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Game Sophistication Analysis: case study using e-Sports Games and TETRIS

By Mingyang Wu

A thesis submitted to
School of Information Science,
Japan Advanced Institute of Science and Technology,
in partial fulfillment of the requirements
for the degree of
Master of Information Science
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Written under the direction of
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March, 2018

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

Game is a structured form of play, usually undertaken for entertainment and sometimes used for education. Any competition or something for fun can be attributed to game. In Katie Salen's book, according to Huizinga's idea [22], the definition of play is described as follow:

> *Summing up the formal characteristics of play we might call it a free activity standing quite consciously outside "ordinary" life as being "not serious", but at the same time absorbing the player intensely and utterly. It is an activity connected with on material interest, and no profit can be gained by it. It proceeds within its own proper boundaries of time and space according to fixed rules and in an orderly manner. It promotes the formation of social groupings which tend to surround themselves with secrecy and to stress their difference from the common world by disguise or other means.*

Typically, the academic concept in "game theory" comes from management and business. Sports such as soccer, basketball, table tennis etc. are classified as to physical games. Of course, the board games and video games are the standard understanding in people's mind. In recent years, the concept of game has been developed and changed into online games, cellphone games and serious games [23].

Unfortunately, in a huge group of people's mind, game is just for enjoyment or associate chair-warmer, that they think the game is soecially for children and teenagers, therefore in university or academic area, few university or laboratory do the work related to the game science. In fact, game called as "Ninth Art" which was a combined name with other 8 different arts - literature, painting, music, dance, sculpture, architecture, theater and film [24] [25]. A fantastic game would need a good story line and background, good structure, mathematical balance and AI algorithm research, complex coding and programming work, and excellent painting and music design, being as complex as operation system. Therefore, this study is dedicated to the development of a new academic research area in "Game Informatics" which has become apparent.

1.1 Game Theory and Game Refinement Theory

Game theory is a fascinating subject [26]. Many entertaining games, such as chess, poker, tic-tac-toe, bridge, baseball and computer games etc. In addtion, there is a vast area of economic games, discussed in Myerson (1991) [27] an Kreps (1990) [28], and the related political games, Ordeshook (1986) [29], Shubik (1982) [30] and Taylor (1995) [31]. The competition between firms, the conflict between management and labor, the fight to get bills through congress, the power of the judiciary, war and peace negotiations between countries, that provide example of games in action. There are also psychological games played on a personal level, where the weapons are words, and the payoffs are good or bad feelings, Berne (1964) [32]. There are biological games, the competition between species, where natural selection can be modeled as a game played with genes, Smith (1982) [33]. There is a connection between game theory and the mathematical areas of logic and computer science. One may view theoretical statistics as a two person game in which mature takes the the role of one of the players, as discussed in Blackwell and Girshick (1954) [34] and Ferguson (1968) [35] [26].

Games are characterized by a number of players or decision makers who interact, possibly threaten each other and form coalitions, take actions under uncertain conditions, and finally receive some benefit or reward or possibly some punishment or monetary loss [26] [36].

In general game progress, typically involves several players; a game with only one player is usually called a decision problem. The formal definition lays out the players, their preferences, their information, the strategic actions available to them, and how these influence the outcome. Games can be described formally at various levels of detail [37]. A cooperative game is a high-level description, specifying only what payoffs each potential group, or coalition, can be obtained by the cooperation of its members. What is not made explicit is the process by which the coalition forms. As an example, the players may be several parties in parliament. Each party has a different strength, based upon the number of seats occupied by party members. The game describes which coalitions of parties can form a majority, but does not delineate, for example, the negotiation process through which an agreement to vote en bloc is achieved [37].

Cooperative game theory investigates such coalitions games with respect to the relative amounts of power held by various players, or how a successful coalition should divide its proceeds. This is most naturally applied to situations arising in political science or international relations, where concepts like power are most important. For example, Nash proposed a solution for the division of gains from agreement in a bargaining problem which depends solely on the the relative strengths of the two parties bargaining position [26] [38]. The amount of power a side has is determined by the usually inefficient outcome that results when negotiations break down. Nash model fits within the cooperative framework in that it does not delineate a specific timeline of offers and counteroffers, but rather focuses solely on the outcome of the bargaining process [26]. In contrast, non-cooperative game theory is concerned with the analysis of strategic choices. The paradigm of non-cooperative game theory is that the details of the ordering and timing of players choices are crucial to determine the outcome of a game. In contrast to Nash's cooperative model, a non-cooperative model of bargaining would posit a specific process in which it is prespecified to make an offer at a given time. The term "non-cooperative" means that this branch of game theory explicitly models the process of players making choices out of their own interest. Cooperation can, and often does, arise in non-cooperative models of games, when players find it in their own best interests [26] [38] [37].

Branches of game theory also differ in their assumptions. A central assumption in many variants of game theory is that the players are rational. A rational player is one who always chooses an action which gives the outcome he most prefers, given what he expects his opponents to do. The goal of game-theoretic analysis in these branches, then, is to predict how the game will be played by rational player, or, relatedly, to give advice on how best to play the game against opponents who are rational [26] [39].

1.1.1 Nash Equilibrium and Prisoner's Dilemma

In game theory, the Nash Equilibrium is a non-cooperative game involving two or more than two game players [40]. Each game player should know the concept of equilibrium solutions. While each player choose a strategy, no player can change the strategy, while the other players keep their income unchanged, then the current strategic choices and the corresponding payoffs is a Nash Equilibrium. The reality of a game's Nash Equilibrium can be tested with an experimental economics approach [26].

Simply to say, *Player A* and *Player B* are in Nash Equilibrium if *Player A* is making the best choice what he can, taking into account *Player B's* decision while *Player B's* decision remains unchanged, and *Player B* is making the best decision he can, taking into account *Player A's* decision while *Player A's* decision remains unchanged [26] [37]. The most typical case of Nash Equilibrium is the Prisoner's Dilemma [26].

