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Analyse of Logic Puzzle

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As a student who learning about logic, it is frequently to be asked some question as 'what can your major do?' or 'what exactly is your researching?'. Mostly being asked by my mother. That is why I consider to write an article, which is easy enough for my famliy can understand what is modal logic. And it will be even better if any audience got some fun while reading it. So the first goal of this report is giving a explain of non-classical logic, mainly the modal logic and epistemic logic to college students. It is hard for non-mathematical major student to understand what logic implies means, and why a classical implication of 'P implies Q' can transform into as relation of 'not P or Q'. Not even talk about the necessarily or the possibly in modal logic. But once we understand the fascination of non-classical logic, we can identify possible worlds which would be frequently mentioned in this report. One benefit of learning logic is that it can helps us to solve logic puzzles. There are two logic puzzles as example of explaining the connection between complex logic theory and joyful logic puzzle. After reading this paper hopefully you would have a deeper understand of what logic can do and enjoy the puzzles. This paper present basic knowledge of modal logic begin with classic logic, provide a previous studying before dynamic epistemic logic. Also introduced how to apply Kripke's semantics to some logic puzzles.

The first chapter is introduction of classic logic. Assume we are nonmathematical major college student, many of us have heard about logic without systematically learned it. We actually are using logic reasoning in our daily life without notice it. Depends on the natural language and cultural environments, the logic people using might be different. Still, there is something we can all agree about. The collection of the agreement of logic might be the origin of classic logic. This is the first step to logic, contained with messive information. In this chapter we defined the symbol to use in classic logic, the proposition p ,conjunction $\wedge$, disjunction $\vee$, negation $\neg$ and implication $\rightarrow$. Mainly, this chapter collected many symbols in classic logic in from[2][5][7]. Focusing on the notation differences. For example, as the symbol stand for negation, there are $\sim$, $\neg$ and $-$ in different references. The logician like to invent new symbol, which may cause students confused to the formulae written by different author. As classic logic can be consider as the basic of logic, a deep understanding of it is meaningful. Funny thing is, even if we have defined the only symbol to use. The classic logic may still make students confused. For example, the 'or' we use in English is not always stand for disjunction. Sometimes the 'or' we mean could be exclusive. (By the way there is a symbol defined as exclusive or the symbol of $\oplus$.) Not only about disjunction, some people may complain that the implication is not really meaning 'if ... then ...', because it let a false proposition can implies anything. Some people may say that the rule of excluded middle is weird, there should be something is not true nor false. All the complaints are meaningful, actually, because of these complaints, people who do not like classic logic invented new logic systems. Like the complaint of implication and excluded middle, leads us to intuitionistic logic and modal logic. The people who do not like only two truth value, invented many-valued logic. After all all the logic can be connected together, at the center is the most basic logic, classic logic or say the classic propositional logic. When we get a sentence, we want to judge it is true or false. As an atom it is easy to decide which is which, But things becomes complicate when atoms connect with each other. After syntax, introduce semantics and valuation of formulae. If we defined true is 1, false is 0, change conjunction to $\times$, disjunction to $+$, negation to 1-. In the same time defined $1+1 = 1$, $1+0 = 1$, $1\times1 = 1$, $1\times0 = 0$, 1-1 = 0, 1-0 = 1. We can translate classic logic into calculate problem of math. It also show us that logic systems has the potential of translation. There will be a translation of one logic translate to another one, in the chapter of intuitionistic logic. We collect tautology in classic logic and call them as theorem, aware that a tautology is just like a algebraic equation, we do not have to give each element value to tell a sentence is true or false. We can prove them. So the following section introduces proof theory. In this section focus on the tableaux tree and natural deduction. While tableaux give us 9 trees that corresponding to logic connective, the natural deduction offered us the 10 rules of introduce and eliminated connectives. Briefly mentioned soundness $\vdash \varphi \Rightarrow \Vdash \varphi$ and completeness $\Vdash \varphi \Rightarrow \vdash \varphi$. Soundness means: for whatever is provable, it is true. Completeness means: for whatever is true, it is provable.

After learning the strengths and weaknesses of classic logic, we can understand the motivation of inventing new logic. As the implication $\rightarrow$ of classic logic did not satisfied Lewis, who invented Lewis systems, which then developed into modal logic systems.[1][3][4] So the second chapter is introducing modal logic. Modal logic can be simply regared as classic logic add two new with operators. $\Box$ is respond to necessary and the other $\Diamond$ stands for probability. $\Box p$ is read as 'it is necessarily p', $\Diamond p$ is read as 'it is possible p'. Then introduce the modal logic with Kripke's possible world semantics. Modal logic was invented on the motivation of capturing possibly and necessary. As we mentioned the implication of classic logic allows people write sentence like 'The sun rise up from west, therefore human can lay eggs.' With the operator of necessary, we can make sentences to claim a requirement is necessary to lead to the result. Adding figure of possible worlds also gives visualized explanation of Kripke's semantics. The intuitionistic logic is similar to modal logic, it has it's own negation and implication. There exist a translation called Gödel–McKinsey–Tarski translation, let us be able to translate intuitionistic logic to modal logic system S4. The translation so called Gödel–McKinsey–Tarski translation [3].

The third chapter is for intuitionistic logic, which introduce another two operator $\sqsupset$ and $\rightharpoondown$. The $p \sqsupset q$ means a proof of p can also prove q. The $\rightharpoondown p$ means there is no proof of p. Latter we can see that intuitionistic logic can be translate into S4. The $p \sqsupset q$ translate to $\Box(p \rightarrow q)$, $\rightharpoondown p$ translate to $\Box(\neg p)$.

The forth chapter is introduction of epistemic logic, which is developed on modal logic, adding a new set A of agents to model $\mathfrak{M}$.[6] The model $\mathfrak{M}$ is a structure $< \mathfrak{F}, V >$, which $\mathfrak{F}$ is a frame, V is a valuation. The frame $\mathfrak{F}$ is $< W, R >$, which W is the set of possible worlds, R is the accessibility between two worlds. Introduced the basic syntax, semantics which is the same as system S5. The differences are in modal logic we use operator $\Box$ and $\Diamond$, here in epistemic logic we use K and B, which stand are the first letter of Know and Belief. But all in all, it just a syntax difference, they still share the same Kripke's semantics.

Finally, we reach to logic puzzles. Two every classic puzzle when logician talking about modal logic, the 'Muddy children' and 'Sum and Product'. Showing that without communicate with information directly. Inform each with their own statement, or playing with knowledge and ignorance can also lead us to the result.

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