Possible Interpretation of Mass-in-Mind: A Case Study Using SCRABBLE

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***Abstract*—This paper explores the possible interpretation of 'mass-in-mind,' which describes a shift in a perceived challenge due to experience. This work involves a measurement called 'game refinement,' which has been used to quantify the engagement of a game. It establishes an incomplete link between real-world physics and physics-in-mind, in which the acceleration and the distance, also known as game progress, have been identified. An existence of mass-in-mind, however, has just been established. The mathematical model of mass-in-mind is constructed based on the data interpretation, in which SCRABBLE matches between computer players were analyzed. The results reveal a significant gap between prior models, which is explainable once the mass-in-mind is considered.**

Keywords–Scrabble; game refinement theory; game progress model; boardgame model; physics-in-mind.

I. INTRODUCTION

Quantifying emotional excitement and mental engagement in games is the subject of game refinement theory [1]. Early work in this direction has been carried out by Iida *et al.* [1] while constructing a logistic model based on game outcome uncertainty to measure the attractiveness and sophistication of games [2]. Efforts have been devoted to the study of the acceleration of the game progress [3], [4], [5]. The mass part has just recently discussed [6], however, still remains unknown even it has been one of the most fundamental concept in classical mechanics [7].

SCRABBLE [8] has been played for several decades in various situations [9], for instance, as a competitive match between professional players [10], [11] or as a friendly match among family members or students [12]. Different players may have different vocabulary knowledge and supposed to have distinct playing experience. Besides, players can play SCRABBLE either for entertaining or educational purpose [12]. This paper focuses on an evaluation of SCRABBLE from the game designer's point-of-view, in which two original game refinement models are considered. However, the differences are observed then being discussed.

The term 'mass' originally came from Latin word 'Massa', which means accumulation, body, crowd or heap [13], [14]. Mass is one of the most fundamental concepts in both classical and modern physics [7]. Initially, the notion of inertial mass was brought to the consideration by Isaac Newton [15], [16]. The superficial definition is the tendency of a body to resist changes of acceleration. However, this might subject to misinterpretation in a case of modern physics [17]. The motion of an object is indeterminable without the consideration of mass [7]. Similarly, the acceleration of the game progress is unobtainable without the mass-in-mind.

Earlier works in game refinement theory successfully established two mathematical models, known as the boardgame model [1] and the game progress model [5], which correspond to the boardgame and the scoring game respectively. SCRABBLE, however, is the particular case where two models are applicable. The results are compared, then lead to the reconsideration of the precedent theory. This work is expected to enhance the completeness of game refinement theory and become one of the standard assessment tools in the future.

The paper structure is as follows. In Section II, we describe the brief history of the study of Game Refinement Theory. Section III explains the mass-in-mind concept, how it is established and its affect to the mathematical model of game refinement. Section IV presents related prior works. Section V presents the assessment and corresponding error from applying the newly proposed model, thus discusses the results of the analysis. Concluding remarks are given in Section VI.

A. *Scrabble*

SCRABBLE® is a registered trademark. All intellectual property rights in and to the game are owned in the United States of America by Hasbro Incorporated, in Canada by Hasbro Canada Corporation, and throughout the rest of the world by J.W. Spear & Sons Limited of Maidenhead, Berkshire, England, a subsidiary of Mattel Incorporated [8], [18], [19].

SCRABBLE has been used as the main test-bed of this study. It is a word anagram game which published in 1938 by Hasbro [18], one of the famous toys game company in the United States.

In opposition to a typical boardgame, SCRABBLE players should possess not only strategic skill but also a sufficient vocabulary size. This is because they are required to form legit words from randomly tiles given.

From earlier work [20], it is known that SCRABBLE possesses the stronger entertaining aspect compared to educational aspect. While many players generally play SCRABBLE for enjoyment purpose, only few players play it in an educational way. By developing artificial intelligent player, the direction to improve SCRABBLE is proposed using the feedback from the artificial intelligence [21].

