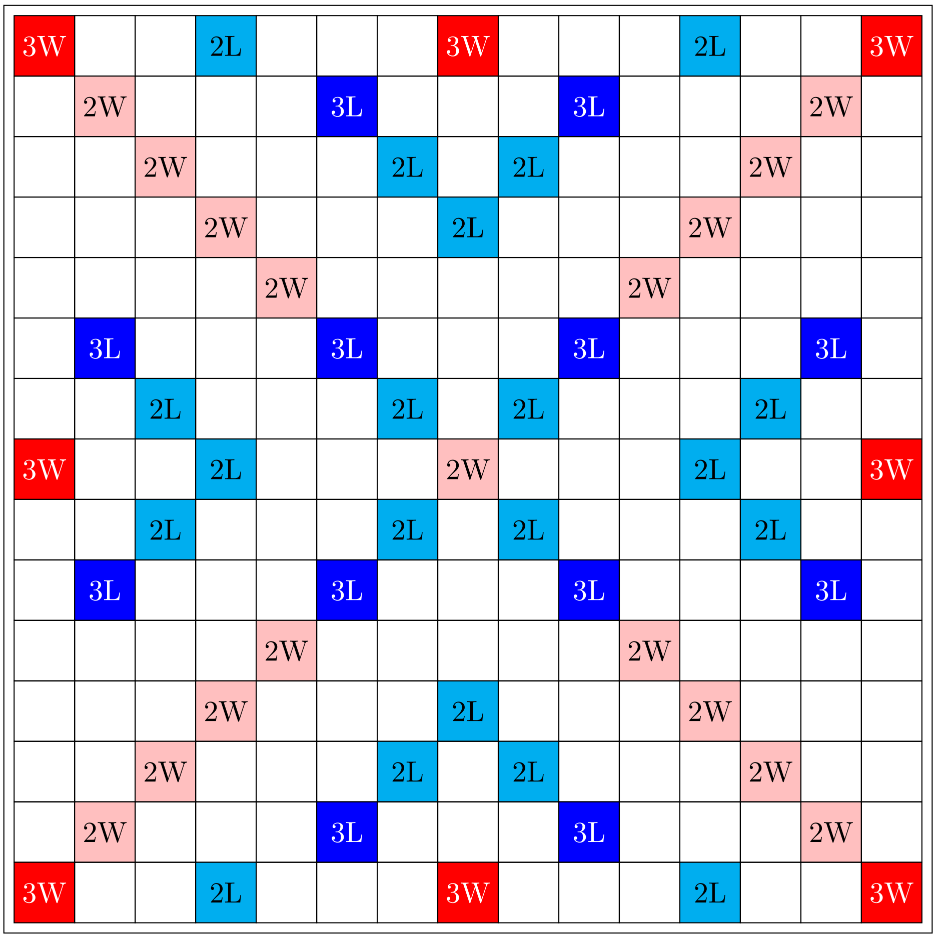

FIGURE 1.1: Standard SCRABBLE board

There are two general sets of acceptable words made explicitly for SCRABBLE, known as 'OCTWL' and 'SOWPODS'. They stand for 'Official Tournament and Club Word List' and 'Collins SCRABBLE Words' respectively [29]. These are shown in Table 1.2.

TABLE 1.2: Legal words in SCRABBLE

Set of words	*OCTWL*	*SOWPODS*
Effective countries	USA, Canada, Thailand	Others
Total words	187,632	267,751

The games end when either any player no longer has a tile to play or continuously pass twice. The remaining tiles will deduct the final score by their respective points [23].

1.1.2 History

As of the typical case in the game industry, SCRABBLE faced against struggling growth era in the first four years [24]. Only 2,400 copies were made in 1949. Later, the president of Macy discovered the game in 1950 during his holidays, then ordered some for his store. Since then, Scrabble has become a must-have game in a year, and it was rumored that a copy of Scrabble could be found in every three American households. It is growing in popularity as well as the frequency of competitions. Every year, the National Scrabble Championship is held in the USA, and also the World Scrabble Championship in alternate years. Also, the National Scrabble Association supports over 180 tournaments and more than 200 clubs in the USA and Canada [24].

Tile distributions in Scrabble were manually designed by analyzing the letter frequency found in newspapers [24].

1.1.3 Popularity

Due to its popularity, there are various resembled reproductions [31, 32], which are either authorized or unauthorized. Some of them have the different parameters, e.g., size of the board, a formation of the board, and the point distribution.

Super Scrabble is another official version which played on the 21 by 21 board or 96% larger than the original [33]. International editions are available in various languages, as shown in Table 1.3. They are also available in computers and smart devices, which have more than ten million accumulated installs.

Table 1.3: National editions of Scrabble

English	Afrikaans	Anglo-Saxon	Arabic	Armenian
Bambara	Basque	Breton	Bulgarian	Catalan
Croatian	Czech	Dakelh	Danish	Dutch
Esperanto	Estonian	Faroese	Filipino	Finnish
French	Galician	German	Greek	Haitian Creole
Hawaiian	Hebrew	Hungarian	Icelandic	Indonesian
IPA English	Irish	Italian	Japanese Hiragana	Japanese Romaji
Klingon	Latin	Latvian	L33t	Lithuanian
Lojban	Malagasy	Malaysian	Mori	Math
Norwegian	Nuxalk	Polish	Portuguese	Romanian
Russian	Scottish Gaelic	Slovak	Slovenian	Spanish
Swedish	Tswana	Turkish	Tuvan	Ukrainian
Welsh	Zhuyin			

1.1.4 Computer Scrabble

In the two-player variant, there are many techniques regards playing Scrabble. It is a game with moderate randomness, due to the process of drawing tiles. During the game, Scrabble is considered as an incomplete information game. However, the game turns into perfect information game during the end game period. From the end game period, a result of a match between professional players is known.

It is known that the rack management is as important as scoring [34]. Upper intermediate players play a word with a decent score while keeping proper remaining tiles on the rack. Since the bingo and hot-spot are the dominant sources of scoring, a player needs to take the advantage from them and prevent an opponent from doing so.

As of 2017, MAVEN [34] is the currently best known artificial intelligence SCRABBLE player presented by Brian Shepperd. It is integrated with all techniques previously mentioned and efficiently makes use of them. Even so, there are several enhancements, which are possible to strengthen MAVEN further. According to the record, it has 32 wins and 17 losses against the champion caliber players. Table 1.4 shows the SCRABBLE players statistics, which implies that MAVEN is significantly stronger than professional players.

TABLE 1.4: SCRABBLE players statistics

	MAVEN	*Experts*	*Intermediates*
Average bingo in a game	1.9	1.5	< 1.5
Average player tiles in a turn	4.762	4.348	< 4.348
Average game length	21.0	23.0	> 23.0
Chance to miss a bingo	0.0%	15.0%	$> 15.0\%$

Despite the different purpose from that of MAVEN, the artificial intelligent SCRABBLE player is developed to study various factors that may impact the game attractiveness and usefulness. For a strategic board game, the representatives are chess and Go that does not much involve with a chance. On the contrary, it is impossible to find the global optimum in the game tree search in a case of SCRABBLE due to unpredictable randomness. Instead, a local optimum has been satisfyingly considered as an acceptable solution in practice.

Trie, a data structure, which is usually used to store a set of strings, is a tree of nodes each bearing necessary information and links to subordinate nodes [35]. In this case, the node stores a Boolean value specifying the validity of a word. An edge shows an alphabet, which is the condition to travel to the corresponding node, so all descendant nodes share the common prefix of the strings considered. Trie can be interpreted as a tree-shaped deterministic finite automaton, as shown in Figure 1.2. An algorithm involved with Trie is used as a fast move generator in our implementation, which is relatively fast but occupies additional memory. Trie is integrated into our AI implementation to increase its performance.

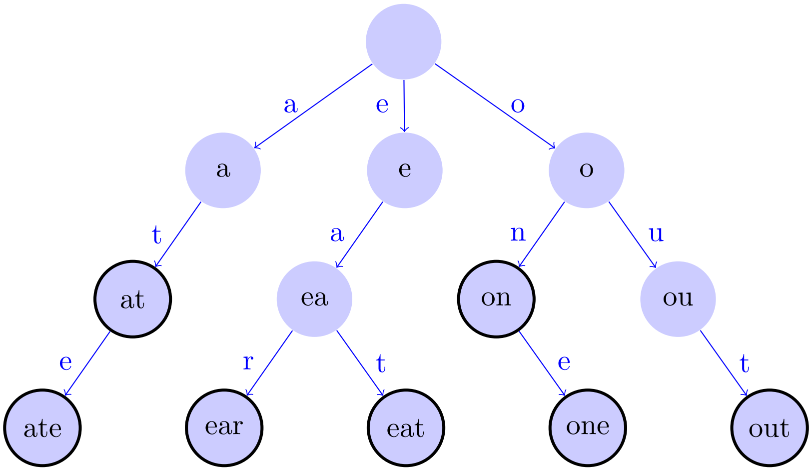

FIGURE 1.2: Sample trie which represents 'at', 'ate', 'ear', 'eat', 'on', 'one' and 'out'

1.1.5 Education

Many competitive SCRABBLE tournaments were held in the United States and Canada, attracting professional players worldwide to join [36]. Besides competitive purpose, SCRABBLE is also playable as a friendly game, which can strengthen the bond among family or faculty members. Meanwhile, SCRABBLE is usable as a medium for learning the language. Playing SCRABBLE is a way to improve vocabulary size, which is less direct but more enjoyable. Regularly playing SCRABBLE will enhance size if one's vocabulary pool and speed up the mental arithmetic skill [37]. Also, the proficiency in using English depends on vocabulary size. It is necessary to know a decent amount of vocabulary, which enough to coverage 95% of the text to understand the reading comprehension [38].

It is undeniable that becoming multilingual grants one more job opportunities and easily accessible to foreigners. Also, going on vacation will become less complicated. Besides, there are several ways that it directly improve the functionality of human's brain [39]. For instance, it may enhance the memory and decrease the rate of experiencing Alzheimer's symptom [13].

Chapter 2

Related Theory

This chapter presents related theories which are the primary concern in this study. They consist of game theory [40], game refinement theory [18], flow theory [41] and physics-in-mind [42]. In fact, those are not independent but genuinely related.

Flow theory is the study of mental state when one is immersively focused on a specific action [41]. Physics-in-mind is an extension of physics to explain the mechanism within a human brain [42]. Game theory concerns on maximizing the profit in decision-making [40]. On the contrary, game refinement theory is an attempt to quantifying engagement in which physics-in-mind is applied to optimize the user enjoyment on a specific domain [18].

2.1 Game Theory

This section gives a short introduction to game theory. Game theory is the study of cooperation and conflict between decision makers in a competitive circumstance [40]. It has been applied to various contexts in economics, political science, psychology, logic, computer science and biology [43]. In computer science, minimax is known as the prior algorithm used in decision making for maximizing the minimum gain, while minimizing the maximum loss. It has been an essential principle for succeeding AI research in games [44].

AI research has been swiftly developed in the past decades [45]. The success was due to increases in computational power, which is from high-end terminals with improved network infrastructure and advancement of the algorithm. In the game research, one of the challenge questions is how to win a game. Therefore, computer players have been developed until the point they are capable of winning regardless of the opponent. The first goal is the victory against world championship calibers, while the second one is to comprehend all sophistication within the game, which is called solving the game [46, 47]. Currently, various games had been completely solved [48], while computer player of some games outplayed world championship [34, 49]. According to the history, the development time of AI tends to be related to the complexity of the corresponding domain.

2.2 Flow Theory

This section gives a short introduction to flow theory. In 1976, Csikszentmihalyi was trying to figure out the phenomenon experienced by the artists who immersed in their work, disregarding daily necessity and losing track of time [50]. The flow was defined to describe that experience. The term 'flow' initially comes from an analogy to water current carrying people along [41].

Flow is known as the zone where one is fully concentrating on a specific activity, which is in the balance between difficulty and skill. During flow, the performance and the creativity are increased, while decision making becomes automatic. All people may experience flow in various activities, e.g., sports, games, studying, working, and even daily routines. Those may conduce to flow as long as the following conditions are fulfilled [51].

- Focused concentration
- Merged action and awareness
- A disappearance of self-consciousness
- A sense of control over the activity
- A distortion of perceived time
- Clear goal in every single step

Creativity brings human to more satisfying life than other wilds. Flow is one of the components of individual and culture development. There are three general ways to measure flow [52].

- Flow questionnaire
- Experience sampling method
- Standardized scales

There is a slight difference between flow and hyperfocus, a mental concentration when the only action is in one's attention [53]. However, flow refers to more positive effect. The dangers of flow are stated that it may lead one to addictive and be being controlled at some point, e.g., playing too much video games [54].

2.3 Physics-in-Mind

This section gives a short explanation of physics-in-mind. Physics is the study of physical phenomena which consists of many sub-fields, e.g., mechanics, thermodynamics, and electronics. Physics-in-mind is the contemporary terminology that studies the system of a human's brain [42, 55].

The computational mechanism and how data is transferred within a human's brain are explained using information theory, biology, and quantum physics [42]. The arrow of time was introduced to describe the awareness of the time. In classical physics, time is a scalar quantity that represents the irreversible change from past to present and from the present to future. However, the time measurement in the most scientific calculation is not necessarily equivalent to the time perceived by a human. The perceived time is distorted while concentrating on a particular subject.

In this study, the phenomenon of physics-in-mind is explained in another manner, which is the classical Newtonian physics [55]. The impact from a specific subject is encrypted into a series of information progress and is being sent into human's brain as the way force is acting upon an object. A player is expected to lose track of time when the impact is resonant and in excellent balance with his/her preference.

