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論　文　題　目	Independent Set Reconfiguration and Related Problems for Some Restricted Graphs
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論文の内容の要旨

For the last decade, a collection of combinatorial problems called *reconfiguration problems* has been studied extensively. Roughly speaking, a reconfiguration problem is specified in terms of a given collection of *configurations* and a *reconfiguration rule* that describes how to transform one configuration into another. Several real-world situations involving movement and change can be modeled as reconfiguration problems. As an example, consider a network that delivers electricity from suppliers to consumers. In such a network, it may happen that some devices at a power station *S* is broken and one need to temporarily shut down *S* for replacing these broken devices with the new ones. Before shutting down *S*, one may need to reroute the transmission lines that go through *S* to some other power station in order to maintain the availability of the network. A technician may wonder which station he/she needs to pick for replacing *S* such that the network remains active and, for saving resources when *S* becomes active again, the chosen station should be as near *S* as possible. Such a situation can be modeled as a Path Reconfiguration problem, where each configuration is a path (transmission line) from the main supplier to customers, and the rule is to change a node (power station) such that the network remains connected (active). Another real-world situation where reconfiguration problems arise is the motion planning of moving objects. For instance, in a 3D printer model where multiple printing heads have been used, one need to plan the printing paths (which the heads will follow) to avoid collisions and other unwanted interactions, as well as making the distance traveled by each head as small as possible. Another situation involves multiple robots moving in an environment and one need to plan their movements such that they can avoid obstacles and each other. Such problems can be modeled as different variants of the Token Reconfiguration problem on graphs. A classic variant of Token Reconfiguration is the so-called 15-puzzle – a research topic since 1879. A configuration of 15-puzzle consists of 15 tokens labeled 1, 2, . . ., 15, placed on a 4 x 4 grid. The rule is that a token can only be slid to an unoccupied adjacent vertex. The 15-puzzle problem asks whether one can transform one configuration into another. 15-puzzle and its generalized versions can be

used as models for the Multi-robot Path Planning problem. For an overview on both theoretical and practical perspectives of reconfiguration problems, the readers are referred to the surveys by van den Heuvel [*Surveys in Combinatorics 2013*, 127–160, 2013] and Nishimura [*Algorithms*, 11:4, 52, 2018].

Among several reconfiguration problems, the reconfiguration variants of Independent Set are of particular interest. In such variants, an independent set (a set of pairwise non-adjacent vertices of a graph) is often viewed as a set of tokens placing on vertices of the input graph. In this viewpoint, Independent Set Reconfiguration can be seen as a restricted version of the Token Reconfiguration problem where distinct unlabeled tokens are placed on the vertices of a graph and no two tokens are adjacent. Among different reconfiguration rules, the following three models have attracted the attention of many theoretical computer scientists: *Token Sliding* (TS), *Token Jumping* (TJ), and *Token Addition and Removal* (TAR). A TS-step involves moving a token to one of its adjacent vertices. A TJ-step involves moving a token to any other vertex (not necessarily in its neighbors) of the graph. A TAR-step involves either adding or removing a token such that the number of remaining tokens is at least some threshold k. Typically, we are interested in determining whether one can transform an independent set I into another independent set J using TS/TJ/TAR rule such that each intermediate result is also an independent set. For all three rules, the problem is PSPACE-complete even for planar graphs of maximum degree 3 and bounded bandwidth/treewidth/pathwidth/cliquewidth. This raises an open question on whether there exist efficient algorithms for solving the problem (under TS/TJ/TAR) when the bandwidth/treewidth/pathwidth/cliquewidth of the input graph is bounded by some practical (small) constant. Interestingly, when comparing the three rules, TJ and TAR are equivalent, in the sense that for any sequence of p TJ-steps between two independent sets I and J of size k, there is also a sequence of $2p$ TAR-steps between them whose number of tokens in each member is either k or k - 1, and vice versa. The TS model seems to be more "restricted", in the sense that any sequence of TS-steps can be seen as a sequence of TJ-steps. However, the reverse direction does not hold. This motivates our study for the TS rule. As a result, in this thesis, we made a significant contribution to the computational complexity of Independent Set Reconfiguration under TS rule via designing polynomial-time algorithms for solving the problem for different restricted graphs, namely trees, and cactus graphs (whose treewidth is at most 2). As consequences of our algorithms, we show that one can construct an actual sequence of TS-steps (if exists) between two given independent sets using a polynomial number of token-slides.

Key Words: polynomial-time algorithm, computational complexity, combinatorial reconfiguration, sliding token, independent set, tree, cactus graph

論文審査の結果の要旨

Reconfiguration problem は，理論計算機科学の中でも比較的新しい研究分野である．具体的には，問題と，その問題の解が二つと，解の変形規則が与えられたときに，一方の解

から他方の解に，与えられた規則で遷移できるかどうかを判定する問題である．例えばルービック・キューブのようなパズルから，電気が途切れることなく電力網を配置変えするといった問題まで，さまざまな場面で現れる問題の枠組である．その一方で，クラス NP や PSPACE といった，計算量的に困難な問題のクラスに対して，新しい特徴づけや視点を与えてくれるといった，理論的な興味からもよく研究がなされている．

HOANG 氏の研究はこの中でもグラフ上の独立点集合の Reconfiguration に関するものである．独立点が 2 つ与えられたときに，この点をトークンとみなし，グラフの上でこのトークンをスライドしつつ，独立点であるという性質を壊さないままで，ある独立点から別の独立点に遷移しうるかどうかを判定する問題である．この問題は，2005 年に一般の場合が PSPACE 完全であることが証明されており，非常に困難な問題であることが知られている．

HOANG 氏はまず，この独立点集合の Reconfiguration が，グラフを木構造に制限したときに線形時間で解けることを示した．それまで主に「困難性」が示されてきがちであったこの問題に対して，特に「木」という自然な構造の上で，線形時間で解けることを示した業績は高く評価されている．この分野の近年の論文では，この HOANG 氏の業績は，ほぼ必ず引用されており，国際会議での発表後，速やかに一流ジャーナルの Theoretical Computer Science に掲載されたことも，この結果が高く評価されていることの証左であろう．その後，HOANG 氏はこの結果を別のグラフクラスに拡張し，いくつかの国際会議で発表している．

以上，本論文は Reconfiguration Problem の中でも独立点集合に注目し，グラフを制限することで多項式時間アルゴリズムを示したものであり，学術的に貢献するところが大きい．よって博士（情報科学）の学位論文として十分価値あるものと認めた．