Logarithmic Utilities for Aggregator Based Demand Response

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***Abstract*—This paper proposes a distributed scheme for demand response and user adaptation in smart grid networks. Our system model considers scarce distributed power sources and loads. User preference is modelled as 'willingness to pay' parameter and logarithmic utility functions are used to model the behaviour of users. The energy management problem is cast as an optimization problem, where the objective is to maximize the utility services to the clients based on price-based demand response scheme. We have addressed the issue concerning the allocation of power among users from multiple sources/utilities within a distributed power network based on users' demands and willingness to pay. We envision a central entity providing a coordinated response to the huge number of scattered consumers, collecting power from all generators and assigning the power flow to the interested users. We propose a two layer price-based demand response architecture. The lower level energy management scheme deals with the power allocation from aggregator to the consumers, and the upper level deals with the distribution of power from utilities to aggregators to ensure the demand-supply balance.**

I. Introduction

Power systems around the world are facing challenges. At the supply side, traditional power generation portfolios are complemented with renewable energy resources (RES). Power generation from RES is characterized by limited controllability, limited predictability, and variability. At the demand side, an electrification of energy is occurring with time, i.e, the integration of battery electric vehicles (BEVs) and heat pumps. The integration of renewable sources and the electrification of energy complicate the power system operation. The variability and limited predictability of power generation introduces difficulties in maintaining the demand-supply balance. Moreover, as the demand grows due to the electrification of energy use, additional generation capacity is needed to cover the peak demand. Traditionally, the supply side provides flexibility to safeguard the demand-supply balance and to cover increased peak demand, while ignoring demand side flexibility. During the last decade, interest in flexibility at the demand side has grown. Hereby, consumers react to system conditions by adapting their consumption patterns, referred to as demand response (DR) [1]. DR not only contributes in mitigating the intermittent effects of renewable energy resources but also can be used either to lower high energy prices, occurring in the wholesale electricity markets, or when the security of power systems is at risk.

DR programs are categorized into two categories including incentive-based program and price-based program. In incentive-based program, participating users are paid with load modification incentives [2]. Direct load control (DLC), interruptible/curtailable load, demand bidding and buyback and emergency demand reduction are the programs in incentive-based category. Price-based program takes advantage of smart pricing, where the users are provided with different electricity prices at different times, resulting in the reduction of electricity usage by the users at the time of high prices [3], thus reduces the demands at peak hours. Time-of-use (ToU) pricing [4], critical peak pricing (CPP), real-time pricing (RTP) [5], inclining block rate (IBR) are the price-based DR programs.

For DR, the behaviour of users is modelled by the utility function, which is the representation of comfort or satisfaction level of users as the function of power consumption. These utility functions are non-decreasing and concave in nature. Most commonly considered functions are the quadratic utility functions [6]. Based on these models, DR is mostly formulated as the optimization problem, where it is solved by using convex optimization [7], game theory, dynamic programming [8], Markov decision process [9], stochastic programming [10] and particle swarm optimization methods [11].

Proportionally fair pricing (PFP) scheme in the IP networks is proposed in [12], where each user declares a charge per unit time that the user is willing to pay and in return network capacity is shared among the flows of all users in proportion to the prices paid by the users. This results in the maximized utility of the network, where utility is modelled by logarithmic function. Motivated by congestion pricing in IP network [12], authors in [13] proposed a framework for DR and user adaptation based on price feedbacks in smart grids for single source. Willingness to pay parameter is used to model the user preference and price function of [14] is considered. The use of logarithmic functions is also reported and the utility of users is maximized. However, in this work, we extend the analysis for the case of multiple scarce sources and multiple users. We consider the case where there is more than one source and each source can assign power to any user and any user can