Superficially, SCRABBLE might be considered as a boardgame. However, it contains the aspect of competitive scoring as well. Hence, we can observe the scoring rate, branching factor, and game length. Those are essential for the game progress model and the boardgame model of the game refinement measure.

II. GAME REFINEMENT MEASURE

This section gives a short description of game refinement theory. A general model of game refinement was proposed based on the concept of the rate of change in game information progress [5]. This model bridges a gap between boardgames and scoring sports games.

A. Game Progress Model

The term 'game progress' is twofold. One criterion is the game speed or scoring rate, while the other is game information progress, which focuses on the game outcome. Game information progress presents the degree of certainty of the game's results in time or steps. Having full information of the game progress i.e., after its conclusion, game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq t_k$ and $0 \leq x(t) \leq x(t_k)$, as shown in (1).

$$x(t) = \frac{x(t_k)}{t_k}\ t \tag{1}$$

However, the game information progress given by (1) is unknown during the in-game period. The presence of uncertainty during the game, often until the final moments of a game, reasonably renders game progress exponential. Hence, a realistic model of game information progress is given by (2).

$$x(t) = x(t_k)(\frac{t}{t_k})^n \tag{2}$$

Here n stands for a constant parameter, which is given based on the perspective of an observer of the game that is considered. Then the acceleration of the game information progress is obtained by deriving (2) twice. Solving it for $t = t_k$, the equation becomes (3).

$$x''(t_k) = \frac{x(t_k)}{(t_k)^n} t^{n-2}\ n(n-1)\ |_{t=t_k} = \frac{x(t_k)}{(t_k)^2}\ n(n-1) \tag{3}$$

It is assumed in the current model that game information progress in any game is transported into and encoded in our brains. We do not yet know about the physics of information in the brain, but it is likely that the acceleration of information progress is subject to the forces and laws of physics. Therefore, we expect that the larger the value $\frac{x(t_k)}{(t_k)^2}$, the more exciting the game becomes, due in part to the uncertainty of the game outcome. Thus, we use its square root, $\frac{\sqrt{x(t_k)}}{t_k}$, as a game refinement measure for the game under consideration. We call it GR value for short, we also call $x(t_k)$ and t_k G and T respectively, as shown in (4).

$$GR = \frac{\sqrt{G}}{T} \tag{4}$$

The tendency of game refinement theory has been explained in [22]. We consider the trend between game refinement theory and the player skill. While an increasing relation leads to the entertaining experience, a decreasing relation leads to the serious or educational experience. However, two above ways may be utilized together to attract customers as shown in a case of business [23].

B. Early Works with Scrabble

The swing model [20], a derivation of the game progress model, is defined to solve the nonidentical scoring system in SCRABBLE. In this study, swing denotes a notion of phase transition in mind from advantage to disadvantage and vice-versa. Let DS be the dictionary size represented in the zero-to-one normalized scale. The data are shown in Table I and Figure 1.

TABLE I. EARLIER GAME REFINEMENT MEASURE OF SCRABBLE

DS	G	B	$T = D$	$\frac{\sqrt{G}}{T}$	$\frac{\sqrt{B}}{D}$	*Difference*
0.1	7.30	14.202	35.79	0.075	0.105	-39.48%
0.2	13.21	28.468	43.77	0.083	0.122	-46.0.8
0.3	12.04	51.093	43.83	0.079	0.163	-106.00%
0.4	11.54	81.300	42.38	0.080	0.213	-165.43%
0.5	13.76	124.393	39.66	0.094	0.281	-200.67%
0.6	13.22	158.125	39.88	0.091	0.315	-245.85%
0.7	10.30	200.321	37.58	0.085	0.377	-341.01%
0.8	11.33	254.111	36.67	0.092	0.435	-373.58%
0.9	10.14	333.689	35.87	0.089	0.509	-473.65%
1.0	10.78	361.805	35.85	0.092	0.531	-479.33%

