Master's Thesis

An Application of Game Refinement Theory in Popular Activities

MAO Xinting

Supervisor Iida Hiroyuki

Graduate School of Advanced Science and Technology
Japan Advanced Institute of Science and Technology
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Abstract

With the development of game development, game theories have sprung up in recent years. Games are intensively studied for their entertaining or educational functions, or the characteristics of game itself. However, the existing theories are mainly about traditional games. Popular games are in lack of scientific study because of their variability and complexity. Moreover, there are very few research about crossing field between game and art, though art is always concerned a pastime way. Therefore, I choose Mafia game and Music as two targets of popular activities, to fill the blank within entertainment field. In this study, I am focusing on the average length and outcome of games. Through the method of game refinement theory, progress model is built, and game acceleration and risk ratio can be calculated. In Mafia game, we suggested that for the citizens group, sheriff is much more important than doctor from the viewpoint of game balancing. The balance setting with $N = 12$, $MFG(12, m, s, d)$ is the players number with $m = 5, s = 1, d = 2$ or $m = 4, s = 1, d = 1$ while minimizing the difference of winning ratio between mafia and citizen groups. Also, the acceleration of game will reduce as the number of players increases. This may imply that Mafia game would become boring as the players number becomes too large. $MFG(N, m, s, d)$ can be played reasonably with $N \in \{14, 15, 16\}$, $m \in \{5, 6\}$, $s = 1$ and $d \in \{1, 2\}$. In particular, $MFG(15, 5, 1, 1)$ or $MFG(15, 6, 1, 2)$ is the best under the assumption that its GR value is within the sophisticated zone. Moreover, the level of players affects game balance and game sophistication. For example, mafia would dominate citizens if all players are weak, which implies the decreasing in game sophistication. Besides, the result of game refinement value and risk ratio of music shows that game and art shares common factor in acceleration (0.07 - 0.08). Yet different genres of music differ from each other. The Out-of-Japan music is unexpectedly like Enka, with a "Zone" value in acceleration. It implies a proper length in general. While Japanese popular music and Rock has a lower GR value. which means they are more simple to listeners.

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Chapter 1

Introduction

Game is an organized form of play [1], usually for entertainment or educational purpose. Game is considered differ from jobs and art, yet the gap between them is not always clear. For example, a professional athlete's job is obviously a sports game, and games like puzzle has both a game and art component.

Game is an old entertainment though, game theories are developing rapidly in recent years, with the development of computer science. However, the studies are mainly about traditional games, for their clear game progress and strategies. There is still a lack of research on popular games. It may because popular games change rapidly and always have a large variety. Thus, popular games can be a good research target to fill the gap in game theories. Here we choose Mafia as the objective, because even through Mafia game is a complex party game, the balance of the game can be affected by simple settings, such as player numbers. This thesis is mainly focusing on the balance setting of Mafia game. Since there is no research on the balance of game setting, this analysis is more essential and tell more about game nature.

Moreover, the gap between game and art remains a question. Although we knew that they may share common points with each other, it hasn't been described scientifically rather than intuitively. Here I choose music as the research target because music is closely related to mathematics and physics. As an art form with science elements, music may be analyzed more scientifically than other types of art forms. Music holds a mathematical beauty, which brings about comfort and harmony. Thus, this research is mainly about musical comfort.

In this research, I would like to use game refinement theory to analyze research objectives. This game theory emphasizes more on progression and

[1] `en.wikipedia.org/wiki/game`

outcomes of games. Since music unfolding can also be known through musical theories, common method will find links between art and game more convincingly.

I suppose that there would be common factors shared by popular games and traditional games, and also by games and art. This standpoint will fill in more blanks within daily entertainment.

Chapter 2

Literature Review

2.1 Game Refinement Measure

Game refinement theory [18] gives a measure to quantify the sophistication of a game. This enables to obtain the deep insight into the current game and improve the quality of the game [27]. Game process will be divided into two elements. One is game speed or scoring rate. Another one is game information progress with a focus on the game outcome, which presents the degree of certainty of a game's result in time or in steps. Then, we use a physical model to obtain the acceleration value (say GR) of game [38]. The value of GR is derived from the average number of goal scores (say $x(t_k)$) over the average number of goal attempts (say t_k), where game information certainty $x(t)$ is a function of time t with $0 \leq t \leq t_k$ and $0 \leq x(t) \leq x(t_k)$ which corresponds to the average number of possible moves and game length in boardgames. For the continuous movement games such as sports and video games, a game progress model can be figured out to find the measure of game refinement [32]. Various types of games such as boardgames and sports have been analyzed while applying game refinement measures [39], summarized in Table 2.1.

The results reasonably enable us to assume an important characteristic of sophisticated games, as stated below.

Remark 1 *Sophisticated games have a common factor (i.e., same or nearly same degree of informatical acceleration value, say* 0.07-0.08*) to feel engaged or excited regardless of different type of games.*

Table 2.1: Measures of game refinement for various types of games

	$x(t_k)$	t_k	GR
Chess	35	80	0.074
Shogi	80	115	0.078
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073
Badminton	46.336	79.344	0.086
Table tennis	54.863	96.465	0.077
DotA ver 6.80	68.6	106.2	0.078
StarCraft II Terran	1.64	16	0.081
The king of the fighters 98	14.6	36.7	0.104

2.2 Model of Game Outcome Process

Let us consider the progress of a task to accomplish within a given time framework. The 'progress' of a task is twofold. One is speed of the task, while the other one is the task's information progress which focuses on the result of the task. The task's information progress presents the degree of certainty of the task's results in time. Having full information of the task's progress, i.e., after its conclusion, the task's progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq T$ and $0 \leq x(t) \leq x(T)$. shown in Eq. (2.1).

$$x(t) = \frac{x(T)}{T}t \tag{2.1}$$

Here T and $x(T)$ corresponds to the end of the task and the goal to achieve in the task respectively. Eq. (2.1) indicates that the goal of the task under consideration can be achieved within the time T, meanwhile it may not be so within the shorter time than T. Under the given time constraint that the task should be completed within the time τ with $0 < \tau < T$ where $x(\tau) = x(T)$, the task's progress $x(t)$ is given as a linear function, as shown in Eq. (2.2).

$$x(t) = \frac{x(\tau)}{\tau}t \tag{2.2}$$

However, the task's information progress given by Eq. (2.2) is unknown during the in-task period since $\tau < T$. The presence of uncertainty during the task, often until the final moments of a task, reasonably renders the task's progress exponential. Hence, a realistic model of information progress of a

given task is given by Eq. (2.3).