2.4 Game Refinement Theory

Many efforts have been devoted to the study of game theory so that it is successfully developed to figure out how to identify the sophisticated decision and strategy. However, how attractive and balance is the game is another challenging question, and little is known about them. Those are believed to depend on various determinants, e.g., game mechanics [56], duration of the game, game complexity, the proficiency and preference of a player.

Conjecture 1. From the perspective of a neutral observer, an identical game with unpredictable outcome tends to be more interesting.

Supposing partiality is not considered, it is known that a game with uncertainty outcome is more attractive than which is predictable during the game [19], as described in Conjecture 1.By applying this conjecture, game refinement theory, the active research area was founded by Iida *et al.* in 2004 [18]. It is firmly believed that attractiveness of a subject can be measured in the same way as done previously in a case of player strength [57].

Emotional excitement and measurement of attractiveness in games are the subjects of game refinement theory [18]. By considering the game outcome uncertainty, the mathematical models of game refinement were proposed as early works, known as game progress model and board game model. Various descendant works have clarified the effectiveness of this method [18, 58–60].

2.4.1 Game progress model

For a scoring game, the game progress is considered as a scoring rate or an information progress, which focuses on the game outcome. The information progress presents the degree of certainty of game results in a specified time frame. Let $x(t)$ be the information progress at time t, $x(t_k)$ is the perfect information at the conclusion time t_k. Assuming the outcome constantly becomes apparent, the model of game progress is given in Equation (2.1).

$$\begin{aligned} x(t) &= \frac{x(t_k)}{t_k}\,t \\ 0 \leq t &\leq t_k \\ 0 \leq x(t) &\leq x(t_k) \end{aligned} \tag{2.1}$$

However, the outcome of an exciting game usually remains uncertain till the very end, thus renders the game progress exponential. Therefore, the more realistic model of game information progress becomes Equation (2.2).

$$x(t) = x(t_k)(\frac{t}{t_k})^n \tag{2.2}$$

Here n stands for a parameter based on the perspective of an observer of the game that is considered. It is assumed that the game information progress is transported in our brains. One is expected to be excited when the rate of change in game progress is proper. This is analogous to the real-world physics, where one is expected to be excited while feeling the gravity, e.g., free falling. Hence, the second derivative of game information progress is considered. After solving at $t = t_k$, the equation becomes Equation (2.3).

$$x''(t_k) = \frac{x(t_k)}{(t_k)^n} t^{n-2}\ n(n-1) = \frac{x(t_k)}{(t_k)^2}\ n(n-1) \tag{2.3}$$

The value $\frac{x(t_k)}{(t_k)^2}$ presents the uncertainty of the game outcome. While deficiency may lead to boredom, an extreme difficulty may lead to frustration. The highly perceived challenge is one of the flow conditions [51], which results in a loss of self-consciousness and track of the time. The average amount of successful shoot G and the average amount of attempt T are introduced to keep the simplicity of the equation. The game refinement measure GR is defined by using its root square, as shown in Equation (2.4).

$$GR = \frac{\sqrt{G}}{T} \tag{2.4}$$

2.4.2 Board Game Model

For a board game, the definition of branching factor and game length are given in Definition 2.1 and Definition 2.2 respectively. A game tree is constructed by recursively attaching all possible transitions to the initial position.

Definition 2.1. Branching Factor For a board game, the branching factor is the amount of all possible instances in a single decision.

Definition 2.2. Game Length For a board game, the game length is the number of steps from the beginning to the ending or the resignation.

Let B and D be the average branching factor and the average game length respectively. A single decision can be illustrated in Figure 2.1

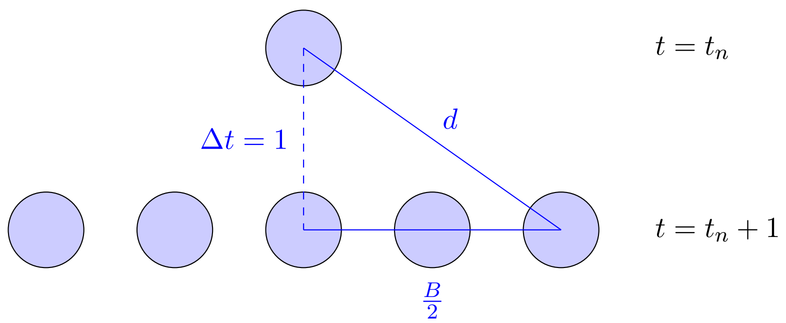

FIGURE 2.1: A generic single decision in a game tree

By considering the geometry, the distance d is obtained by $\sqrt{(\frac{B}{2})^2 + 1}$ according to the Pythagorean theorem [61]. However, 1 is much smaller than B and left from the consideration. Hence, the distance d becomes $\frac{B}{2}$. Assuming the outcome continuously becomes evident, the model of game progress $x(t)$ is determined by the proportion of the d and the game length D. Therefore, the model of game progress becomes $x(t) = \frac{t}{D} \cdot d = \frac{Bt}{2D}$. In general, we have Equation (2.5).

$$x(t) = B(\frac{t}{D}) \tag{2.5}$$

Following the game progress model, the uncertainty of the outcome renders the $x(t)$ exponential. Therefore, the more realistic model of game information progress becomes Equation (2.6).

$$x(t) = B(\frac{t}{D})^n \tag{2.6}$$

The game refinement measure GR of the board game model is obtained similarly by the root square, as shown in Equation (2.7).

$$GR = \frac{\sqrt{B}}{D} \tag{2.7}$$

2.4.3 Swing Model

The game progress model supports only a game with uniformed scoring rate, e.g., Soccer. In Soccer, a successful shoot is particularly challenging to obtain and regarded as one score. Supposing that the gained score is multiplied, the measure of game progress model may alter, but the real essence of the game remains unchanged [30]. However, Scrabble players earn several marks in turn. This incident happens in a case of the games with the non-uniformed scoring system. Therefore, Scrabble has non-uniformed scoring rate and not directly compatible with the game progress model. Instead, the swing model is introduced by defining swing turnover in Definition 2.3.

Definition 2.3. Swing Turnover is a state transition in mind during the game progress among some possible states.

In a game with non-uniformed scoring rate, the average amount of swing turnover S is proposed as a measure for counting the actual successful shoot. Although the transition among possible states may differ for a different domain, we consider two cases: advantage and disadvantage.

It is rumored that maximizing own profit while minimizing others have been the general principle of the intelligent decision maker [40]. Obtaining the highest score is the goal of playing SCRABBLE. In each step, players are taking their turn to attempt to have the advantage over the opponent, which is considered as the actual successful shoot G if successful. Let D' be the average turn that player potentially turn the swing. Due to difficulty in measuring D', the game length D is used as its approximation. The game refinement GR of the swing model is obtained by Equation (2.8).

$$GR = \frac{\sqrt{S}}{D'} \approx \frac{\sqrt{S}}{D} \tag{2.8}$$

Swing model is an appropriate approximation of game progress model as an exciting game would have a proper amount of swing turnover as opposed to a single-sided game [30].

The game refinement measure reflects the balance between player strength and surrounding randomness in a game considered [62]. While a superior value implies that a chance becomes a stronger factor, a game with an extreme game refinement measure might flood player with the information, which results in frustration.

The prior game refinement research indicates the following game refinement measures [18, 20, 28, 58–60, 63, 64]. Interestingly, most of them relate to the same region between 0.07 and 0.08, which we called it 'refined zone' or 'sophisticated zone' [20]. These are shown in Table 2.1.

TABLE 2.1: Correlative measures of legacy game refinement

Subject	G	T	B	D	GR
Chinese chess			38	95	0.065
Soccer	2.64	22			0.073
Basketball	36.38	82.01			0.073
Western chess			35	80	0.074
Go			250	208	0.076
Table tennis	54.863	96.465			0.077
UNO®	0.976	12.684			0.078
DotA®	68.6	106.2			0.078
Shogi			80	115	0.078
Badminton	46.336	79.344			0.086
SCRABBLE (swing)	10.78	35.85			0.092
SCRABBLE (board game)			361.8	35.85	0.531

In addition to the fundamental value, the relation between the game refinement measure and the player strength has been discussed earlier [65]. It is suspected that it can describe the characteristic of the game, in which increasing and decreasing tendency express enjoyable and serious experience respectively. Nevertheless, both can be utilized together in a single domain to maintain the user engagement, as in the case of businesses [65].

In software development life-cycle, the unified process is an iterative and incremental software development framework, which allows greater flexibility [66]. This methodology entirely takes advantage of Unified Modeling Language, which has been an industry standard in software engineering [67].

From the viewpoint of video game development, game refinement theory allows more agile and straightforward process for the game assessment. However, its mathematical models are based on the arguable hypotheses, which may lead to misinterpretation and less reliability. Game refinement is currently not a common practice broadly. Therefore we intend to increase its efficiency.

2.4.4 An Application to Scrabble

This section presents an application of legacy game refinement to SCRABBLE. Game refinement measure has been used to quantify the engagement of the games regardless of their category. The board game model and the scoring game are fit to board games and scoring games respectively. However, SCRABBLE is the remarkable domain, which has compatibility among both.

The excessive branching factor B of SCRABBLE is acceptable for a player with decent vocabulary knowledge. However, this might not be a case for the contrary. Therefore, SCRABBLE is favorable on a hand of the native speakers, but possibly frustrate the language learners. This fact is the cause of unbalanced player population, which is shown in Table 2.2.

TABLE 2.2: Population distribution of SCRABBLE players in CROSS-TABLES[2]

Country	*Official language(s)*	*Player count*	*Percentage*
Barbados	English, Bajan	2	0.149%
Canada	English, French	293	21.833%
Israel	Hebrew, Arabic	1	0.075%
Thailand	Thai	3	0.224%
USA	English	1041	77.571%
Unknown	Unknown	2	0.149%

The variety of words amount in the dictionary are mainly concerned. The reason is that SCRABBLE with limited dictionary size would shrink the searching space and branching factor B efficiently, thus results in more reachable to language learners. Let LV and DS be a player strength and a dictionary size in a normalized scale from 0.0 to 1.0 respectively.

The application of swing model to Scrabble is illustrated in Figure 2.2 and Figure 2.3 with circumstances. The data using the board game model and fully visualized data are

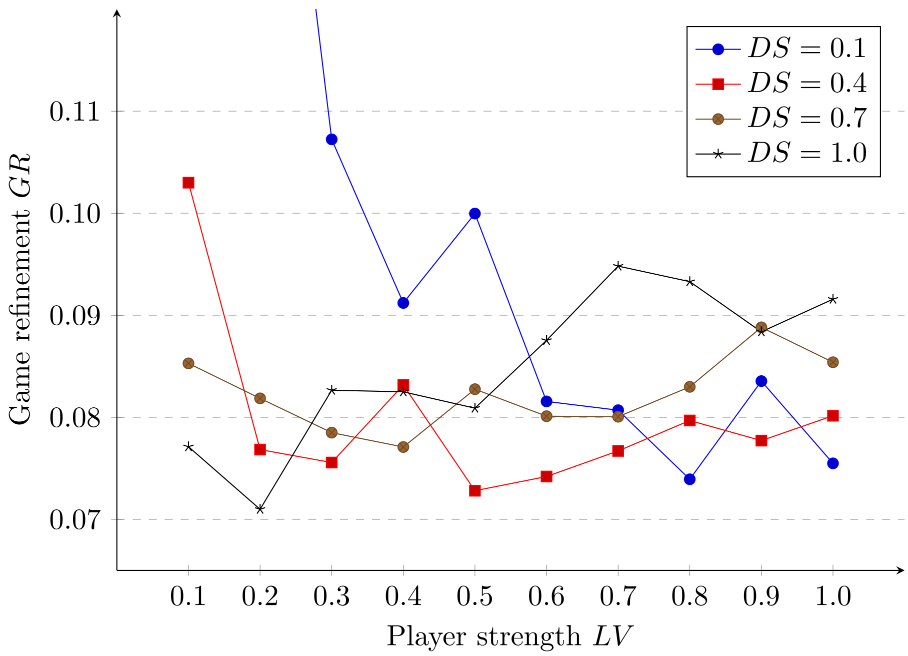

FIGURE 2.2: Impact of player strength on game refinement using legacy swing model

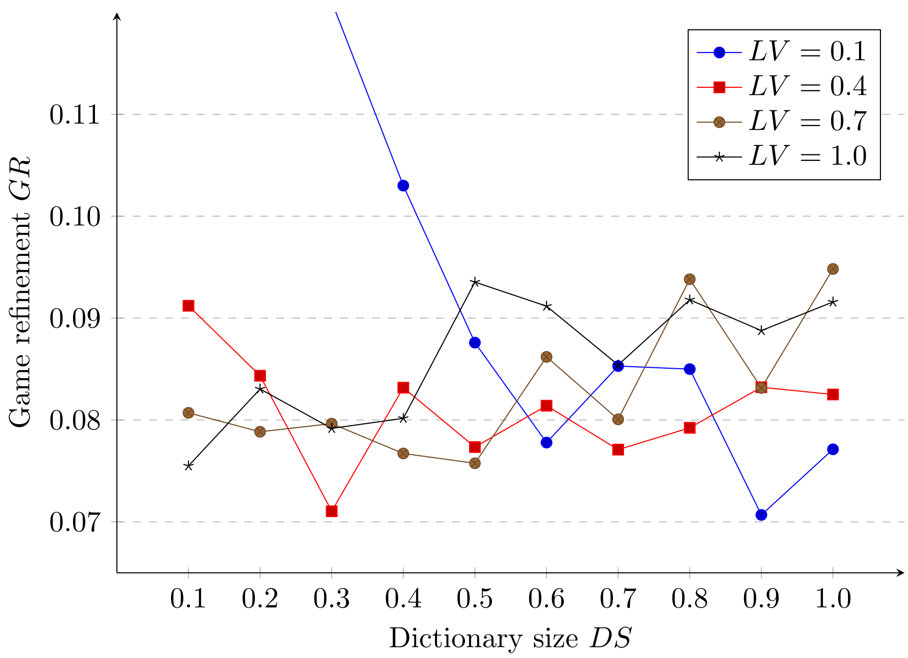

FIGURE 2.3: Impact of dictionary size on game refinement using legacy swing model

given in Appendix A.1. The comparison of two different approaches is shown in Figure 2.4.

