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Title	Binary Data Gathering with a Helper in Internet of Things: Distortion Analysis and Performance Evaluation	
Author(s)	Lin, Wensheng; He, Xin; Juntti, Markku; Matsumoto, Tad	
Citation	IEEE Access, 7: 12855-12867	
Issue Date	2019-01-15	
Туре	Journal Article	
Text version	author	
URL	http://hdl.handle.net/10119/15724	
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Description		





Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.

Digital Object Identifier xx.xxxx/ACCESS.xxxx.DOI

# Binary Data Gathering with a Helper in Internet of Things: Distortion Analysis and Performance Evaluation

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This work is funded in part by China Scholarship Council (CSC) and in part by JAIST Core-to-Core Program. This work is also in part funded by National Natural Science Foundation of China (NSFC), No. 61702011 and Anhui Provincial Natural Science Foundation, No. 1808085QF191.

**ABSTRACT** This paper focuses on one-helper assisted binary data gathering networks, for example, such as in Internet of Things (IoT), where a destination makes estimates of binary data relying on a number of agents and one helper. Due to the noise, corrupting errors already exist in the agent observations. To analyze performance of this system, we formulate this system as a binary chief executive officer (CEO) problem with a helper. Initially, we use a successive decoding scheme to decompose the binary CEO problem with a helper into the multiterminal source coding and final decision problems. Then, we present an outer bound on the rate-distortion region for multiterminal source coding with binary sources and a helper. After solving a convex optimization problem formulated from the derived outer bound, we obtain the final distortion by substituting the minimized distortions of observation into the distortion propagating function (DPF), which is derived to bridge the relationship between the joint decoding results and final decision. Finally, we analyze the trade-off of rate-distortion through theoretical calculation and simulations. Both the theoretical and simulation results demonstrate that a helper can obviously reduce the signal-to-noise ratio (SNR) threshold. We also have an in-depth discussion on the differences of system performance improvement between locating a helper and including an additional agent.

**INDEX TERMS** Binary CEO problem, binary data gathering, Internet of Things, rate-distortion, side information

#### I. INTRODUCTION

Internet of Things (IoT) attracts increasing attentions of academia and industry, owing to the significant role of IoT in the smart society. Due to the frequent use of binary signalling, in general, binary data gathering plays fundamental roles of IoT in the big data era. Generally, binary data gathering is based on direct detection and/or with the aid of agents. However, direct transmissions are almost impossible in some strict communications environment, e.g., path loss due to the long distance between the original binary data and the destination, and/or time varying nature of the channels due to fading. Therefore, the destination has to only rely on the data received from the agents and make estimations of the original binary data. Essentially, this scenario can be formulated as the chief executive officer (CEO) problem [1] with binary sources. In the ordinary case, the agent observations of the original data may contain errors when the agents suffer from noise. Consequently, the estimate of the target information could be a lossy recovery, if the rates supported by the channels are not large enough due to the harsh communications environment.

Nowadays, to make connections more robust and enhance the system performance, several wireless communications systems introduce the concept of helper [2]–[5]. These applications of the helper inspire us to refine the performance of binary data gathering in IoT system by the assistance of a helper. As illustrated in Fig. 1, there are many agents observing the same binary data while errors corrupt the data

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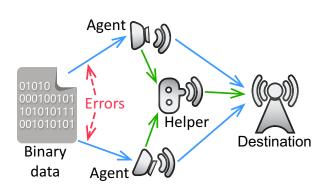


FIGURE 1. A scenario of binary data gathering with a helper in IoT.

sequences. Subsequently, the agents independently encode and transmit the error-corrupted sequences to the destination; meanwhile, a helper generates the helper sequence by monitoring the agents and then send it to the destination. After decoding the received sequences from all agents and the helper, the destination makes the final estimation of the original binary data.

Since the system model shown in Fig. 1 can be formulated as a binary CEO problem with a helper, we can analyze the system performance inspired by the previous achievements related to the binary CEO problem. He et al. [6] presented a lower bound of Hamming distortion for the binary CEO problem with two sources, and the result was further extended to solve the binary CEO problem with many sources in [7]. Razi and Abedi [8] developed a method to analyze the convergence of iterative decoding for the binary CEO problem. An iterative joint decoding algorithm was implemented into the wireless sensor networks (WSNs) with binary sources according to the model of binary CEO problem by Haghighat et al. in [9]. It is noticed that the binary CEO problem is solved in [6], [7] by decomposing the problem into multiterminal source coding and final decision, i.e., a successive decoding process. Likewise, for the performance analysis of the binary CEO problem with a helper, we can start from the problem of multiterminal source coding with a helper and then investigate the distortion of final decision.

Regarding the theoretical work related to multiterminal source coding with a helper or side information, Ahlswede and Korner [10] derived the rate region for source coding with a helper providing side information. In [11], Wyner and Ziv characterized the rate-distortion function for lossy compression with side information, where only one source needs to be recovered and another one provides assistance without rate limit. Slepian and Wolf [12] presented the fundamental theorem of lossless multiterminal source coding, where each source can be viewed as the side information for another one. The Slepian-Wolf theorem asserts a surprising result that distributed compression has the same achievable rate region as joint source coding. For the system without necessary requirements of the full source recoveries, Berger [13] and Tung [14] derived the inner and outer bounds on the rate-distortion region of the lossy multiterminal source coding problem. In [15], Wagner and Anantharam extended the Berger-Tung bounds to the case with multiple sources and one link of uncompressed side information.

In the final decision step, the bit error probability of binary data gathering was analyzed in [16], where many correlated sources in a WSN have diverse bit-flipping probabilities, and soft combining is implemented as the final decision rule. Based on these previous achievements, we establish the framework of the binary CEO problem with a helper as a successive encoding/decoding process, i.e., encoding/decoding the multiple sources with the assistance of a helper and then combining the joint decoding results. By this means, we have finished the performance analysis for binary data gathering with a helper in IoT.

The contributions of this paper are summarized as follows:

- To theoretically analyze the system performance, we formulate binary data gathering by a one-helper assisted IoT as a binary CEO problem with a helper. Based on the Berger-Tung outer bound, we derive an outer bound on the rate-distortion region of multiterminal source coding problem with many agents and a helper for binary sources. Then, the outer bound is utilized to formulate a convex optimization problem, for the purpose of minimizing distortions when reconstructing observations.
- Moreover, we analyze the distortion propagating from the estimate of agent sequences to the final decision. By substituting the solution of the convex optimization problem, we investigate the trade-off of rate-distortion for binary data gathering with a helper in IoT.
- Finally, we propose a practical coding scheme and evaluate the system performance through simulations for binary data gathering with a helper in IoT. Besides, we make a comparison of performance improvement between the system with a helper and that with an additional agent.

The rest of this paper is organized as follows. Section II formulates binary data gathering with a helper in IoT as the binary CEO problem with a helper. In Section III, we analyze the rate-distortion performance of the formulated binary CEO problem with a helper. Then, Section IV evaluates the system performance through computer simulations for binary data gathering with a helper in IoT. Finally, we conclude this work in Section V.

