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A study of dynamic logic

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1 Introduction

Modal logics have been studied in order to discuss ordinary reasonings which we cannot explain adequately in propositional clasical logic. Modal logics are usually obtained from classical logic by adding modal operators expressing "necessity" and "possibility". By giving various interpretations to modal operators, we can get logical systems like temporal logics treating time and deontic logic relation to law, etc.

Dynamic logic is a logic obtained by introducing the modal operator which expresses the execution state for every program, and is used for verification of description of the specification in procedural program language and for justification of programs.

Study of logics about programs and computation was started by Thiele and Engeler late in the 1960s. Among studies which preceded dynamic logic, algorithmic logic introduced by Salwicki and monadic programming logic introduced by Constable are typical ones. Dynamic logic was introduced by Pratt in 1976, and therefore is regarded as a comparatively new research field. Further, dynamic logic can be applied to software science, and hence has been studied from both and computer science.

Existence of the *iteration operator* * gives rise to many difficulities in the study of dynamic logic. This operator is both reflexive and transitive, and is defined in an inductive way. Because of this, it makes the logic very complicated.

In 1996, Castilho and Herzig introduced the logic $\mathcal{A}$, in which the iteration operator of propositional dynamic logic is replaced by a simpler modal operator $\Box$ for modal logic S4, and showed that to a certain extent, the iteration operator can be simulated by $\Box$.

In this paper, we will develop a further study of the logic $\mathcal{A}$. We will show the finite model property of $\mathcal{A}$, and introduce a sequent calculus for it, for which cut elimination

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theorem holds. Moreover, by applying Maehara's method, we succeeded to prove Craig's interpolation theorem for $\mathcal{A}$.

2 Dynamic logic

We will give here a brief explanation of the original dynamic logic. The operators in dynamic logic are as follows: logical operators, $\wedge$, $\vee$, $\supset$, $\neg$; program operators, $\cup$(nondeterministic choice), ;(sequential composition), *(iteration); mixed operators, [ ](necessity), and ?(test). By using these operators, basic programs are defined as follows.

$$\begin{array}{rcl}
\text{if } A_1 \to \alpha_1 \mid \cdots \mid A_n \to \alpha_n \text{ fi} & \overset{\text{def}}{=} & A_1?; \alpha_1 \cup \cdots \cup A_n?; \alpha_n \\
\text{do } A_1 \to \alpha_1 \mid \cdots \mid A_n \to \alpha_n \text{ od} & \overset{\text{def}}{=} & (A_1?; \alpha_1 \cup \cdots \cup A_n?; \alpha_n)^*; (\neg A_1 \wedge \cdots \wedge \neg A_n)? \\
\text{if } A \text{ then } \alpha \text{ else} \beta & \overset{\text{def}}{=} & \text{if } A \to \alpha \mid \neg A \to \beta \text{ fi} \\
 & = & A?; \alpha \cup \neg A?; \beta \\
\text{while } A \text{ do } \alpha & \overset{\text{def}}{=} & \text{do } A \to \alpha \text{ od} \\
 & = & (A?; \alpha)^*; \neg A?
\end{array}$$

We can show that the following formulas are provable in dynamic logic.

- $[\alpha^*]A \supset A$
- $[\alpha^*]A \supset [\alpha]A$
- $[\alpha^*]A \supset [\alpha^*][\alpha^*]A$

These results mean that the iteration operator * is reflexive and transitive.

3 Logic $\mathcal{A}$ and system MA

Logic $\mathcal{A}$, which was introduced in reference[1], is designed to be a logic which removes difficulties caused by the iteration operator * of dynamic logic, by replacing it with a necessity operator $\Box$ of modal logic S4. We will develop a study of $\mathcal{A}$ in this paper. Below, a atomic program is fixed only to α in order to simplify an argument. As an axiom of $\mathcal{A}$ which shows the relation between $\Box$ and $[\alpha]$, we will take only $\Box A \supset [\alpha]A$. Tableau system of the logic $\mathcal{A}$, which was introduced in [1] has the following rules, in addition to tableau rules for classical propositional logic. The completeness of this tableau system is shown in [1].

- $\mathcal{V}^{\Box}$-rule
$$\frac{\mathcal{V}^{\Box}}{\mathcal{V}_0^{\Box}}$$
- $\pi^{\Box}$-rule
$$\begin{array}{c} S, \pi^{\Box} \\ \hline\hline S^{\Box} \\ \pi^{\Box} \end{array}$$
- π^{α}-rule
$$\begin{array}{c} S, \pi^{\alpha} \\ \hline\hline S^{\Box} \\ S^{\alpha} \\ \pi_0^{\alpha} \end{array}$$

In our thesis, we introduce the system MA of Gentzen style sequent calculus for $\mathcal{A}$. This system MA is a system which is obtained from the sequent calculus LK for classical propositional logic, by adding the following inference rules. Each of them corresponds to one of rules of $\mathcal{A}$-tableau system mentioned before.

$$\frac{A, \Gamma \to \Delta}{\Box A, \Gamma \to \Delta}\ (\mathcal{V}^{\Box})$$

$$\frac{\Box\Gamma \to A}{\Box\Gamma \to \Box A}\ (\pi^{\Box})$$

$$\frac{\Box\Gamma, \Sigma \to A}{\Box\Gamma, [\alpha]\Sigma \to [\alpha]A}\ (\pi^{\alpha})$$

Our main results on MA are the cut elimination theorem, and Craig's interpolation theorem, which follows from the cut elimination theorem.

4 Semantical approach

The completeness by the tableau system for the logic $\mathcal{A}$ is shown already in [1]. However, the proof is complicated. Here, we give a proof of the completeness with respect to Kripke semantics, by using canonical model constructions. As a corollary, the equivalence of the $\mathcal{A}$-tableau system to the sequent culculus MA is shown. Moreover, we show the finite model property of the logic $\mathcal{A}$ by using filtration method. Thus, the decidability of $\mathcal{A}$ follows.

5 Conclusions

In our paper, we prove completeness and finite model property of the logic $\mathcal{A}$ which is obtained from dynamic logic by replacing the iteration operator * by a modal operator $\Box$. Further, we introduce a sequent calculus MA for it, and prove cut elimination theorem and Craig's interpolation theorem.

As future works, approaches by algebraic methods and by Mu-calculus will be necessary.

References

[1] M. Castilho and A. Herzig, An alternative to the iteration operator of propositional dynamic logic, Technical report, IRIT - Universite Paul Sabatier, 1996.

[2] M. Fitting. Proof methods for modal and intuitionistic logics, Reidel Publishing Company, 1983

[3] R. Goldblatt, Logics of Time and Computation, Center for the study of language and information, 1992.

[4] D. Harel, D. Kozen and J. Tiuryn, Dynamic Logic, The MIT Press, 2000.

[5] 小野 寛晰, 情報科学における論理, 情報科学セミナー, 日本評論社, 1994.