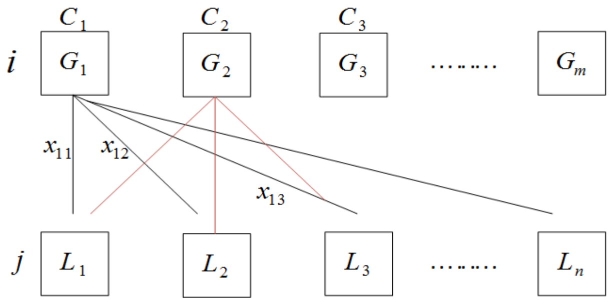

Fig. 1: Distributed Approach

receive power from any source, resulting in the formation of a distributed network. Optimization framework in our work leads to a decomposition of overall system problem into a separate problem of each user, where the user chooses its willingness to pay, and one for the network: the primal and dual formulation of network's problem.

From the system's perspective, it is well established that DR is only beneficial if a huge number of scattered consumers can provide a coordinated response to their requirements. Thus, the coordination of DR resources by an aggregating entity/DR aggregator/local aggregator (LA), is essential to facilitate the aforementioned interaction between the consumers and the utility. In this context, a LA can participate in the electricity markets as an intermediary between the utility and flexible consumers [15], [16], offering aggregated DR resources from a large number of consumers and coordinating their response. More specifically, a LA keeps the balance between demands and supply. Thus, we envision a central entity collecting power from all generators and assigning the power flow to the interested users. We extend our proposed distributed power allocation approach and propose a two level power allocation algorithm, where first level scheme assigns power from scattered sources to the LA in distributed manner and second level scheme allocates the available power form LA among competing users based on their willingness to pay factor.

The organization of the paper is following: In the Section II, we propose optimization based DR scheme for multiple power sources and loads. In Section III, we propose a two layer price based DR scheme for the smart grid architecture with aggregators. In Section IV, we present results of simulations and finally in Section V, we offer conclusion and future considerations.

II. Distributed Approach

With the integration of renewable energy generation, prosumer functionality and many utility generations, the power network is highly distributed in nature, where one source may supply power to more than one loads and vice versa as shown in the Fig. 1. Thus, there is a need of an algorithm for power assignment in distributed manner fulfilling the constraints of total power available and demands. It can be illustrated by formation of smart communities, which are characterized by scarce distributed power sources and loads, and so, an effective energy management policy is vital to offer maximal utility services to the community clients. Consider a power network with generator (G) being a source. Let $C(t)$ be the finite capacity of the generator. Our objective is to allocate the limited time varying available power (which is equal to capacity of source) among all the users based on their willingness to pay factor and demands. Let there are i generators, where $i = 1, 2, 3...m$ and j loads (users) where $j = 1, 2, 3...n$. Each generator has power generation capacity of $C_i(t)$ and x_{ij} is the power flow from i generator to the j user such that the total power supplied by generator G_i is $\sum_{j=1}^{n} x_{ij}$, then the distributed power flow problem takes the following optimization form:

$$\begin{aligned} &\max_{\bar{x}_j \geq 0} \sum_{j=1}^{n} U_j(\bar{x}_j) \\ s.t: &\sum_{j=1}^{n} x_{1j} \leq C_1(t) \\ &\vdots \\ &\sum_{j=1}^{n} x_{ij} \leq C_i(t) \end{aligned} \tag{1}$$

where $\bar{x}_j$ is a vector and is defined as $[x_{1j} \quad x_{2j}.........x_{nj}]$. It maybe noted that throughout the paper the term x is used interchangeably for the demands of user or power allocated to the users. $U_j(\bar{x}_j)$ is the utility function associated with each user j and it is assumed to be a non-decreasing, concave and continuously differentiable function of x. The utility function is of the form $U(\bar{x}_j) = w_j \sum_{i=1}^{m}(log(x_{ij}))$, where x is the assigned power and w is a factor referred to as the willingness to pay (In practical terms a household says that I am willing to pay 180Euros per month or a factory says that I am willing to pay 4000Euros per month). Then $w = \mu x$, where μ is the price and x is the allocated power. It has been pointed out in the literature of networks [12], that such weighted logarithmic functions lead to the proportional fairness. The w factor may be dictated by the user or can be estimated by the provider based on power load models. The parameters of the power load models can be determined using recently proposed online parameter identification techniques [17].