#### **II. SYSTEM MODEL**

*Notation.* The random variables and their realizations are denoted by uppercase and lowercase letters, respectively. Calligraphic letters  $\mathcal{X}, \mathcal{Y}, \cdots$  denote the finite alphabets of a random variable. The superscript of a random vector and its realization represent the length of the vector. We use t to denote the time index and i to denote the index of an agent. The random variable with a finite alphabet as subscript denotes a set of all random variables with index in the finite alphabet, such as  $X_{\mathcal{L}} = \{X_i | i \in \mathcal{L}\}$ . For a subset  $\mathcal{S} \subseteq \mathcal{L}, \mathcal{S}^c$  represents the corresponding complementary set.

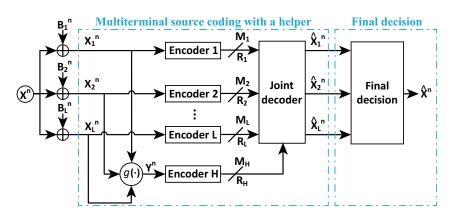


FIGURE 2. A binary CEO problem with a helper.

As mentioned above, the IoT system depicted in Fig. 1 can be formulated as the binary CEO problem with a helper in Fig. 2, where a binary source X acts as a common source. The discrete memoryless source (DMS) X generates independent and identically distributed (i.i.d.) sequence  $x^n = \{x(t)\}_{t=1}^n$  by taking values from the binary alphabet  $\mathcal{X} = \{0, 1\}$  for each time slot. The source X is observed by L agents at the same time. Due to the influence of noise, the observation  $x_i^n = \{x_i(t)\}_{t=1}^n$ , where  $i \in \mathcal{L} = \{1, 2, \cdots, L\}$ , may contains errors  $b_i^n = \{b_i(t)\}_{t=1}^n$  [1]. Hence, the error probability  $\Pr\{x_i(t) \neq x(t)\} = p_i$  for  $B_i \sim \text{Bern}(p_i)$ . The erroneous sequences are still forwarded to the destination, which is referred to as lossy-forward [17], [18]. Simultaneously, a helper generates the helper sequence  $y^n = \{y(t)\}_{t=1}^n$ from the agent sequences bit by bit, and then transmits the helper sequence to the destination after compressing it. Therefore, all the  $X_i$  and Y can be also regarded as DMS. The sequences  $x_i^n$  and  $y^n$  are encoded at rates  $R_i$  and  $R_{\rm H}$ by encoder i and encoder H, respectively. Encoder i and encoder H assign an index to each sequence according to the following mapping rules:

$$\varphi_i: \mathcal{X}_i^n \mapsto \mathcal{M}_i = \{1, 2, \cdots, 2^{nR_i}\},\tag{1}$$

$$\varphi_{\mathrm{H}}: \mathcal{Y}^n \mapsto \mathcal{M}_{\mathrm{H}} = \{1, 2, \cdots, 2^{nR_{\mathrm{H}}}\}.$$
 (2)

Then, the encoder outputs  $M_{\mathcal{L}}$  and  $M_{\rm H}$  are transmitted to a joint decoder. The joint decoder constructs the estimates  $\hat{x}_{\mathcal{L}}^n$  from indices  $M_{\mathcal{L}}$  and  $M_{\rm H}$  by utilizing the mapping rule, as:

$$\psi: \mathcal{M}_1 \times \mathcal{M}_2 \times \dots \times \mathcal{M}_L \times \mathcal{M}_H \\ \mapsto \mathcal{X}_1^n \times \mathcal{X}_2^n \times \dots \times \mathcal{X}_L^n.$$
(3)

Since the estimate  $\hat{x}_i^n$  may occasionally deviate from the observation  $x_i^n$ , the Hamming distortion measure is defined to describe the distortion level between  $x_i(t)$  and  $\hat{x}_i(t)$ , as

$$d_i(x_i(t), \hat{x}_i(t)) = \begin{cases} 1, & \text{if } x_i(t) \neq \hat{x}_i(t), \\ 0, & \text{if } x_i(t) = \hat{x}_i(t). \end{cases}$$
(4)

Hence, the expected distortion between the sequences  $x_i^n$  and  $\hat{x}_i^n$  is

$$d_i(x_i^n, \hat{x}_i^n) = \frac{1}{n} \sum_{t=1}^n d_i(x_i(t), \hat{x}_i(t)).$$
 (5)

For given distortion values  $D_{\mathcal{L}}$ , the rate-distortion region  $\mathcal{R}(D_{\mathcal{L}})$ , consisting of all achievable rate tuple  $(R_{\mathcal{L}}, R_{\rm H})$ , is defined as

$$\mathcal{R}(D_{\mathcal{L}}) = \{ (R_{\mathcal{L}}, R_{\mathrm{H}}) : (R_{\mathcal{L}}, R_{\mathrm{H}}) \text{ is admissible such that} \\ \lim_{n \to \infty} \mathrm{E}(d_i(x_i^n, \hat{x}_i^n)) \le D_i + \epsilon, \\ \text{for } i \in \mathcal{L}, \text{ and any } \epsilon > 0 \}.$$
(6)

Finally, the destination reconstructs the estimate of  $x^n$  from  $\hat{x}_{\mathcal{L}}^n$ . Obviously, the distortion between  $x_i^n$  and  $\hat{x}_i^n$  will determine the final estimate  $\hat{x}^n$ . Hence, the final distortion

$$\mathbf{E}\left[\frac{1}{n}\sum_{t=1}^{n}d(x(t),\hat{x}(t))\right] \le D+\epsilon,\tag{7}$$

can be formulated as a function  $F_{DP}(\cdot)$  of  $D_{\mathcal{L}}$ , where  $F_{DP}(\cdot)$  is referred to as distortion propagating function (DPF). The DPF is defined as  $D = F_{DP}(D_{\mathcal{L}})$ , which highly depends on the decision rule. It should be emphasized here that the DPF limits the decoding scheme to successive decoding, i.e., first reconstructing  $\hat{x}_{\mathcal{L}}^n$  and then making the final decision.

#### **III. RATE-DISTORTION ANALYSIS**

The first step of performance analysis for binary data gathering with a helper in IoT is to derive an outer bound on the achievable rate-distortion region of multiterminal source coding with a helper. By utilizing the derived outer bound, we can formulate a convex optimization problem to minimize the distortions in the step of multiterminal source coding. Then, by investigating the DPF with respect to specified decision rule, we can obtain the distortion of final decision and finish the performance analysis.

# A. MULTITERMINAL SOURCE CODING WITH A HELPER 1) Outer Bound of Rate-Distortion Region

From the extended Berger-Tung outer bound [15] with multiple sources and one link of side information, we can obtain

VOLUME x, 2019

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the outer bound for multiterminal source coding with a helper as presented in the following proposition.