The Prisoner's Dilemma is a standard example of a game analyzed in game theory that shows why two completely "rational" individuals might not cooperate, even if it appears that it is in their best interests to do so. It was originally framed by Merrill Flood and Melvin Dresher working at RAND in 1950. Albert W. Tucker formalized the game with prison sentence reward and name it [41], presenting it as follows:

> Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge. They hope to get both sentenced to a year in prison on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either to: betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent. The offer is:
>
> - If A and B each betray the other, each of them serves 2 years in prison
> - If A betrays B but B remains silent, A will be set free and B will serve 3 years in prison (and vice versa)

- If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge)

It is implied that the prisoners will have no opportunity to reward or punish their partner other than the prison sentences they get, and that their decision will not affect their reputation in the future. Because betraying a partner offers a greater reward than cooperating with them, all purely rational self-interested prisoners would betray the other, and so the only possible outcome for two purely rational prisoners is for them to betray each other. The interesting part of this result is that pursuing individual reward logically leads both of the prisoners to betray, when they would get a better reward if they both kept silent. In reality, humans display a systemic bias towards cooperative behavior in this and similar games, much more so than predicted by simple models of ”rational” self-interested action. A model based on a different kind of rationality, where people forecast how the game would be played if they formed coalitions and then maximized their forecasts, has been shown to make better predictions of the rate of cooperation in this and similar games, given only the payoffs of the game.

1.1.2 Game Refinement Theory

Previously, we introduced the concept of game theory. However, game theory only can solve the problem of “how to win the game”, it is the mathematical method with focus on the players' side. In order to develop a new game theory from game designers' side, Iida et al. created game refinement theory in 2003[17].

Game theory and game refinement theory have played an important role in the development of computer playing game and general games. So what is the difference between game theory and game refinement theory? How could those ideas be applied in our real life? As we have known that von Neumann is one of the man that formed the background for the modern game theory. From his idea of minimax, one of the most effective chess playing algorithm the minimax game-tree search algorithm was born. And what we can see in his theory is to find the best outcome in a game no matter what the other player does, also how to ensure the possibility of winning a game based on the understanding of current positions. On the other hand, in game refinement theory, the focus is not on how to win a game but how much attraction of a game to players. In particularly, in game refinement theory, they try to quantify the engagement of players to games and based on those values, games are classified and analyzed to improve the attractiveness of the game itself.

Moreover, game refinement theory could be used to gain more understanding about the development of game history. Therefore, it gives us a more general and reasonable look on the evolution of specific game variants. In another way, game refinement theory provides us with another viewpoint of games from the entertainment aspect while game theory helps us understand about the game's mechanism itself. From that viewpoint, we can extend the idea of game refinement into other domains in human life such as sport games, video games, education or business. The possibility of extension that comes from the core idea of game refinement theory is quantifying the engagement. In many human activities, the engagement is usually used as one of the important standards to evaluate the effectiveness of those activities. In my opinion, we can extend models of game refinement and apply it into many fields as mentioned above.

Although there are differences between game theory and game refinement theory, both have contributed as a firmed base for the development of computer chess. Moreover, I believe that their potential would not only be limited to game informatics field but could also be useful for many other domains from scientific research to daily life improvement.

1.2 Problem Statement

TETRIS is the best-selling video game, which is released in 1984. It has been popular for over 30 years[43]. And it is almost the only one video game being popular for so long. But TETRIS is a very simple game, also a little bit boring. Because one can never win this game. There is only failure at the end of every game.

In later variant version of Tetris games, competition mechanisms were added to attract more players. Score system and punish system of Tetris games were improved to make the games fairer and more interesting. In Classic Tetris World Championship held every year since 2010, any stochastic elements are removed from the game to keep the competition impartial. Even the sequence of Tetriminos for two competitors is same.

Recent years, the video game became more and more popular over the world, some video games were defined as e-sports for competition [5]. In this case, fairness is a very important element to attract more people to play the target game. But whether such fair like in Classic Tetris World Championship is necessary?

Research Question 1: Can game refinement theory be used in TETRIS? How do we use it to explain why the "boring" TETRIS can be so popular? In Chapter 2 will try to solve this research question.

Research Question 2: How the e-sports games keep fair? What is the relationship between fairness and attractiveness of a game? In Chapter 3 will try to solve this research question with the case study using MOBA games.

1.3 Structure of The Thesis

In Chapter 2, we introduce game refinement theory and TETRIS briefly, and we analyze TETRIS using the game refinement theory. In Chapter 3, we introduce the concept and development of MOBA games and their mechanism used to maintain fairness in several MOBA examples. Then we compare these examples for their fairness and game refinement value. In Chapter 4, we present our conclusion and some future works.

Chapter 2

An Analysis of TETRIS using Game Refinement Measure

2.1 Chapter Introduction

Game refinement theory has been proposed earlier by Iida et al. [17] to determine level of sophistication of games. Some applications have already been done, such as in domain of board games [17], for example Mah Jong [18], and sports games [19]. Although there are still many types of games to cover, this theory has already performed well, and generalized fundamental concept. By using sophistication measurement, many facts have been revealed regarding changes of attractiveness of games in decades.

TETRIS[1] is a tile-matching puzzle video game, originally designed and programmed by Russian game designer Alexey Pajitnov. It was released on June 6, 1984, while he was working for the Dorodnitsyn Computing Centre of the Academy of Science of the USSR in Moscow. He derived its name from the Greek numerical prefix tetra- (all of the game's pieces contain four segments) and tennis, Pajitnov's favorite sport.

The Tetris game requires players to strategically rotate, move, and drop a procession of Tetriminos that fall into the rectangular Matrix at increasing speeds. Players attempt to clear as many lines as possible by completing horizontal rows of blocks without empty space, but if the Tetriminos surpass the Skyline the game is over[20]!

To analyze a continuous movement game like Tetris, with Game Refinement Theory, we usually construct a game progress module at first. But it is hard to construct a perfect game progress module. We observed that at the first several levels in Tetris, Tetriminos fall slowly. Players have enough time to think carefully about each Tetrimino just like in board games. So we considered whether we can use board game approach to analyze Tetris. Although with speed increasing, player will do not have enough time to consider all options for each Tetrimino, if we can figure out the relationship between the speeds and the number of options can be considered, we can construct a bridge to transform the higher levels of Tetris to a changing branching factor board game.

