Title	多項式制約解消のためのSMTソルバ
Author(s)	Vu, Tung Xuan
Citation	
Issue Date	2018-06
Type	Thesis or Dissertation
Text version	ETD
URL	http://hdl.handle.net/10119/15430
Rights	
Description	Supervisor:小川　瑞史, 情報科学研究科, 博士

Abstract

The need for solving non-linear arithmetic arises from many applications in artificial intelligence and formal methods. Although the full first-order theory of real numbers is decidable, the best well-known decision procedure for it, namely quantifier elimination by cylindrical algebraic decomposition, has the complexity of double exponential with respect to the number of variables. This remains as an impediment for a solver supporting non-linear arithmetic. This thesis aims at an efficient complete framework for solving existential fragment of polynomial constraints by first developing (incomplete) efficient procedures as heuristics and then combine them with a complete procedure. Two efficient procedures proposed are

- an extension of the raSAT loop, which is, in turn, an extension of interval constraint propagation (ICP) with testing, with the application of the intermediate value theorem (IVT), and
- subtropical satisfiability.

Distinct procedures and their combinations are further integrated into a satisfiability modulo theories (SMT) framework by supporting features of SMT solving such as unsat core computation.

While raSAT loop (an extension of the ICP with testing) aims at quickly detecting satisfiability of inequalities, the application of the IVT aims at showing satisfiability of equations. We propose a scheme to combine interval arithmetic, testing, and the IVT to show satisfiability of combinations of inequalities and equations. SAT-directed heuristics are also proposed for the framework to quickly detect satisfiability while not affecting performances of detecting unsatisfiability. Experimental data shows that the proposed extensions increase the number of solved problems and the heuristics improve the running time on solved problems and also increase the number of solved benchmarks. Comparing with other SMT solvers, except for weaknesses in completeness, **raSAT** shows comparable running time on problems it solved.

The second procedure, i.e. subtropical satisfiability, aims at finding an assignment for variables which satisfies inequalities by examining the exponent vectors of polynomials appearing in the inequalities. From those exponent vectors, the method generates linear arithmetic constraints such that if they are satisfiable, then the original inequalities are also satisfiable. The solution of the generated linear constraints is further used to provide a witness for satisfiability of non-linear inequalities. Experimental results show that the procedure is quite fast at either detecting satisfiability or failing. In particular, it finds solutions for problems where other state-of-the-art non-linear arithmetic SMT solvers times out.

Both proposed procedures are incomplete and in order to produce a decision framework, we utilize quantifier elimination methods implemented in the computer algebra system Redlog/Reduce. We propose two kinds for combining the ICP-based methods

and the quantifier elimination, namely lazy and less lazy approaches. While the lazy approach uses ICP-based methods as pre-processing steps for the quantifier elimination, the less lazy one invokes the quantifier elimination on every box generated by the ICP framework. In both approaches, subtropical satisfiability is utilized first as an attempt to find a model for inequalities. Experimentally, the lazy method performs better than the less lazy one but we expect some future improvements for the less lazy approach so that several unsatisfiable boxes can be all discarded once the quantifier elimination method detects the unsatisfiability of one box. Experimental results also show that our framework is an efficient decision procedure to solve non-linear arithmetic SMT problems and complementary to implementations in other SMT solvers.
Keywords: *SMT solving, non-linear arithmetic, interval constraint propagation, subtropical satisfiability, complete efficient framework.*