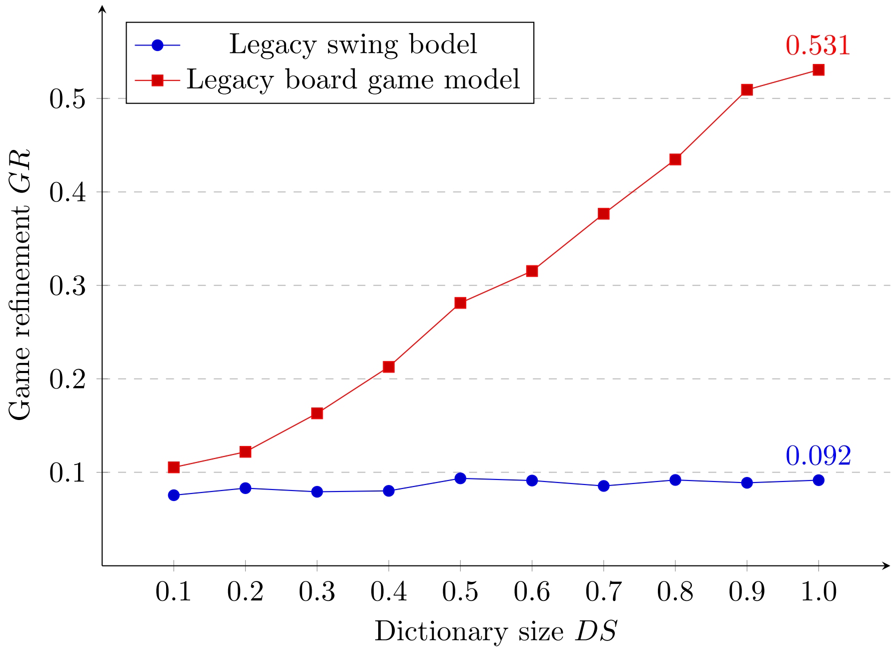

FIGURE 2.4: Comparison of 2 legacy game refinement measures, supposing $LV = 1.0$

The game refinement GR of SCRABBLE is 0.092 and 0.531 for the swing model and the board game model respectively, so randomness takes priority over player strength in a case of SCRABBLE. However, this indicates an inconsistency between two legacy models. This inequality is because game refinement GR slightly shifts in the swing model, but the changes are significant for the board game model. The explanation is that the branching factor B is escalated as the dictionary size DS increased, but the swing turnover S remains invariable.

While considering the history, successful games tend to have an appropriate game refinement measure or adapted toward sophisticated zone. However, game refinement GR exposes only one aspect of the domain. Thus, it is not necessarily the case that a game with an appropriate game refinement will become popular. Although the real essence and the actual interpretation of the game refinement GR is still a broad question, the practical use of game refinement has become more tangible. The subsequent works have shown the compatibility in an application of game refinement theory to other domains, e.g., video games [28], serious games, educations [17, 30], and businesses [65].

In SCRABBLE, the tendency between game refinement and player strength considerably depends on the dictionary size, as shown in Table 2.3.

TABLE 2.3: Impact of dictionary size on GR tendency

Dictionary size	*GR tendency*
$DS < 0.2$	Decrease
$0.2 \leq DS < 0.6$	Decrease then increase
$0.6 \leq DS < 0.9$	Increase
$0.9 < DS$	Increase then slightly decrease

According to the prior study [65], here implies that SCRABBLE with standard dictionary size tends to be a fun game, then continually transforms into a serious game as dictionary size shrinks.

Chapter 3

Entertaining Aspect

This chapter presents an entertaining aspect in SCRABBLE using an extension of game refinement theory [1], which we call game refinement considering mass.

3.1 Personal Decision

We firstly explain a personal decision process, a process in mind which all possibilities are reduced to only one solution. This process commonly involves both skill and chance. In a case of a board game, an experienced player may identify only a few decent moves out of all possible instances. However, only one solution has to be decided as the final solution. This idea has been expanded to establish the mass-in-mind model, which later being integrated into game refinement considering mass.

While all possible instances are relatively large, some of them are out of an experienced player's consideration as they might lead to deficiency or failure. The effective branching factor b of a player is a subset of the branching factor, in which only acceptable solutions are concerned. Definition 3.1 describes its property.

Definition 3.1. Effective Branching Factor For a board game, the effective branching factor of a player is the number of instances, which are satisfyingly perceived by that player in a single decision.

The effective branching factor b is significantly smaller than the branching factor B but not underneath 1, so $1 \leq b \leq B$. Figure 3.1 presents the generic selection process in a player's mind.

[1] Afterward, original game refinement will be called 'legacy game refinement' to prevent ambiguity.

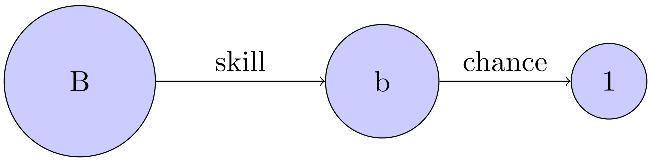

FIGURE 3.1: Process through the player selection

For beginners and experts, the effective branching factor is expected to be close to B and 1 respectively. However, that of intermediate players is a challenging issue. Prior study shows that $\log B$ is a reasonable approximation [68]. Figure 3.2 shows the relation between them.

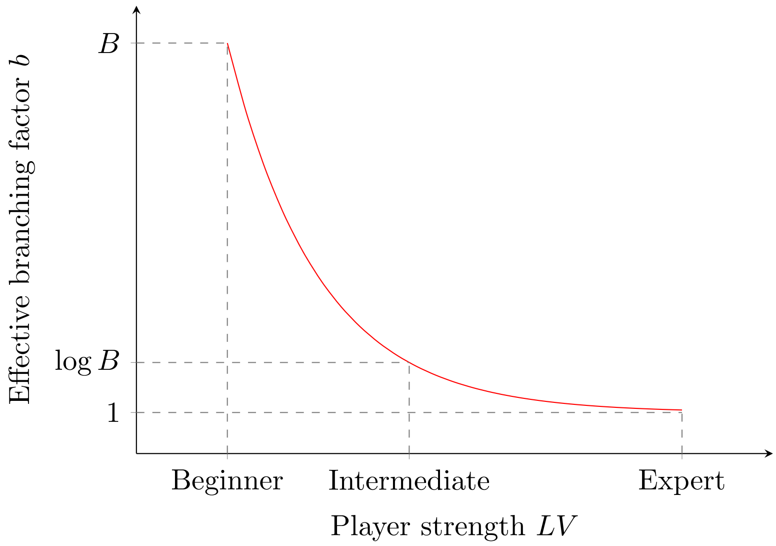

FIGURE 3.2: Impact of player strength on effective branching factor

3.2 Kinetics

In physics, kinetics is the branch of classical mechanics, which focuses on a motion. In classical physics, the relationship between a body and the forces acting upon it are described in 3 fundamental laws, known as Newton's laws of motion. Particularly, Law 1 defines the force quantitatively, then Law 2 offers a quantitative measure of the force and Law 3 claims that there exists no single isolated force.

Law 1. **Newton's First Law** In an inertial frame, an object either remains at rest or continues to move at a constant velocity in a straight line, unless acted upon by an external force.

Law 2. **Newton's Second Law** In an inertial frame, the summation of the forces F acting on an object is equal to the multiplication of its mass m and acceleration a, as shown in the following.

$$\mathbf{\Sigma F} = m\mathbf{a} \tag{3.1}$$

It is assumed that the mass m is a constant.

Law 3. **Newton's Third Law** When one object exerts a force on a second object, the second object simultaneously exerts a force equal in magnitude and opposite in direction on the first object.

Newton's second law is the primary attention of this study as it describes the nature of the mass, resistance to either acceleration or inertia when a net force is applied.

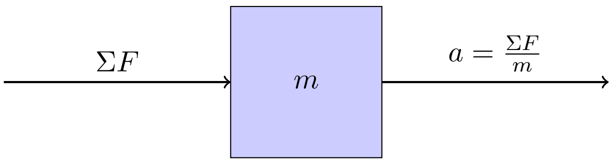

FIGURE 3.3: Newton's second law of motion

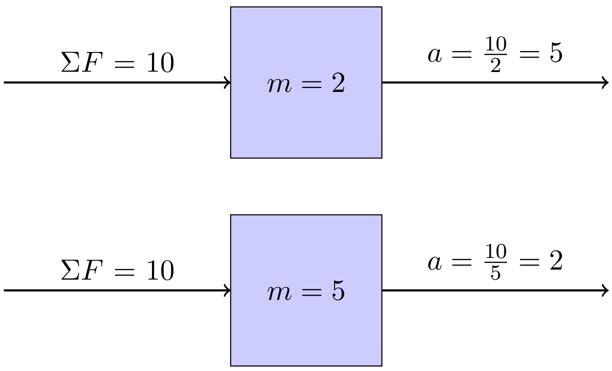

FIGURE 3.4: Newton's second law of motion in action

Newton's second law and its application are illustrated in Figure 3.3 and Figure 3.4 respectively. Due to their respective mass, different objects may react differently to the same net force applied. The Newtonian physics-in-mind is established identically, in which an actual intuition from the same subject on different players is diverse, depending on their corresponding mass-in-mind, as illustrated in Figure 3.5.

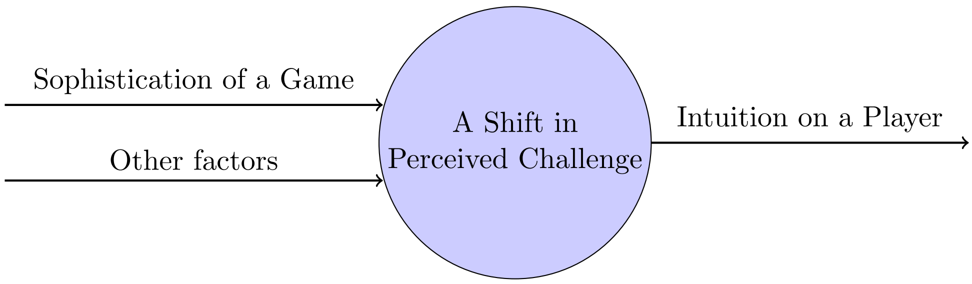

FIGURE 3.5: Newtonian physics-in-mind

A net force consists typically of other sources of force that are external factors. For instance, friction is a force resisting a motion due to contacting solid surfaces. Drag is a form of resistance from a surrounding fluid, either a liquid or a gas. The sophistication of a game, known as game refinement measure GR is a part of the net force-in-mind ΣF, which also consists of other several factors, e.g., a personal preference, an experience, and a present emotion. They are regarded as f and left as zero for the average case until further discovery. Then, the real intuition on a player a is derived by the net force-in-mind and the mass-in-mind, as shown in Equation (3.2).

$$\begin{aligned} \Sigma F &= ma \\ GR^{2}{}^{2} - f &= ma \\ a = \frac{GR^2 - f}{m} &= \frac{GR^2}{m} \end{aligned} \tag{3.2}$$

3.3 Mass-in-Mind

Game refinement theory was established on the hypothesis of the correspondence between Newtonian physics and physics-in-mind [20]. The game refinement measure GR itself represents an acceleration of the game information progress. However, other physical units including mass, one of the most fundamental concepts in the motion physics are not yet concerned.

The term 'mass' originally came from Latin word 'Massa' [69], which means accumulation, body, crowd or heap. In this study, we refer to the definition in classical physics, which mass of an object is described as a property to resist a change in its motion when a net force is applied. This fact was brought into consideration by Isacc Newton. Without the mass, it is absurd to determine the movement of an interested object [70]. Similarly, the actual interpretation of an acceleration of the game information progress must involve with the mass-in-mind [55].

As corresponding with the case of Newtonian physics, the mass-in-mind of a player is defined as a property of a player, which represents resistance to a change in his/her perceived game information progress. The attractiveness of the game depends not only on the game itself but also the preference and the proficiency of a player. A game could be recognized as an amusing game for beginners. However, the identical game may commit an intense competition on the hand of expert players. Several aspects, e.g., the importance of a match and accumulated experience of himself/herself may intensify the degree of perceived challenge [55]. The mass-in-mind, or the decision complexity perceived by a player, is defined in order to describe this incident. The mathematical

[2] According to the history, GR is formerly obtained by the square root of the acceleration of the game progress. We intentionally unfolded the square root to retrieve the original formula.

model of the mass-in-mind involves the selection possibility. Their definitions are given in Definition 3.2 and Definition 3.3.

Definition 3.2. Selection Possibility p In a subject considered, selection possibility is given as a proportion between selectable instances which are satisfyingly perceived and the entire.

Definition 3.3. Mass-in-mind m In a subject considered, the mass-in-mind is given as an inversion of the selection possibility of a player.

The mass-in-mind is regarded as a probability concerning a selection of the personal optimal solution, which represents a shift of the perceived challenge. In practice, a concrete mathematical model of the mass-in-mind may slightly differ based on the game considered. In this study, we proposed two models, which are optimized for the board game and the scoring game.