**Proposition 1:** Let  $(X_{\mathcal{L}}, Y)$  be a (L + 1)-DMS and  $d_i(x_i, \hat{x}_i)$  for  $i \in \mathcal{L}$  be distortion measures. If a rate tuple  $(R_{\mathcal{L}}, R_{\mathrm{H}})$  is achievable with distortion tuple  $D_{\mathcal{L}}$  for distributed lossy source coding with a helper observing Y, then it must satisfy the inequalities

$$\sum_{i \in \mathcal{S}} R_i \ge I(X_{\mathcal{L}}; U_{\mathcal{S}} | U_{\mathcal{S}^c}, V), \tag{8}$$

$$R_{\rm H} \ge I(Y;V),\tag{9}$$

for some conditional probability mass function  $p(u_{\mathcal{L}}, v|$  $x_{\mathcal{L}}, y)$ , and functions  $\hat{x}_i(u_{\mathcal{L}}, v)$  such that  $U_i \to X_i \to X_j$  and  $X_i \to X_j \to U_j$  form Markov chains and  $E(d_i(X_i, \hat{X}_i)) \leq D_i$ , where  $i, j \in \mathcal{L}$  and  $i \neq j$ .

It is easy to understand that the constraint of (9) results from the rate limit on the helper instead of uncompressed side information. Then, we introduce the definitions to be used in the derivation of the outer bound for binary sources.

Definition 1: We define the following variables:

$$\alpha_i = p_i * h^{-1} (1 - [R_i]^-), \tag{10}$$

$$\beta_i = p_i * D_i,\tag{11}$$

where the operation \* denotes the binary convolution process;  $[R_i]^- = \min\{1, R_i\}$ ;  $h(\cdot)$  and  $h^{-1}(\cdot)$  denote the binary entropy function and its inverse, respectively.

Definition 2: According to [7], given a set of crossover probabilities  $\{\mathcal{P}\}$  with a common source X, the joint entropy  $f(\cdot)$  of the outputs from independent binary symmetric channels (BSCs) is calculated as

$$f(\{\mathcal{P}\}) = -\sum_{j=1}^{2^{|\mathcal{P}|}} q_j \log_2(q_j),$$
(12)

where  $|\cdot|$  denotes the cardinality of the set, and

$$q_j = 0.5 \left( \prod_{k \in \mathcal{A}_i} p_k \prod_{k' \in \mathcal{A}_i^c} \bar{p}_{k'} + \prod_{k \in \mathcal{A}_i} \bar{p}_k \prod_{k' \in \mathcal{A}_i^c} p_{k'} \right), \quad (13)$$

with  $\bar{p} = 1 - p$  and  $\mathcal{A}_i \subseteq \{1, 2, \cdots, |\mathcal{P}|\}.$ 

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Now, we calculate the outer bound for binary sources. Consider

$$\sum_{i \in S} R_i \ge I(X_{\mathcal{L}}; U_{\mathcal{S}} | U_{\mathcal{S}^c}, V)$$

$$= H(X_{\mathcal{L}} | U_{\mathcal{S}^c}, V) - H(X_{\mathcal{L}} | U_{\mathcal{S}}, U_{\mathcal{S}^c}, V)$$

$$= H(X_{\mathcal{L}} | U_{\mathcal{S}^c}) - I(X_{\mathcal{L}}; V | U_{\mathcal{S}^c})$$

$$- H(X_{\mathcal{L}} | U_{\mathcal{S}}, U_{\mathcal{S}^c}) + I(X_{\mathcal{L}}; V | U_{\mathcal{S}}, U_{\mathcal{S}^c})$$

$$= I(X_{\mathcal{L}}; U_{\mathcal{S}} | U_{\mathcal{S}^c}) - I(X_{\mathcal{L}}; V | U_{\mathcal{S}^c})$$

$$+ I(X_{\mathcal{L}}; V | U_{\mathcal{S}}, U_{\mathcal{S}^c})$$

$$= I(X_{\mathcal{L}}; U_{\mathcal{S}} | U_{\mathcal{S}^c}) + I(X_{\mathcal{S}}; V | U_{\mathcal{L}})$$

$$- I(X_{\mathcal{S}}; V | U_{\mathcal{L}}) - I(X_{\mathcal{L}}; V | U_{\mathcal{S}^c})$$

$$+ I(X_{\mathcal{L}}; V | U_{\mathcal{S}}, U_{\mathcal{S}^c}).$$
(14)

Then, we calculate  $I(X_{\mathcal{L}}; U_{\mathcal{S}}|U_{\mathcal{S}^c}) + I(X_{\mathcal{S}}; V|U_{\mathcal{L}})$  and  $-I(X_{\mathcal{S}}; V|U_{\mathcal{L}}) - I(X_{\mathcal{L}}; V|U_{\mathcal{S}^c}) + I(X_{\mathcal{L}}; V|U_{\mathcal{S}}, U_{\mathcal{S}^c})$  separately. Consider

$$\begin{split} I(X_{\mathcal{L}}; U_{\mathcal{S}} | U_{\mathcal{S}^{c}}) + I(X_{\mathcal{S}}; V | U_{\mathcal{L}}) \\ &= I(X_{\mathcal{S}}; U_{\mathcal{S}} | U_{\mathcal{S}^{c}}) + I(X_{\mathcal{S}^{c}}; U_{\mathcal{S}} | U_{\mathcal{S}^{c}}, X_{\mathcal{S}}) \\ &+ I(X_{\mathcal{S}}; V | U_{\mathcal{L}}) \\ &= I(X_{\mathcal{S}}; U_{\mathcal{S}} | U_{\mathcal{S}^{c}}) + H(X_{\mathcal{S}^{c}} | U_{\mathcal{S}^{c}}, X_{\mathcal{S}}) \\ &- H(X_{\mathcal{S}^{c}} | U_{\mathcal{S}}, U_{\mathcal{S}^{c}}, X_{\mathcal{S}}) + I(X_{\mathcal{S}}; V | U_{\mathcal{L}}) \\ &= I(X_{\mathcal{S}}; U_{\mathcal{S}} | U_{\mathcal{S}^{c}}) + H(X_{\mathcal{S}^{c}} | U_{\mathcal{S}^{c}}, X_{\mathcal{S}}) \\ &- H(X_{\mathcal{S}^{c}} | U_{\mathcal{S}^{c}}, X_{\mathcal{S}}) + I(X_{\mathcal{S}}; V | U_{\mathcal{L}}) \\ &= I(X_{\mathcal{S}}; U_{\mathcal{S}} | U_{\mathcal{S}^{c}}) + I(X_{\mathcal{S}}; V | U_{\mathcal{L}}) \\ &= I(X_{\mathcal{S}}; U_{\mathcal{S}} | U_{\mathcal{S}^{c}}) - I(X_{\mathcal{S}}; U_{\mathcal{S}^{c}}) \\ &+ I(X_{\mathcal{S}}; V | U_{\mathcal{S}}, U_{\mathcal{S}^{c}}) \\ &= I(X_{\mathcal{S}}; U_{\mathcal{L}}, V) - I(X_{\mathcal{S}}; U_{\mathcal{S}^{c}}) \\ &\geq I(X_{\mathcal{S}}; \hat{X}_{\mathcal{S}}) - I(X_{\mathcal{S}}; U_{\mathcal{S}^{c}}), \end{split}$$
(16)