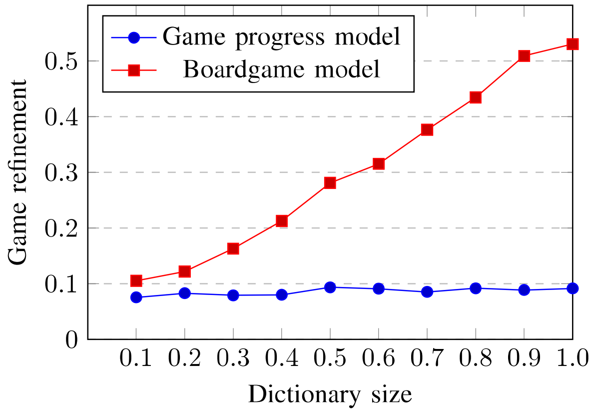

Figure 1. Comparison of two original game refinement measures

Next, we compare the value of the two approaches, thus display the observable difference. While the branching factor grows with the dictionary size as expected, there is only slight change in the number of swing occurrence and the game length.

Since game refinement theory has been used to quantify the engagement of the game regardless of the type of the game, we expected that the measures using two different approaches are identical. However, this is not necessarily true in a case of SCRABBLE.

The simple explanation is that the total branching factor B is overwhelmingly excessive in the case of SCRABBLE. Generally, SCRABBLE players cannot recognize all possible instances to human limitations. They might not be able to remember all the words in the standard dictionary or use them efficiently within the limited time. In particular, we must take the effective branching factor b into account. It was previously introduced in [24].

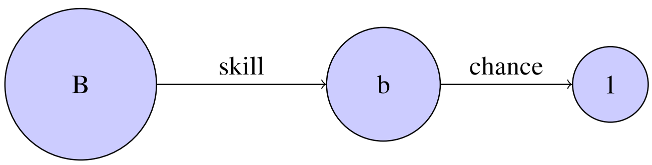

Figure 2. Player selection process

The branching factor B represents a number of all possible moves [1]. The effective branching factor b is the subset of the branching factor B which contains only instances satisfyingly perceived by a player [22]. The process to identify effective solutions among all possible moves is involved with individual skill. The generic single-step selection process is illustrated in Figure 2.

III. Mass-in-mind

A. Real-world Physics

In physics, kinetics is the branch of classical mechanics regarding motion. Newton's laws of motion [25], [26], [27], [28] are three fundamental laws which describe the relationship between forces acting upon a body, and its movement in response to those, as shown in Law 1, Law 2 and Law 3. More precisely, the first law defines the force qualitatively, the second law offers a quantitative measure of the force and the third postulates that a single isolated force does not exist.

In this study, we mainly focus on the second law, which describes the nature of mass, resistance to acceleration, or inertia, when a net force is applied.

Law 1: **Newton's first law** In an inertial frame of reference, an object either remains at rest or continues to move at a constant velocity in a straight line, unless acted upon by an external force.

Law 2: **Newton's second law** In an inertial reference frame, the vector sum of the forces F acting on an object is equal to the mass m of that object multiplied by the acceleration a of the object:

$$\mathbf{F} = m\mathbf{a} \tag{5}$$

It is assumed that the mass m is a constant.

Law 3: **Newton's third law** When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

B. Game Refinement Theory Revisited

Although the study of the game refinement measure and the attempt to construct a link between real-world physics and physics-in-mind has been made, currently only the acceleration of the game progress is identified. However, mass, the essential part of real-world physics is not yet mentioned.

SCRABBLE is a scoring game played on a board, so it is the first domain, which two different game refinement approaches are applicable. Once two procedures were applied to SCRABBLE, we identified a significant gap between them, then realized that there might be an inconsistency in the original game refinement measure.

C. Establishment of Mass-in-Mind

Different objects react differently to the same net force due to their respective mass. For the same net force applied, an object with a higher mass will have a lower acceleration. We suppose that force-in-mind is the property of the game, and the mass-in-mind is the property of the player, then acceleration-in-mind is obtained by those two factors. Table II shows an intended mapping between real-world physics and physics-in-mind.