$$x(t) = x(\tau)(\frac{t}{\tau})^n \tag{2.3}$$

Here $n \in N$ stands for a parameter which is given based on the task's progress patterns or hardness of completing the task. Larger n corresponds to more hardness of the goal achievement. When $1 \leq n$,w the first derivative of the task's information progress model $x(t)$ in Eq. (2.3) or velocity in the sense of dynamics is given in Eq. (2.4).

$$x'(t) = \frac{x(\tau)}{\tau^n} t^{n-1} n \tag{2.4}$$

The second derivative of the task's information progress or acceleration in the sense of dynamics is obtained by deriving Eq. (2.3) twice. Solving it at $t = \tau$, we have Eq. (2.5).

$$x''(\tau) = \frac{x(\tau)}{\tau^n} t^{n-2} n(n-1)|_{t=\tau} = \frac{x(\tau)}{\tau^2} n(n-1) \tag{2.5}$$

Remark 2 *Under the original condition that the uncertainty of a game outcome can be eliminated within the time T, the certainty of game outcome are obtained proportional as the time goes on, given in Eq.(2.1) thus the experience of these kind of games are more likely be boring since no surprise. Under the new condition when $1 \leq n$, uncertainty of game outcome would be like what happen in see-saw game, i.e.,the experience in of these kind of games are excited.*

In arcade games, the total length of the game is significantly different for players with different level. Imagine what will happen in the Tetris game. Theoretically there will be no end for the perfect player. Therefore, in arcade game case, the total uncertainty of a game initially is regarded as 100 percent, i.e. $x(T) = 1$, the velocity of a game outcome certainty obtained at time t in Eq.(2.1) is shown as $x(t) = \frac{t}{T}$, while in Eq. (2.3) is shown as $x(t) = (\frac{t}{\tau})^n$. So from the perspective of achievement in game process, solve Eq.(2.3) we can get Eq.(2.6).

$$x''(\tau) = \frac{t^{n-2}}{\tau^n} n(n-1)|_{t=\tau} = \frac{n(n-1)}{\tau^2} \tag{2.6}$$

Definition We call the first and second derivative given in Eq.(2.4) and Eq.(2.6) *velocity-in-mind* and *acceleration-in-mind* in arcade games.

selection model.jpg

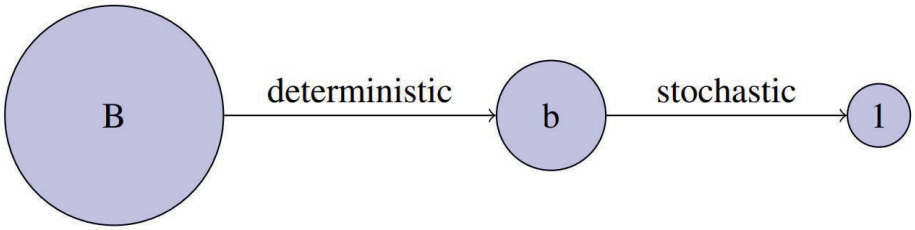

Figure 2.1: A model of move selection in most games[15]

Eq.(2.6) is divided into two parts, where the term $\frac{1}{\tau^2}$ can be used as the measurement of game refinement to examine the balance between deterministic and stochastic aspects of the arcade game outcome[17]. It has been found that sophisticated games have a similar value of game refinement located at the zone between 0.07 to 0.08[33][40][19] using the term in Eq.(2.5). Another term $n(n-1)$ may correspond to the game progress patterns such as one-side game and seesaw game, or the difficulty of completing the task. The game progress pattern or its difficulty is important with respect to engagement. Former research have found that the most exciting setting , either for boardgame with a focus on average branching factor, or the progress approach with a focus on the average scoring points in a scoring sport game, is given by n=2 [16][40] in the sense that the outcome is highly uncertain. People would feel gamified experience if GR is a zone value and n=2.

The model of move selection process in game playing is shown in Fig.(2.1). A move to play would be selected first in a deterministic way, which is followed by a stochastic way. Usually, amateurs are not skilled at making a move selection. They are likely to eliminate choices based on instinction, which is inefficient and unreliable, hence their b would be relatively large. On the other hand, experts can always reduce choices from B to relatively small b based on their skills and make high quality of decision. In boardgame case, $b = \sqrt{B}$ according to $\alpha - \beta$ pruning method.

However for arcade game case the process model is slightly different from boardgame and sports game. Mostly the rules of arcade games would be simple and just repeat same operations, the choices player need to select for each step usually less than 5. As we mentioned, the term $n(n-1)$ are mightily related to player skill level. The higher level the player skill would be, the less option they need to face, the value of n would be nearer to 1 . Theoretically most of the perfect expert players can directly find the best move, which means all their decision making are deterministic, hence their move selection would be like in Fig.2.2, where $n = 1$.

selection model for arcade experts.jpg

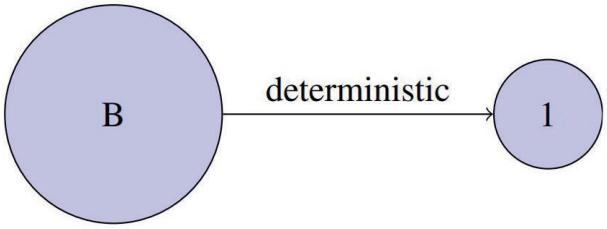

Figure 2.2: A model of move selection for arcade games experts

Chapter 3

Balance Setting in Mafia using Game Refinement Theory

This chapter is an updated and abridged version of work previously published in

- Xiong S., Li W., Mao X., Iida H. (2018) Mafia Game Setting Research Using Game Refinement Measurement. In: Cheok A., Inami M., Romão T. (eds) Advances in Computer Entertainment Technology. ACE 2017. Lecture Notes in Computer Science, vol 10714, pp 830-846, Springer.

3.1 Introduction

Mafia is a party game, which is also called Werewolf. It can be held face to face in reality or just through the internet. In the typical version of Mafia game, players gather together and separate into two groups- Citizen group and Mafia group. The rule of this game is simple. What players have to do is to 'kill' citizens and find out who is the hidden Mafia (killer) within several rounds. The game ends when all Mafia players are found - it represents the succeed of Citizen group. Otherwise, Mafia players will win, while all the citizens are 'killed' by them.

