Robustness Test Method of Power Flow System Containing Controllable and Fluctuating Power Devices

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***Abstract*—The electricity generated by renewable energy sources fluctuates depending on its intermittent nature, and change of weather conditions. Similarly, power demand also vary dynamically due to change of operation mode, user preferences etc. To mitigate the effects of power fluctuations caused by fluctuating power sources and loads, a power flow control is introduced which assigns power levels for controllable power devices and connections between power sources and loads to absorb the power fluctuations of fluctuating power devices. This paper introduces a new robustness test method for a power flow system consisting of controllable and fluctuating power devices which can guarantee the existence of feasible solution for any power level of fluctuating power devices. The proposed test method can be formulated as a linear programming problem, and can be solved with a polynomial time complexity.**

***Index Terms*—Power flow control, power fluctuations, renewable energy, demand uncertainty, robustness test.**

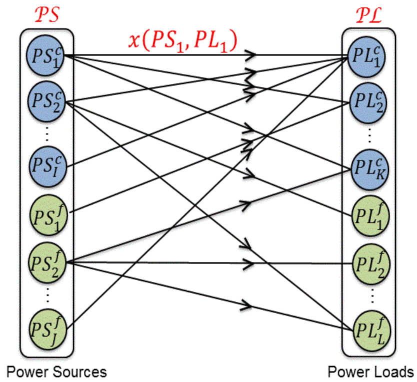

Fig. 1. Representation of power devices and connections.

I. Introduction

The shift towards modern power systems is achieved by taking advantage of research innovations in terms of smart grids, distributed power generation, smart power sensing and controlling, micro-grids etc. [1]. Conventional power plants have been large, centralized units. A new trend is developing towards small-scale distributed power generation, which means that the power generating sources are located close to energy consumers, and large power generation systems are replaced with smaller ones. A distributed power generation system is reliable, efficient, and environment friendly alternative to the traditional power generation system. Additionally, these power generation systems can effectively utilize local power generation sources and power network [2].

The electricity generated by renewable energy sources fluctuates depending on its intermittent nature, and change of weather conditions. Similarly, power demand also vary dynamically due to change of operation mode, user preferences [3]. Therefore, the critical task of electrical power management system is to keep balance between dynamic changing power supply and consumption patterns.

The increasing penetration of renewable power sources along with uncertain power demand necessitates power flow management studies. In order to manage power flow streams between these fluctuating power sources and loads, a real-time power flow control is required. This paper introduces a power flow control problem which can handle such uncertainties in load demand, wind generated power and solar generated power by cooperation with controllable power devices [4], [5], [6]. The goal of this power control problem is to find the power level for controllable power devices, and connections (i.e., power flow streams) between power sources and loads based on the measured power levels of fluctuating power devices.

The real physical power system changes at each time instance, therefore, the issue whether the system (i.e., power flow control problem) has a feasible solution or not is an important issue to solve. In our previous work [7], we discussed two types of solvability conditions for a system (i) with controllable power sources and loads with given generated power and demand of fluctuating power sources and load, and (ii) with controllable power sources and loads along with any situation/value of fluctuating power sources and loads to have a feasible solution.

The power flow management has been discussed in past with respect to different objectives and power sharing methods [8]–[12]. One of the major challenges for integration of renewable energy systems remains in the balancing of the intermittent energy production with the dynamic power demand but there is no discussion about solvability even though it is important issue which is the focus of our studies.

This paper introduces a new robustness test method for a power flow system consisting of controllable and fluctuating

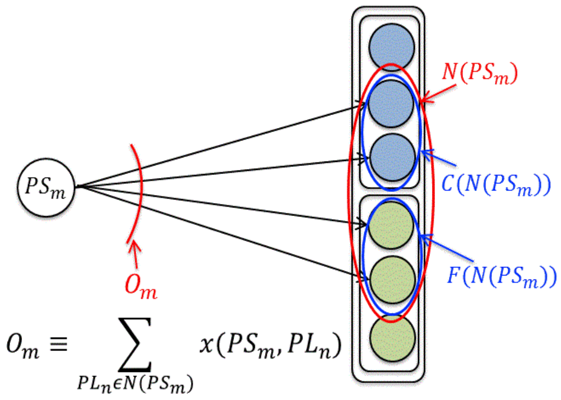

Fig. 2. A Power source with connections.