In Section 2, we will briefly introduce the Game Refinement Measure. Then we will

[1]Tetris logos, and Tetriminos are trademarks of Tetris Holding.

focus on TETRIS and using a personal test to explain the methodology to transform Tetris to a changing branching factor board game in Section 3. At last we will explain the reason why TETRIS been popular for so many years.

2.2 Assessment Methodology

We give a short sketch of the basic idea of game refinement theory[19]. A measure of game refinement will be derived from the game progress model. "Game progress" has twofold. One is game speed or scoring rate, while another one is game information progress with focus on the game outcome.

Game information progress itself is how certain is the result of the game in a certain time or steps. Let G and T be the average number of successful shoots and the average number of shoots per game, respectively. If one knows the game information progress, for example after the game, the game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq T$ and $0 \leq x(t) \leq G$, as shown in Equation 2.1.

$$x(t) = \frac{G}{T}t \tag{2.1}$$

However, the game information progress given by Equation 2.1 is usually not known during the in-game period. Hence, the game information progress should be not linear but something like exponential. This is because the game outcome is uncertain until the very end of game in many games. Hence, a realistic model of game information progress is like Equation 2.2.

$$x(t) = G(\frac{t}{T})^n \tag{2.2}$$

Here n stands for a constant parameter which will depend on the perspective of an observer in the game considered. Then acceleration of game information progress could be obtained by deriving Equation 2.2 twice. Solving it at $t = T$, the equation becomes

$$x''(T) = \frac{Gn(n-1)}{T^n}t^{n-2} = \frac{G}{T^2}n(n-1).$$

It is assumed in this study that the game information progress in any type of games is happening in our minds. We do not know yet about the physics in our minds, but it is likely that the acceleration of information progress is related to the force in mind. Hence, it is reasonably expected that the larger the value $\frac{G}{T^2}$ is, the more the game becomes exciting due to the uncertainty of game outcome. Thus, we propose to use its root square, $\frac{\sqrt{G}}{T}$, as a game refinement measure for the game considered.

As the cases of board games Let B and D be average branching factor (number of possible options) and game length (depth of whole game tree), respectively. One round in board games can be illustrated as decision tree. At each depth of the game tree, one will choose a move and the game will progress. One level of game tree is illustrated in Fig. 2.1. The distance d, which has been shown in Fig. 2.1, can be found by using simple Pythagoras theorem, thus resulting in $d = \sqrt{\Delta l^2 + 1}$.

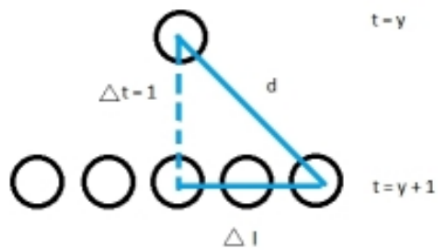

Figure 2.1: Illustration of one level of game tree

Assuming that the approximate value of horizontal difference between nodes is $\frac{B}{2}$, then we can make a substitution and get $d = \sqrt{(\frac{B}{2})^2 + 1}$. The game progress for one game is the total level of game tree times d. For the meantime, we do not consider Δt^2 because the value ($\Delta t^2 = 1$) is assumed to be much smaller compared to B. The game length will be normalized by the average game length D, then the game progress $x(t)$ is given by $x(t) = \frac{t}{D} \cdot d = \frac{t}{D}\sqrt{(\frac{B}{2})^2} = \frac{Bt}{2D}$.Then, in general we have, $x(t) = c\frac{B}{D}t$, where c is a different constant which depends on the game considered. However, we manage to explain how to obtain the game information progress value itself. The game progress in the domain of board games forms a linear graph with the maximum value $x(t)$ of B. Assuming $c = 1$, then we have a realistic game progress model for board games, which is given by

$$x(t) = B(\frac{t}{D})^n \tag{2.3}$$

Equation 2.3 shows that the game progress in board games corresponds to that of sports games as shown in 2.2.

2.3 Dynamic of Branching Factor Case Study using TETRIS

In the Tetris game, there are 7 kinds of different tetrominos, we call them “I”, “O”, “T”, “L”, “J”, “S”, and “Z”.

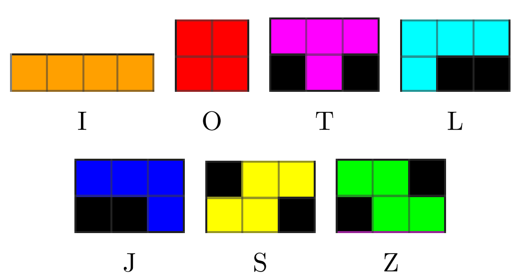

Figure 2.2: Tetrominos

"I" has 2 rotations. One is 1 block wide and another is 4 blocks wide. As the board is 10 blocks wide, "I" has $10 + (10 - 4 + 1) = 17$ possible mobilities.

The other tetrominoes can be calculated by this way too. "O" has 9 possible mobilities, "T" has 34 possible mobilities, "L" has 34 possible mobilities, "J" has 34 possible mobilities, "S" has 17 possible mobilities, and "Z" has 17 possible mobilities.

So the branching factor of TETRIS is shown as

$$B = \frac{17 + 9 + 34 + 34 + 34 + 34 + 17 + 17}{7} = 23.14 \tag{2.4}$$

From a personal test, we got my own velocity of reflex nerve is 0.287 second per move. The velocity of pushing keyboard is 0.133 second per time. We assume every tetromino needs to be moved 4 times, we can result it will take $(0.287 + 0.133) \times 4 = 1.68$ seconds. In an ordinary Tetris games, velocity of Tetriminos falling is $0.05 \times (11 - level)s/row$. So at level 1, it takes 5 seconds for Tetriminos to fall 10 rows. It means I have $5 - 1.68 = 3.32$ seconds to think about strategy. But when it comes to level 6, it takes 2.5 seconds for Tetriminos to fall 10 rows. There will be only $2.5 - 1.68 = 0.82$ second to think about strategy. We assume 1 second as a proper thinking time, then since level 6 the branching factors will be limited.