A. Equilibrium Analysis

In this subsection, we analyse the solution of primal problem defined in the (1). We substitute the $U(x_j) = w_j \sum_{i=1}^{m}(log(x_{ij}))$ into (1) and represent the Lagrange multipliers for the constraints in (1) as μ_i. Lagrangian is defined as:

$$L(x_{ij}, \mu_i) = -\sum_{j=1}^{n} U_j(\bar{x}_j) + \sum_{i=1}^{m} \mu_i (\sum_{j=1}^{n} x_{ij} - C_i(t)) \tag{2}$$

Let $x_{ij}{}^*$ denotes the minimizer of corresponding minimization problem of (1). The gradient of (2) with respect to x_{ij} for a particular j yields following expression:

$$x_{ij}{}^* = \frac{w_j}{\mu_i{}^*} \tag{3}$$

Additionally the gradient of (2) with respect to μ_i yields following expression:

$$\mu_i{}^* = \frac{1}{C_i}\sum_{j=1}^{n} w_j \tag{4}$$

B. Dual Problem

In this subsection, we use a duality-based approach where the objective function of the dual problem is defined as:

$$q^*(\mu) = -(\sum_{j=1}^{n} w_j \sum_{i=1}^{m} log(\frac{w_j}{\mu_i})) + \sum_{i=1}^{m}(\sum_{j=1}^{n} w_j - \mu_i C_i) \tag{5}$$

The dual problem then corresponds to maximizing $q^*(\mu_i)$ over the dual variables μ_i and is defined to be:

$$D : \max_{\mu_i \geq 0} q^*(\mu_i) \tag{6}$$

The problem defined in (6) can then be solved by gradient descent iterations on the dual variables μ_i as following.

$$\mu_i(k+1) = \mu_i(k) - \alpha(\frac{1}{\mu_i(k)}\sum_{j=1}^{n} w_j - C_i) \tag{7}$$

III. Aggregator Based Approach

In this section, we envision aggregator, which collects power from different sources and distribute the power to the registered clients. The architecture is depicted in Fig. 2. We propose a two layer price-based demand response architecture, where lower level energy management scheme deals with the power allocation from aggregator to the users and upper level deals with the distribution of power from utilities to aggregators. Let there are i generators, where $i = 1, 2, 3...m$, j agreggators where $j = 1, 2, 3...n$ and k users where $k = 1, 2, 3...p$. Each generator has a power generation capacity of $C_i(t)$. Let y_{ij} is the power flow from i generator to the j agreggator such that the total power supplied by generator G_i is $\sum_{j=1}^{n} y_{ij}$, and x_{jk} is the power flow from j agreggator to the k user / load such that the total power supplied by agregattor A_j is $\sum_{k=1}^{p} x_{ij}$. Let $\bar{x}_j$ is a vector which is defined as $\bar{x}_j = [x_{j1} \quad x_{j2}.....x_{jp}]$. Let utilization function is defined as following:

$$U_j(\bar{x}_j) = \sum_{k=1}^{p} w_{jk} log(x_{jk}) \tag{8}$$

where w_{jk} is the willingness of user k to get power from aggregator j. The power flow from generators to users based