For a board game, the selection possibility p is obtained by $\frac{b}{B}$, where b and B stand for the average effective branching factor and the average branching factor respectively. Hence, the mass-in-mind m is obtained by its inversion, as shown in Equation (3.3).

$$\begin{aligned} p &= \frac{b}{B} \\ m = \frac{1}{p} &= \frac{B}{b} \end{aligned} \tag{3.3}$$

However, there is a difficulty identifying an optimal solution for the scoring game. Thus, an approximation model is presented. Supposing that a player obtains z points out of Z total points at the end game, one point has $\frac{z}{Z}$ probability to be distributed to the player. Therefore, $\frac{g}{\Sigma g}$ is considered as the selection possibility, then the mass-in-mind is obtained by its inversion, as shown in Equation (3.4).

$$\begin{aligned} p &= \frac{z}{Z} \\ m = \frac{1}{p} &= \frac{Z}{z} \end{aligned} \tag{3.4}$$

Table 3.1 shows the established link between real-world physics and physics-in-mind.

TABLE 3.1: Correspondence between real-world physics and physics-in-mind

Notation	*Newtonian physics*	*Physics-in-mind*
F	Force	Sophistication of a game
m	Mass	A shift in perceived challenge
a	Acceleration	Intuition on a player

Therefore, the mathematical model of game refinement considering mass is given in Table 3.2.

TABLE 3.2: Game Refinement considering the mass

Notation	*Game progress model*	*board game model*	*Swing model*
F	$\frac{G}{T^2}$	$\frac{B}{D^2}$	$\frac{G}{T^2}$
m	$\frac{Z}{z}$	$\frac{B}{b}$	$\frac{Z}{z}$
a	$\frac{Gz}{T^2Z}$	$\frac{b}{D^2}$	$\frac{Sz}{D^2Z}$

3.4 An Application to Scrabble

This section shows the application of the game refinement models considering mass to SCRABBLE with their result.

3.4.1 Swing Model Considering Mass

In a two-player competitive game, it is rumored that a winner ordinarily enjoys a game than a loser. The variation of the average score z can explain the difference between the intuition of a winner. Data from both perspectives are considered then visualized in Figure 3.6 and Figure 3.7.

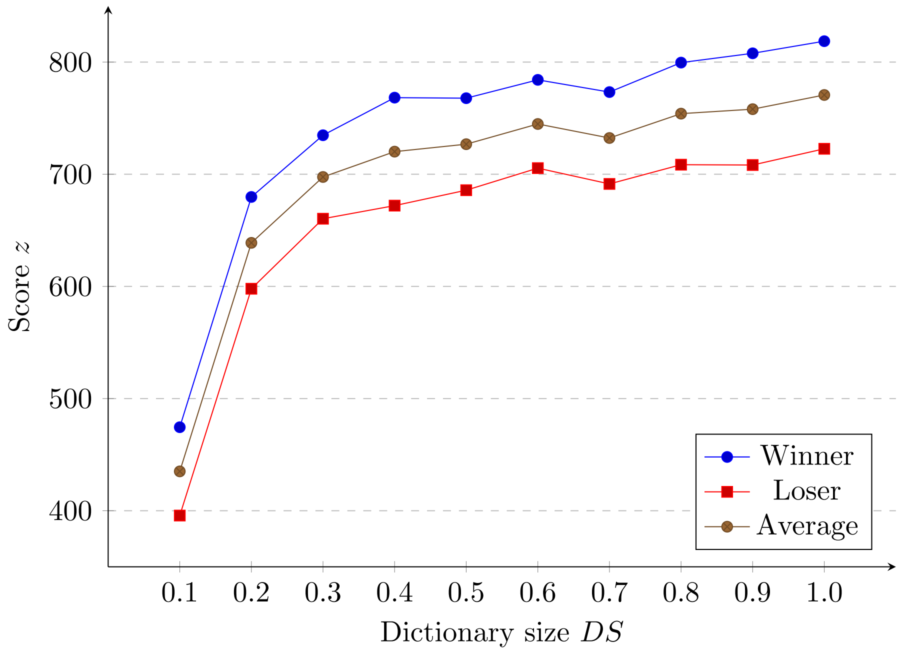

FIGURE 3.6: Impact of dictionary size on score, supposing $LV = 1.0$

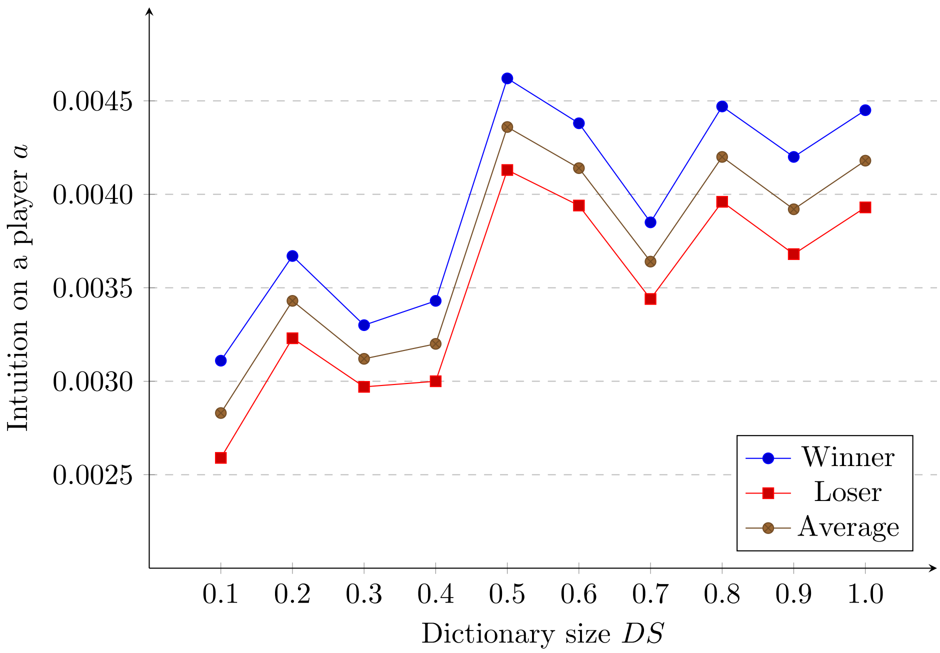

FIGURE 3.7: Impact of dictionary Size on intuition on a player using swing model considering the mass, supposing $LV = 1.0$

3.4.2 Board Game Model Considering Mass

The data is analyzed using the board game model considering mass with three different levels of players, which represent beginners, intermediates, and experts. We refer to the prior approximation of the effective branching factor b for each case [68]. Results are given in Figure 3.8 and 3.9.

Then, the comparison between two game refinement models considering mass is presented. The average is given in Figure 3.10 while the fully visualized data is provided in Appendix A.2.

3.5 Discussion

This section presents a discussion of game refinement considering mass.

Table 3.2 shows the generic formula of game refinement considering mass. We consider the average case to simplify the formula. In a match between players with same strength, $\frac{z}{Z}$ becomes $\frac{1}{2}$. On the other hand, the effective branching factor b becomes $\log B$. Therefore, the simplified formula are shown in Equation 3.5

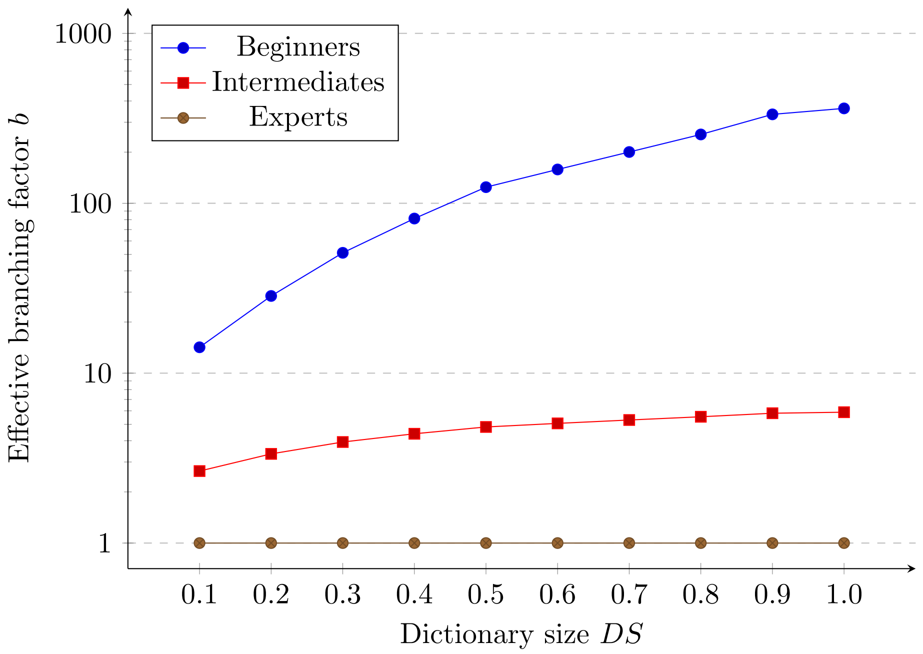

FIGURE 3.8: Impact of dictionary size on effective branching factor using board game model considering the mass

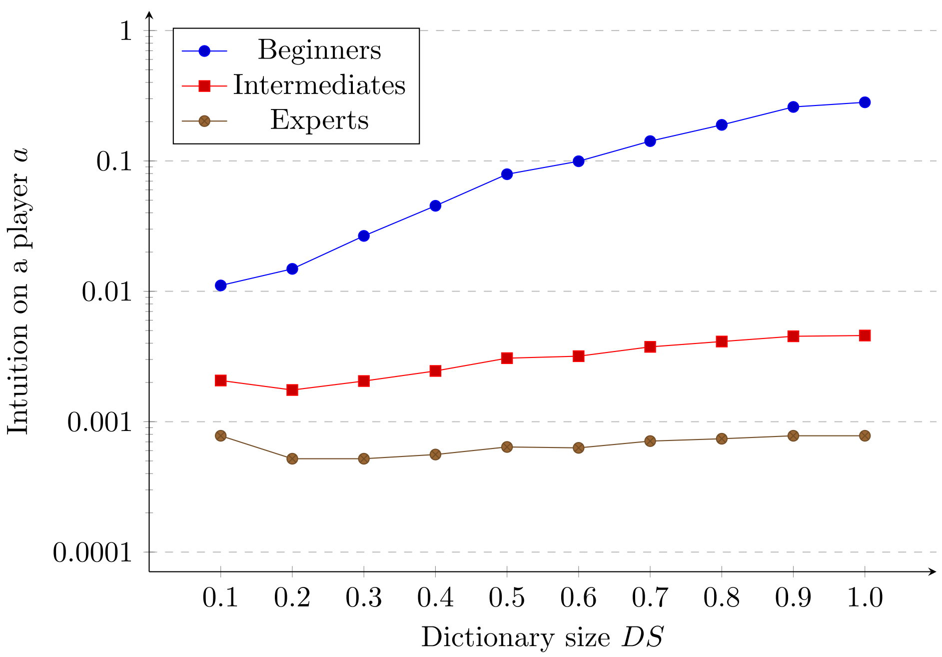

FIGURE 3.9: Impact of dictionary Size on intuition on a player using board game model considering the mass

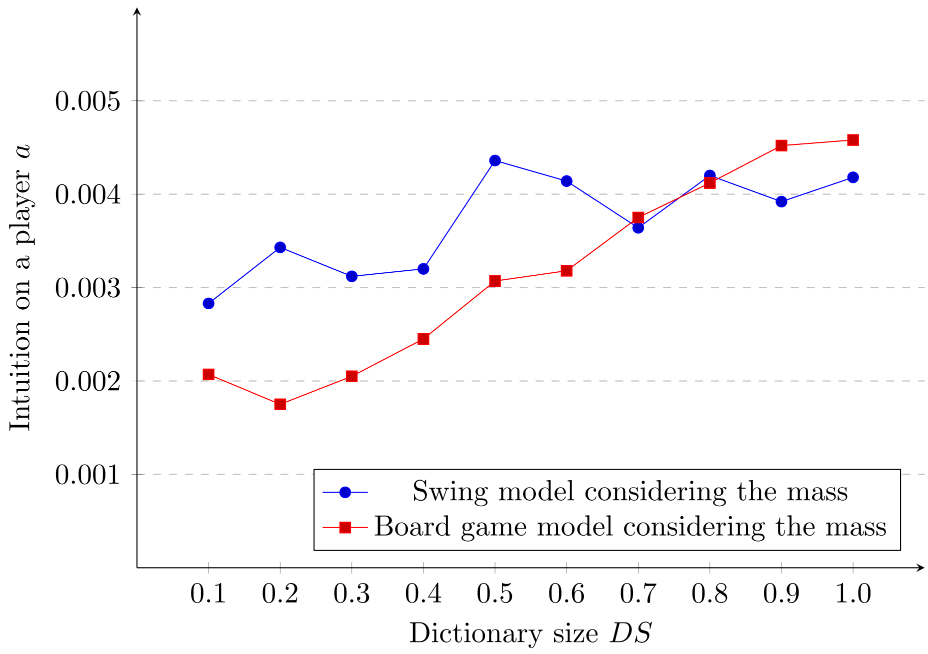

FIGURE 3.10: Comparison of swing Model considering the mass and board game model considering the mass, supposing average case

$$\begin{aligned} GRCM_{scoringgame} &= \frac{G}{2T^2} \\ GRCM_{boardgame} &= \frac{\log B}{D^2} \end{aligned} \tag{3.5}$$

After the existence of mass-in-mind is identified, the real intuition of specific domains are compared in Table 3.3. The refined zone of game refinement considering mass is suggested to be from 0.00245 to 0.00320. Besides, it is observed that actual intuition in a case of board game is much less than the refined zone, which means that it is more competitive but less entertaining.