Obviously if we could consider more options, we can eliminate more rows. So how many rows we can eliminate at a certain velocity can represent how many options we can think at that velocity indirectly. Since level 6 is the level that branching factors beginning to be limited, we did some experiments for level 5 through level 10. We modified the Tetris game to play at a certain speed level for an entire game. And we repeated playing at each level for 5 times and calculated the average rows be eliminated at each level. The result is showed in Table 2.1:

Table 2.1: Eliminated rows at different speed Level

Level	5	6	7	8	9	10
E.R.	108	75	32	15	3	0

These points follow the trend line $y = -65.01 \ln(x) + 110.12$. We assume the branching factor follow the same trend to eliminated rows at each level. As (5, 23.14) point is on the line, the trend line of branching factors from level 5 should be $y = -13.93 \ln(x) + 23.594$. Then we can get predicting branching factors of each level like in Table 2.2:

Table 2.2: Predicting branching factors at different speed Level

Level	5	6	7	8	9	10
B_i	23.14	16.069	6.856	3.213	0.643	0

Since the velocity only changes after 10 rows been eliminated, the change of branching factors is also discrete. So we can calculate the average branching factor of each game like below:

$$B(s) = \frac{\sum_{i=1}^{n}(B_i(s) \times R_i(s))}{D} \tag{2.5}$$

$B_i(s)$ is the branching factor at level i. $R_i(s)$ is how many Tetriminos fall at level i. Both $B_i(s)$ and $R_i(s)$ are related to the strength of players. D is the total number of Tetriminos fall down. All numbers of Tetriminos are normalized to the number of rows can be eliminated with all these Tetriminos. The normalized formula is:

$$D \ or \ R_i(s) = \frac{\text{number of Tetriminos} \times 4}{\text{width of the board}} \tag{2.6}$$

2.4 Discussion

Based on an open source Tetris AI test platform[21], we modified an AI to match my level to collect data.

In one sample, there are 168 Tetriminos fell down and 51 rows been eliminated. It means the game ended at level 6.

First, we normalize 168 Tetriminos to the number of rows,

$$D = \frac{168 \times 4}{10} = 67.2.$$

Here, we do not stack Tetriminos to eliminate multiple rows at once. So at the first 5 levels we can assume that there are only 10 rows of Tetriminos fall down at each level. The dynamic branching factor of this sample can be

$$B = \frac{\sum_{i=1}^{6}(B_i(s) \times R_i(s))}{D} = \frac{23.14 \times 10 \times 5 + 16.069 \times (67.2 - 50)}{67.2} = 21.33.$$

Then we can get the game refinement value is

$$GR = \frac{\sqrt{B}}{D} = \frac{\sqrt{21.33}}{67.2} = 0.068.$$

We calculate in this way for 30 samples the average game refinement value is 0.077. It is close to the game refinement values of other games had been invested with game refinement theory, like Go, soccer, etc. But in fact, we do not consider all the possibilities all the time. Based on our experience or skill, we only consider about several optimal choices, represented by b. Obviously more experience we have, smaller b will become, and longer time of a game we will play, which means larger D. So the game refinement value $GR = \frac{\sqrt{b}}{D}$ will be much smaller than 0.077.

From [44], we know **the mass of game is related to the decision complexity of a game with rules and mechanism and force is the elaborative faculty.** Then we have table 2.3.

Decision complexity [45]: This is the complexity of the problem to find the optimal move in a given situation. So we can assume the mass of game $m = \frac{B}{b}$.

Table 2.3: Newton's 2nd law in motion and game compared

Symbol	Physical concept in motion	Physical concept in game
F	Force	Elaborative faculty
m	Mass	Decision complexity
a	Acceleration	Related to game refinement value

From Equation 2.3 we know

$$a = x''(D) = \frac{b}{D^2} n(n-1).$$

Hence, we get

$$F = m \times a = \frac{B}{b} \times \frac{b}{D^2} n(n-1) = \frac{B}{D^2} n(n-1) = GR^2 n(n-1).$$

Then, from Newton's second law , we could get the "momentum" of game is

$$P = F \times t = GR^2 n(n-1)t.$$

In physics, the larger momentum an object have, it is harder to stop it. Therefore we conjecture in game informatics field, momentum is a factor shows the addictiveness of a game. And in psychology, there are Zeigarnik effect. It states that **people remember uncompleted or interrupted tasks better than completed tasks**[46]. In other games, when players reach some checkpoint or achieved some goal, players will get the fill of achievement. And it is a good point to save and quit the game. But in Tetris games players can hardly eliminate all blocks from the board. And there are always new Tetriminos fall from above. This means player can never "complete" the game. The game always gives players pressure. So although the game refinement value of TETRIS is not as high as other popular games, it gives more momentum to players. Therefore TETRIS keeps popular for many years even though it is not so interesting.

2.5 Chapter Summary

This chapter presented the basic idea of game refinement theory, then used it to analyze TETRIS. Following the approach of dynamic branching factor, we got the game refinement value of TETRIS is 0.077. But in real games the value will be much smaller than 0.077. Hence we extended to the concept of "momentum" in game. Because the TETRIS always give players pressure, even if the game refinement value of TETRIS is not as high as other popular games, it also gives more momentum to players. Therefore TETRIS keeps popular for many years.

Chapter 3

Fairness Mechanism in Multiplayer Online Battle Arena Games

This chapter is an updated and abridged version of the following publication.

- Mingyang Wu, Shuo Xiong, Hiroyuki Iida. Fairness mechanism in multiplayer online battle arena games. In Systems and Informatics (ICSAI), 2016 3rd International Conference on. IEEE. Shanghai, China. Nov. 2014

3.1 Chapter Introduction

Fairness is important in games as well as in our society. Games without fairness would lose their charm. Lives without fairness would become miserable disasters. There are many examples about people seeking for fairness. Last century John von Neumann created a new academy branch which was called game theory, it can be widely used in various cases. von Neumann [11] proposed the notion of minimax equilibrium in n-person games, which provides an insightful aspect of fairness.