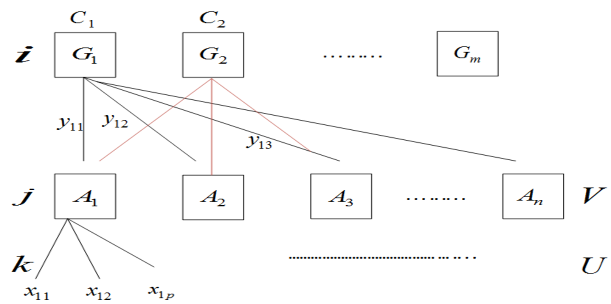

Fig. 2: Aggregator Based Approach

on above utilization function takes the following form:

$$\begin{aligned} &\max_{\bar{x}_j \geq 0} \sum_{j=1}^{n} U_j(\bar{x}_j) \\ s.t :& \sum_{i=1}^{m} y_{i1} = \sum_{k=1}^{p} x_{1k} \\ &\vdots \qquad \vdots \\ &\sum_{i=1}^{m} y_{in} = \sum_{k=1}^{p} x_{nk} \\ &\sum_{j=1}^{n} y_{1j} = C_1(t) \\ &\vdots \qquad \vdots \\ &\sum_{j=1}^{n} y_{mj} = C_m(t) \end{aligned} \tag{9}$$

A. Equilibrium Analysis

In this subsection, we analyse the solution of primal problem defined in the (9). We substitute the $U_j(\bar{x}_j) = \sum_{k=1}^{p} w_{jk} log(x_{jk})$ into (9) and represent the Lagrange multipliers for the constraints in (9) as λ_i (prices set by generators / utility) and μ_j (prices set by aggregator). Lagrangian is defined as:

$$\begin{aligned} L(\bar{x}_j, \lambda, \mu) = -\sum_{j=1}^{n} U_j(\bar{x}_j) + \sum_{j=1}^{n} \mu_j(-\sum_{i=1}^{m} y_{ij} + \sum_{k=1}^{p} x_{jk}) \\ + \sum_{i=1}^{m} \lambda_i(\sum_{j=1}^{n} y_{ij} - C_i(t)) \end{aligned} \tag{10}$$

Let $x_{jk}{}^*$ denotes the minimizer of corresponding minimization problem of (9). The gradient of (10) with respect to x_{jk} yields following expression:

$$x_{jk}{}^* = \frac{w_{jk}}{\mu_j{}^*} \tag{11}$$

Additionally the gradient of (10) with respect to μ_j yields following expression:

$$\mu_j{}^* = \frac{\sum_{k=1}^{p} w_{jk}}{\sum_{i=1}^{m} y_{ij}} \tag{12}$$

Further the gradient of (10) with respect to λ_i yields the following expression:

$$\sum_{j=1}^{n} y_{ij} = C_i(t) \tag{13}$$

However, the equation (13) yields the solution which is not unique. To tackle with this problem the following assumption is made:

$$\sum_{i+1}^{m} \lambda_i{}^* y_{ij}{}^* = \sum_{k=1}^{p} w_{jk} \tag{14}$$

Above assumption is reasonable, it corresponds to the balance between supply and demand. However, in the following section, we decompose the optimization problem of (9) into two sub-problems. We find the solution of two sub-problems, correlate them and prove that the solution obtained from two decoupled problems is also the solution of original coupled problem.

B. Sub-Problem 1

Sub-problem 1 involves the power flow from aggragtor to the users. Let U_1, U_2, ..., U_k are user utility functions, w_{jk} is the willingness of user k to get power from aggregator j and x_{jk} is the power flow from j agregattor to the k user. Let $\bar{x}_k$ is a vector and is defined as $\bar{x}_k = [x_{1k} \quad x_{2k} ....x_{nk}]$. We consider the following utility function.

$$U(\bar{x}_k) = \sum_{j=1}^{n} w_{jk}(log(x_{jk})) \tag{15}$$

Utility maximization problem takes the following form:

$$\begin{aligned} &\max_{\bar{x}_k \geq 0} \sum_{k=1}^{p} U_k(\bar{x}_k) \\ s.t: &\sum_{k=1}^{p} x_{1k} = \sum_{i=1}^{m} y_{i1} = Z_1 \\ &\quad\vdots \qquad\qquad \vdots \\ &\sum_{k=1}^{p} x_{nk} = \sum_{i=1}^{m} y_{in} = Z_j \end{aligned} \tag{16}$$

where Z_j is the total supply of power from all generators to the j aggregator and is considered as the capacity of the aggregator.