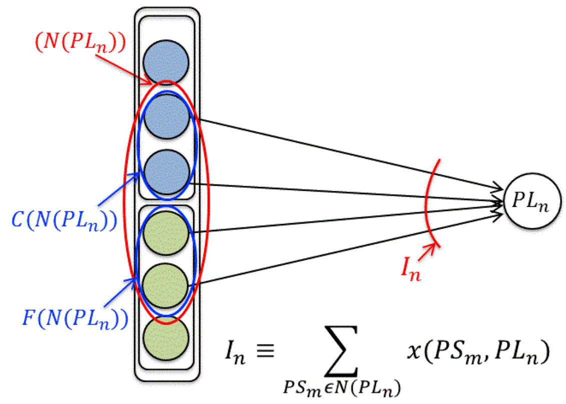

Fig. 3. A Power load with connections.

power devices which can guarantee the existence of feasible solution for any power level of fluctuating power devices. The proposed test method can be formulated as a linear programming problem, and can be solved with a polynomial time complexity. This test method can identify the system which can satisfy the third solvability condition which has robustness against power fluctuations.

This paper is organized as follows: Section II shows system overview with representation, and categorization of power devices and connections between power devices. Section III describes the solvability issues of our power flow control problem. Section IV introduces a new solvability theorem for a system for any power levels of fluctuating devices to show the robustness of system against power fluctuations. Finally, concluding remarks are given in Section V.

II. System Overview

In this section, we consider a system which consists of power sources, power loads and connections between them. All power devices are categorized into two types as, fluctuating power sources/loads and controllable power sources/loads, where the latter can work for managing (i.e., absorbing) the power fluctuations of power generation/demand in the former and for making an entire system robust against the effects caused by fluctuating power devices.

This section explains the details of our system model and *Power Flow Control Problem*.

A. Representation and Categorization of Power Devices

A power source (PS) is an electric device which can supply electric power to power loads, e.g., photo-voltaic, wind turbine, utility grid, etc. A power load (PL) is an electric device which consumes electric power supplied by power sources. Since all power devices are divided into two categories based on their characteristics and functionality, such as *Controllable* PS^c/PL^c and *Fluctuating* PS^f/PL^f.

A controllable power device can control its power, whereas fluctuating power device cannot control its power. All power sources with both types can be represented as, $\mathcal{PS} = \{PS_1^c, PS_2^c, \cdots, PS_I^c, PS_1^f, PS_2^f, \cdots, PS_J^f\} = \{PS_1, PS_2, PS_3, \ldots, PS_{I+J}\}$, where I and J show the total numbers of controllable and fluctuating power sources, respectively. Similarly, all power loads can be indexed as, $\mathcal{PL} = \{PL_1^c, PL_2^c, \cdots, PL_K^c, PL_1^f, PL_2^f, \cdots, PL_L^f\} = \{PL_1, PL_2, PL_3, \ldots, PL_{K+L}\}$ where K and L show the total numbers of controllable and fluctuating power loads.

The actual power generation and consumption levels of power sources and loads can be represented as ps_i^c, ps_j^f, $p\ell_k^c$ and $p\ell_\ell^f$, respectively for PS_i^c, PS_j^f, PL_k^c and PL_ℓ^f.

Each power device PS/PL has a minimum and maximum power generation/consumption limitation, which shows the range of operation and performance of that power device. The minimum power generation limit ps_i^{c-min} and maximum power generation limit ps_i^{c-max} show the capacity of a controllable power source PS_i^c and the power ps_i^c generated by PS_i^c is assumed to be bounded as,

$$ps_i^{c-min} \leq ps_i^c \leq ps_i^{c-max} \tag{1}$$

Similarly, the minimum and maximum power generation limits will be given as ps_j^{f-min} and ps_j^{f-max} respectively, for PS_j^f and the power generation ps_j^f is limited as,