Mafia game is a very good research target because the balance of the game can be affected by different settings.[24] For example, changing numbers of players in a multi-player game can maintain the attractiveness.[29]

Although Mafia game is popular, there is no research on the balance of the game setting. Thus, a measurement using game refinement theory is put forward here, in order to achieve the goal to the balance setting of Mafia game.

3.1.1 What is Mafia Game?

The game Mafia was not a game at first. In 1970s, it was created for a psychology research.[41]. It becomes a popular party game through times and the game rules are roughly fixed. In the typical version of this game, Several players play together. They get together and divided into two groups randomly. It means players have two kinds of identities: Mafia and Citizen. The number of players is not always fixed, neither does the size of each group. At the beginning, Mafia players know the identities of both groups, yet Citizen players know only themselves. The game starts with two alternating steps which are called day and night. During the night time, all the players close their eyes and Mafia players 'wake up'. They choose and 'kill' one of the Citizen players. Then comes to the day time, all the players 'wake up' and discuss and vote for one possible Mafia player.

During the game, the point is that Mafia players should hide their identities and 'kill' Citizen players as many as possible. And for the Citizen players, their target is to find out all the Mafia players. At the end of the game, Mafia players win when there are no one left in Citizen group, or Citizen players win when all the Mafia players are found out.

In the improved version, characters with special skills in the Citizen groups are invented.For example, Sheriff, who can investigate at night to figure out one certain player is citizen or not, and Doctor, who can save anyone's life at daytime.

3.1.2 Related Work

At the beginning, Mafia game was created for psychology research in 1970s [41]. Then it evolved into a party game for fun. Dmitry Davidoff is generally acknowledged as the game's creator. He dates the first game to spring 1987 at the Psychology Department of Moscow State University, spreading to classrooms, dorms, and summer camps of Moscow University, he developed the game to combine psychology research with his duties teaching high school students [22]. In 1998 the Kaliningrad Higher school of the Internal Affairs Ministry published the methodical textbook Nonverbal communications. Developing role-playing games "Mafia" and "Murderer" for a course on Visual psychodiagnostics, to teach various methods of reading body language and nonverbal signals. Therefore, Mafia game not only can make human feel relax, but also the game has certain strong academic meaning in the psychology and society communication.

Mafia game as an interesting research target from the multi-player game domain has been investigated from various perspectives. Basically we focus

on four areas: psychology and game theory, artificial intelligence, team work strategy, and design of the game.

There are several works focused on the human behavior and the psychological aspects of playing a werewolf that used various features for determining whether a player is a werewolf. For example, they include a study that used each player's utterances, utterance lengths, and the number of interruptions [20]; a study that used hand and head movements [3]; and a study that used the number of words in each utterance [37] to determine whether a player was a werewolf.

Braverman et al. [2] performed a theoretical study on Mafia game to find the optimal strategies for different groups and calculate the winning rate. They were interested in the best strategies for the different groups in such scenarios and in evaluating their relative power, then analyzed two variants that are with or without a Sheriff - which is also called a Detective - and found out the best strategy in the absence of a Sheriff, i.e., for the total number of players (say R) the number of Mafia members is $O(\sqrt{R})$, there will be an equal rate for each group to win. Conversely, when the number of Mafia members is linear in R, it is proved to be fair when there are Sheriffs in the game, which means that even a single Sheriff could change the qualitative behavior of the game.

There are other Mafia game studies that concentrated on other aspects than strategies. Katagami et al. [20] focused on the nonverbal information in Mafia game. They studied on how nonverbal information, like gestures and facial expressions, impacts the winning rate. After the investigation, they found out that nonverbal information is important for the victory of the game. Furthermore, Kobayashi et al. [21] developed a match system for humans with life-like agents. With this system, they tried to analyze nonverbal information from movies of games played by human to verify whether a life-like agent can give impressions like a human if they mount the analyzed movement on a life-like agent. They found that the movement which is felt doubtful is also doubtful even if a life-like agent expressed, though there was a difference in an impression that is influenced by the contents of utterances.

Furthermore, several audio–visual corpora containing dialogue data in the Mafia game were constructed to analyze group communication [14] [28]. Hirata [13] constructed a behavioral model by obtaining behavioral information from play logs describing play between humans. The proposed model identifies an action selection probability to realize an agent that can behave like humans.

Later Bi [1] concentrated on a werewolf-side strategy called the "stealth werewolf" strategy with which each of the werewolf-side players (Mafia members) behaves like a citizen without showing their special roles. They limited

some of the human-side strategies and they calculated the ϵ-Nash equilibrium of strategies for both sides under the limitation. They found out that this strategy is not friendly to the Mafia group.

From the game design's point of view, Revenant [5] created a custom game in Blizzard's StarCraft II which we call "SC2Mafia". Similar to other web-based Mafia games, SC2Mafia is inspired by the classic party game of the same name but features a faster pace and a wide range of different role options. The only requirement to play SC2Mafia is a copy of StarCraft II. People who have not purchased the game can also download StarCraft II Starter Edition, which has been made available by Blizzard for free. SC2Mafia features a strong player base and loyal community of users who are playing games at almost any time during the day. The most interesting feature in SC2Mafia game is that every player has his/her own special ability, and room number, e.g., Spy can wiretap the talking of Mafia or Triad group in night, Lookout can observe a player and see who "visited" them during a particular night, etc.

Generally, academical research approaches have been focusing mainly on the human behaviors, artificial intelligence or natural language processing. However, there are hardly any studies about Mafia game settings or the game balancing issue. As a base of the game, this paper will therefore propose a method to examine the balancing and sophisticated (or comfortable) settings of Mafia game.

The results reasonably enable us to assume an important characteristic of sophisticated games, as stated below.

Remark 3 *Sophisticated games have a common factor (i.e., same or nearly same degree of informatical acceleration value, say* 0.07-0.08*) to feel engaged or excited regardless of different type of games.*

Therefore, it is highly expected that a sophisticated version or comfortable setting of Mafia game would have the nearly same degree of game sophistication.

3.2 Assessment Methodology

This section presents our assessment methodology for analyzing Mafia game in this study. It presents a simple version of Mafia game as a benchmark, whereas a short sketch of game progress model for Mafia game is given to apply the measure of game refinement for the assessment.