TABLE 3.3: Correlative measures of game refinement considering the mass

Subject	*Legacy GR*	*GR considering mass*
Chinese chess	0.065	0.00056
Soccer	0.073	0.00266
Basketball	0.073	0.00266
Western chess	0.074	0.00056
Go	0.076	0.00013
Table tennis	0.077	0.00296
UNO®	0.078	0.00304
DotA®	0.078	0.00304
Shogi	0.078	0.00033
Badminton	0.086	0.00370
SCRABBLE (swing)	0.092	0.00418
SCRABBLE (board game)	0.531	0.00458

The possible interpretation of the $n(n-1)$ in Equation (2.3) has been discussed earlier [62]. In particular, they are called as C_s and C_b in a case of scoring game and board game respectively. Their mathematical models are given in Equation (3.6).

$$\begin{gathered} C_s = 1 \\ C_b = \frac{b}{B} \quad (\frac{1}{B} \leq C_b \leq 1) \end{gathered} \tag{3.6}$$

By the effect of the C_s and C_p, the game progress of a scoring game GR_s and a board game GR_b becomes Equation (3.7).

$$\begin{aligned} GR_s &= \frac{\sqrt{G}}{T} \\ GR_b &= \frac{\sqrt{b}}{D} \end{aligned} \tag{3.7}$$

By considering the mathematical formula, the C_b is as an inversion of the mass-in-mind. Hence, GR_b is identical to the root square of the acceleration from the board game model considering mass. However, the mass-in-mind of game progress model is not equivalent to C_s as it is defined as a constant. This method could enhance the completeness of the interpretation of the C parameter by redefining C_s, which would have some value, instead of being always 1. [62]. Although our interpretations are not exactly the same, we have the same results.

In reality, the branching factor B is not necessarily a constant for every player but expands with player strength. For instance, a novice board game player might not be able to recognize much possible branching factors as an expert does. Supposing the branching factor B is the complete branching factor regardless of a player considered, the perceived branching factor B' is defined as 'entire branching factor perceived by a player', in which $b \leq B' \leq B$. However, this is not yet regarded and $B' \approx B$ is assumed in this study.

Table 3.4: Comparison of mean percentage error MPE and mean absolute percentage error $MAPE$

Model	MPE	$MAPE$
Legacy GR	133.31%	133.31%
GR considering the mass	21.06%	26.47%

Compared to the prior work, the game refinement considering mass are greatly more accurate regardless of the error model used. However, the margin of errors is still noticeable, as shown in Table 3.4 and the possible causes are listed as follows.

- Theoretical approximation of the effective branching factor b
- The assumption that $B' \approx B$
- The assumption that $D' \approx D$
- Difference in AI and human characteristics
- The randomness within the simulation
- Insufficient sampling size

Although the accurate measurement has not been made, we suspect that the maximum rating of our artificial intelligence is about from 1,600 to 1,800, which approximately equates to 370th to 170th rank in WESPA international player ranking [71]. Nevertheless, the actual purpose of this study is not to develop the strongest artificial intelligent player, but to study the characteristics of SCRABBLE.

Chapter 4

Educational Aspect

This chapter presents an educational aspect in SCRABBLE.

Despite the fact that entertainment is the primary factor in playing a game, but there are still possibilities to consider the other factors in an extraordinary game, e.g., SCRABBLE. In this study, the educational essence is theoretically discussed with 'learning coefficient'.

Learning coefficient LC is a methodology to quantify an educational essence of a specific domain [21]. Its mathematical model involves complexity C and dictionary size DS.

4.1 Complexity

Let B and D be the average branching factor and average game length respectively. The measure of the search-space complexity [72] which specifying the total possible instances in the game is determined by B^D. The complexity C is defined identically to that but given in the natural logarithm formula. Hence, it is obtained by Equation (4.1).

$$C = D \log B \tag{4.1}$$

While complexity C expresses the complicatedness of the domain, the complexity from the perception of a player C_p expresses the comprehension degree of that player in that domain [21]. A player with higher complexity C_p tends to understand the domain clearly and profoundly, thus apparently makes a better decision. Supposing that B_p and D_p represent the average branching factor from the player perspective and the game length from the player perspective respectively, the C_p is obtained similarly in Equation (4.2).

$$C_p = D_p \log B_p \tag{4.2}$$

According to the game AI development history, the dominant AI of several games were released from time to time. That of Tic-tac-toe is undoubtedly simple to implement, thus was released very early [72]. However, that of chess was far more challenging therefore took many years after. In May 1997, Chess computer Deep Blue [73] won a world champion, Garry Kasparov. Yet, that of Go, which is known to be one of the most complicated board game toward history, was even more challenging. In May 2017, Go computer AlphaGo [49] won a world champion Ke Jie. Even so, Go was still not yet completely solved, which means that there is a possibility to develop an AI which is even stronger. Therefore, the difficulty in AI development implies the fact that the complexity of Tic-tac-toe is far less than that of chess. Similarly, the complexity of chess is far less than that of Go, as shown in Table 4.1.

TABLE 4.1: Correlative measures of complexity

Game	*Branching factor*	*Game length*	*Complexity*
Tic-tac-toe	≤ 9	≤ 9	≤ 19.775
Chess	35	80	284.428
Go	250	208	1148.464

4.2 Learning Coefficient

During the experiments performed with various dictionary size DS and player strength LV, we observed that the complexity measure conforms a linear relation with player strength, thus a slope M can be measured. Supposing T is the total words in the standard dictionary, the whole words in the modified dictionary T' can be obtained by Equation (4.3).

$$T' = DS \times T \tag{4.3}$$

By learning x new words, the player strength will increase. By applying the above equation, the advanced player strength LV' becomes Equation (4.4).

$$LV' = LV + \frac{x}{T'} = LV + \frac{x}{DS \times T} \tag{4.4}$$

By the definition of the slope M, it can be obtained by the proportion of the difference of complexity and the variation of player strength, as shown in Equation (4.5).

$$M = \frac{\Delta C}{\Delta LV} = \frac{\Delta C}{LV' - LV} = \frac{\Delta C}{LV + \frac{x}{DS \times T} - LV} = \frac{\Delta C}{\frac{x}{DS \times T}} = \frac{\Delta C \times DS \times T}{x} \tag{4.5}$$

However, the newly learned words x and total words T are uncontrollable constants. Therefore, we attempt to maximize ΔC as it represents the improvement of a learner for the same condition. In short, the measure $\frac{M}{DS}$, which we call 'learning coefficient' or LC, is considered, as shown in Equation (4.6).

$$LC = \frac{M}{DS} \tag{4.6}$$

Figure 4.1 and Figure 4.2 illustrate the result of applying complexity and learning coefficient to SCRABBLE for some circumstances.

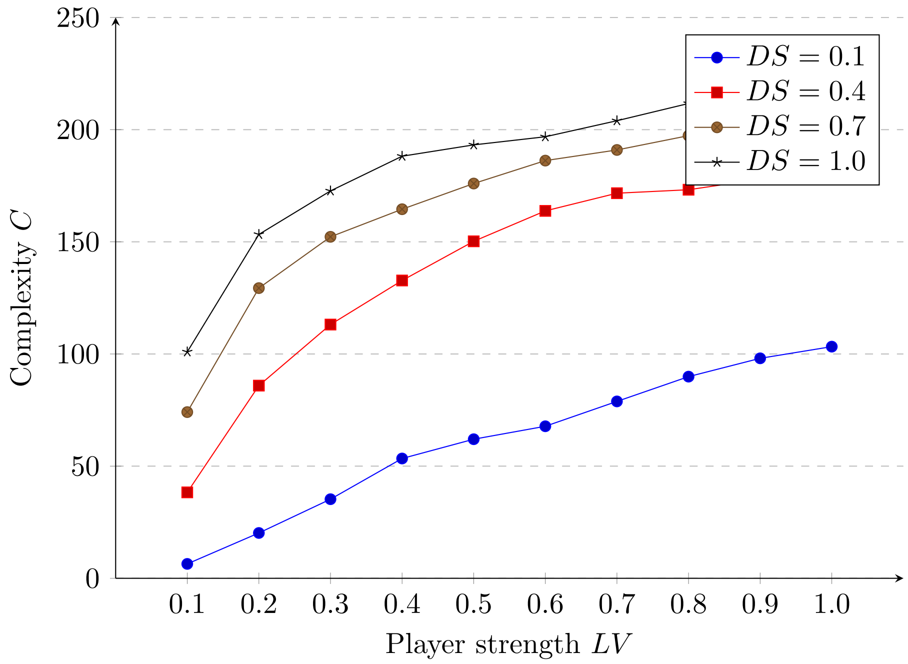

FIGURE 4.1: Impact of player strength on complexity

By using only 4% to 6% size of the standard dictionary, we obtain SCRABBLE with the highest learning coefficient. These are equivalent to 7,200 to 10,700 words respectively.

4.3 Discussion

This section presents a discussion of learning coefficient.

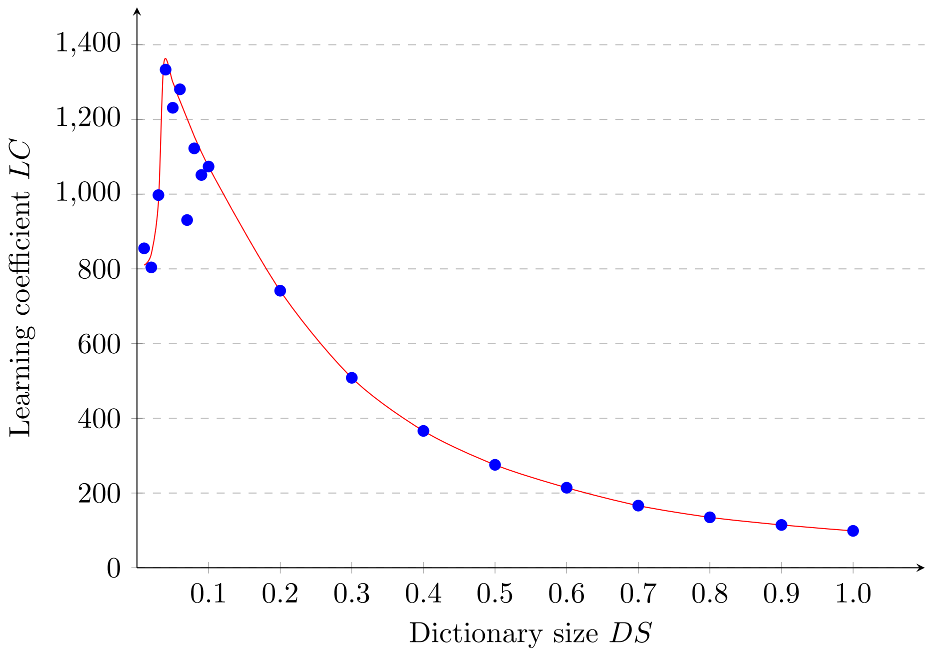

FIGURE 4.2: Impact of player strength on learning coefficient

The prior study indicates the success in using an online game as a core part of teaching English as a foreign language [6]. Vocabulary plays an essential role in learning language as it describes the meaning of the word and enhance four language skills, or listening, reading, writing, and speaking [74, 75].

English as a foreign language learners know 4,500 words [76], which covers 86.8% to 88.7% of the text. The modified dictionary will cover up to 94%, which is almost identical to the minimum requirement for understanding reading comprehension or 95% [38]. Besides, it is close to the vocabulary size the foreign test-takers have, which tends to reach over 10,000 words by living abroad [76]. Figure 4.3 shows the relation between vocabulary size and text coverage percentage.

However, there are several limitations regarding the proposed model. Currently, it is usable only for SCRABBLE or other domains with dictionary system integrated. Also, we did not know yet about the upper limit of this measure. Besides, this lacks experimental data, which is to be investigated in the future.

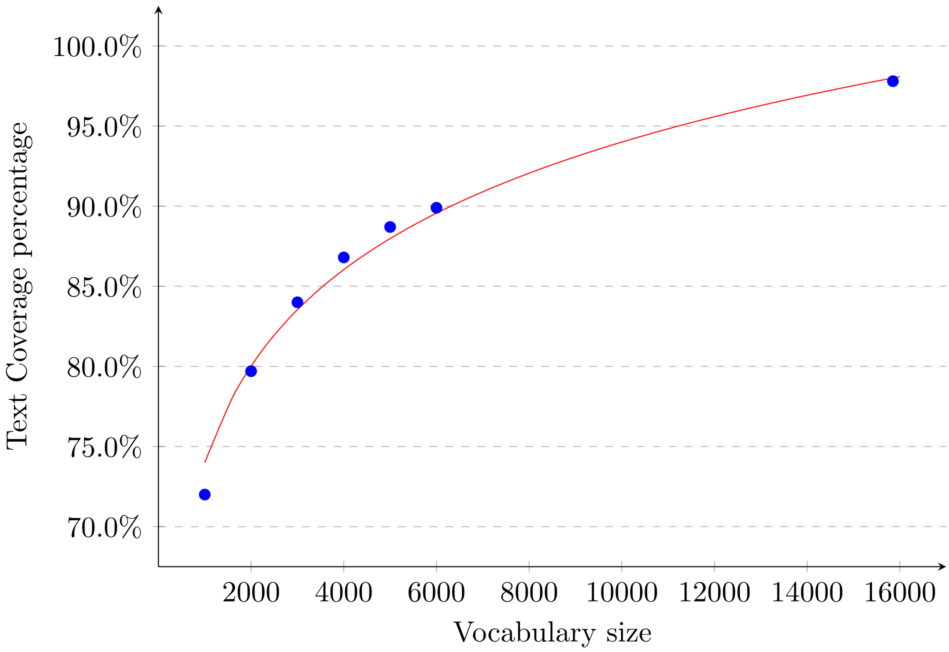

FIGURE 4.3: Impact of vocabulary Size on text coverage [1]

Chapter 5

Conclusion

This chapter presents an overall summary and the possible direction for future work.

5.1 Concluding Remarks

Scrabble is a scoring game played on the board, which involves utilization of English alphabets. An attempt to find its possible enhancement is the primary concern of this study. The goal is to indicate a direction to achieve entertaining and beneficial game based on Scrabble settings.