For example, Cutting-a-Cake game is an interesting and simple example. In this game, two persons (say A and B) are given a cake and they will have to share the cake and they shall receive a fair share in the cake. Here the problem is that if A is asked to cut the cake, there is a chance that he reserves the bigger piece for himself and gives B the smaller piece. So, the best solution to this problem is given as the person who is not cutting the cake will reserve the right to choose first. Here, the person will choose the bigger piece available and leave the smaller piece to the one who had cut the cake. So, the person cutting the cake will cut the cake in such a way that neither of them will get the maximum possible portion of the cake.

Recent years, the video game became more and more popular over the world, some video games were defined as e-sports for competition [5]. In this case, fairness is a very important element to attract more people to play the target game. If a game loses fairness and equality, then it cannot survive for a long time. Therefore, in this paper we focus on Multiplayer Online Battle Arena (MOBA) games which are popular and full of competitive, and study the fairness mechanism and its evolution process to understand how it works.

Fairness in the domain of two-player board games such as chess has been studied [6]. The definition was given that a game is fair if and only if the winning ratio for White and Black is statistically equal or nearly so. A sophisticated game would have the game-theoretic value as draw.

In MOBA games, the definition of fairness should also be like this: two similar level teams have a statistically equal or nearly equal winning ratio.

We show, in Table 3.1, some differences among 3 types of games: mind sports like strategic games, physical sports like soccer and e-sports like MOBA games.

- In mind sports games just like board games, players are mostly individual. While in physical sports and e-sports, there are both individual and team work.
- In mind sports games, players focus on mental competitions. In physical sports games, players mostly focus on physical competitions, and in e-sports games, both mental and physical competitions are important.
- Mind sports games and physical sports games have very long histories with slow changes. Physical sports games changed a lot especially in recent years. Compared with mind sports and physical sport, e-sports games have a short history with high frequency development, like Dota always updates even every week.
- Rules of mind and physical sports games can be easily changed temporarily with players' agreement. However, e-sports games cannot be changed only by players because they are based on computer programs.
- Lives of mind sports games and physical sports games survived for long histories. But lives of e-sports games are completely different. A game may fade out quickly. Therefore, fairness mechanism is much more important in e-sports games.

Table 3.1: Differences among 3 types of sports games

	Mind sports	**Physical sports**	**E-sports**
Players	Individual	Individual or Team	Individual or Team
Focus	Mental	Physical	Mental and Physical
Tempo	Slow	Medium	Fast
Rules	Flexible	Flexible	Strict
Life	Long	Long	Short

In order to analyze the issue and show a good countermeasure, we present some examples of fairness mechanism in board games and sports games in Section 3.2. Section 3.3 introduces the concept and development of MOBA and Section 3.4 explains the mechanism used to maintain fairness in several MOBA examples. In Section 3.5 and Section 3.6, we present our discussions, conclusion and some future works.

3.2 Fairness in Board Games and Sports Games

There are many examples in board games to show how they maintain fairness. For example, in Connect5 or Gomoku, Allis [12] has shown that on a 15x15 board first player must win with perfect play. In order to keep fairness, the rules have been changed as below. First player was forbidden to play some moves, and second player gets a chance to swap color with first player after first player's second move in the game of Renju. However, there is still some advantage for one side under this rule, hence Connect6 [8] was born, which is fair (more than Gomoku) in some sense.

Another example is Go in which the rules have been changed many times to maintain fairness in its long history. For the advantage of initiative [10], Black (first player) needs to subsidize some value to White. We show the winning ratio for different versions of *komi* or *komidashi* in Go in Table 3.2. In the history of Chinese and Western chess, the rules have been changed many times to make the games more fair as well. As a result, draw ending became more and more in competitive tournaments.

Table 3.2: Winning ratio and different komi in Go

Komi	**Winning ratio (Black)**
5.5 point	52.42
6.5 point	50.78
7.5 point	52.72

For the sports games like soccer, we consider the sunlight and grass quality, two teams then need to exchange their fields after 45 minutes. In 1996, FIFA (The Federation Internationale de Football Association) modified the advantageous principle to avoid players fouling to get advantage. For basketball, two teams vie the ball at first quarter, the winner gets the ball at first and last quarters and the loser gets the ball at second and third quarter, teams also need exchange courts after second quarter.

For club tournament, player can transfer from one club to others. The transfer windows can limit some behaviors to avoid the games becoming a competition of money. For volleyball, teams change serving after losing one point, and one match contains several games, two teams need to exchange courts after one game. For race games, there is a pre-race called qualifying to decide the order of setting off. This makes every racer get advantage only by their skill. Also some technologies are forbidden to use during the races.

It is difficult to maintain fairness while a game was created. Games survived in the long history by changing the rules to seek fairness and become more attractive. MOBA games follow the same logic.

3.3 Historical Overview of MOBA Games

In this section, we show the overview of MOBA games, then present three phrases in the evolution of MOBA games.

3.3.1 MOBA games

Multiplayer On-line Battle Arena (MOBA) originated in a sub-genre of the Real-Time Strategy (RTS) genre of video games, in which one controls a single character in one of two teams. The objective is to destroy the opposing team's main structure with the assistance of periodically spawned computer-controlled units that march forward along set paths. Player characters typically have various abilities and advantages that improve over the course of a game and that contribute to the team's overall strategy. Unlike RTS, players usually do not construct either buildings or units. We show, in Figure 3.1, an example of typical map of MOBA game [1].

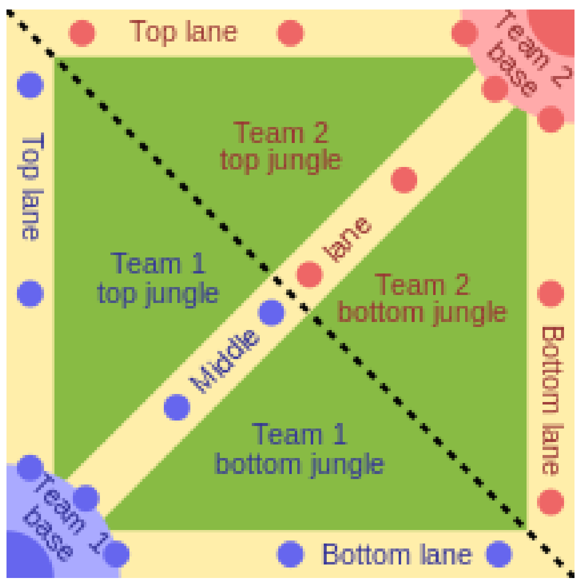

Figure 3.1: General setup of map of MOBA game

3.3.2 Evolution of MOBA games

Here we present some phrases in the evolution of MOBA games.