3.2.1 Simple Version of Mafia Game

We introduce the rules of a simple version of Mafia game (or "Werewolf" game). We need some participants collected in a circle, and a game coordinator assigns each player to one of the two groups: Mafia or Citizens. Citizens know only their own identity, whereas Mafia members know not only the identity of themselves but also of their fellows. The target is to defeat the other group. The game consists of two alternating phases – day and night. During the night the Mafia members make a decision to kill one of the Citizens. Then during the day time, all the players discuss together and vote for a possible mafia that they want to execute. There can also be some special characters of the Citizens who can use their skills to achieve the success, such as Sheriff (to investigate whether a player is mafia member or not at night) and Doctors (to save a killed character at night) [24].

There can be many variations of Mafia game. A typical modification of the game is to add characters with special skills as many as possible. For example, in the "Werewolf" game, besides Prophet (the same as Sheriff in Mafia) and Guarder (the same as Doctor in Mafia), a number of characters can be added, such as Hunter (when a hunter is killed, he can kill anyone of the alive characters for revenge), Witch (she has a poison to kill any character and a panacea to save a partner) and Twins (two citizens knowing the identity of each other) etc. "Werewolf" game also adds some event cards to adjust the game process, which makes the game more interesting [2].

Mafia game is played in a party to enjoy or just to kill time. Currently, there are two types of Mafia game – one is the traditional card game, in which players get together and pick cards to decide their roles. They also need to choose one person as the compere master, who has to control the game period such as to remind Mafia, Sheriff and Doctor to do their works. Another one is the Mafia video game in which players use computers to play. Players are assigned to their characters by the game system, and can use skills by pressing a simple button. The game system can also judge automatically which group is the winner. The most famous Mafia video game platform is the custom map in StarCraft II as shown in Figure 3.1. Later we introduce the simple Mafia game in detail.

Notation 1 *$MFG(N, m, s, d)$ denotes a game of Mafia that has totally N players which consists of m Mafia group members and $N - m$ citizen group members including s sheriffs and d doctors. $MFG(N, m)$ simply stands for a game of Mafia that has totally N players including m Mafia group members and $N - m$ citizen group members.*

Figure 3.1: A screenshot of Mafia game map in StarCraft II

3.2.2 Methodology

Game refinement theory was established in 2004 by Iida et al. [18]. This theory gives a measure to quantify the sophistication of the game under consideration. It provides us with a deep insight into current games and we are therefore enabled to improve the quality of the game [27]. In game refinement theory, the game progress is divided into two elements. One is called game speed or scoring rate, while another one is game information progress with a focus on the game outcome. Game information progress presents the degree of certainty of a game's result in time or in steps. Finally, we use a physical model to get the second derivative of game [38], which we call game refinement value. Many games have been analyzed by this measure as shown in Table 3.1 [39]. Here GR stands for the measure of game refinement, where $GR = \frac{\sqrt{G}}{T}$ which is derived from the game information progress model.

From the results, we conjecture the relation between the measure of game refinement and game sophistication, as stated below.

Conjecture 1 *Sophisticated games have a common factor (i.e., same degree of informatical acceleration value, say* 0.07-0.08*) to feel engaged or excited regardless of different type of games.*

Therefore, it is expected that Mafia game can be analyzed by this method. In Section 3.2.3, the detail of mathematical process will be described.

Table 3.1: Measures of game refinement for various types of games

	$x(t_k)$	t_k	GR
Chess	35	80	0.074
Shogi	80	115	0.078
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073
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DotA ver 6.80	68.6	106.2	0.078
StarCraft II Terran	1.64	16	0.081
The king of the fighters 98	14.6	36.7	0.104

3.2.3 Game Progress Model

Let us consider the game progress of Mafia game. In every day and night, players will use their skills and logic to kill citizens or execute the mafia member, while the number of players is reduced to a certain value, then game is over. Therefore, let K and L be the average number of players killed and average game length, respectively. If one knows the game information progress, for example, after the game, the game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq L$ and $0 \leq x(t) \leq K$, as shown in Eq. (3.1).

$$x(t) = \frac{K}{L}\, t \tag{3.1}$$

However, the game information progress given by Eq. (3.1) is usually unknown during the in-game period. Hence, the game information progress is reasonably assumed to be exponential. This is because the game outcome is uncertain until the very end of game in many games. Hence, a realistic model of game information progress is given by Eq. (3.2).

$$x(t) = K(\frac{t}{L})^n \tag{3.2}$$

Here n stands for a constant parameter which is given based on the perspective of an observer in the game considered. Then the acceleration of game information progress is obtained by deriving Eq. (3.2) twice. Solving it at $t = L$, the equation becomes

$$x''(L) = \frac{Kn(n-1)}{L^n} t^{n-2} = \frac{K}{L^2} n(n-1)$$

Similarly, we get the refinement value $GR = \frac{\sqrt{K}}{L}$.

The average game length L is given by $L = QT$ where Q and T stands for the average number of survivals and the average number of turns, respectively. Thus, the final form of game refinement is given by $GR = \frac{\sqrt{K}}{QT}$.

$$GR = \frac{\sqrt{K}}{QT} \tag{3.3}$$

3.3 Simulations and Results

3.3.1 Simulations

In order to simulate the simple version of Mafia game, details are given below.

* The game needs N players and one more person to coordinate it.
* At the beginning, players are randomly divided into $(N - m)$ citizens and m mafia members.
* Mafia members know the identity of each other, while citizens know only the identity of themselves but do not know others.
* There are two alternating phases: day and night.
* During the day, there are two consecutive subphases:
  - Debate. Everyone still alive can say anything related to accusing or defending.
  - Vote. Everyone has a chance to vote for whom should be executed. The player who gets the highest number of votes is eliminated (in case of a tie, random again). The victim's faction is revealed.
* During the night:
  - Sheriff decides whom he wants to inspect, then the game compere will tell Sheriff if the object who was inspected belongs to Mafia or citizens.
  - Mafia members jointly decide whom they want to kill.
  - Doctor decides whom he wants to save.
  - Game compere announces the night finished, and all the players open their eyes and are mentioned who has been killed or that nobody has been killed and that it was a peace night.

* Repeat the process. The game continues until there is only one group (either the citizens or mafia) left and that group wins.

We create a program to simulate the above process of Mafia game - $MFG(N, m, 1, 1)$ where we assume that there is a sheriff and a doctor. Every round is divided into 4 parts – 1. Night citizens' strategy; 2. Night Mafia's strategy; 3. Day discussion; 4. Day vote. Then, we follow the Nash equilibrium strategy and the algorithm as Table3.2.