TABLE II. Correspondence between Real-World Physics and Physics-in-Mind

Notation	*Real-World Physics*	*Physics-in-Mind*
F	Force	Game Sophistication
m	Mass	Decision Complexity perceived by a player
$a = \frac{F}{m}$	Acceleration	Intuition of a player

Based on the perception described above, the definition of the mass-in-mind, or decision complexity perceived by a player, is given in Definition 2.

Definition 1: **Selection possibility** p is given as a proportion between selective instances which are satisfyingly perceived and the entire.

Definition 2: **Mass-in-mind** m is the inversion of the selection possibility of a player in a specific subject.

According to the definition given, we construct the mathematical model to make it more concretely for both game progress model and the boardgame model. Considering a boardgame, the possibility among a personal optimal selection is $\frac{b}{B}$, thus its inversion is $\frac{B}{b}$. In a case of the scoring game, however, is not directly obtainable. Therefore, the approximate model is introduced. By supposing that a player gets g scores out of Σg total score at the endgame, one score has the $\frac{g}{\Sigma g}$ possibility to be distributed to that player. Hence, the selection possibility is obtained by $\frac{g}{\Sigma g}$, thus its inversion is $\frac{\Sigma g}{g}$. Table III summarizes the mathematical model of game refinement considering mass.

TABLE III. Game Refinement Model Considering Mass

Notation	*Game Progress Model*	*Boardgame Model*
F	$\frac{G}{T^2}$	$\frac{B}{D^2}$
m	$\frac{\Sigma g}{g}$	$\frac{B}{b}$
$a = \frac{F}{m}$	$\frac{Gg}{T^2 \Sigma g}$	$\frac{b}{D^2}$

IV. Related Works

This section presents prior works done in this direction. How data is transferred within the human brain was explained using physics [29]. As opposed to our Newtonian physics analogy, the computational mechanism in the human brain is explained by quantum physics and information theory.

The arrow which points from the past to the future, known as the Time's arrow, was introduced to explain the consciousness and the awareness of the time. The time quantity used in most physics equations dealing with events is measurable. However, that is not necessarily equal to the time we sense. A human is subjected to lose track of time when concentrating on some medium.

In game refinement theory, the uncertainty of the game outcome is described with classical physics model. Game refinement measure reflects attractiveness of a game from the viewpoint of designers. A game is enjoyable when its challenge matches with preferences and skills of a player [30]. While deficiency leads to a tiresomeness, an extreme difficulty may lead to frustration. The high perceived challenge is one of the conditions in flow theory [31], which results in a loss of self-consciousness and track of the time.

The study of the $n(n-1)$ in (3) is explained in [22] as the C parameter. C_b and C_s are used for the boardgame and the scoring game respectively. They are defined in (6).

$$C_b = \frac{b}{B} \quad (\frac{1}{B} \leq C_b \leq 1) \qquad C_s = 1 \tag{6}$$

Through the effect of the C parameter, the acceleration part of the game refinement measure becomes (7)

$$R_b = \frac{\sqrt{b}}{D} \qquad R_s = \frac{\sqrt{G}}{T} \tag{7}$$

The result shows a similarity with this study. According to the mathematical formula, the C_b is as an inversion of the mass-in-mind. Thus R_b is the exact value of the square root of the acceleration from the boardgame model considering the mass. However, the mass-in-mind of game progress model does not share the same definition with C_s. Instead, it is likely constant. This method could enhance the completeness of the interpretation of the C parameter by redefining C_s, which would have some value, instead of being always 1.

V. Assessment and Discussion

We developed an artificial intelligent player to simulate multiple Scrabble matches. A hundred of distinct match settings are simulated with two hundred iterations each. Essential data, including individual score, total score, branching factor, game length were collected.