[1]https://commons.wikimedia.org/wiki/Category:MOBAs, CC BY-SA 3.0

Pioneering works

The roots of the genre can be traced back decades to one of the earliest real-time strategy titles, the 1989 Sega Mega Drive/Genesis game "Herzog Zwei"[2] [1] [3]. It was different from nowadays MOBA games. But it used a similar formula, where each player controls a single command unit in one of two opposing sides on a battlefield [4]. In 1998, computer

Table 3.3: Historical overview of MOBA games

MOBA game	Map (year)	Author	Remarks
StarCraft	Aeon of Strife (1998)	Aeon64	First MOBA and the original term used to classify such games
War3: ROC	Defense of the Ancients (2002)	Eul	More complex than AoS
War3: TFT	Dota: Allstars (2003)	Meian	Combined heroes from many other versions of Dota
War3: TFT	Dota: Allstars (2005)	Guinsoo	Added item combine system and "Roshan"
War3: TFT	Dota: Allstars(until 2008)	IceFrog	Initiated large changes to the mechanics
League of Legends (2009)		Guinsoo	Factors outside matches affect characters
Dota 2 (2013)		IceFrog	Added in-game talk system and great OB system
Smite (2014)		Hi-Rez	Third-person perspective MOBA game
Heroes of the storm (2015)		Blizzard	Experience share and no item

game company Blizzard Entertainment released its best-selling real-time strategy game "StarCraft" with a suite of game editing tools called "StarEdit". The tools allowed members of the public to design and create custom maps that allowed play very different from the normal maps. A modder known as Aeon64 made a custom map named "Aeon of Strife" (AoS) that became very popular. In the "Aeon of Strife" map, players control a single powerful hero unit fighting amidst three lanes, though terrain outside these lanes was nearly vacant [2].

Following Works

In 2002, Blizzard released "Warcraft III: Reign of Chaos" (ROC), with the accompanying "Warcraft III World Editor". A modder named Eul began converting "Aeon of Strife"

[2] All products, company names, brand names, trademarks, and pictures are properties of their respective owners.

into the “Warcraft III” engine, calling the map “Defense of the Ancients” (Dota) and Eul substantially improved the complexity of playing from the original “Aeon of Strife” mod. But, shortly after creating the custom “Dota” map, Eul left the modding scene. With no clear successor, Warcraft III modders created a variety of maps based on Dota and featuring different heroes. In 2003, after the release of “Warcraft III: The Frozen Throne” (TFT), a map creator named Meian created a Dota variant closely modeled on Eul’s map, but combining heroes from many other versions of Dota that existed at the time. It was “Dota: Allstars” and it was inherited after a few months by a MOD marker called Steve “Guinsoo” Feak, under his guidance it became the dominant map of the genre.

After more than a year of maintaining the “Dota: Allstars” map, with the impending release of an update that significantly changed the map layout, Guinsoo left the project. After some weeks of development and some versions released, the latter turned over responsibility to a modder named IceFrog, who initiated large changes to the mechanics that deepened its complexity and capacity for innovative gameplay. The changes conducted by IceFrog were well-received and the number of users on the “Dota: Allstars” forum is thought to have peaked at over one million [2].

By 2008, the popularity of Dota had attracted commercial attention. Gas Powered Games also released the first stand-alone commercial title in the genre, “Demigod”. In late 2009, Riot Games’ debut title, “League of Legends” initially designed by Feak, was released. Riot began to refer to the game’s genre as a Multiplayer Online Battle Arena (MOBA). Also in 2009, IceFrog, who had continued to develop “Dota: Allstars”, was hired by Valve Corporation, in order to design a sequel to the original map [2].

Recent works

In 2010, S2 Games released “Heroes of Newerth”, with a large portion of its gameplay and aesthetics based on “Dota: Allstars” [16]. The same year, Valve announced “Dota 2” and subsequently secured the franchise’s intellectual property rights, after being contested by Riot Games for the Dota trademark. Dota 2 was released in 2013, and was referred to by Valve as an “action real-time strategy” game. In 2012, Activision Blizzard settled a trademark dispute with Valve over the usage of the DOTA trademark and announced their own standalone game, which was eventually named “Heroes of the Storm” [13] [9] [14]. Blizzard adopted their own personal dictation for their game’s genre with “hero brawler”, citing its focus on action [15]. In 2014, Hi-Rez Studios released “Smite”, a MOBA with third-person perspective. moreover, Microsoft China announced “King Of Wushu” to be a cooperation game on Xbox One on July 30, 2014. It became the first MOBA game on home video game console. The evolution process of MOBA game was shown in Table 3.3.

3.4 Fairness Mechanism in MOBA Games

Fairness and equality are essential components of games. Without them games will lose their charm and be forgotten in the history. This is also suitable for MOBA games. In a game with the concept of turn to move, they may exist the advantage of the initiative [7]. In this sense the second player in a two-person game deserves compassion from the first player in order to maintain fairness [6]. Here we observe some fairness mechanisms in MOBA games.

3.4.1 Ban and pick system

Although in MOBA games, victories mostly depend on the performances of players, before games begin, selecting characters plays a very important role to games' results, especially in tournaments. So game designers, inspired by minimax equilibrium, created the ban and pick system. The ban and pick is a system that each team chooses several characters that cannot be used in the following game for both teams and chooses several characters to use in the following game for their own team. Teams should minimize their opponents' power by banning characters they may want to choose, and maximize their own power by picking suitable characters. Therefore, teams who go first to ban and pick have the advantage of the initiative and the second teams deserve compassion to keep the game fair.