Table 3.2: Nash equilibrium in Mafia game for doctor and mafia

Doctor \ Mafia	Kill Sheriff	Kill Randomly
Save Sheriff	$(1, 0)$	$(0 \leq v \leq 1, 0 \leq v \leq 1)$
Save Randomly	$(0 \leq v \leq 1, 0 \leq v \leq 1)$	$(0 \leq v \leq 1, 0 \leq v \leq 1)$

* Sheriff will check the identity of a specific target player at night, then write his identity in "last word". If he finds out the mafia member and can be alive until the next day, he will show his sheriff status and ask citizens to execute the mafia.
* Doctor will save himself at night, while he knows who is the sheriff. The doctor will always protect sheriff, unless he was killed.
* Mafia members will kill a player each night, because mafia members do not know whether the doctor is alive or not and his decision, even if they know who is sheriff, they still keep killing a player randomly. The detail of Nash equilibrium strategy is given in Figure 3.2.
* Citizens will vote and execute a player at the day time. If sheriff has information, then they will vote somebody based on the sheriff's suggestion. Otherwise, they will vote a player (who was not investigated by sheriff) randomly. If several players have the same poll, randomly vote again until someone can be executed.
* All the players repeat this process, until one group wins the game.

Then, we use Python to create a simulation program to simulate the game process. For each case, the number of simulation runs is 10000, then the results are shown in Table 3.3. The pseudo code is given in Algorithm 1.

Algorithm 1 Computer simulation of Mafia game

```
1: function Assignment(N, m)
2:     count(mafia)=m, count(sheriff)=1, count(doctor)=1, count(players)=n
3:     Random(n,m);
4:         N ← mafia, sheriff, doctor, citizen
5: end function
6: function Investigate(sheriff)
7:     Random(left players)
8:         if checked
9:             return -1
10:        else
11:            get character of target player i
12:            i ← m,d,c
13:        return 0
14: end function
15: function Kill(mafia)
16:     Random(left players)
17:         if mafia member
18:             return -1
19:         else
20:             kill target player j
21:         return 0
22: end function
23: function Save(doctor)
24:     Random(left players)
25:         if Sheriff=1
26:             protect sheriff k
27:         else
28:             protect (left players) k
29:         return 0
30:         if j =!k
31:             n=n-1;
32: end function
33: function Discussion(players)
34:     if i = m
35:         show sheriff's status
36:     else
37:         keep silence
38: end function
39: function Vote(players)
40:     if i = 1
41:         vote i
42:     else
43:         vote N-i-sheriff
44:     n=n-1;
45:         return 0
46: end function
47: function Main(players)
48:     while m > n/2
49:         return 0
50:     else
51:         repeat
52: end function
```

3.3.2 Data collection and Analysis

In this section we analyze the data which are obtained by computer simulations and we discuss the results.

Data Analysis

In this study, the game refinement measure is employed for the assessment of game sophistication. However, game balance or fairness for both sides (citizens group and mafia group) is another important aspect. For example, Table 3.3 shows that the winning ratio of the citizens group is significantly low in $MFG(12, 6)$ while the winning ratio of the mafia group is also low in $MFG(12, 1)$. Therefore, we need to identify the balanceable setting.

Table 3.3: The results of simulation for $MFG(12, m)$

N	m	Win for citizens	Rounds	Death	Win for mafia	Rounds	Death
12	6	267	6.07	10.63	9733	1.12	1.22
12	5	3284	5.38	10.01	6716	3.44	5.82
12	4	5975	5.04	9.52	4025	4.64	8.21
12	3	7859	4.60	8.76	2141	5.36	9.63
12	2	9003	3.97	7.60	997	5.78	10.46
12	1	9578	2.91	5.58	422	6.07	11.00

Considering the game played by human with full of emotional behaviors, mafia members know each other and hence they have the information advantage over the citizens group, here we pick up the setting in which the wining ratio of the citizens group is a little higher than the mafia group, just like $MFG(12, 4)$ and $MFG(12, 3)$. We show, in Table 3.4, the results of simulations for various settings denoted by $MFG(12, m, s, d)$. The results indicate two remarks:

(1) For the citizens group, sheriff is much more important than doctor from the viewpoint of game balancing, and (2) $MFG(12, 5, 1, 2)$ among four different settings is the best to play from the viewpoint of game balancing while minimizing the difference of winning ratio between citizens and mafia.

Table 3.4: The results of simulation for $MFG(12, m, s, d)$

Setting	Win for citizens	Rounds	Death	Win for mafia	Rounds	Death
MFG(12,5,1,2)	5216	5.21	9.86	4784	3.21	5.41
MFG(12,4,1,1)	6716	4.96	9.49	3284	4.49	7.93
MFG(12,3,0,1)	7722	4.44	8.88	2278	5.29	9.52
MFG(12,3,1,0)	2170	5.15	9.65	7830	5.05	8.97

Further simulations are conducted to find the comfortable settings based on the measure of game refinement and game balancing for various settings $MFG(N, m, s, d)$ with $8 \leq N \leq 20$.

The average number of players killed (say K) is calculated for the setting $MFG(12, 5, 1, 2)$ as $K = \frac{5216*9.86+4784*5.41}{10000} = 7.73$. Similarly, the average number of turns or game length (say T) is obtained. Thus we can find the refinement value for each setting described in Table 3.5.

In order to make the game more interesting, we not only consider the game refinement value, but also firstly need to analyze the setting balance. For example, Table 3.3 shows that the winning ratio of citizens group is significantly low in $MFG(12, 6)$ while the winning ratio of the mafia group is low in $MFG(12, 1)$. Therefore, we need to choose the balancing setting. Considering the game played by human with full of emotional behaviors, also mafia members know each other and they have the information advantage, here we pick out the settings in which the wining ratio of the citizens group is a little higher than the mafia side, such as $MFG(12, 4)$ and $MFG(12, 3)$. We show, in Table 3.4, the results of simulation for various settings with 12 players.

Conjecture 2 *For the citizens group, sheriff is much more important than doctor from the viewpoint of game balancing.*

Conjecture 3 $MFG(12, m, s, d)$ *is the best to play with* $m = 5, s = 1, d = 2$ *or* $m = 4, s = 1, d = 1$ *from the viewpoint of game balancing while minimizing the difference of winning ratio between mafia and citizens.*

Further simulations will be conducted to find comfortable settings with a focus on game refinement and game balancing for $MFG(N, m, s, d)$ with $9 \leq N \leq 20$.