1) Equilibrium Analysis: In this subsection, we analyse the solution of primal problem defined in the (16). We substitute the $U(\bar{x}_k) = \sum_{j=1}^{n} w_{jk}(log(x_{jk}))$ into (16) and represent the Lagrange multipliers for the constraints in (16) as μ_j (prices set by aggregator). Lagrangian is defined as:

$$L(x,\mu) = -\sum_{k=1}^{p} U_k(\bar{x}_k) + \sum_{j=1}^{n} \mu_j (\sum_{k=1}^{p} x_{jk} - \sum_{i=1}^{m} y_{ij}) \tag{17}$$

$$L(x,\mu) = -\sum_{k=1}^{p} U_k(\bar{x}_k) + \sum_{j=1}^{n} \mu_j (\sum_{k=1}^{p} x_{jk} - Z_j) \tag{18}$$

Let $x_{jk}{}^*$ denotes the minimizer of corresponding minimization problem of (16). The gradient of (18) with respect to x_{jk} for a particular k yields following expression:

$$x_{jk}{}^* = \frac{w_{jk}}{\mu_j{}^*} \tag{19}$$

Additionally the gradient of (18) with respect to μ_j yields the following expression:

$$\mu_j{}^* = \frac{1}{Z_j} \sum_{k=1}^{p} w_{jk} \tag{20}$$

2) Dual Problem: In this section, we use a duality based approach where the objective function of the dual problem is defined as:

$$q^*(\mu) = -(\sum_{k=1}^{p}\sum_{j=1}^{n} w_{jk} log(\frac{w_j k}{\mu_j})) + \sum_{j=1}^{n}(\sum_{k=1}^{p} w_{jk} - \mu_j Z_j) \tag{21}$$

The dual problem then corresponds to maximizing $q^*(\mu_j)$ over the dual variables μ_j and is defined to be:

$$D : \max_{\mu_j \geq 0} q^*(\mu_j) \tag{22}$$

The problem defined in (22) can then be solved by gradient descent iterations on the dual variables μ_j as following.

$$\mu_j(k+1) = \mu_j(k) - \alpha(\frac{1}{\mu_j(k)} \sum_{k=1}^{p} w_{jk} - Z_j) \tag{23}$$

where α is the step size which affects the speed of convergence and is kept sufficiently small normally.

C. Sub-Problem 2

Sub-problem 2 involves the power flow from generator to the aggregator. Let V_1, V_2, ..., V_j are aggregator utility functions, v_{ij} is the willingness of aggregator j to get power from generator i. Values of v_{ij} are obtained according to the following relation:

$$\sum_{j=1}^{n} v_{ij} = \sum_{k=1}^{p} w_{jk} \tag{24}$$

Above relation is based on the supply-demand balance of power at aggregator and values of w_{jk} are already known from the sub-problem 1. Initially some values of v_{ij} are set such that above relation is satisfied. Let y_{ij} is the power flow from i generator to the j aggregator and $\bar{y}_j$ is a vector which is defined as $\bar{y}_j = [y_{1j} \quad y_{2j}.........y_{mj}]$. We consider the following utility function.

$$V(\bar{y}_j) = \sum_{i=1}^{m} v_{ij}(log(y_{ij})) \tag{25}$$

Utility maximization problem takes the following form:

$$\begin{aligned} &\max_{\bar{y}_j \geq 0} \sum_{j=1}^{n} V_j(\bar{y}_j) \\ &s.t: \sum_{j=1}^{n} y_{1j} = C_1 \\ &\vdots \qquad\qquad \vdots \\ &\sum_{j=1}^{n} y_{mj} = C_i \end{aligned} \tag{26}$$

where C_i is the total generation capacity of the i generator.

