On the Development of Algorithms for Optimal Placements of Polygonal Objects in a Given Frame

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February 15, 2002

Keywords: optimum arrangment, bin-packing, BSG, Delaunay trianglation, Minkowski-sum.

Optimal arrangement of objects has been studied in various fields as a problem of cutting out parts of clothes from fixed cloth and that of arranging chip elements in VLSI design. Although they are very difficult problems that do not seem to be solved in polynomial time, practically reasonable results are obtained for them. In this study we consider a problem of packing many polygons closely and efficiency inside a frame with fixed aspect ratio unlike those existing studies. Since there are few studies on the problem, a new idea is required. In this study we realize short computation time and efficient arrangement by arranging objects hierarchically using more than one arrangement technique. Furthermore, to apply it to automatic advertisement generation, we adjust arrangement considering the appearance toward a more practical system.

Although the target objects are arbitrary convex polygons, it is very difficult to arrange a convex polygon in a fixed frame in a direct way. Therefore, in this study, to ease the argument, we approximate convex polygons by rectangles and start with an initial arrangement. Moreover, we perform compaction of those rectangles to increase the ratio occupied by rectangles. Then, we restore those rectangles to original convex polygons and adjust the resulting arrangement to improve the impression.

For the initial arrangement of rectangles we apply bin-packing techniques. Using them, we have a rough arrangement of rectangles. Although the bin-packing technique is originally developed for one-dimensional packing of objects that have only widths, we extend it to apply for two-dimensional objects with widths and heights. Typical bin-packing techniques are the Next-Fit method, the First-Fit method, and the Best-Fit method. In this study we use the Best-Fit method after sorting rectangles to be packed in the decreasing or increasing order of their heights or after ordering them randomly. In the Best-Fit method we put each object in a bin with the least remaining capacity, unlike the First-Fit method in which we choose the first available bin. Bin packing cannot be carried out unless the maximum length of a bin is specified. We determine the length of a bin by binary search.

Two-dimensional bin packing may generate some dead space in the vertical direction because it creates bins by the largest height of rectangles. Now, if we perform two-dimensional compaction to remove the dead space, we may be able to increase the occupancy ratio. In this study we implement the compaction using the BSG(Bounded-Slice line Grid) structure developed for module arrangement of VLSI. Using the BSG structure, we can decrease the dead space of those rectangles, taking the direction of length and width into consideration. Moreover, when we have any dead space after a BSG compaction, we may be able to increase the occupancy ratio as follows. We adjust those polygons by stuffing horizontally and vertically alternately.

After the compaction is completed, we restore those rectangles to original convex polygons. So far we have paid attention only to increase the ratio occupied by rectangles, but if we think of applying this method to advertisement we don't have good appearance when a polygon leans to a part too much. Therefore, in this study we improve the impression of appearance by adjusting the arrangement so that the dead space of those polygons becomes uniform. We need to know how much space is in the surrounding area of those polygons to adjust arrangement. For this purpose we use a concept of the Voronoi diagram and the Delaunay triangulation of those polygons. Concretely, we calculate the Delaunay triangulation for a set of the line segments defined by the edges of those polygons and find dead space. To move polygons appropriately we implement two methods. In the first method we calculate the maximum inscribed circle of the space that encloses the polygon to be moved and move the polygon to the center of the inscribed circle. Concretely, we calculate the inscribed circles of those polygons and the maximum inscribed circle of the space that encloses those polygons and move a polygon with the largest distance between two circles. Then we repeat it until this operation is converged to some extent. In the second method we calculate the Minkowski-sum with the polygon and the dead space, and move a polygon using it. Concretely, we find the maximum space that encloses the vertex of those polygons, and calculate the Minkowski-sum of the space and those polygons. And we move a polygon with the largest Minkowski-sum. We repeat this operation several times. The advantage of using Minkowski-sum is that we can prevent polygons of similar sizes from getting together in one place. Using the above two adjustment methods, we can realize polygons arrangement that takes balance as a whole, though it takes some time to find the polygon that it moves.

Finally, in this study we moved only one polygon at a time to adjust the polygon arrangement. However, if we could simultaneously move two or more polygons, it would lead to still better polygon arrangement.