(20,9,4,4), (20,8,1,2), (19,7,1,1), (19,6,1,1), (18,7,1,2), (18,6,1,1), (17,7,1,2)
(17,7,1,1), (16,6,1,2), (16,6,1,1), (15,6,1,2), (15,5,1,1), (14,5,1,2), (14,5,1,1)
(13,6,1,2), (13,5,1,1), (12,5,1,2), (12,4,1,1), (11,4,1,1), (10,3,1,1), (9,4,2,1),
(8,3,1,1)

As Table 3.4 shows, we also can get the average number of player killed. In the case of $MFG(12, 5, 1, 2)$, the average number of player killed K is calculated by $K = \frac{5216*9.86+4784*5.41}{10000} = 7.73$. Similarly we can get the average number of turn T, then we can simulate to get the refinement value of previous 21 balance settings as shown in Table 3.5.

From Table 3.5 the following remarks are given.

Table 3.5: The results of simulation for $MFG(N, m, s, d)$ and its game refinement measure

Setting	Win for citizens	Average death	Round	Average length	GR
MFG(20,9,4,4)	6635	13.20	7.61	76.09	0.0478
MFG(20,8,1,2)	5782	14.74	7.80	78.03	0.0492
MFG(19,7,1,1)	6532	15.12	7.83	74.40	0.0523
MFG(19,6,1,1)	7798	15.56	8.03	76.24	0.0517
MFG(18,7,1,2)	6347	13.56	7.17	64.55	0.0571
MFG(18,6,1,1)	6541	14.12	7.43	66.88	0.0562
MFG(17,7,1,2)	6938	13.27	6.89	58.58	0.0622
MFG(17,7,1,1)	5128	12.27	6.37	54.15	0.0647
MFG(16,6,1,2)	6980	12.22	6.46	51.72	0.0676
MFG(16,6,1,1)	5241	11.62	6.23	49.84	0.0684
MFG(15,6,1,2)	7548	11.95	6.21	46.61	0.0742
MFG(15,5,1,1)	7575	12.06	6.25	46.90	0.0740
MFG(14,5,1,2)	7612	10.72	5.67	39.66	0.0825
MFG(14,5,1,1)	5838	10.34	5.55	38.87	0.0827
MFG(13,6,1,2)	5117	8.35	4.38	28.45	0.1016
MFG(13,5,1,1)	6540	9.96	5.20	33.80	0.0934
MFG(12,5,1,2)	5292	7.75	4.27	25.59	0.1088
MFG(12,4,1,1)	6554	8.93	4.80	28.79	0.1038
MFG(11,4,1,1)	7269	8.51	4.46	24.54	0.1189
MFG(10,3,1,1)	7464	7.33	3.94	19.71	0.1373
MFG(9,4,2,1)	5424	5.74	3.27	14.69	0.1630
MFG(8,3,1,1)	5393	5.29	3.00	12.01	0.1916

Remark 4 *The measure of game refinement would reduce as the number of players increases. This may imply that Mafia game would become boring as the number of players becomes too large.*

Remark 5 *$MFG(N, m, s, d)$ can be played reasonably with $N \in \{14, 15, 16\}$, $m \in \{5, 6\}$, $s = 1$ and $d \in \{1, 2\}$. In particular, $MFG(15, 5, 1, 1)$ or $MFG(15, 6, 1, 2)$ is the best to play under the assumption that its GR value is within the sophisticated zone 0.07 - 0.08.*

Remark 5 suggests that the most suitable number of players is 15. It is interesting to note that for all the defeat setting in StarCrart II complex Mafia game map, the number of players also equals to 15, which we can see at the bottom right corner in the screenshot (see Figure 3.2). Therefore, the game designers used their experience to set 15 players, which coincides with our research results.

To observe the relation between the performance quality and game characteristics, further simulations are conducted using three different level of AIs: strong, fair and weak.

Figure 3.2: A screenshot of StarCraft II

- The strong AI always follows the Nash equilibrium strategy as described in Algorithm 1.
- In the fair AI, doctor will protect randomly and mafia member will kill randomly. We change the save function in Algorithm 1.
- In the weak AI, doctor will protect randomly, mafia will kill randomly and sheriff will not show their identity. We change the save and discussion function in Algorithm 1.

Then, computer simulations are performed using these different AIs for the two settings: $MFG(15, 5, 1, 1)$ and $MFG(20, 9, 4, 4)$. The results are presented in Table 3.6.

Table 3.6: The results of simulation using different AIs for two settings

Setting	AI level	Win for citizens	Average death	Round	Average length	GR
$MFG(15, 5, 1, 1)$	Strong	7575	12.06	6.25	46.90	0.0740
	Fair	7552	12.11	6.16	46.20	0.0753
	Weak	1781	10.34	7.93	59.49	0.0541
$MFG(20, 9, 4, 4)$	Strong	6635	13.20	7.61	76.09	0.0478
	Fair	4691	11.51	6.29	62.9	0.0554
	Weak	380	8.60	5.79	57.6	0.0509

Remark 6 *The level of players affects the game balancing and game sophistication. For example, mafia would dominate citizens if all the players are weak, in case the game sophistication would be reduced, which implies that game would be boring.*

3.3.3 Mafia Game and SC2Mafia Compared

The analysis of the simple version of Mafia game suggests that the number of players $N \in \{14, 15, 16\}$ will make players feel more enjoyable or comfortable, whereas the number of citizens $N - m \in \{9, 10\}$ will be a balance setting. Interestingly, this observation concerning the game setting can be seen in the SC2Mafia game. A short sketch of the SC2Mafia game features is outlined below [5].

* At the beginning, there are always 15 participants. Then, the host computer player chooses a rule or setting from system, or custom-defined.
* Every player has a "room number", some characters' skills are related to their room numbers, and all behaviours in the night period need to be pointed to a certain number (or a certain player).
* Not only Mafia and citizen group, but also Triad and some other neutral characters exist.
* Every player has a special skill. For example, there is a leader – Godfather – in Mafia group, whom Sheriff cannot investigate whether he belongs to Mafia or not. Moreover, Godfather is invincible at night, which means that he cannot be killed by Citizens or Triad characters' attack. Another example is Mass Murderer – a neutral character, who can perform a killing spree at someone's house every night, killing anyone who visits that player at night. Mass Murderer will win if he is the last player remained alive.