$$ps_j^{f-min} \leq ps_j^f \leq ps_j^{f-max} \tag{2}$$

For the power demand $p\ell_k^c$ of controllable power load PL_k^c with given minimum and maximum consumption levels $p\ell_k^{c-min}$ and $p\ell_k^{c-max}$, and for the power demand $p\ell_\ell^f$ of fluctuating load PL_ℓ^f with given minimum and maximum consumption levels $p\ell_\ell^{f-min}$ and $p\ell_\ell^{f-max}$ are bounded as,

$$p\ell_k^{c-min} \leq p\ell_k^c \leq p\ell_k^{c-max} \tag{3}$$

$$p\ell_\ell^{f-min} \leq p\ell_\ell^f \leq p\ell_\ell^{f-max} \tag{4}$$

B. Connections between Power Sources and Loads

A connection is a pair of a power source and a power load, (PS_m, PL_n). In order to represent connections between power devices, a bipartite graph is introduced as shown in Fig. 1, which consists of a set of power sources $(\mathcal{PS})$, a set of power loads $(\mathcal{PL})$, and a set $\mathcal{X}$ of connections between power sources and loads as, $\mathcal{X} \subseteq \mathcal{PS} \times \mathcal{PL}$. Each connection

(PS_m, PL_n) is associated with some power level in Watt $x(PS_m, PL_n)$ to show the amount of power supplied from a power source PS_m to a power load PL_n via this connection, which is always non-negative real number.

C. Power Flow Control Problem

As the physical power by a fluctuating power device (i.e., PS/PL) varies a lot due to its nature and operation mode, the power level on each connection must be changed according to the fluctuating environment. Here, it is assumed that the power levels of fluctuating power devices are measured with power sensors at each time instance. In order to accommodate power fluctuations caused by fluctuating power devices, a power flow control algorithm is required. This power flow control algorithm uses measured power levels of fluctuating power devices and computes power levels for controllable power devices and connections under the power balance constraint such that the total power generated by all power sources is fully consumed by power loads, and all power loads receive sufficient power from power sources.

Each connection connects a PS to its neighbor on the other side of the connection. The set of neighbors of PS_m is denoted as $N(PS_m)$, which can be separated into $C(N(PS_m))$, and $F(N(PS_m))$, the sets of controllable and fluctuating power devices, respectively. As for the representation of neighboring power devices and the power flows, please refer to Figs. 2, and 3.

The sum of all outgoing power flows, O_m, of power source PS_m can be written as,

$$O_m = \sum_{PL_n \in N(PS_m)} x(PS_m, PL_n)$$

Similarly, the sum of all incoming power flows, I_n, of a power load, PL_n, can be computed as,

$$I_n = \sum_{PS_m \in N(PL_n)} x(PS_m, PL_n)$$

At the end of power flow control, the power generation ps_m of power source PS_m must be equal to the sum of all outgoing power flows, O_m, defined as,

$$O_m = ps_m \tag{5}$$

The power consumption $p\ell_n$ of power load PL_n must be equal to the sum of all incoming power flows to this PL as,

$$I_n = p\ell_n \tag{6}$$

Hence, the goal of this control problem is, for given (i.e., measured) power levels ps_j^f and $p\ell_\ell^f$ of fluctuating power sources and loads, to find the power levels ps_i^c and $p\ell_k^c$ of controllable power sources and loads and power flow assignment $x : \mathcal{X} \to R_+$ such that Eqs. (5) and (6) are satisfied along with the limitations given by Eqs. (1) and (3).

III. Solvability Issues

At each time instance, we need to solve power flow control problem using measured information of fluctuating power devices. In real physical situations, the system controller needs to handle transient behavior, latency of system control, cost efficiency etc., the issue whether the system (i.e., power flow control problem) has a feasible solution or not is one of the most important issues.

In our previous work [7], we discussed two types of solvability conditions for a system (i) with controllable power sources and loads along with given generated power and demand of fluctuating power sources and load, and (iii) with controllable power sources and loads along with any situation/value of fluctuating sources and loads to have a feasible solution.

The objective of this particular paper is to identify the system which can satisfy the solvability condition in our previous paper which is called robustness against power fluctuations caused by fluctuating power devices. The solvability condition of previous paper is presented as "*Theorem-1*" in this paper.

Theorem- 1

The power flow control problem always has a feasible solution if and only if the following two conditions are satisfied.