1) Equilibrium Analysis: In this section, we analyse the solution of primal problem defined in the (26). We substitute the $V(\bar{y}_j) = \sum_{i=1}^{m} v_{ij}(log(y_{ij}))$ into (26) and represent the Lagrange multipliers for the constraints in (26) as λ_i (prices set by generator). Lagrangian is defined as:

$$L(y,\lambda) = -\sum_{j=1}^{n} V_j(\bar{y}_j) + \sum_{i=1}^{m} \lambda_i(\sum_{j=1}^{n} y_{ij} - C_i) \tag{27}$$

Let ${y_{ij}}^*$ denotes the minimizer of corresponding minimization problem of (26). The gradient of (27) with respect to y_{ij} for a particular j yields following expression:

$${y_{ij}}^* = \frac{v_{ij}}{{\lambda_i}^*} \tag{28}$$

Additionally the gradient of (27) with respect to λ_i yields the following expression:

$${\lambda_i}^* = \frac{1}{C_i} \sum_{j=1}^{n} v_{ij} \tag{29}$$

2) Dual Problem: In this subsection, we use a duality based approach where the objective function of the dual problem is defined as:

$$q^*(\lambda) = -(\sum_{j=1}^{n}\sum_{i=1}^{m} v_{ij} log(\frac{v_i j}{\lambda_i})) + \sum_{i=1}^{m}(\sum_{j=1}^{n} v_{ij} - \lambda_i C_i) \tag{30}$$

The dual problem then corresponds to maximizing $q^*(\lambda_i)$ over the dual variables λ_i and is defined to be:

$$D: \max_{\lambda_i \geq 0} q^*(\lambda_i) \tag{31}$$

The problem defined in (31) can then be solved by gradient descent iterations on the dual variables λ_i as following.

$$\lambda_i(k+1) = \lambda_i(k) - \alpha(\frac{1}{\lambda_i(k)} \sum_{j=1}^{n} v_{ij} - C_i) \tag{32}$$

where α is the step size which affects the speed of convergence and is kept sufficiently small normally.

Remarks: Once a load assignment has been determined (sub-problem 1), it is important to check the feasibility and quality of the resulting power flow assignment. This can be done by solving the power flow problem (sub-problem 2) in the presence of the load assignment decided by the utility optimization problem. If the solution is not good, there is a need for updating the value of v_{ij} and the utility optimization problem must be resolved by changing for example the weights v_{ij}. The weights can be reduced at flows which are not feasible, where the following condition should be satisfied.

$$\sum_{j=1}^{n} v_{ij} = \sum_{k=1}^{p} w_{jk} \tag{33}$$

where (33) demonstrates the balance between supply and demand of power at aggregators. It correlates the two sub-problems of optimization and correspond to the assumption made in (14). In the following, we demonstrate that the solution of two sub-problems is also the solution of original coupled problem and two decoupled problems with correlation, corresponds to the original coupled problem. ${x_{jk}}^*$ obtained from the original problem represented by equation (11) is the same as obtained by sub-problem 1 and represented by equation (19). Similarly, ${\mu_j}^*$ obtained from the original problem represented by equation (12) is the same as obtained by sub-problem 1 and represented by equation (20). Further, equation (13) of the original coupled problem yields the following expression:

$$\sum_{j=1}^{n} y_{ij} = C_i(t) \tag{34}$$

Substituting the solution ${y_{ij}}^*$ from the equation (28) of sub-problem 2 into the above equation yields the following.

$${\lambda_i}^* = \frac{1}{C_i} \sum_{j=1}^{n} v_{ij} \tag{35}$$

where above value of ${\lambda_i}^*$ is same as given by equation (29) of sub problem 2. Hence, solution of two sub-problems is also the solution of the original coupled problem and original problem corresponds to the two sub-problems.