SC2Mafia is too complex and it is related with natural language processing, we cannot use program or coding to simulate all the situations in our research. Therefore, in this subsection, we just focus on a simple research question: How powerful is the Godfather. In order to solve it, we can use a program to simulate the winning ratio as Table 3.7.

Table 3.7: The results of simulation for Godfather case

Setting	Win for citizens	Average death	Average length	GR
Normal(15,5,1,1)	7531	12.05	46.94	0.0739
Normal(15,6,1,1)	5844	11.21	43.83	0.0764
Godfather(15,1+0,1,1)	7687	10.35	40.39	0.0802
Godfather(15,1+1,1,1)	5506	12.35	48.36	0.0727

From Table 3.7 two conclusion could be gotten:

1 Compared with the winning ratio, we believe 1 Godfather has same power as 5 normal Mafia killers. Therefore, while Mafia group has a

Godfather, Citizens group needs to set more characters that has special abilities to keep balance.

2 While the total number of players is decided, changing the setting will not change the interesting level to a large degree if the setting is still in balance.

However, the research of SC2Mafia is not only limited to this issue, in order to analyze the different setting rules in SC2Mafia, we need to collect the human's replay of SC2Mafia and analyze the refinement values. Some classic balance setting rules can be followed as Table 3.8 shows.

Table 3.8: Some setting rules in SC2Mafia game

Name	Setting Rules
Traditional	4 Mafia members 10 normal Citizens (1 doctor+1 Sheriff) 1 Neutral Benign
Little Italy	Godfather+3 Mafia members 9 Specially Skill Citizens (emphasis on Town Protective) 1 Neutral Benign, 1 Neutral Evil
China Town	Dragon Head+3 Triad members 9 Specially Skill Citizens (emphasis on Town Killing) 1 Neutral Benign, 1 Neutral Evil
Red & Blue	Godfather, Dragon Head, 3 Random (Mafia or Triad) 9 Specially Skill Citizens 1 Neutral Benign
Heresy	9 Specially Skill Citizens (1 Mason+1 Mason Leader) 3 Original (1 Witch Doctor+2 Cultist) 1 Neutral Killing, 1 Neutral Evil, 1 Neutral Benign
Blood Rust	Godfather+1 Mafia member 2 Mass Murderers, 1 Neutral Benign 10 Specially Skill Citizens
Family Glory	Godfather+2 Mafia members 1 Neutral Killing, 1 Neutral Evil, 1 Neutral Benign 9 Specially Skill Citizens

Except traditional mode, every player has a specially identity and skill during the game setting. Here we introduce several identities to enable reader to understand the SC2Mafia system well. We need to mention that, the types of character are not only limited as seen in the list, according to the game developed, more and more characters and rules setting will be created [5].

Citizen Group

- Bodyguard. Guard one player each night. If someone attacks a guarded player, both the attacker and the Bodyguard die instead of the guarded player.
- Bus Driver. Swap two players at night, making actions that target the first player instead target the second player.
- Detective. Track one person's activity each night.
- Doctor. Visit someone at night to save them if someone tries to kill them.
- Block someone's role at night, canceling their night abilities.
- Investigator. Check one player each night for that player's criminal record.
- Jailor. Jail and roleblock one player each night following a day where no lynch occurred. Speak anonymously with the jailed player at night, and optionally execute that player.
- Lookout. See everyone who visits his target each night.
- Mason Leader. May to try to convert one person to a Mason each night. This ability only works on Citizens. Also collaborates with the other Masons at night in the secret Mason chat.
- Mayor. May reveal himself during the day and thereafter, have additional votes
- Sheriff. Check one player each night for criminal activity.
- Spy. Hears what people say during chats at night (Mafia, Triad, Cultist, and Masons).
- Veteran. May go on alert during the night. If he goes on alert, will automatically kills any player who targets him that night.
- Vigilante. Kill one target at night.

Mafia/Triad

- Godfather/Dragon Head: The leader of Mafia/Triad. Kill one target at night, investigation immunization, invincible in night.

- Mafioso/Enforcer: The normal killer of Mafia/Triad. Kill one target at night.
- Consigliere/Administrator. Investigate one player each night
- Agent/Vanguard. See who one person visits and is visited by each night.
- Framer/Forger. Frame one player each night.
- Janitor/Incense Master. Sanitize one player each night.
- Disguiser/Informant. Kill a player and steal their identity. Consort/

Neutral

- Amnesiac. Convert to any role from the graveyard.
- Arsonist. Need to be the last person left alive. Either douse a person in gasoline or kill all previously doused targets at night. May also undouse himself by taking no action.
- Cultist. Wins if the Town, Mafia, and all other killing roles are dead or converted to Cultists. Speak together at night with other Cultists and attempt to convert a player to a cultist.
- Executioner. He need live to see his target lynched during the day, invincible in night. While his target was killed in night, Executioner will change to the Jester.
- Jester. Need die by being lynched during the day, the player who vote guilt will suicide in the next night. Annoy another player at night, indicating to that player that he was visited by a Jester.
- Judge. Need Survive and see Town loses the game. May call court during the day, stopping all discussion and forcing an anonymous ballot vote where the Judge has additional votes. Also, can speak to the entire town at night, anonymously.
- Mass Murderer. Need to be the last person left alive. Perform a killing spree at someone's house each night, killing anyone who visits that player at night, invincible in night.
- Serial Killer. Need to be the last person left alive. Kill one target at night, invincible in night.

- Survivor. Survive to the end of the game, regardless of whether or not the Town or Mafia win.
- Witch. Survive and see the town lose the game. Force one person to use their night ability on a target of the Witch's choice.
- Witch Doctor. Save a player, protecting him from death and converting him to the Cult if he was attacked that night. Also, may speak with the Cult at night in the Cultist night chat.

Then, we played the SC2Mafia to collect data. For each setting, we have 10 samples, also the research supported by Python simulation. Table 3.9 show the each parameter in SC2Mafia.