where (15) follows the fact that  $U_i \to X_i \to X_j$  form a Markov chain for  $i \in S$  and  $j \in S^c$ ; (16) follows data processing inequality when estimating  $\hat{X}_S$  from  $(U_L, V)$ . By applying the result in [7] into (16), we have

$$I(X_{\mathcal{L}}; U_{\mathcal{S}}|U_{\mathcal{S}^c}) + I(X_{\mathcal{S}}; V|U_{\mathcal{L}})$$
  

$$\geq f(\{p_{\mathcal{S}}, \alpha_{\mathcal{S}^c}\}) - f(\{\alpha_{\mathcal{S}^c}\}) - \sum_{i \in \mathcal{S}} h(D_i), \quad (17)$$

Next, consider

$$- I(X_{\mathcal{S}}; V|U_{\mathcal{L}}) - I(X_{\mathcal{L}}; V|U_{\mathcal{S}^{c}}) + I(X_{\mathcal{L}}; V|U_{\mathcal{S}}, U_{\mathcal{S}^{c}}) 
= -I(X_{\mathcal{S}}; V|U_{\mathcal{L}}) - H(V|U_{\mathcal{S}^{c}}) 
+ H(V|X_{\mathcal{L}}, U_{\mathcal{S}^{c}}) + H(V|U_{\mathcal{S}}, U_{\mathcal{S}^{c}}) 
- H(V|X_{\mathcal{L}}, U_{\mathcal{S}}, U_{\mathcal{S}^{c}}) 
= -I(X_{\mathcal{S}}; V|U_{\mathcal{L}}) - H(V|U_{\mathcal{S}^{c}}) + H(V|X_{\mathcal{L}}) 
+ H(V|U_{\mathcal{S}}, U_{\mathcal{S}^{c}}) - H(V|X_{\mathcal{L}})$$
(18)  
= H(V|U\_{\mathcal{L}}, X\_{\mathcal{S}}) - H(V|U\_{\mathcal{L}}) - H(V|U\_{\mathcal{S}^{c}})   
+ H(V|U\_{\mathcal{S}}, U\_{\mathcal{S}^{c}})   
= -I(V; U\_{\mathcal{S}}, X\_{\mathcal{S}}|U\_{\mathcal{S}^{c}})   
= -I(V; X\_{\mathcal{S}}|U\_{\mathcal{S}^{c}}), (19)

where (18) and (19) follow since  $U_i \to X_i \to V$  form a Markov chain for  $i \in \mathcal{L}$ . To further calculate (19), consider

<

$$I(V; X_{\mathcal{S}} | U_{\mathcal{S}^c}) \le I(V; X_{\mathcal{S}}) \tag{20}$$

$$\leq I(V;Y) \tag{21}$$

$$\leq R_{\rm H},$$
 (22)

where (20) follows since condition reduces entropy, and (21) follows since  $X_S \to Y \to V$  form a Markov chain. Notice that the equality of (20) holds when no mutual information exists between V and  $U_{S^c}$ , i.e., the helper allocates full rate to help recovering  $X_S$ . The equality of (21) holds when the helper only utilize  $X_S$  to generate the helper information Y. Moreover, the equality of (22) holds when the helper rate is completely exploited for compression. Consequently, there is

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no waste of the helper rate if the conditions for the equality of (20-22) are satisfied. In this case, it is obvious that the structure of the helper is optimal. By assuming that we have an optimal helper and substituting (17), (19) and (22) into (14), we can finally obtain

$$\sum_{e \in \mathcal{S}} R_i$$

$$\geq f(\{p_{\mathcal{S}}, \alpha_{\mathcal{S}^e}\}) - f(\{\alpha_{\mathcal{S}^e}\}) - \sum_{i \in \mathcal{S}} h(D_i) - R_{\mathrm{H}}. \quad (23)$$

*Remark 1*: Since we assume that the structure of the helper is optimal, the constraint on the helper link, i.e., the inequality (9), is satisfied by the helper encoder, which finds a proper codeword  $M_{\rm H}$  to make I(Y; V) as large as possible.

*Remark 2*: If we set  $R_{\rm H} = 0$ , i.e., the helper link is equivalently cut off, (23) reduces to the outer bound without a helper in [7].

#### 2) Distortion Minimization

1

Since the final distortion D is a function of  $D_{\mathcal{L}}$  by the successive decoding for given  $R_{\mathcal{L}}$  and  $R_{\mathrm{H}}$ , we can first minimize the  $l_2$ -norm of the vector  $[D_1, D_2, \dots, D_L]$  by solving a convex optimization problem [6], [7], which is formulated from the outer bound. Then, we calculate the minimum distortion  $D^*$  by substituting the solution of the convex optimization problem into DPF. Notice that for a practical communications system with channels, the channel capacity should also be taken into consideration. According to Shannon's lossy source-channel separation theorem [19], [20], the distortion tuple  $D_{\mathcal{L}}$  is achievable if the following inequalities hold:

$$\begin{cases} R_i(D_i) \cdot r_i \le C(\gamma_i), \text{ for } i \in \mathcal{L}, \\ R_{\rm H} \cdot r_{\rm H} \le C(\gamma_{\rm H}), \end{cases}$$
(24)

where  $C(\gamma_i)$  and  $C(\gamma_{\rm H})$  are the Shannon capacity using Gaussian codebook with  $\gamma$  as the signal-to-noise ratio (SNR) of the channel;  $r_i$  and  $r_{\rm H}$  represent the end-to-end coding rates [21] of each agent link and the helper link, respectively. By applying the outer bound derived above, we can formulate the convex optimization problem for the system with an optimal helper as

$$\min_{\substack{D_1, D_2, \cdots, D_L}} \| [D_1, D_2, \cdots, D_L] \|_2$$
s.t.
$$\sum_{i \in S} h(D_i) \ge f(\{p_S, \alpha_{S^c}\}) - f(\{\alpha_{S^c}\}) - \sum_{i \in S} \frac{C(\gamma_i)}{r_i}$$

$$- \frac{C(\gamma_H)}{r_H},$$

$$0 \le D_i \le 0.5, i \in \mathcal{L}.$$
(25)

After solving the convex optimization problem, we can obtain the minimum value of distortion  $D_i^*$  for  $i \in \mathcal{L}$ . Then, we use the estimates  $\hat{X}_{\mathcal{L}}^n$  with minimum distortions  $D_{\mathcal{L}}^*$  to make final decision.