1-1 $\forall S \subseteq \mathcal{PS}$,

$$\sum_{PS_i^c \in C(S)} ps_i^{c-min} + \sum_{PS_j^f \in F(S)} ps_j^{f-max} \leq \sum_{PL_k^c \in C(N(S))} p\ell_k^{c-max} + \sum_{PL_\ell^f \in F(N(S))} p\ell_\ell^{f-min}$$

1-2 $\forall T \subseteq \mathcal{PL}$,

$$\sum_{PS_i^c \in C(N(T))} ps_i^{c-max} + \sum_{PS_j^f \in F(N(T))} ps_j^{f-min} \geq \sum_{PL_k^c \in C(T)} p\ell_k^{c-min} + \sum_{PL_\ell^f \in F(T)} p\ell_\ell^{f-max}$$

The above solvability condition for a system consists of controllable and fluctuating power devices with any power level of fluctuating power devices within its power capacity range. The condition can guarantee the robustness property of a particular system which consists of power sources, loads, and connections between them. In order to ensure the continuity of operation of the given system with uncertainty of power generation and demand caused by fluctuating power devices, the system must have this property.

Since the direct application of Theorem-1 to a system needs to generate all possible subsets of power sources and power loads, its time complexity is an exponential order with respect to the numbers of power sources and power loads. Therefore, we need to find another way which can reduce time complexity of testing whether a given system always has a feasible solution for any power level of fluctuating power devices or not.

IV. New Solvability Theorem

The new solvability theorem can guarantee the existence of feasible solution.

Theorem- 2

The system has always a feasible solution for any given power levels of fluctuating power devices, if and only if

2-1 There exists a power flow assignment $x : \mathcal{X} \rightarrow R_+$ which satisfies following constraints,

$$\forall PS_i^c, \quad ps_i^{c-min} = O_i^c \tag{7}$$

$$\forall PS_j^f, \quad ps_j^{f-max} = O_j^f \tag{8}$$

$$\forall PL_k^c, \quad I_k^c \leq p\ell_k^{c-max} \tag{9}$$

$$\forall PL_\ell^f, \quad I_\ell^f \leq p\ell_\ell^{f-min} \tag{10}$$

2-2 There exists a power flow assignment $x : \mathcal{X} \rightarrow R_+$ which satisfies following constraints,

$$\forall PS_i^c, \quad ps_i^{c-max} \geq O_i^c \tag{11}$$

$$\forall PS_j^f, \quad ps_j^{f-min} \geq O_j^f \tag{12}$$

$$\forall PL_k^c, \quad I_k^c = p\ell_k^{c-min} \tag{13}$$

$$\forall PL_\ell^f, \quad I_\ell^f = p\ell_\ell^{f-max} \tag{14}$$

Proof

In order to prove this theorem, we will show the equivalences between condition *2-1* and condition *1-1* in theorem-1 and between condition *2-2* and condition *1-2*.

At first, we will prove the sufficiency of condition *2-1* to condition *1-1*. Let $x : \mathcal{X} \rightarrow R_+$ be a feasible solution which can satisfies the following conditions for every PS and PL.

$$ps_i^{c-min} = O_i^c, \qquad \text{for each } PS_i^c \tag{15}$$

$$ps_j^{f-max} = O_j^f, \qquad \text{for each } PS_j^f \tag{16}$$

$$p\ell_k^{c-max} \geq I_k^c, \qquad \text{for each } PL_k^c \tag{17}$$

$$p\ell_\ell^{f-min} \geq I_\ell^f, \qquad \text{for each } PL_\ell^f \tag{18}$$

Let S be an arbitrary subset of power sources and $N(S)$ be the set of neighboring power loads, then the following condition holds as,

$$\sum_{PS_m \in (S)} ps_m = \sum_{PS_m \in (S)} O_m \tag{19}$$

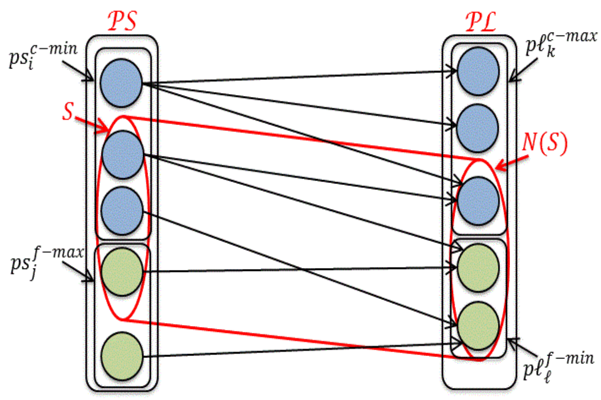

Fig. 4. Illustration of a subset S of power sources and its neighbor set $N(S)$.