Example in Dota

In Dota2, captain mode is the standard format for tournament games. The captain selects bans for certain heroes, preventing either team from picking the hero. The captain also chooses five heroes for his team. After the captains choose five heroes, each player chooses a hero from their captain's selections. Each captain has 110 seconds in total bonus time that can be used throughout any selection with 30 seconds are allotted for bans and 40 seconds for picks. If time runs out during a ban selection, no hero will be banned; if time runs out during a pick selection, a random hero will be chosen. The player that clicks the "become captain" button first is the captain. The starting team is randomly selected in matchmaking; if playing in a private lobby a starting team may be specified. Some heroes that were recently created or tweaked are unavailable in Captain's Mode; they will be added eventually. An example of Captains Mode picking order is shown in Figure 3.2 [3].

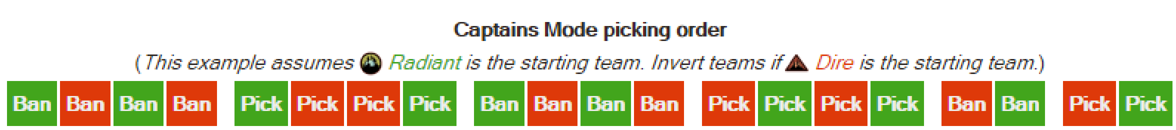

Figure 3.2: An illustration of an example process of ban and pick mechanism

This captain mode is somehow like Jury selection. Each captain bans the biggest threats to his team, and picks proper character for his team. But, the second team often has to ban some IMBA (imbalance) characters, depending on game versions to prevent

[3]http://dota2.gamepedia.com/Game_modes, CC BY-NC-SA 3.0

the starting team from first picking them. So the second team gets a chance to pick 2 characters continually as compassion. In the later phrase, the second team will get first pick and ban, which is also for compassion.

Example in League of Legends

In League of Legends (LoL), a similar ban-pick system is used for tournament games. At the start of champion selection, team captains have the opportunity to ban a total of 6 champions. The bans alternate between the teams, blue team starting. During the banning phase, each team can see their opponent's available champions in order to ban viable options. Then the team that receives first ban gets the first pick. After the initial pick, teams alternate and pick two at a time until each summoner has chosen a champion. No champion can be selected more than once in a game. The second team gets the first two pick can be treated as compassion.

Example in Heroes of the Storm

In Heroes of the Storm (HotS), a system similar to Dota2 is used. But, in HotS each team bans only two characters, and each team will pick several characters after baned one character. Teams who go first ban will pick 3 characters and teams who go second ban will pick 2 characters. Then teams will exchange their order of ban and pick. But, there is another element used for compassion with maps. In HotS, the team who does not get first pick will get the chance to pick the map to play. Because of the difference of maps in HotS, picking map is also important for the results of games. Dividing it from the advantage of starting teams is compassion to keep the game impartial. In fact, in Dota2, there are similar dividing. The team who does not get first pick will get the chance to pick the side of the map to occupy.

3.4.2 Balancing of characters

Since there are differences among characters in MOBA games, balancing of characters is also important to maintain fairness. It can also cooperate with the ban and pick system. Once a character is too powerful, it must consume a seat in the ban phrase to prevent the opponent from choosing it. But, if the number of powerful characters does not match the number of ban and pick, there may be a situation, every powerful character is banned except one. Only the team first to pick can get it. This will be a great advantage. If there are too many powerful characters, teams cannot ban all characters which their opponents are good at. If they get one, they will get a high possibility to win. So to modify the parameters of characters to keep them balance and match the amount of ban and pick is also important.

The states of characters can influence appearance rate of characters. Range characters have advantages against melee characters. The attack ranges, skill ranges and damages of range characters should be modified to make compassion to melee characters.

3.4.3 Maps

After the ban and pick, MOBA games are continuous time games, not turn based. So there is no advantage of initiative. To maintain fairness after the ban and pick phrase, maps are very important. In order to avoid advantages for any side, maps are mostly point or line symmetrical. At early era, with the limitation of mini-map UI, maps of MOBA games are line or point symmetrical squares divided into top-right and bottom-left sections. Because, only this way can fully use the whole map. Nowadays, without this kind of limitation, there are more and more irregular maps. However, they are still line or point symmetrical. Especially in HotS, which has many special map mechanisms that can easily change situations of games, which are more necessarily to be separated symmetrically on maps.

Maps also influence the appearance rate of characters. As we know, the range characters has advantage against melee characters while competing on the lane. Therefore, if maps are simple and clear, the range characters will suppress melee characters easily. In addition, no one will like to choose melee characters. In the case of LoL, the map is a point symmetrical square, as shown in Figure 3.3 [4]. Both sides have the same length of lanes and there are few roundabout ways for melee characters to ambush range characters. Therefore, melee characters are hardly chosen to be cores of team against range cores, only half of characters (depending on version) can appear in LoL tournament games. In HotS, there are both point and line symmetrical maps, as shown in Figure 3.4 and Figure 3.5 [5]. Although lanes are almost same length for both sides, lacking of wards (things can provide view nearby) in the game provides many roundabout ways for melee characters to ambush range characters. In Dota2, the map is symmetrical rectangle, as shown in Figure 3.6 [6], and each side has a hard lane. On these lanes, the range characters can be easily ambushed by the opponents. Therefore, in HotS and Dota2, almost all characters appear in tournament.

Table 3.4: Statistical data from three MOBA games

Game	**Ban and Pick**			**Map**			**Chara.**	**Counted Matches**
	First	Second	Diff.	Left	Right	Diff.		
HotS	52.33%	47.67%	4.66%	55.81%	44.19%	11.62%	86.27%	86
Dota2	54.54%	45.46%	9.08%	39.39%	60.61%	21.22%	76.58%	99
LoL	58.33%	41.67%	16.66%	58.33%	41.67%	16.66%	54.96%	72

Figure 3.3: An example of LoL map.

Figure 3.4: A line symmetrical example of HotS map.

Figure 3.5: A point symmetrical example of HotS map.

Figure 3.6: An example of Dota2 map.