IV. Performance Evaluation

In the previous sections, we proposed price based DR schemes and user adaptation in a smart grid network with and without aggregators. In this section, we use MATLAB simulation tool to analyze the behaviour of the proposed schemes. For the first scenario, we consider a smart grid network without aggregator, having 3 generators and 5 users. Without the loss of generality, we consider that capacity of generator G_1, G_2 and G_3 is 30, 50 and 10 respectively. It maybe noted that our objective is to analyze the stability and illustrate the concept of proposed schemes and frameworks, we omit the units of parameters (demand, price, allocated power) because the units are not that significant. In the scenario

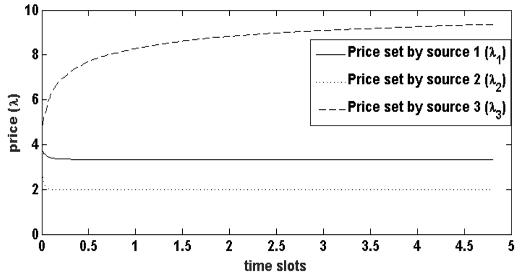

Fig. 3: Price change

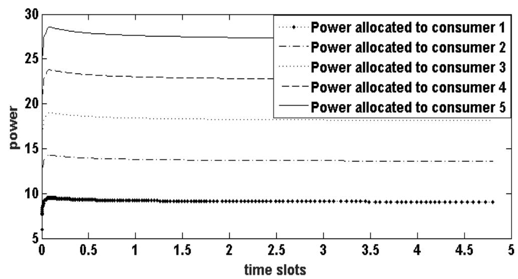

Fig. 4: Demand adaptation

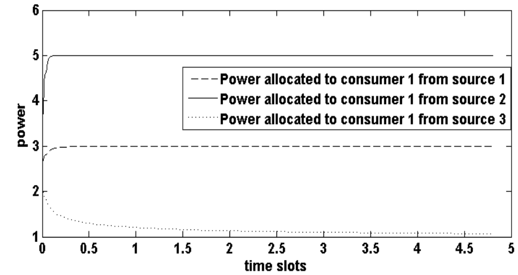

Fig. 5: Power allocation

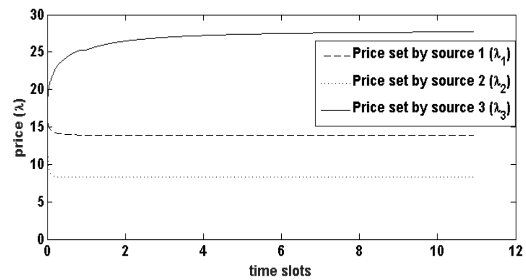

Fig. 6: Price change by generators for scenario 2

under consideration, which corresponds to the section II, all the 5 users initiate their demands with 2. Willingness to pay parameter for user 1, 2, 3, 4 and 5 is set to 10, 15, 20, 25 and 30 respectively. Adaptation parameter or step size (α) is set to 0.01 indicatively.

Fig. 3 depicts the change of price by source based on the demands from all users and Fig. 4 demonstrates the corresponding demand adaptation of the all 5 users. It may be noted that (the total demands of a user) total power allocated to a user is sum of the power from all the sources/generators to that user as shown in the Fig. 4. It is clear from the behaviour that after the small transient time, demands of users converge to stable values which are determined by willingness to pay factor. Indicatively, the users were assigned different values of willingness to pay factor and hence demands of every user converge to different value accordingly which demonstrates that willingness to pay parameter is a crucial factor in determining how aggressive a user need to respond to change in price. Since, we consider the scenario, where each user is able to receive power from any source and each source can assign power to any user, hence, to demonstrate this, indicatively we selected the user 1 and in Fig. 5 we show the power assignment from all the sources to user 1.