For the game progress model, we measure the individual score of a winner side and a loser side separately because they are obviously different. The data involving the force-in-mind is given in Table IV. Then, the acceleration-in-mind of each side are obtainable by considering their respective mass-in-mind, as shown in Table V and Table VI. Hence, the average is calculated and shown in Table VII and Figure 3.

TABLE IV. Game Progress Model Considering Mass

DS	G	T	F
0.1	7.30	35.79	5.70×10^{-3}
0.2	13.21	43.77	6.90×10^{-3}
0.3	12.04	43.83	6.27×10^{-3}
0.4	11.54	42.38	6.43×10^{-3}
0.5	13.76	39.66	8.75×10^{-3}
0.6	13.22	39.88	8.31×10^{-3}
0.7	10.30	37.58	7.29×10^{-3}
0.8	11.33	36.67	8.43×10^{-3}
0.9	10.14	35.87	7.88×10^{-3}
1.0	10.78	35.85	8.39×10^{-3}

For the boardgame model, the effective branching factor is considered. It was suspected to be close to B and 1 for beginners and experts respectively. However, determining the effective branching factor b for intermediate players is a challenging question. We then introduce approximate models, as shown in Table VIII.

Each approximate model is used with the boardgame model considering mass. The comparative results with the game progress model considering mass are shown in Figure 4.

TABLE V. Game Progress Model Considering Mass (Winner's Intuition)

DS	F	g_{winner}	Σg	m_{winner}	a_{winner}
0.1	5.70×10^{-3}	474.47	870.17	1.83	3.11×10^{-3}
0.2	6.90×10^{-3}	679.75	1277.66	1.88	3.67×10^{-3}
0.3	6.27×10^{-3}	734.77	1395.11	1.90	3.30×10^{-3}
0.4	6.43×10^{-3}	768.29	1440.30	1.87	3.43×10^{-3}
0.5	8.75×10^{-3}	767.80	1453.58	1.89	4.62×10^{-3}
0.6	8.31×10^{-3}	784.15	1489.60	1.90	4.38×10^{-3}
0.7	7.29×10^{-3}	773.26	1464.62	1.89	3.85×10^{-3}
0.8	8.43×10^{-3}	799.52	1508.04	1.89	4.47×10^{-3}
0.9	7.88×10^{-3}	807.81	1516.03	1.88	4.20×10^{-3}
1.0	8.39×10^{-3}	818.62	1541.31	1.88	4.45×10^{-3}

TABLE VI. Game Progress Model Considering Mass (Loser's Intuition)

DS	F	g_{loser}	Σg	m_{loser}	a_{loser}
0.1	5.70×10^{-3}	395.70	870.17	2.20	2.59×10^{-3}
0.2	6.90×10^{-3}	597.91	1277.66	2.14	3.23×10^{-3}
0.3	6.27×10^{-3}	660.33	1395.11	2.11	2.97×10^{-3}
0.4	6.43×10^{-3}	672.01	1440.30	2.14	3.00×10^{-3}
0.5	8.75×10^{-3}	685.77	1453.58	2.12	4.13×10^{-3}
0.6	8.31×10^{-3}	705.45	1489.60	2.11	3.94×10^{-3}
0.7	7.29×10^{-3}	691.36	1464.62	2.12	3.44×10^{-3}
0.8	8.43×10^{-3}	708.52	1508.04	2.13	3.96×10^{-3}
0.9	7.88×10^{-3}	708.22	1516.03	2.14	3.68×10^{-3}
1.0	8.39×10^{-3}	722.69	1541.31	2.13	3.93×10^{-3}

TABLE VII. Game Progress Model Considering Mass (Average Intuition)