Since, each power source in S is connected to only power loads in $N(S)$, but the power loads in $N(S)$ could have connections with power sources outside S (see Fig. 4). Therefore, if we compare the total outgoing power from S with the total incoming power to $N(S)$, the former must not larger than the the latter, i.e.,

$$\sum_{PS_m \in (S)} O_m \leq \sum_{PL_n \in N(S)} I_n \tag{20}$$

From Eqs. (17) and (18), we have

$$\sum_{PS_m \in (S)} O_m \leq \sum_{PL_n \in N(S)} I_n \leq \sum_{PL_n \in N(S)} p\ell_n \tag{21}$$

This shows the sufficiency of condition *2-1* to condition *1-1*.

Now, in order to show the necessity of condition *2-1* to condition *1-1*, we will introduce Optimization Problem and some definitions.

Optimization Problem

For minimum and maximum power levels of fluctuating and controllable power devices, find the power flow assignment the power flow $x : \mathcal{X} \rightarrow R_+$ such that

$$min \quad \sum_{i=1}^{I} (ps_i^{c-min} - O_i^c) + \sum_{j=1}^{J} \left(ps_j^{f-max} - O_j^f\right)$$

with following constraints,

$$O_i^c \leq ps_i^{c-min}, \quad 1 \leq i \leq I \tag{22}$$

$$O_j^f \leq ps_j^{f-max}, \quad 1 \leq j \leq J \tag{23}$$

$$I_k^c \leq p\ell_k^{c-max}, \quad 1 \leq k \leq K \tag{24}$$

$$I_\ell^f \leq p\ell_\ell^{f-min}, \quad 1 \leq \ell \leq L \tag{25}$$

A solution which satisfies all constraints is called a feasible solution of the optimization problem, and a feasible solution that minimizes the objective function is called an optimum solution.

1) Definition- 1:

- POWER-HIGH : When $ps_i^{c-min} > O_i^c$ and $ps_j^{f-max} > O_j^f$ hold for PS_i^c and PS_j^f respectively, these power sources are called "power-high" nodes.
  On the other hand, when $I_k^c > p\ell_k^{c-max}$ and $I_\ell^f > p\ell_\ell^{f-min}$ hold for PL_k^c and PL_ℓ^f respectively, such power loads are also called "power-high" nodes.
- POWER-BALANCED : When the total sum of all outgoing/incoming power flows from/to a power source/load is same with the specified value, $ps_i^{c-min} = O_i^c$, $ps_j^{f-max} = O_j^f$, $I_k^c = p\ell_k^{c-max}$ and $I_\ell^f = p\ell_\ell^{f-min}$, the node is called "power-balanced".
- POWER-LOW : When $ps_i^{c-min} < O_i^c$ and $ps_j^{f-max} < O_j^f$ hold for PS_i^c and PS_j^f respectively, and when $I_k^c < p\ell_k^{c-max}$ and $I_\ell^f < p\ell_\ell^f$ hold for PL_k^c, and PL_ℓ^f respectively, such power devices are called "power-low" nodes.

2) Definition- 2: A path is an alternate sequence of nodes and connections, where each node in a path is either a starting node followed by a connection incident to this node, an intermediate node which is incident to the preceding and the following connections or a terminating node which is incident to the preceding connection. A path may contain "forward edges" having same direction with path direction as well as "backward edges" having the opposite direction with the path direction. Every backward edge in a path has positive power flow then the path is called "alternating path". The power flow requirement on each connection of an alternating path is shown in Fig. 5.

Definition- 3: An alternating path which starts from "power-high" node and terminates on "power-low" node is called an augmenting path (Fig. 6).