This work directly involves the handy framework to quantify engagement of a subject interested, known as game refinement theory. A mathematical model was proposed on the concept of game outcome uncertainty and Newtonian physics-in-mind. It bridges a gap between the board game and the scoring game. Prior works suggest that several famous games shared the related amount between 0.07 and 0.08, which we call 'sophisticated zone'. Although game refinement theory is imperfect and not be able to explain everything, many things could be described with it.

Scrabble is an exceptional test-bed that both aspects: the scoring game and the board game, are fit with game refinement measure. However, the scoring system in Scrabble is non-uniformed and not directly compatible with game progress model. Instead, the swing model is proposed by deriving from it.

According to game refinement theory, Scrabble is considered as an amusing game, which is more dependable on luck. Although the original setting is comfortable for native speakers, the excessive branching factor may discourage language learners, which results in unbalanced players distribution.

Several methods were successfully proposed to shrink the branching factor. These include dictionary size and board size reduction, which is given in Appendix A.3. It is expected that the modification is more favorable for language learners.

After analyzing the data from the application, we then realized the inconsistency between two legacy models. This error inspired us to identify the mass, an essential property in physics. The possible interpretation of the mass-in-mind is given as 'a shift in the perceived challenge'. The concrete mathematical models are constructed for the practical use. The formula for the swing model and the board game model are obtained by the proportion of gained score and the branching factor reduction rate respectively. We applied the state-of-the-art models to SCRABBLE to verify the consistency. The margin of error significantly decreased compared to the prior model.

To measure the educational benefit, the mathematical model learning coefficient is constructed. The interpretation is given as a capability of utilizing knowledge.

Result shows that SCRABBLE with only 4% to 6% yield the highest learning coefficient. Currently, higher learning coefficient is suspected to be better. However, we do not know yet the upper limit of learning coefficient, which may lead to frustration as well.

By supposing Scrabble is implemented as a computer game, it is possible to apply various gamification techniques into it. For example, by taking advantage of artificial intelligence, we can deliver appropriate daily or weekly vocabulary learning goal to each individual player. Also, the approximate vocabulary size would be made visible to player, which will update automatically after matches as the way rating system does. These are called 'sub-goal' [77] and 'immediate feedback' [78] techniques, which can encourage player to play and learn more.

TABLE 5.1: Summary of SCRABBLE modifications based on legacy *GR* and *LC*

Focus	*Size*	*DS*	*GR*	*GR Tendency*	*LC*
Standard	15x15	1.00	0.0751 - 0.0926	Inc then dec	98.7323
Entertainment	13x13	1.00	0.0771 - 0.0808	Dec then inc	140.9987
Education	15x15	0.04 - 0.06	0.0951 - 0.3843	Dec	1333.335
Balance	15x15	0.10	0.0731 - 0.2204	Dec	1073.907

In conclusion, Table 5.1 presents the possible enhancement to SCRABBLE for each particular use.

5.2 Future Works

This section presents the future work raised from this study.

In this study, the assessment of SCRABBLE primarily relies on the computer simulation and theoretical hypotheses. This cannot be completed without any evidence from human data. The modified SCRABBLE need to be concretely developed, then being evaluated with the traditional approaches, e.g., questionnaire. After all, the SCRABBLE improvement process is summed up then can be generalized to other domains as well. Besides, it is possible to design a new game, which entirely taking advantage of educating and entertaining.

This study is a first attempt to identify the mathematical model of the mass-in-mind in SCRABBLE. The interpretation may subject to change when further investigation and verification from applications to other domains are done.

As previously mentioned, the game refinement considering mass still has the remarkable error, which is possible to reduce by increasing the quality of the simulation. First, the study of the actual branching factor B' is necessary to be explored to achieve a better outcome. Second, the current implementation of the AI does not consider rack evaluation as the way an experienced player does. This fact causes the computer player to behave closer to an apprentice, then results in increasing inconsistency. Third, the more accurate of the approximation model for effective branching factor b and potential-to-swing count D' should be considered as well. Fourth, increasing the sampling size would improve the output stability and minimize the possible randomness.

Once the mathematical model of the mass-in-mind is established, it is possible to discuss other physical quantities e.g. power, energy, and momentum. The energy and the momentum are interesting arguments as they have particular characteristics and are in the law of conservation.

Chapter 6

Curriculum Vitae

Suwanviwatana Kananat is a software developer who holds strong interests in the game creation. He was born in September 1992 in the territory of Thailand. He began learning to develop a game in a secondary school. Meanwhile, his major was mathematics and science program. Then, he enrolled in the department of computer engineering, faculty of engineering, Chulalongkorn University to sharpen his skills. After graduation, he employed as a game developer for Donuts Bangkok, a branch office from a company in Japan. During that, he had experienced many valuable things, including practical use, self-improvement and working in a team. He is familiar with using the Unity3d game engine and C# programming language. Later, he had a chance to meet and being interviewed by Ph.D. Iida Hiroyuki, a specialist from a game research area. After the decision has been made, he went to study a master degree at Japan Advanced Institute of Science and Technology. He mainly focuses on an analysis of gamification on an educational domain, in which Scrabble was considered a test-bed. Upon assistance from former company and advisor, he had an internship at Donuts Japan Headquarters and finally obtained the job offer, which scheduled to begin from April 2018. Besides, he is also interested in becoming multilingual, thus study Japanese and Chinese enthusiastically.

Chapter 7

Publications

Kananat, Suwanviwatana, Jean-Christophe Terrillon, and Hiroyuki Iida. "Gamification and Scrabble." International Conference on Games and Learning Alliance. Springer, Cham, 2016.

Suwanviwatana, Kananat, and Hiroyuki Iida. "First Results from Using Game Refinement Measure and Learning Coefficient in SCrABBlE." arXiv preprint arXiv:1711.03580 (2017).

Kananat, Suwanviwatana, Jean-Christophe Terrillon, and Hiroyuki Iida. "Possible Interpretation of Mass-in-Mind: A Case Study Using Scrabble." (Accepted as regular paper, January 2018)

Appendix A

An Appendix

A.1 An Application of Legacy Game Refinement to Scrabble

This appendix section presents additional information of an application of legacy game refinement to SCRABBLE.

The data using legacy board game in several circumstances are shown in Figure A.1 and Figure A.2.

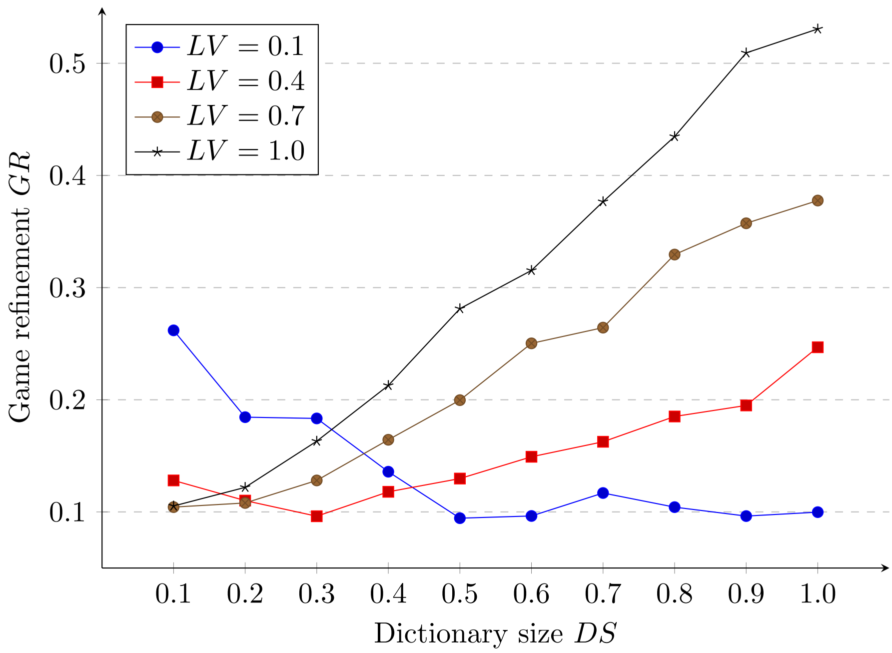

FIGURE A.1: Impact of dictionary size on game refinement using legacy board game model

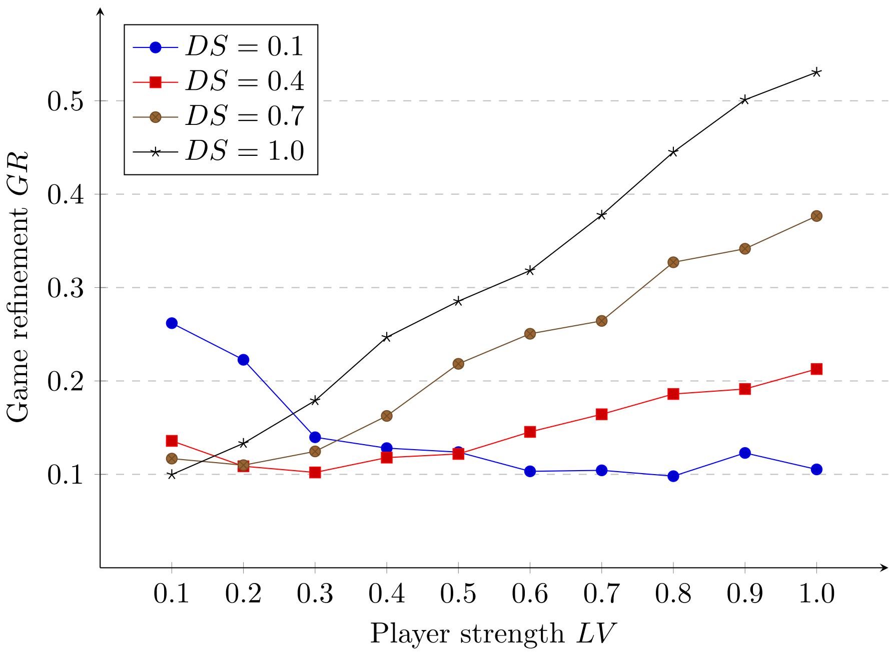

FIGURE A.2: Impact of player strength on game refinement using legacy board game model

The fully visualized data using legacy swing model and board game model are shown in Figure A.3 and Figure A.4 respectively.

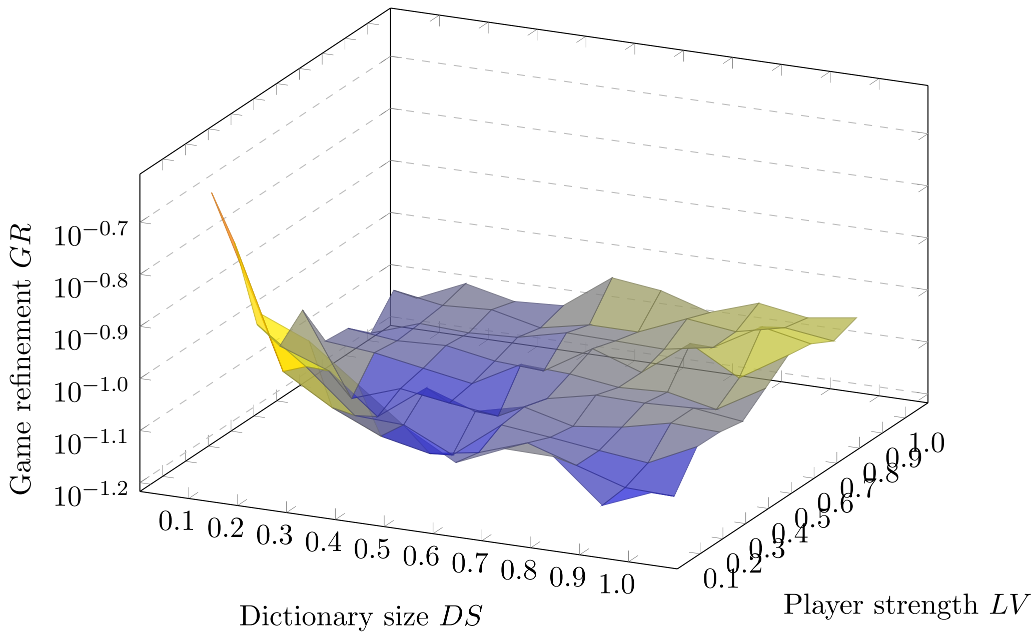

FIGURE A.3: Fully visualized data of an application of legacy swing model

Percentage error [79] PE has been one of the standard models to estimate the error in the scientific research and numerical analysis. The lower percentage error PE indicates that the estimator is more precise. The formula is given in Equation (A.1).