In the next scenario, we consider the case where we have aggregators. Aggregators provide a coordinated response to the number of scattered consumers, collect power from all generators and assign the power flow to the interested users. We consider 3 generators with capacities 30, 50 and 15 respectively. There are 5 aggregators and each aggregator is responsible to provide power to 4 users. It is valid to note that each aggregator can demand power from any generator but each user can receive power from only one aggregator which makes the scenario more realistic. All the users initiate the demands with value 6. In every iteration, an aggregator calculates its willingness to pay factor by summing willingness to pay factor of all users associated with it. Fig. 6 shows the change of price by the generators and Fig. 7 depicts the adaptation of demands by the aggregators in response to the change in price. Fig. 8 illustrates the power assignment from all the sources to an aggregator (aggregator 1 is chosen indicatively for demonstration). Fig. 9 depicts the change of prices by aggregators in response to change in demands from associated users. To demonstrate the dynamics in better way, we are presenting the behaviour of only one aggregator (aggregator 5). Fig. 10 shows the power assignment from aggregator 5 to its associated users. From the simulation results, it is evident that after a small transient period, price signal of all the generators and aggregators converge to stable value and users adapts their demands accordingly.

V. Conclusion

This paper proposes a distributed scheme for DR and user adaptation in smart grid networks for multiple power sources and loads. Individual user adapts to the price signal to maximize its own benefits. We have modelled the user's preference as willingness to pay factor and logarithmic utility functions are used to model behaviour of users. Further, we envision a central entity providing a coordinated response to the huge number of scattered consumers, collecting power from all generators and assigning the power flow to the interested users.

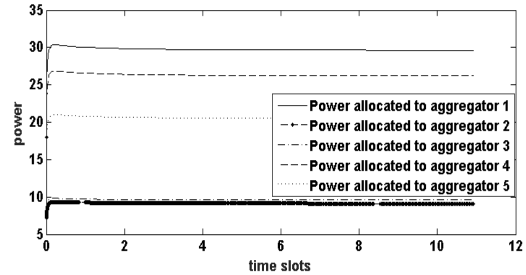

Fig. 7: Demand adaptation of aggregators

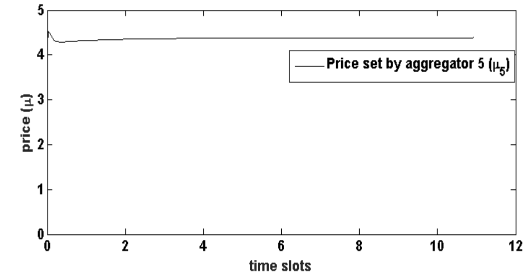

Fig. 9: Price change by aggregator

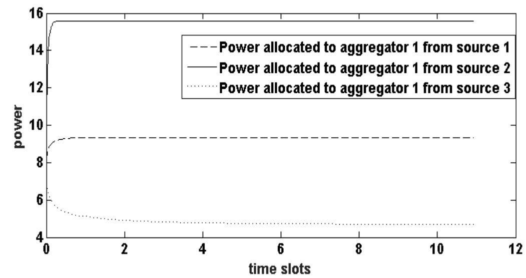

Fig. 8: Power allocation to aggregators

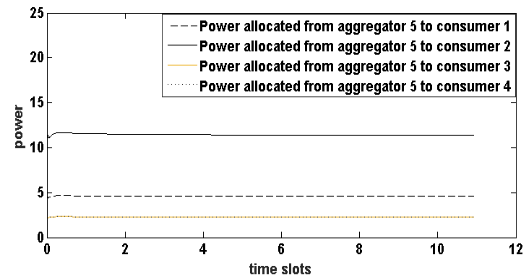

Fig. 10: Power allocation from aggregator to users

We propose a two layer DR scheme whose combined objective is to maximize the utility services of both aggregator and end users based on willingness of users. A natural extension of this work is the investigation of prediction for user's willingness factor based on power load models. Further, in the future, we aim at establishing analytically the stability of the proposed algorithms and we aim at applying the methods in a smart community setting where the performance is going to be evaluated using simulations on a dedicated simulator available to us.

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