3.5 Discussion

There are some statistical data from Dota2 Frankfurt major 2015. Teams who got first to ban and pick won 54 out of 99 games (54.54%); teams who got second to ban and pick won 45 out of 99 games (45.46%). Radiant side, bottom left side has won 39 out of 99 games (39.39%). Dire side, top right side has won 60 out of 99 games (60.61%). Above all, 85 different characters out of 111 characters appeared in the games (76.58%).

From LoL World Championship 2015, blue side who got first to ban and pick, has won 42 out of 72 games (58.33%). Purple side has won 30 out of 72 games (41.67%). Above all, 72 different characters of 131 appeared in games (54.96%).

From HotS Global Championship Spring and Summer 2016, teams who got first to ban and pick won 45 out of 86 games (52.33%); teams who got second to ban and pick won 41 out of 86 games (47.67%.) Left side teams won 48 out of 86 games (55.81%). Right side teams won 38 out of 86 games (44.19%). Above all, 33 different characters out of 51 appeared in the games (86.27%).

The data was shown in Table 3.4 for three MOBA games. According to the data, it seems that HotS is the fairest games among the three. The difference of winning percentage between different Ban-Pick order is 4.66%. The difference of winning percentage between different side of map is 11.62%. Note that the maps in HotS are fully symmetric. 86.27% of all characters appeared in the tournament games.

For Dota2, the difference of winning percentage between different Ban-Pick order is 9.08%. The difference of winning percentage between different side of map is 21.22% and 76.58% of all characters appeared in tournament games. It seems that the Dire (top right) side has some advantage.

For LoL, since the order of ban and pick is bound to the side of map, the only difference of winning percentage between different side of map is 16.66%. Only 54.96% of all characters appeared in tournament games, which means that the balancing of characters in LoL is not so well.

3.6 Chapter Summary

In this study we observed the fairness mechanism in MOBA games with focus on three factors: winning ratio between the different sides of a map, different orders of ban and pick, and the number of characters which appeared in the game. These three factors are equally important with scoring 100 points for each. Then we calculate the total scores for the three games.

Dota2 has 9.08% different winning rate between different order of ban and pick, it gets $100 - 9.08 = 90.92$ points. Difference between different sides of map is 21.22%, then it gets $100 - 21.22 = 78.78$ points. 76.58% of all characters appeared in the tournament games, so it gets 76.58 points. Dota2 scores $90.92 + 78.78 + 76.58 = 246.28$ points in total.

LoL has 16.66% different winning rate between different sides of map. Since the ban and pick of LoL is bound to the side of map, it gets $(100 - 16.66) \times 2 = 166.68$ points. 54.96% of all characters appeared in the tournament games, so it gets 54.96 points. LoL scores $166.68 + 54.96 = 221.64$ points in total.

HotS has 4.66% different winning rate between different order of ban and pick, it gets $100 - 4.66 = 95.34$ points. Difference between different sides of map is 11.62%, then it gets $100 - 11.62 = 88.38$ points. 86.27% of all characters appeared in the tournament games, so it gets 86.27 points. HotS scores $95.34 + 88.38 + 86.27 = 269.99$ points in total.

Therefore, these scores proved that HotS is the fairest game among the three, whereas Dota2 is the second and LoL is the last one. But the audiences number of these games is reverse. According to Twitch which is the most famous live stream site, it shows that, LoL has 73,098 audiences, Dota2 has 20,398 audiences, and HotS has only 1,546 audiences at the same time (9:41 PM, July 6, 2016, PT). The fairest one is the least popular one. This may imply that fairer games would become less popular. However, a totally (too much) unfair game obviously cannot be a competition game. Further investigation of the lower limit of fairness is needed.

Chapter 4

Conclusion and Future Work

In this chapter, we give the conclusion in this thesis. We answer our research questions. Then, some future works are discussed.

4.1 Summary

We summarize our research result of each chapter as below:

- **Chapter 2: An Analysis of TETRIS**
  In this chapter we presented the basic idea of game refinement theory, then used it to analyze TETRIS. Following the approach of dynamic branching factor, we got the game refinement value of TETRIS is not as high as other popular games. Hence we extended to the concept of "momentum" in game. Because the TETRIS always give players pressure, even if the game refinement value of TETRIS is not as high as other popular games, it gives more momentum to players. Therefore TETRIS keeps popular for many years.

- **Chapter 3: Fairness Mechanism in MOBA Games**
  In this chapter we reviewed fairness mechanism of board games and sports games. Then we observed the fairness mechanism in MOBA games with focus on three factors: winning ratio between the different sides of a map, different orders of ban and pick, and the number of characters which appeared in the game. And we figured out that in the three example MOBA games HotS is the fairest game among the three, whereas Dota2 is the second and LoL is the last one. But the audiences number of these games is reverse. So we conjecture a proper fairness is needed for competition games not too much or too little.

4.2 Answer to Research Question

Above all of the contents in this thesis, we answer our research questions as below.

- **Research Question 1**
  From Chapter 2, we know that the game refinement value of TETRIS is not as high as other popular games. But it keeps giving psychological pressure to players all the time which means more momentum than other games. Hence TETRIS is addictive, and popular for so many years.

- **Research Question 2**
  From Chapter 3, we know that MOBA games keep fair with three main factors: maps, ban and pick system and characters. And we found that the attractiveness and fairness of MOBA games are in reverse. So we conjecture a proper fairness is needed for competition games. But too much fairness of a game may reduce the appreciation of a game.

4.3 Future Works

Future works are given below.

- **Physics in game**
  In Chapter 2, we presented the "momentum" concept of a game simply. But there are more quantitative research should be done to the "momentum" concept. In the equation $P = F \times t = GR^2 n(n-1)t$, we assume n is a parameter related to the players' level and the difficulty of games. Since there is a zone value of game refinement value, if we can determine n, we can result a zone value of game momentum.

- **Fairness and game refinement value**
  From some early research[47], we have known the game refinement value of the three MOBA games. Dota is about 0.076 within 0.07 to 0.08, while each map of HotS is larger than 0.08. And LoL is even larger. This order follows neither the order of fairness nor the order of numbers of audiences. Therefore, further research is needed for the relationship between game refinement value and fairness and attractiveness of game.

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