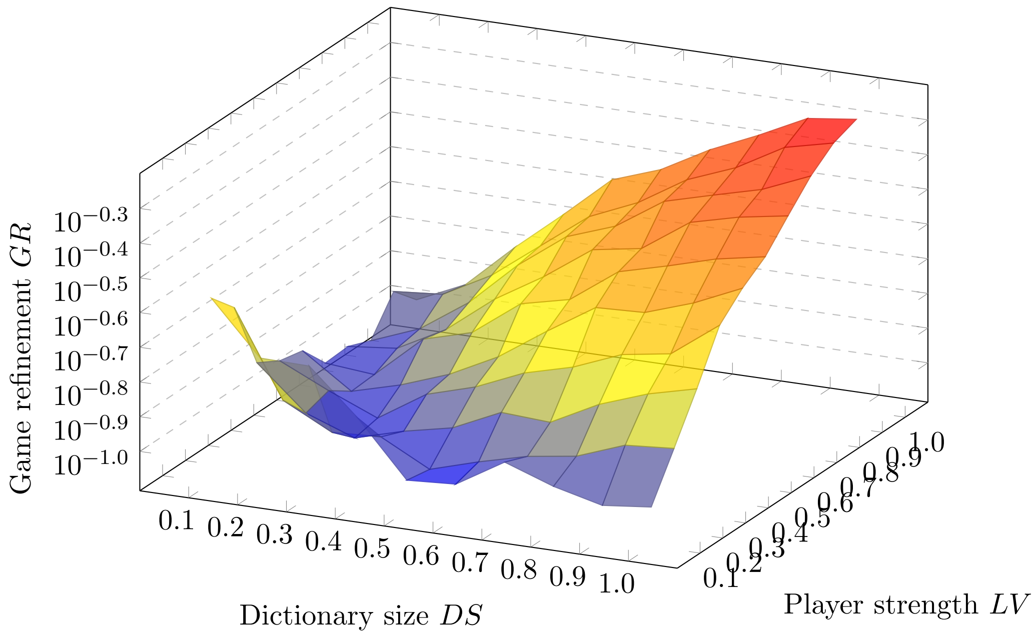

FIGURE A.4: Fully visualized data of an application of legacy board game model

$$PE = 100\% \times |1 - \frac{v_{experimental}}{v_{theoretical}}| \tag{A.1}$$

In this study, PE is used to calculate the error between 2 models. Let v_{swm} and v_{bgm} are the results from the swing model and the board game model respectively, the percentage error PE is obtained by Equation (A.2)

$$PE = 100\% \times |1 - \frac{v_{bgm}}{v_{swm}}| \tag{A.2}$$

Then, the percentage error PE is illustrated in Figure A.5.

A.2 An Application of Game Refinement considering Mass to Scrabble

The fully visualized data using swing model considering mass and board game model considering mass are shown in Figure A.6 and Figure A.7.

Then, the percentage error PE is illustrated in Figure A.8.

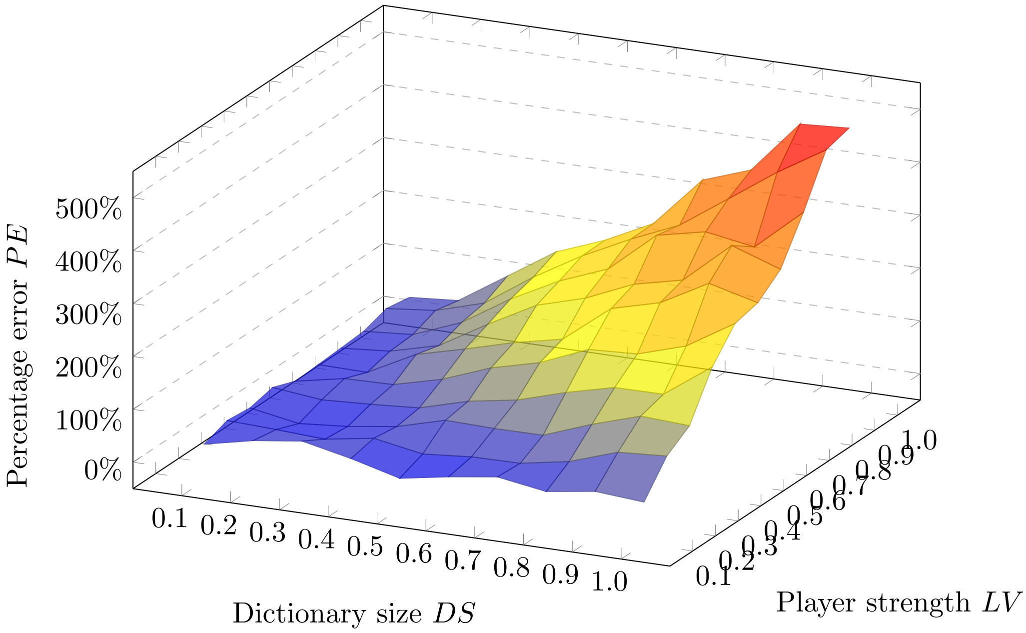

FIGURE A.5: Percentage error of 2 legacy game refinement measures

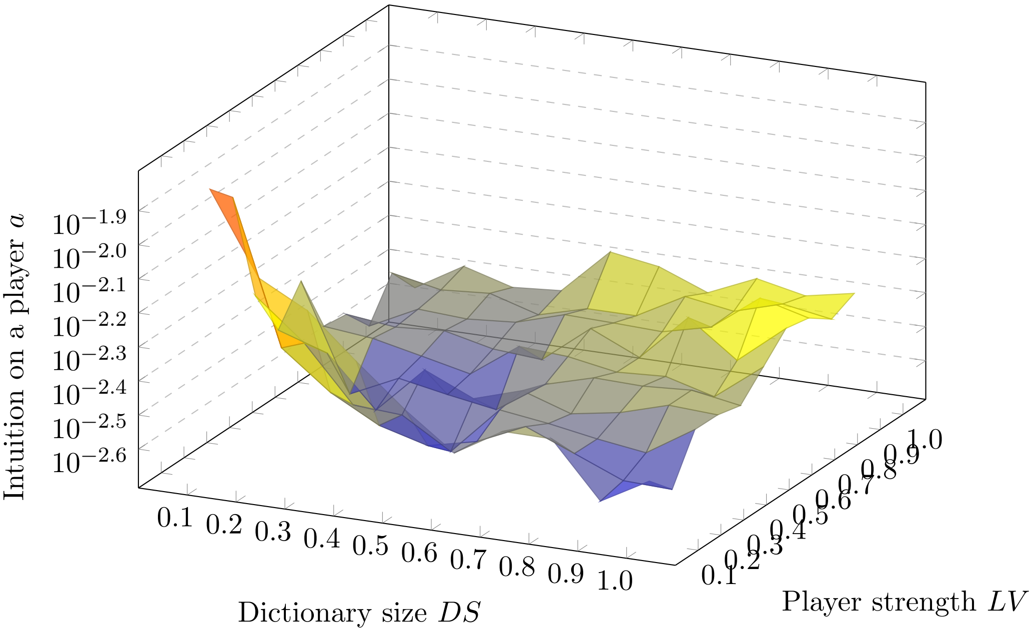

FIGURE A.6: Fully visualized data of an application of swing model considering the mass

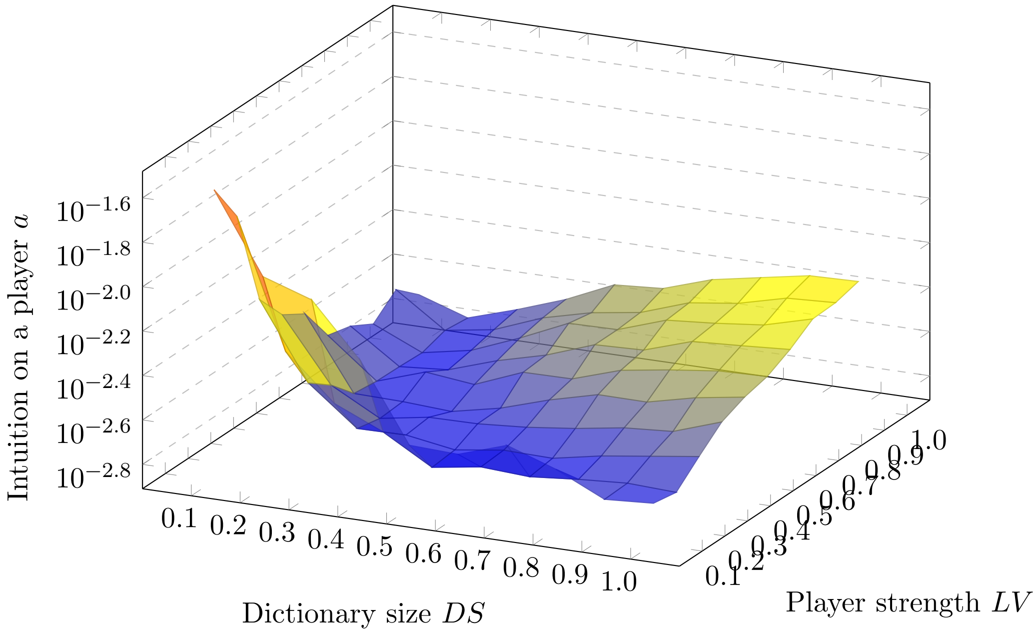

FIGURE A.7: Fully visualized data of an application of board game model considering the mass

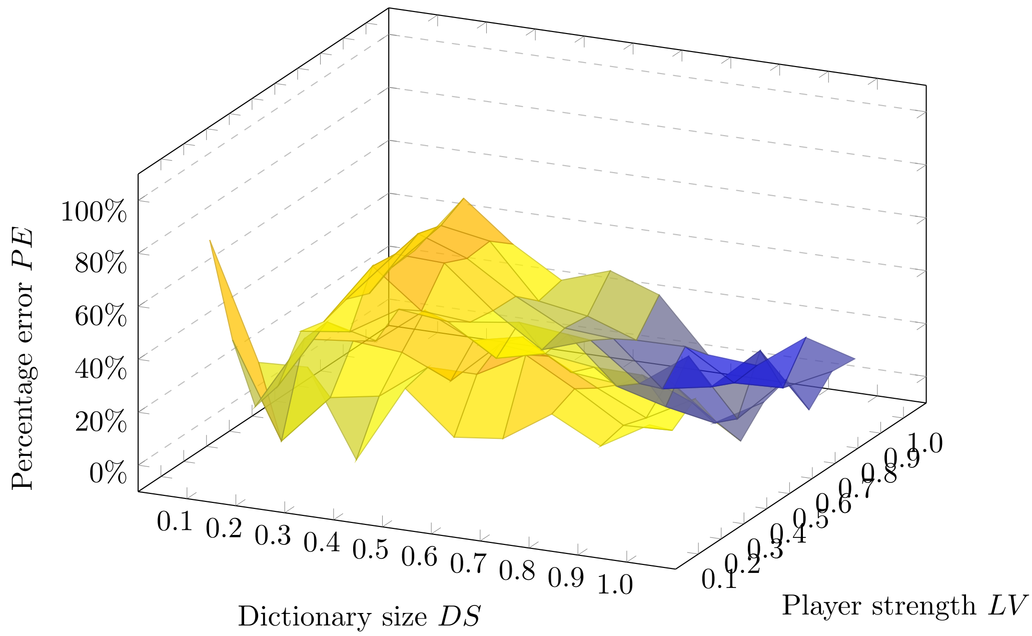

FIGURE A.8: Percentage error of 2 game refinement measures considering the mass

A.3 Shrunk Scrabble

This appendix section presents the analysis of using 13x13 Scrabble, which results in 24.89% smaller than the original.

We knew from the earlier discussion that massive branching factor in Scrabble may frustrate beginners and language learners. Previously, we showed the possible enhancement, which focuses on decreasing the dictionary size as well as the branching factor. However, there is another direction, in which size of the board is concerned. This is a common technique as seen in a case of the board game where its complexity is excessively high. For a beginner, 9x9 Go is used instead of the original 19x19. In this study, 11x11 and 13x13 version of Scrabble are constructed and discussed. The 13x13 Scrabble board is designed with the similar pattern, shown in A.9.

2W				3L				3L				2W
	2W				2L		2L				2W	
		2W				2L				2W		
			2W						2W			
3L				3L				3L				3L
	2L				2L		2L				2L	
		2L				2W				2L		
	2L				2L		2L				2L	
3L				3L				3L				3L
			2W						2W			
		2W				2L				2W		
	2W				2L		2L				2W	
2W				3L				3L				2W

Figure A.9: 13x13 Scrabble board

Interestingly, the 13x13 Scrabble yields an attractive output as it is much close to the sophisticated zone, as shown in Figure A.10.

However, the 11x11 Scrabble does not show any interesting result and left off from the consideration.

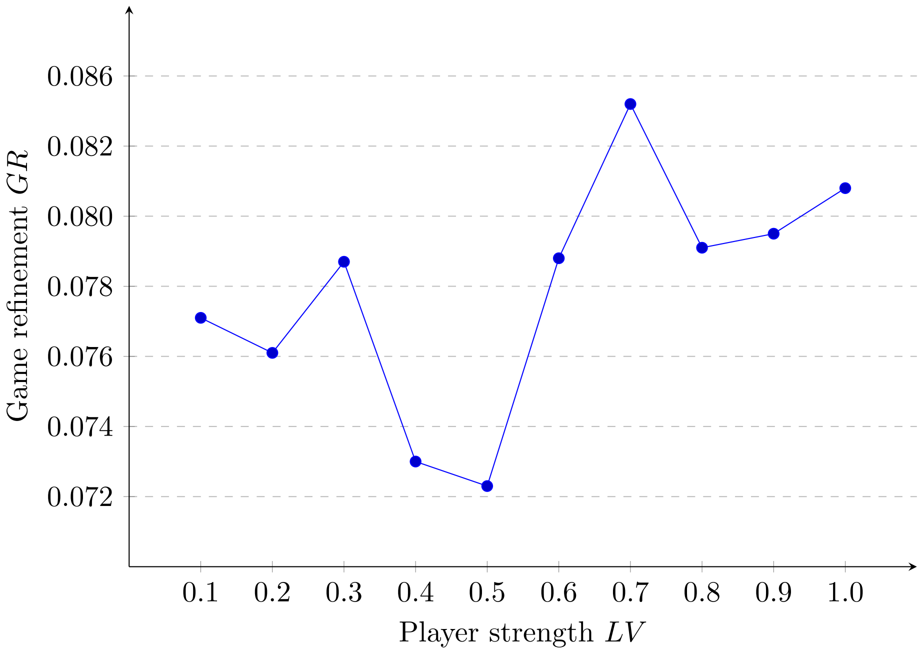

FIGURE A.10: Impact of player strength on legacy game refinement in 13x13 SCRABBLE

Bibliography

[1] Paul Nation and Robert Waring. Vocabulary size, text coverage and word lists. *Vocabulary: Description, acquisition and pedagogy*, 14:6–19, 1997.