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#### **B. FINAL DECISION**

Notice that the compressed side information  $V^n$  is not used in the final decision to minimize the distortion between  $X^n$ and  $\hat{X}^n$ . Hence, it is necessary to discuss the influence of the compressed side information on final decision. We consider the theoretically optimal case and a practical case, i.e., majority voting decision, respectively. For the final decision with compressed side information, i.e., the system model shown in Fig. 3, when the final decision is optimal, consider

$$H(X) - h(\tilde{d})$$

$$\leq I(X; \hat{X})$$

$$\leq I(X; \hat{X}_{\mathcal{L}}, V)$$

$$= I(X; \hat{X}_{\mathcal{L}}) + I(X; V | \hat{X}_{\mathcal{L}})$$
(26)

$$I(\underline{X}, \underline{\hat{Y}}) + I(\underline{X}, \underline{\hat{Y}}) + I(\underline{X}, \underline{\hat{Y}})$$

$$= I(\Lambda;\Lambda_{\mathcal{L}}) + H(V|\Lambda_{\mathcal{L}}) - H(V|\Lambda_{\mathcal{L}},\Lambda)$$
(27)

$$=I(X;X_{\mathcal{L}}),\tag{28}$$

where d is a dummy variable, and the steps are justified as: (26) according to data processing inequality, information loss may happen in the final decision,

(27) notice that V is a function of Y and Y is a function of  $X_{\mathcal{L}}$ , and hence V is a function of  $X_{\mathcal{L}}$ , i.e.,  $V = g'(X_{\mathcal{L}})$ . If  $H(V|\hat{X}_{\mathcal{L}}) = H(g'(X_{\mathcal{L}})|\hat{X}_{\mathcal{L}}) > 0$ , it means that there is still some information of  $X_{\mathcal{L}}$  not utilized in joint decoding. Hence, some distortions  $D_i$  for  $i \in \mathcal{L}$  can be further reduced in the convex optimization problem. Consequently, we have  $H(V|\hat{X}_{\mathcal{L}}) = 0$  and  $H(V|\hat{X}_{\mathcal{L}}, X) = 0$  for the best utilization of helper information in multiterminal source coding.

It is obvious that (26) is equal to (28), i.e., the compressed side information does not change the minimum final distortion. Similarly, for the majority voting decision, since we use the successive decoding scheme, the distortions  $D_{\mathcal{L}}$ must be minimized after joint decoding. This means that the helper information has been completely utilized in joint decoding step. Hence, the final distortion will not further reduce, whether the compressed side information is taken into consideration or not in the majority voting decision. It is sufficient to derive the DPF  $F_{DP}(D_{\mathcal{L}})$  for both optimal and majority voting decision without side information.

For the optimal decision rule, consider

$$H(X) - h(\tilde{d})$$

$$\leq I(X; \hat{X})$$

$$\leq I(X; \hat{X}_{\mathcal{L}}) \qquad (29)$$

$$= H(X) + H(\hat{X}_{\mathcal{L}}) - H(X, \hat{X}_{\mathcal{L}})$$

$$= H(X) + f(\{\beta_{\mathcal{L}}\}) - f(\{0, \beta_{\mathcal{L}}\}), \qquad (30)$$

(29) the probable information loss in the final decision,(30) X can be regarded as the output of a BSC with itself as the input and the crossover probability equal to 0.

Consequently, we have

$$\tilde{d} \ge h^{-1}[f(\{0, \beta_{\mathcal{L}}\}) - f(\{\beta_{\mathcal{L}}\})].$$
(31)

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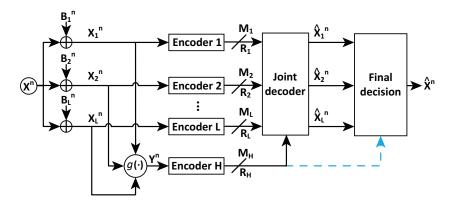


FIGURE 3. The system model for final decision with compressed side information.

Obviously, the minimum final distortion D, i.e., the distortion by optimal decision, is given by

$$D = h^{-1}[f(\{0, \beta_{\mathcal{L}}\}) - f(\{\beta_{\mathcal{L}}\})].$$
(32)

Since the optimal decision specifies a universal lower bound, here, we consider another practical decision scheme, i.e., majority voting. The distortion between  $X^n$  and  $\hat{X}^n$  by majority voting is the sum probability of several events in a Poisson binomial process [16]. We introduce a function to evaluate the distortion in a Poisson binomial process as follows:

Definition 3: Poisson binomial distortion function [16]. The distortion between  $X^n$  and  $\hat{X}^n$ , which is estimated from  $\hat{X}^n_{\mathcal{L}}$  by majority voting, is calculated by  $D = PB(\beta_{\mathcal{L}})$  as

$$PB(\beta_{\mathcal{L}}) = \begin{cases} \sum_{j=\frac{L+1}{2}}^{L} \Pr(J=j), & \text{if } L \text{ is odd,} \\ \frac{1}{2} \Pr(J=\frac{L}{2}) & (33) \\ + \sum_{j=\frac{L}{2}+1}^{L} \Pr(J=j), & \text{if } L \text{ is even,} \end{cases}$$

where

6

$$\Pr(J = j) = \begin{cases} \prod_{i=1}^{L} (1 - \beta_i), & j = 0, \\ \frac{1}{j} \sum_{i=1}^{j} (-1)^{(i-1)} \Pr(J = j - i) \lambda(i), & j > 0, \end{cases}$$
(34)

with  $\lambda(i) = \sum_{k=1}^{L} \left(\frac{\beta_k}{1-\beta_k}\right)^i$  for  $0 \le j \le L$ . By utilizing the Poisson binomial distortion function, we can calculate the distortion D by majority voting among  $X_{\mathcal{L}}$ as

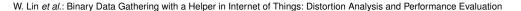
$$D = PB(\beta_{\mathcal{L}}). \tag{35}$$

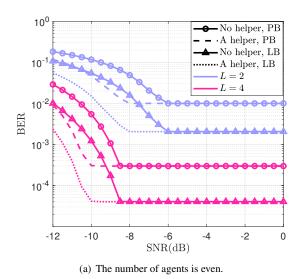
*Remark*: If  $p_i$  are various among all links, weighted majority voting should be implemented to generate more accurate estimate of  $X^n$ . The error probability by weighted majority voting is presented in Appendix A. In this paper, we focus on the system with homogeneous agents for simplicity, and the results can be easily extended to the case with heterogeneous agents according to Appendix A.