DS	a_{winner}	a_{loser}	$a_{average}$
0.1	3.11×10^{-3}	2.59×10^{-3}	2.83×10^{-3}
0.2	3.67×10^{-3}	3.23×10^{-3}	3.43×10^{-3}
0.3	3.30×10^{-3}	2.97×10^{-3}	3.12×10^{-3}
0.4	3.43×10^{-3}	3.00×10^{-3}	3.20×10^{-3}
0.5	4.62×10^{-3}	4.13×10^{-3}	4.36×10^{-3}
0.6	4.38×10^{-3}	3.94×10^{-3}	4.14×10^{-3}
0.7	3.85×10^{-3}	3.44×10^{-3}	3.64×10^{-3}
0.8	4.47×10^{-3}	3.96×10^{-3}	4.20×10^{-3}
0.9	4.20×10^{-3}	3.68×10^{-3}	3.92×10^{-3}
1.0	4.45×10^{-3}	3.93×10^{-3}	4.18×10^{-3}

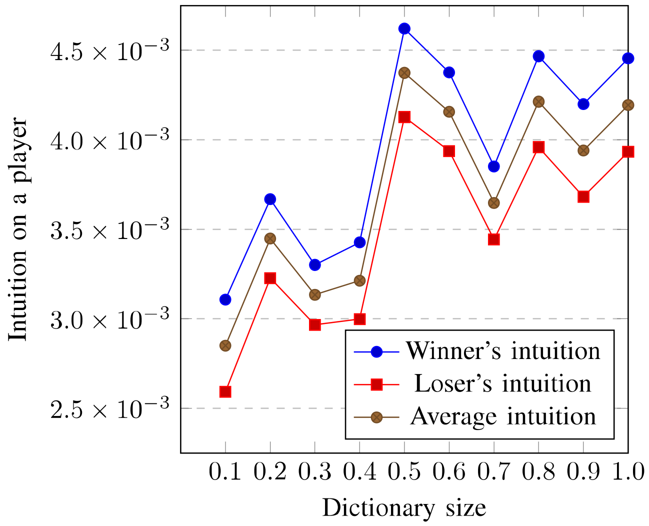

Figure 3. Comparison of the game progress model considering mass

The above figure shows that $\log B$ and $\sqrt[3]{B}$ yield a precise

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TABLE VIII. EFFECTIVE BRANCHING FACTOR APPROXIMATION

Formula	*Interpretation*
1	Experts
$\log B$	Possible approximation for intermediate players
$\sqrt[3]{B}$	Possible approximation for intermediate players
$\sqrt{B}$	Possible approximation for intermediate players
B	Beginners

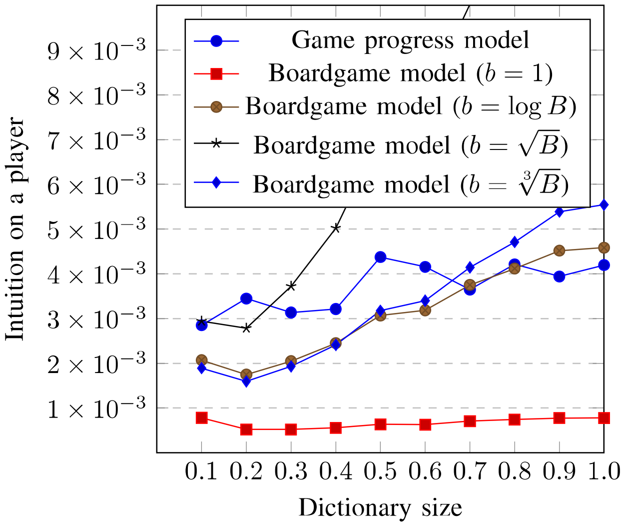

Figure 4. Comparison of GR measures considering mass

approximation for the effective branching factor b. For more precise comparison, the respective mean squared errors between the approximation and the average acceleration from the game progress model considering mass are shown in Table IX. The final comparative results using $\log B$, which is the current best known approximation are shown in Table X.