[2] Cross-tables.com - scrabble®tournaments, statistics, and community, 2018. URL http://www.cross-tables.com. (Accessed on 7/12/2017).

[3] Sebastian Deterding, Dan Dixon, Rilla Khaled, and Lennart Nacke. From game design elements to gamefulness: defining gamification. In *Proceedings of the 15th international academic MindTrek conference: Envisioning future media environments*, pages 9–15. ACM, 2011.

[4] Ray Huling. Gamification: Turning work into play. *H Plus Magazine*, 2010.

[5] Cristina Ioana Muntean. Raising engagement in e-learning through gamification. In *Proc. 6th International Conference on Virtual Learning ICVL*, volume 1. sn, 2011.

[6] Florence WM Yip and Alvin CM Kwan. Online vocabulary games as a tool for teaching and learning english vocabulary. *Educational media international*, 43(3): 233–249, 2006.

[7] Kai Huotari and Juho Hamari. Defining gamification: a service marketing perspective. In *Proceeding of the 16th international academic MindTrek conference*, pages 17–22. ACM, 2012.

[8] Pedro Pereira, Emília Duarte, Francisco Rebelo, and Paulo Noriega. A review of gamification for health-related contexts. In *International conference of design, user experience, and usability*, pages 742–753. Springer, 2014.

[9] MIKHAIL VASILIEVICH VINICHENKO, ALEXANDER VASILIEVICH MELNICHUK, ANDREI VLADIMIROVICH KIRILLOV, SERGEY ANATOLYEVICH MAKUSHKIN, and YULYIA ALEXANDROVNA MELNICHUK. Modern views on the gamification of business. *The Journal of Internet Banking and Commerce*, 1970.

[10] Angela D Friederici. The brain basis of language processing: from structure to function. *Physiological reviews*, 91(4):1357–1392, 2011.

[11] James V Wertsh and Peter Tulviste. Apprenticeship in thinking: Cognitive development in social context. *Science*, 249(4969):684–686, 1990.

[12] Rebecca Piekkari, Denice Welch, and Lawrence S Welch. *Language in international business: The multilingual reality of global business expansion*. Edward Elgar Publishing, 2014.

[13] Anne Merritt. Why learn a foreign language? benefits of bilingualism. URL http://www.telegraph.co.uk/education/educationopinion/10126883/Why-learn-a-foreign-language-Benefits-of-bilingualism.html. (Accessed on 11/24/2017).

[14] Peter F Drucker. The discipline of innovation. *Harvard business review*, 63(3): 67–72, 1985.

[15] Sheila Tucker. Distance education: Better, worse, or as good as traditional education?. *Online journal of distance learning administration*, 4(4):n4, 2001.

[16] Joanne Gikas and Michael M Grant. Mobile computing devices in higher education: Student perspectives on learning with cellphones, smartphones & social media. *The Internet and Higher Education*, 19:18–26, 2013.

[17] Duy Huynh, Long Zuo, and Hiroyuki Iida. Analyzing gamification of duolingo with focus on its course structure. In *Games and Learning Alliance*, pages 268–277. Springer, 2016.

[18] Hiroyuki Iida, Kazutoshi Takahara, Jun Nagashima, Yoichiro Kajihara, and Tsuyoshi Hashimoto. An application of game-refinement theory to mah jong. In *International Conference on Entertainment Computing*, pages 333–338. Springer, 2004.

[19] Peter Májek and Hiroyuki Iida. Uncertainty of game outcome. In *3rd International Conference on Global Research and Education in Intelligent Systems*, pages 171–180, 2004.

[20] Arie Pratama Sutiono, Ayu Purwarianti, and Hiroyuki Iida. A mathematical model of game refinement. In *International Conference on Intelligent Technologies for Interactive Entertainment*, pages 148–151. Springer, 2014.

[21] Kananat Suwanviwatana and Hiroyuki Iida. First results from using game refinement measure and learning coefficient in scrabble. *arXiv preprint arXiv:1711.03580*, 2017.

[22] Scrabble — word games — board games — scrabble online, . URL https://scrabble.hasbro.com/en-us. (Accessed on 1/30/2017).

[23] Scrabble rules — official word game rules — board games, . URL https://scrabble.hasbro.com/en-us/rules. (Accessed on 1/30/2017).

[24] Scrabble history — making of the classic american board game, . URL https://scrabble.hasbro.com/en-us/history. (Accessed on 1/13/2018).

[25] Cosimo Cardellicchio. Evolution for games. *Board Game Studies Journal*, (8):1–2, 2014.

[26] N Sasaki, N Takeshita, T Hashimoto, and H Iida. Decision-complexity estimate in evolutionary changes of games. In *Game Programming Workshop*, pages 140–147, 2001.

[27] Dota 2 - play for free. URL http://www.dota2.com/. (Accessed on 1/31/2018).

[28] Norizan Mat Diah, Arie Pratama Sutiono, Long Zuo, Nathan Nossal, Hiroyuki Iida, Nor Azan, and Mat Zin. Quantifying engagement of video games: Pac-man and dota (defense of the ancients). In *17th International Conference on Mathematical and Computational Methods in Science and Engineering (MACMESE15), Kuala Lumpur*, 2015.

[29] Official tournament rules - naspawiki, . URL http://www.scrabbleplayers.org/w/Rules. (Accessed on 1/30/2017).

[30] Suwanviwatana Kananat, Jean-Christophe Terrillon, and Hiroyuki Iida. Gamification and scrabble. In *Games and Learning Alliance*, pages 405–414. Springer, 2016.

[31] Duplicate scrabble, . URL http://www.poslfit.com/scrabble/variants/duplicate. (Accessed on 1/30/2018).

[32] David Bukszpan. *Is That a Word?: From AA to ZZZ, the Weird and Wonderful Language of SCRABBLE*. Chronicle Books, 2012.

[33] Super scrabble, . URL https://www.boardgamegeek.com/image/52794/super-scrabble. (Accessed on 1/30/2018).

[34] Brian Sheppard. World-championship-caliber scrabble scrabble® is a registered trademark. all intellectual property rights in and to the game are owned in the usa by hasbro inc., in canada by hasbro canada corporation, and throughout the rest of the world by jw spear & sons limited of maidenhead, berkshire, england, a subsidiary of mattel inc. *Artificial Intelligence*, 134(1-2):241–275, 2002.

[35] Rene De La Briandais. File searching using variable length keys. In *Papers presented at the the March 3-5, 1959, western joint computer conference*, pages 295–298. ACM, 1959.

[36] Wespa tournament calendar, . URL http://www.wespa.org/tournaments/index.shtml. (Accessed on 1/30/2018).

[37] Scrabble: An entertaining way to improve your child's vocabulary and spelling skills, . URL http://mathandreadinghelp.org/articles/Scrabble%3A_An_Entertaining_Way_to_Improve_Your_Child%27s_Vocabulary_and_Spelling_Skills.html. (Accessed on 11/24/2017).

[38] Batia Laufer. How much lexis is necessary for reading comprehension? In *Vocabulary and applied linguistics*, pages 126–132. Springer, 1992.

[39] Claudia Dreifus. The bilingual advantage. *The New York Times*, 30:D12, 2011.

[40] John C Harsanyi, Reinhard Selten, et al. A general theory of equilibrium selection in games. *MIT Press Books*, 1, 1988.

[41] Mihaly Csikszentmihalyi. *Beyond boredom and anxiety.* Jossey-Bass, 2000.

[42] Werner Loewenstein. *Physics in mind: a quantum view of the brain.* Basic Books (AZ), 2013.

[43] B Myerson Roger. Game theory: analysis of conflict. *The President and Fellows of Harvard College, USA*, 1991.

[44] Michel Willem. *Minimax theorems*, volume 24. Springer Science & Business Media, 1997.

[45] Stuart Jonathan Russell, Peter Norvig, John F Canny, Jitendra M Malik, and Douglas D Edwards. *Artificial intelligence: a modern approach*, volume 2. Prentice hall Upper Saddle River, 2003.

[46] Julia Robinson. An iterative method of solving a game. *Annals of mathematics*, pages 296–301, 1951.

[47] Philip Henderson. Playing and solving the game of hex. 2010.

[48] Jonathan Schaeffer and Robert Lake. Solving the game of checkers. *Games of no chance*, 29:119–133, 1996.

[49] David Silver, Aja Huang, Chris J Maddison, Arthur Guez, Laurent Sifre, George Van Den Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot, et al. Mastering the game of go with deep neural networks and tree search. *Nature*, 529(7587):484–489, 2016.

[50] Mihaly Csikszentmihalyi. Toward a psychology of optimal experience. In *Flow and the foundations of positive psychology*, pages 209–226. Springer, 2014.

[51] Owen Schaffer. Crafting fun user experiences: A method to facilitate flow. *Human Factors International*, 2013.

[52] Giovanni B Moneta. On the measurement and conceptualization of flow. In *Advances in flow research*, pages 23–50. Springer, 2012.

[53] Todd B Kashdan, William E Breen, and Terri Julian. Everyday strivings in war veterans with posttraumatic stress disorder: Suffering from a hyper-focus on avoidance and emotion regulation. *Behavior therapy*, 41(3):350–363, 2010.

[54] Mihaly Csikzentmihalyi. Flow: The psychology of happiness. *Rider. London*, 1992.

[55] Suwanviwatana Kananat, Jean-Christophe Terrillon, and Hiroyuki Iida. Possible interpretation of mass-in-mind: A case study using scrabble. 2018.

[56] Staffan Bjork and Jussi Holopainen. Patterns in game design (game development series). 2004.

[57] Christof Neumann, Julie Duboscq, Constance Dubuc, Andri Ginting, Ade Maulana Irwan, Muhammad Agil, Anja Widdig, and Antje Engelhardt. Assessing dominance hierarchies: validation and advantages of progressive evaluation with elo-rating. *Animal Behaviour*, 82(4):911–921, 2011.

[58] Alfian Ramadhan, Hiroyuki Iida, and Nur Ulfa Maulidevi. Game refinement theory and multiplayer games: case study using uno. 2015.

[59] Junki Takeuchi, Rido Ramadan, and Hiroyuki Iida. Game refinement theory and its application to volleyball. *Research Report 2014-GI-31 (3), Information Processing Society of Japan*, pages 1–6, 2014.

[60] Nathan Nossal and Hiroyuki Iida. Game refinement theory and its application to score limit games. In *Games Media Entertainment (GEM), 2014 IEEE*, pages 1–3. IEEE, 2014.

[61] Judith D Sally. *Roots to research: a vertical development of mathematical problems.* American Mathematical Soc., 2007.

[62] Shuo Xiong, Long Zuo, and Hiroyuki Iida. Possible interpretations for game refinement measure. In *International Conference on Entertainment Computing*, pages 322–334. Springer, 2017.

[63] Arie Pratama Sutiono, Rido Ramadan, Peetikorn Jarukasetporn, Junki Takeuchi, Ayu Purwarianti, and Hiroyuki Iida. A mathematical model of game refinement and its applications to sports games. 2015.

[64] Shuo Xiong, Long Zuo, Rachaya Chiewvanichakorn, and Hiroyuki Iida. Quantifying engagement of various games. In *The 19th Game Programming Workshop 2014*. Information Processing Society of Japan, 2014.

[65] Long Zuo, Shuo Xiong, and Hiroyuki Iida. An analysis of hotel loyalty program with a focus on the tiers and points system.

[66] Ivar Jacobson, Grady Booch, James Rumbaugh, James Rumbaugh, and Grady Booch. *The unified software development process*, volume 1. Addison-wesley Reading, 1999.

[67] ISO/IEC 19505-2:2012. Iso/iec 19505-2:2012 - information technology – object management group unified modeling language (omg uml) – part 2: Superstructure, 2012. URL https://www.iso.org/standard/52854.html. (Accessed on 1/13/2017).

[68] Hiroyuki Iida, Nobuo Takeshita, and Jin Yoshimura. A metric for entertainment of boardgames: its implication for evolution of chess variants. In *Entertainment Computing*, pages 65–72. Springer, 2003.

[69] Max Jammer. *Concepts of mass in classical and modern physics*. Courier Corporation, 1997.

[70] Lev B Okun. The concept of mass. In *Energy and Mass in Relativity Theory*, pages 11–16. World Scientific, 2009.

[71] Wespa international rankings, . URL http://www.wespa.org/aardvark/cgi-bin/rating.cgi. (Accessed on 1/19/2018).

[72] Louis Victor Allis et al. *Searching for solutions in games and artificial intelligence*. Ponsen & Looijen, 1994.

[73] Murray Campbell, A Joseph Hoane, and Feng-hsiung Hsu. Deep blue. *Artificial intelligence*, 134(1-2):57–83, 2002.

[74] Mofareh Alqahtani et al. The importance of vocabulary in language learning and how to be taught. *International Journal of Teaching and Education*, 3(3):21–34, 2015.

[75] Kenneth Chastain. Developing second language skills. *Theory and practice*, 3, 1988.

[76] Test your vocab - how many words do you know? URL http://testyourvocab.com/blog/2013-05-10-Summary-of-results. (Accessed on 1/21/2018).

[77] Darren C Treasure and Glyn C Roberts. Applications of achievement goal theory to physical education: Implications for enhancing motivation. *Quest*, 47(4):475–489, 1995.

[78] Lizzy Bleumers, Anissa All, Ilse Mariën, Dana Schurmans, Jan Van Looy, An Jacobs, Koen Willaert, and Frederik De Grove. State of play of digital games for empowerment and inclusion: a review of the literature and empirical cases. *European Comission. Doi*, 10:36295, 2012.

[79] Olgierd C Zienkiewicz and Jian Z Zhu. A simple error estimator and adaptive procedure for practical engineering analysis. *International journal for numerical methods in engineering*, 24(2):337–357, 1987.