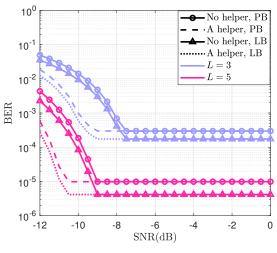
In summary, the DPFs for optimal decision and majority voting are (32) and (35), respectively. Therefore, we can obtain the minimum final distortion between  $X^n$  and  $X^n$ by substituting the solution  $D^*_{\mathcal{L}}$  of the convex optimization problem into DPF as  $D^* = F_{DP}(\beta_{\mathcal{L}}^*)$ , where  $\beta_{\mathcal{L}}^* = \{p_i * \}$  $D_i^* | i \in \mathcal{L} \}.$ 

#### C. NUMERICAL RESULTS

Now, we start investigations on the trade-off of rate-distortion for binary data gathering with a helper in IoT through numerical results. A memoryless source  $X \sim \text{Bern}(0.5)$  is used in the following. Initially, we compare the bit error rate (BER) performance between majority voting and optimal decision. By utilizing the DPF after solving a corresponding convex optimization problem, we can depict the curve of SNR for each link versus BER as shown in Fig. 4. All the crossover probabilities between X and  $X_i$  are set at the same value of 0.01. Moreover, the end-to-end coding rate is set at  $\frac{1}{2}$ , and the SNR is set at the same level for all of the agent and helper links. Notice from the results that whether there is a helper or not, a gap obviously appears between the Poisson binomial (PB) process, i.e., majority voting, and the theoretical lower bound (LB), i.e., optimal decision. The reason for the performance gap is that it is extremely difficult to completely utilize the mutual information between  $X^n$  and  $X^n_{\mathcal{L}}$ . For instance, assuming that there are 2K agents with  $X_t = X_{1,t} = \cdots =$  $X_{K,t} = 0$  and  $X_{K+1,t} = \cdots = X_{2K,t} = 1$  at some time index t, it is obvious that there is some mutual information between  $X_t$  and  $\{X_{1,t}, \dots, X_{K,t}\}$ . However, decision error will still occur in the Poisson binomial process, resulting in the loss of mutual information between  $X_t$  and  $\hat{X}_t$ . It should be also noticed that the gap is sensitive to the number of agents, i.e., the gap is smaller when there are odd number of agents. Because there could be equal number of "0" and "1" at the same bit of the agent sequences, if the number of agents is even. In this draw case, the bit of final decision by majority





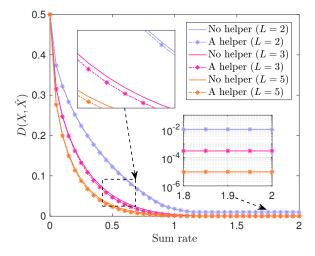


(b) The number of agents is odd.

**FIGURE 4.** Comparison of system performance between majority voting and optimal decision, where  $p_i=0.01$ .

voting is randomly selected, resulting in more performance loss. Hence, the majority voting decision can achieve better performance if odd number of agents are deployed. For the effect of a helper, we find that the helper can reduce BER at small SNR value range, but keep the same BER floor as the case without a helper. Because for sufficiently large SNR, there is already no distortion between  $X_i^n$  and  $\hat{X}_i^n$ , and the side information  $Y^n$  generated from  $X_L^n$  becomes redundant.

We further investigate the effect on rate-distortion by introducing a helper with majority voting decision in Fig. 5. Since the rate allocation scheme is out of the scope in this paper, the rate is evenly allocated to all nodes including agents and helper. Surprisingly, the helper can reduce the final distortion D before achieving the distortion floor, even though the agent rate decreases by sharing the sum rate with a helper. This phenomenon indicates that it is possible to improve the system performance for multiple access channels with the same sum rate by introducing a helper. However, the



**FIGURE 5.** The trade-off of rate-distortion with a majority voting helper, where  $p_i = 0.01$  for all agents.

curve with a helper will finally converge with the no-helper case at the same distortion floor. Notice the fact that the curve with a helper is still valuable, although the improvement is very small.

Finally, we make a comparison of rate-distortion between majority voting and optimal decision with diverse  $p_i$ . Fig. 6 shows an inclination that the more correlated observations are, i.e.,  $p_i$  is smaller, the faster the distortion floor is achieved. Because it is easier to minimize the distortion between  $X_i^n$  and  $X_i^n$  for more correlated observations, owing to more mutual information among the observations. Moreover, as the correlation among the agent observations increases, the distortion floor decreases for both majority voting and optimal decision. We can also find the same phenomenon as Fig. 4 that the gap between majority voting and optimal decision is smaller for odd number of agents, even if the number of agents in Fig. 6(a) is less than that in Fig. 6(b). In addition, since in the situation where the mutual information is lost less frequently with smaller  $p_i$ , the gap between majority voting and optimal decision is smaller with more correlated observations.

#### **IV. PRACTICAL PERFORMANCE EVALUATION**

#### A. SIMULATION DESIGN

In this section, we evaluate the practical performance of binary data gathering with a helper in IoT through simulations. As depicted in Fig. 7, there are L encoders separately encode the observations  $X_i^n$ , which is detected from a common source sequence  $X^n$  and mixed with the error  $B_i^n$ . Simultaneously, the encoder H encodes the side information  $Y^n$  generated from the agent observations. Next, the encoded sequences are sent to a fusion center through additive white Gaussian noise (AWGN) channels after modulation. The fusion center first demodulates the received signals and then jointly decodes them. Finally, the estimate of all sequences in the last round of iteration is used to make final decision.

Regarding the helper sequence  $Y^n$ , since its optimal

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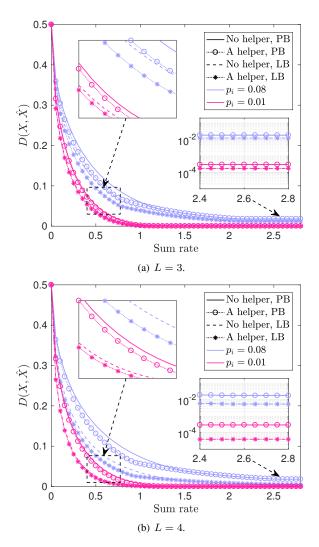


FIGURE 6. The trade-off of rate-distortion with diverse number of agents.

structure is still an open problem, we select two frequently implemented structures in practice as  $g(\cdot)$ , i.e., the helper information generated by modulo-2 sum or majority voting. The  $g(\cdot)$  by modulo-2 sum is illustrated in Fig. 8, where  $X_i^n$ is interleaved by  $\Pi_{i,1}$  for the first step; subsequently,  $Y^n$  is produced by the modulo-2 sum of interleaved  $X_{\mathcal{L}}^n$  bit by bit. By the interleaver  $\Pi_{i,1}$ , the distribution of Y is approximate to Bern(0.5), i.e.,  $Y^n$  can contain side information as much as possible.

Fig. 9 shows the structure of encoders. In order to better exploit the correlation among  $X_{\mathcal{L}}^n$ , the interleaver  $\Pi_{i,1}$  is used to disperse noises into different bits of  $X_i^n$ . Subsequently, the interleaved  $X_i^n$  is outer-encoded with a convolutional code (CC). For the purpose of utilize the principle of turbo code [22] in decoding, an accumulator (ACC) [23] inner-encodes the interleaved outer code processed by another interleaver  $\Pi_{i,2}$ . For the helper, only one interleaver  $\Pi_{\rm H}$  is needed between CC and ACC.