TABLE IX. MEAN SQUARED ERROR COMPARISON

Formula	*Mean Squared Error*
1	9.55×10^{-6}
$\log B$	8.42×10^{-7}
$\sqrt[3]{B}$	1.29×10^{-6}
$\sqrt{B}$	3.42×10^{-5}
B	2.13×10^{-2}

TABLE X. COMPARISON OF TWO GAME REFINEMENT MEASURES CONSIDERING MASS

DS	*Game Progress Model*	*Board Game Model*	*Difference*
0.1	2.83×10^{-3}	2.07×10^{-3}	27.30%
0.2	3.43×10^{-3}	1.75×10^{-3}	49.30%
0.3	3.12×10^{-3}	2.05×10^{-3}	34.66%
0.4	3.20×10^{-3}	2.45×10^{-3}	23.78%
0.5	4.36×10^{-3}	3.07×10^{-3}	29.89%
0.6	4.14×10^{-3}	3.18×10^{-3}	23.40%
0.7	3.64×10^{-3}	3.75×10^{-3}	-2.91%
0.8	4.20×10^{-3}	4.12×10^{-3}	2.25%
0.9	3.92×10^{-3}	4.52×10^{-3}	-14.60%
1.0	4.18×10^{-3}	4.58×10^{-3}	-9.30%

VI. CONCLUSION

Game refinement theory has been used to assess the engagement of a subject. Two earlier models, the game progress model and the boardgame model have been used for the scoring game and the typical boardgame respectively.

SCRABBLE, a game with a scoring system, which is played on a board, is the primary concern of this study. Two earlier models have been applied to this game. However, the significant difference has inspired us to investigate in particular and strengthen the link between real-world physics and physics-in-mind.

We revise game refinement theory after an analogy between real-world physics and physics-in-mind is considered, then the definition of mass-in-mind is established. The concrete mathematical model is constructed based on the type of the subject that is examined. The $\frac{\Sigma g}{g}$ and $\frac{B}{b}$ are introduced for the scoring game and the boardgame respectively. Our study shows that $\log B$ is a good approximation for b. After mass-in-mind is brought into consideration, there is a small difference in the measurement of the acceleration. The randomness of the raw data and the rough approximation of the effective branching factor b are possibly the cause of the apparent error.

In Newtonian mechanics, the concept of mass is based on the self-object only. As opposed to that, mass-in-mind is not just based on the player considered, but also on his experience or skillfulness in that particular subject. In the case of the boardgame, the effective branching factor b depends on the branching factor B and the skill of the player. Professional players tend to have a smaller b, which leads to a higher mass. However, they will have a lower mass in a scoring game. Also, the mass-in-mind of a winner is always less than that of a loser, which leads to a higher acceleration a, which exposes more emotional impact. This appearance is typical behavior as the game usually is more enjoyable to the winner.

Although this paper focuses on the artificial intelligence with perfect vocabulary knowledge, the proposed models fit well with the other cases, which is shown in Table XI.

TABLE XI. COMPARISON OF AVERAGE ERROR

Player Knowledge	*Mean Squared Error*	*Mean Absolute Percent Error*
0.1	1.65×10^{-6}	24.13%
0.2	1.65×10^{-6}	30.09%
0.3	1.19×10^{-6}	32.45%
0.4	1.17×10^{-6}	31.29%
0.5	7.78×10^{-7}	26.59%
0.6	7.68×10^{-7}	24.39%
0.7	8.37×10^{-7}	24.01%
0.8	7.14×10^{-7}	21.06%
0.9	6.26×10^{-7}	19.07%
1.0	8.42×10^{-7}	21.74%

In practice, mass-in-mind is not always a constant but depends on various uncontrollable causes. For instance, current mood and temper may affect the enjoyment of a game. Hence, player may not have the same intuition while playing the same game. We currently do not consider these factors and leave them for future work.

After mass-in-mind is proposed, it is possible to discuss later other physics-in-mind variables, for instance, energy-in-mind and momentum-in-mind, which will enhance the completeness of our theory and the explanation of the phenomenon of emotional impact.

Although the interpretation of mass-in-mind for the case of a time-limited sport is not yet addressed in this paper, we

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strongly affirm that the same result will be obtained as with the game progress model considering mass, which is $\frac{g}{\Sigma g}$. Further investigation and verification are also left for future work.

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