Definition- 4: With respect to an augmenting path, the operation to increase power flow on each connection in the path uniformly by $\triangle > 0$ ($+\triangle$ for a forward edge, and $-\triangle$ for a backward edge) is called "power flow augmentation". Note that, by this power flow augmentation, the total outgoing/incoming power changes only at a starting node and a terminating node.

From now on, we will prove the necessity of condition *2-1* to condition *1-1*. The target of this proof is to show that Optimization Problem has an optimum solution which achieves the objective function equal to zero,

We assume that the optimum solution $x^* : \mathcal{X} \to R_+$, does not achieve the objective function equal to zero. This shows that there exists PS_a such that $O_a < ps_a$ because of the constraints (22) and (23). We consider alternating paths starting from PS_a, and let A be the set of power sources which can be reached from PS_a by alternating paths. Similarly, let B be the set of power loads which can be reached from PS_a by alternating paths. Since an alternating path can be extended from a power source node to a power load node without any restriction, $B = N(A)$. However, power loads in B can have connection (it must have zero power flow) with power sources outside A, i.e., $A \subseteq N(B)$. The power consumption by power

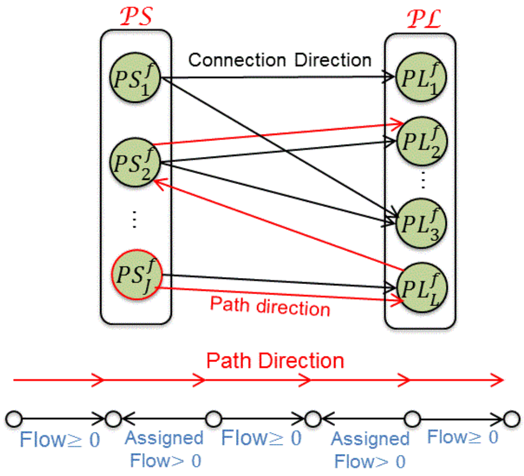

Fig. 5. Alternating Path.

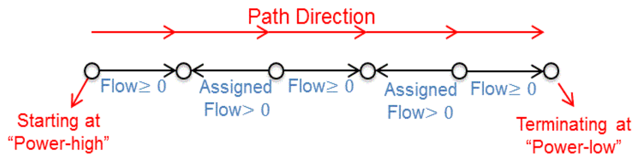

Fig. 6. Augmenting Path.

loads in B is supplied from only power sources in A, since power flows on connections from $\mathcal{PS} \setminus A$ to B are zero.

Now we can consider two possibilities given below.

[Case-1]: At least one node, say PL_b, in B is "power-low". Then the alternating path from PS_a to PL_b is an augmenting path. The power flow can be updated along the path and the difference between $ps_a - O_a$ can be reduced to get a new solution better than the assumed optimum solution $x^* : \mathcal{X} \to R_+$.

[Case-2]: All nodes in B are "power-balanced". If power loads in B are all "power-balanced" nodes, this gives

$$\sum_{PS_m \in A} ps_m > \sum_{PS_m \in A} O_m = \sum_{PL_n \in N(A)} I_n = \sum_{PL_n \in N(A)} p\ell_n$$

which contradicts to condition *1-1* in Theorem 1.

From case-1 and case-2, Optimization Problem-1 has an optimum solution which achieves the objective function equal to zero, and shows the existence of a feasible solution given in condition *2-1*.

The sufficiency and necessity of condition *2-2* to condition *1-2* can be shown in a similar way with appropriate modification of the definitions of "power-high", "power-balanced", and "power-low".

V. CONCLUDING REMARKS

The combination of renewable energy generation connected to grid, and ever increasing power demand have increased the risks of stability and quality of power of the power grid. Considering the increase of the power fluctuation in the future power systems due to uncontrollable power generation sources and in order to manage power fluctuations caused by fluctuating power sources and loads, a power flow control is introduced which assigns power levels for controllable power devices and connections between power devices.

This paper presents a new robustness test method for a power flow system consisting of controllable and fluctuating power devices. New robustness test can be realized by finding a feasible solution of a kind of linear programming problem formulated using the information of a given power system to be tested, which can be easily solved by using a LP solver. In this paper, the existing exponential time order test is reduced into a linear programming problem which can be solved with a polynomial time complexity.

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