The general structure of joint decoder is depicted in Fig. 10. A decoder of ACC (ACC<sup>-1</sup>) and a decoder of CC

 $(CC^{-1})$  decode inner code and outer code, respectively. In local iteration, the extrinsic information is exchanged between  $ACC^{-1}$  and  $CC^{-1}$  via an interleaver II and its corresponding deinterleaver  $\Pi^{-1}$ . After several rounds of local iteration, the  $CC^{-1}$  outputs *a posteriori* log-likelihood ratio (LLR<sup>*p*</sup>) of information bits. In global iteration, an extrinsic information exchanger updates *a priori* LLR (LLR<sup>*a*</sup>) with the extrinsic LLR (LLR<sup>*e*</sup>), which is calculated by (LLR<sup>*p*</sup> – LLR<sup>*a*</sup>). The joint decoder alternately executes local iteration and global iteration, until the mutual information calculated from LLR<sup>*p*</sup><sub>*i*</sub> is large enough or the maximum iteration time is exceeded. Finally, the estimate of  $X_i^n$  is produced by the hard decision of LLR<sup>*p*</sup><sub>*i*</sub> in the last round of local iteration.

The extrinsic information exchanger updates the LLR<sup>a</sup> according to the rules shown in Fig. 11. For the case with helper information generated by modulo-2 sum, the LLR<sup>e</sup> of all agents and the helper is first exchanged based on the same principle for the check node of low-density parity-check (LDPC) codes [24], as

$$LLR_{i}^{tmp} = LLR_{i}^{e} - 2\operatorname{arctanh}\left(\prod_{j \in \mathcal{W}} \tanh \frac{-LLR_{j}^{e}}{2} + \operatorname{tanh} \frac{-LLR_{H}^{e}}{2}\right),$$

$$(36)$$

$$LLR_{H}^{a} = LLR_{H}^{e} - 2\operatorname{arctanh}\left(\prod_{i \in \mathcal{L}} \tanh \frac{-LLR_{i}^{e}}{2}\right), \quad (37)$$

where  $\mathcal{W} = \mathcal{L} \setminus i$ , and  $\text{LLR}_i^{\text{tmp}}$  is the temporary result for agents. Then, according to the correlation model [25], the  $\text{LLR}_i^{\text{tmp}}$  is deinterleaved by  $\Pi_{i,1}^{-1}$  and further calculated by the LLR updating function  $\mu(\cdot)$  for correlated source [26]. Finally,  $\Pi_{i,1}$  interleaves the output of  $\mu(\cdot)$  again to provide  $\text{LLR}_i^a$ . The structure of extrinsic information exchanger is much simpler for  $g(\cdot)$  being majority voting. As illustrated in Fig. 11(b),  $\mu(\cdot)$  directly updates all of the  $\text{LLR}_i^e$  after deinterleaving with the  $\text{LLR}_{\text{H}}^e$ , and then the outputs from  $\mu(\cdot)$ for agents are interleaved into  $\text{LLR}_i^a$ .

#### **B. SIMULATION RESULTS**

TABLE 1. Basic Settings of Simulation Parameters

Parameter	Value
Number of Blocks	1000
Block length	10000 bits
Distribution of X	Bern(0.5)
Generator polynomial of CC	$G = ([3, 2]3)_8$
Rate of CC	1/2
Type of interleaver	random interleaver
Modulation method	BPSK
Maximum iteration time	30

Fig. 12 compares the simulation results between  $g(\cdot)$  being modulo-2 sum (M2S) and majority voting (MV), where the

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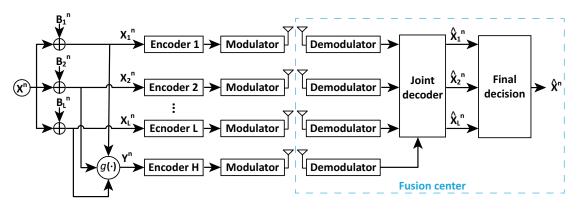


FIGURE 7. Simulation system.

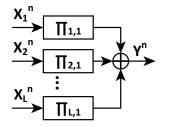


FIGURE 8. Generation of helper information by modulo-2 sum.

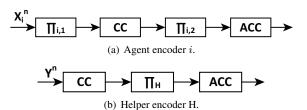


FIGURE 9. The structure of encoders.

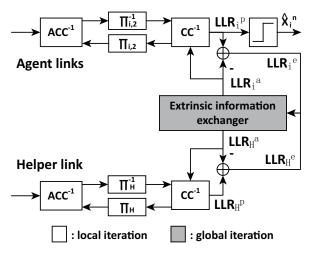
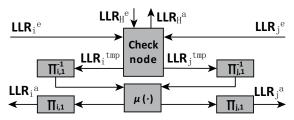


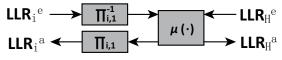
FIGURE 10. The structure of joint decoder.

basic parameter settings are listed in Table 1. Since the majority voting decision rule is implemented in simulations, we use the limit derived from Poisson binomial as the theoretical

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(a) The helper information is generated by modulo-2 sum, where  $i, j \in \mathcal{L}$  and  $i \neq j$ .



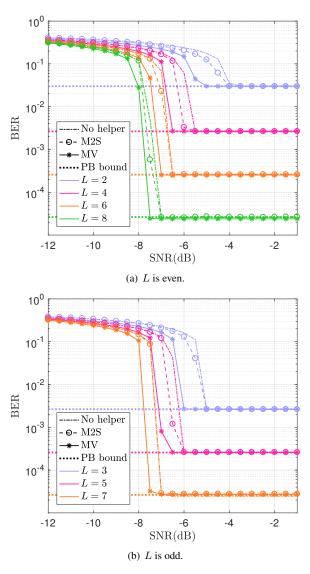
(b) The helper information is generated by majority voting.

FIGURE 11. The structure of extrinsic information exchanger.

bound. Clearly, the trade-off of rate-distortion in simulations perfectly matches that in theoretical analysis, i.e., a helper can shift the turbo cliff to left but cannot reduce the BER floor. It should be highlighted that shifting SNR to left also means eliminating distortions for low SNR level. Moreover, the helper with  $g(\cdot)$  being majority voting can obviously reduce the SNR threshold more than the helper which generates its information by modulo-2 sum. The reason for the difference of performance is that the distortion is included in the helper sequence with modulo-2 sum operation if there is only one check node. If one of the  $LLR_i$  is with the opposite sign, all of the other (L-1) LLRs will get negative helper information. However, such negative helper information cannot be reversed again as in the LDPC codes, because no other check nodes exist in the system with only one helper. Hence, if the SNR is in an extremely low level, the helper with  $q(\cdot)$  being modulo-2 sum will lose its effect due to the large distortion of  $X_i$ . This problem also results in the reduction of helper efficiency for large L, i.e., it is difficult to shift the turbo cliff by further increasing L. Therefore, the system with only one helper for  $g(\cdot)$  being modulo-2 sum cannot obtain enough gains as the LDPC codes with a lot of check nodes. Nevertheless, the helper with its information generated by

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**FIGURE 12.** Comparison of simulation results between different structures of helper, where  $p_i = 0.03$ .

majority voting still can obviously reduce the SNR threshold with larger L. Because not only can the helper information generated by majority voting preserve large enough mutual information among  $X_{\mathcal{L}}$  for lower error-corrupted probability  $p_i$ , but the distortions occurring at the small part of nodes are also not dominant when exchanging extrinsic information. Besides, the simulation results for both structures of helper can achieve the bound derived from Poisson binomial process. Hence, we can draw a conclusion that it is convincible and effective to predict the trade-off of rate-distortion for a practical system by applying the theoretical results.

Finally, we make a comparison between (L + 1) agents and a helper for diverse  $p_i$  as illustrated in Fig. 13 and Fig. 14. Since majority voting shows a better performance than modulo-2 sum as the helper structure, we only plot the curves of majority voting for comparison. It is remarkable in Fig. 13(a) and Fig. 14(a) that the system with (L + 1)

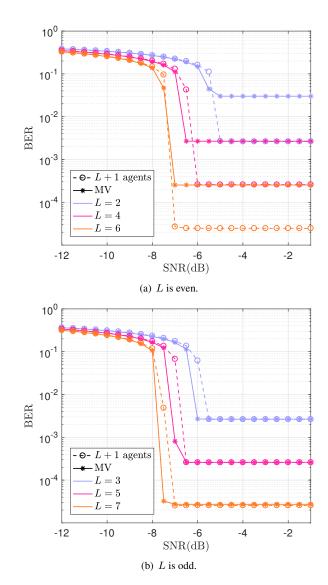


FIGURE 13. Comparison between one additional agent and a helper, where  $p_i = 0.03$ .

agents can achieve the BER floor of the case with (L + 2)agents, when L is an even number. Because one additional agent provides extra information of X, and this avoids the draw case that equal number of "0" and "1" appear in the same bit of agent sequences when L is even. However, the system with a helper has a lower SNR threshold for arbitrary L, due to the larger mutual information between  $Y^n$  and  $X^n_{\mathcal{L}}$ by majority voting. Especially when  $p_i$  becomes larger or L is odd, the gap of turbo cliff between (L + 1) agents and a helper is very conspicuous. We can also find that the system with (L+1) agents keeps the same BER floor as one-helper system when L is odd, even though one more agent provides extra mutual information about X. Consequently, except the additional implementation cost needed, it is better to add a helper than one more agent for a system with odd number of agents. If the wireless channels are good enough, one more agent can make the estimate of source more accurate for the

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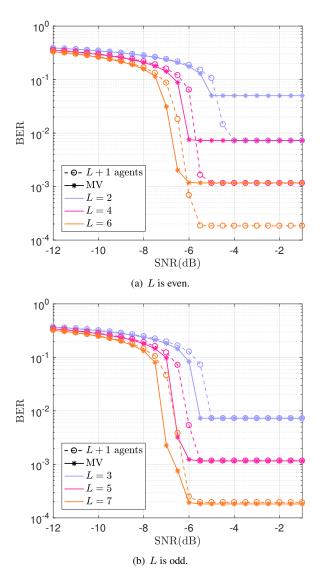


FIGURE 14. Comparison between one additional agent and a helper for more independent sources, where  $p_i=0.05$ .

system already having even number of agents. However, in an extremely noisy environment, i.e., before achieving BER floor and/or  $p_i$  is relatively large, the system performance can benefit more from a helper than adding an additional agent. In addition, there is another noticeable phenomenon that the BER floor is not immediately achieved for relatively larger  $p_i$  and L after turbo cliff appears. Because for the bits at the same time index, they need to obtain enough extrinsic information from each other, so as to decode correctly when SNR is extremely low. Hence, only the bits almost without corrupting errors can accumulate their extrinsic information and be correctly decoded. The SNR threshold decreases as the number of agents increases, while larger SNR is required for large  $p_i$  to correctly decode all bits at the same time index. Once the SNR threshold decreases to the level in which the bits with some corrupting errors decode fail, the BER is not able to achieve the BER floor as soon as the turbo cliff appears.

#### **V. CONCLUSION**

We have analyzed and evaluated the performance of binary data gathering with a helper in IoT. To begin with, we formulate binary data gathering with a helper in IoT as a binary CEO problem with a helper. Then, we decompose the binary CEO problem with a helper into two sub problems as multiterminal source coding with a helper and final decision. Subsequently, we derive an outer bound on the ratedistortion function for the multiterminal source coding step and the DPF for the final decision step. Based on the derived outer bound, a convex optimization problem is formulated to minimize the distortion of observations with given agent and helper link rates. By substituting the minimized distortions into DPF, we investigate the relationship between link rates and final distortion. Although there is an obvious gap between majority voting and optimal decision, they show the same tendency on the trade-off of rate-distortion. Finally, we have risen an encoding/decoding scheme and design a simulation for practical performance evaluation, so as to compare with the theoretical results and analyze the trade-off of ratedistortion for binary data gathering with a helper in IoT. Both the theoretical and simulation results indicate that a helper can reduce the SNR threshold, while the BER floor does not change. Moreover, a helper with its structure as majority voting has a better performance than an additional agent for the system with odd number of agents or in extremely noisy communication environment. These significant observations are extremely useful for the design of practical systems.

#### APPENDIX A ERROR PROBABILITY BY WEIGHTED MAJORITY VOTING

Regarding the error probability by weighted majority voting, the final decision  $\hat{x}$  follows [27]:

$$\hat{x} = \begin{cases} 1, & \text{if } \mathbf{w}\mathbf{z}^T > 0, \\ 0, & \text{otherwise,} \end{cases}$$
(38)

where  $\mathbf{w} = [\log \frac{1-p_1}{p_1}, \cdots, \log \frac{1-p_L}{p_L}]$  and  $\mathbf{z} = 2 \cdot [\hat{x}_1, \cdots, \hat{x}_L] - 1$ . Similarly to the Poisson binomial process, the error probability for the estimate of x is given by

$$p_{e} = \Pr\left\{\sum_{k \in \mathcal{Z}_{+}} w_{k} > \sum_{j \in \mathcal{Z}_{-}} w_{j}\right\}$$
$$+ \frac{1}{2} \Pr\left\{\sum_{k \in \mathcal{Z}_{+}} w_{k} = \sum_{j \in \mathcal{Z}_{-}} w_{j}\right\}, \quad (39)$$

where  $\mathcal{Z}_+ = \{i | z_i = +1\}$  and  $\mathcal{Z}_- = \{i | z_i = -1\}$ . Note that in order to calculate (39), it needs to carry out the search over all the possible combinations of  $w_i$ .

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