Table 3.9: The results of simulation for Godfather case

Setting	Average death	Average length	GR
Traditional	9.2	41.57	0.0730
Little Italy (15)	11.9	44.96	0.0767
China Town (15)	11.5	45.09	0.0752
Red & Blue (15)	12.6	51.68	0.0688
Heresy (15)	13.2	50.54	0.0719
Blood Rust (15)	8.4	44.27	0.0655
Family Glory (15)	10.7	41.56	0.0787

3.4 Chapter Conclusion

The results of our computer simulations using a simple version of Mafia game show some interesting aspects of Mafia game and suggest the reasonable settings from the perspective of game sophistication, as summarized below. For the citizens group, sheriff is much more important than doctor from the viewpoint of game balancing. For $N = 12$, $MFG(12, m, s, d)$ is the best to play with $m = 5, s = 1, d = 2$ or $m = 4, s = 1, d = 1$ from the viewpoint of game balancing while minimizing the difference of winning ratio between two groups of mafia and citizens. The measure of game refinement would reduce as the number of players increases. This may imply that Mafia game would become boring as the number of players becomes too many. $MFG(N, m, s, d)$ can be played reasonably with $N \in \{14, 15, 16\}$, $m \in \{5, 6\}$, $s = 1$ and $d \in \{1, 2\}$. In particular, $MFG(15, 5, 1, 1)$ or $MFG(15, 6, 1, 2)$ is the best under the assumption that its GR value is within the sophisticated zone. Moreover, the level of players affects the game balancing and game sophistication. For

example, mafia would dominate citizens if all players are weak, which implies that the game sophistication would be reduced.

we also analyze the complicated versions of Mafia game in StarCrart II custom map (SC2Mafia). In SC2Mafia, there are many different balance settings with 15 participates, for each setting we collected human's data and support by Python simulation, then the refinement value of human almost closed to the pure AI simulation, what means while the total number of players was decided, the game exciting level and progress almost similar. Therefore, 15 or 16 participates could be considered as the most interesting setting by reasonable evidence. In the future, the more SC2Mafia related work will be researched, also we will focus on the real AI design in Mafia game.

Chapter 4

Dissonant Chord Separation in Music

4.1 Introduction

In this chapter, I would like to apply scientific method to art field. Music is a good target because it doesn't only related to art, but also have close relationship with mathematics. Many famous musicians are both mathematician, and tried to look at music through the lens of science. However, the reason why art is art, just because its mystery. Other than pure science, how to find suitable parameters in music, and how to qualify related parameters, has still been a problem. Thus, I would like to present a new mathematical model to simulate the progression in music unfolding, and find the factors that effects the attractiveness of music.

4.2 Previous work

Theoretical studies on music is generally called musicology. The general task of musicology is to elucidate the nature and laws among music itself, through various musical phenomena. For example, music aesthetics, music history, music ethnology, music psychology, music pedagogy, are studies on the relationship between music and ideology. There are also studies on musical materials such as acoustics, temperament, instrumentality; on the form and composition of music including melody theory, harmony theory, form theory and other composition techniques; on the aspect of performance, like performance theory, command method and so on.

The most authoritative and systematic analysis method of music is 'The New Grove Dictionary of Music and Musicians' [10] written by British musi-

cians George Grove. He mentioned that the object of music analysis seriously examined, because it can be the music text itself, or the sound effect projected by music, or the inner emotion that the composer at the time of creation, or the performance, or the reaction of listeners.

Music work analysis is the mainstream in the field of art. These works are mainly based on existing musical theory and methods. For example, Tagg.P [34] introduced a method to analyze popular music and give examples on how to practice the method to certain music work besides textbooks, and there are countless studies on a certain piece of music work or a certain composer, about how they create attractive music.

Research on music also covers other disciplines. Music therapy is a hit topic in the field of psychology recent years, referring to 'An Introduction to music therapy: Theory and Practice'. [6] Music therapy is mainly focusing on the effects on human's state of mind and emotion. Meyer focuses on the meaning and emotion in music. [23]

Music is also connected to mathematics and physics and information science. For example, N. Wiener [36] generalized harmony theory by mathematical methods, through the knowledge of acoustics. Or using signal processing to analyze music melody. [25]. Especially, information in music is considered by the way of calculating musical entropy [8], to prove that melodic information can be calculated, although the calculation method is not fixed, say using neural networks [4] or Markov chains [7].

However, there is little research covering both art and scientific. Existing artistic study are mainly focusing on musical theory rather than music nature itself, while scientific method is lack of overall outlook for the artistic work. Hence I try to present a new method to combine both musical theory and scientific model, to analyze the nature of music attractiveness.

4.3 Musical Theories and Musical Structures

Music is closely related to mathematics and physics. It looks like art but full of science. In this section, music theories will help us to analyze the deep structure of music scientifically, and to find out crucial elements in music work. I also come up with a musical progressing model to explore why harmonious music rather inseparable from dissonance.

4.3.1 Music, Melody and Chord

Music, to be broadly spoken, can be any kind of art composed of sound. Different with sound, music needs to be imagined and created through human

mind. Generally, music contains several elements, such as melody, rhythm, and dynamics. [1]

Music has two factors, one of which is time. In a music work, individual notes are organized in time and last for a certain length. While The other factor is memory. The unfolding and reception of a music work relies on audiences' memory. Only by relating what is heard to memory in the mind can audiences get a complete impression of music. [30]

The concept of 'what is heard' is called melody. It is the most important musical element in expression, because it consolidates the contents that a music work presents in your mind. In monophonic (only one musical line) music, melody is the only sound that can be heard, thus it contains all the information of a music work. Even in polyphonic (several musical lines) music, melodic lines can draw more attention of listeners than non-melodic lines [8] . Melody is more than continuous notes. Indeed, it is principle and stand as figures in front of background.

Melody is formed by chord tones and non-chord tones. Compared each melodic note with its simultaneous chord, notes that belongs to the triad structure of the chord are called chord tones, otherwise are non-chord tones. Non-chord tones are almost in weak beats, less prominent than chord tones. The only one type of strong-beat non-chord tone is 'Suspension', which is the note of the previous chord delayed to the next one. Thus, 'Suspension' can be considered as a dependent note and it doesn't affect general expression. Meanwhile, while creating harmony with a piece of given melody, composers usually take chord tones as the framework. Then non-chord tones are added to promote the melodic fluidity and enrich the rhythm combination. In other words, chord closely ties to melodic notes, thereby expresses the unfolding of melody.

In addition, chords also have structural functions. Music is considered a kind of language [7]. According to the theory of musical forms [9], a complete musical piece has structures like paragraphs and sentences in human languages, called sections and phrases. Section is defined as a basic piece with relatively independent musical idea. It lasts a certain length (often 8 bars), and contains no less than two phrases. The smaller level is called passage, and then motif. Motif (also motive) is considered as the smallest structural unit possessing thematic identity [35]. That is to say, musical structures are sections, phrases, passages and motif, from larger to smaller. Certain chords are used as marks at the end of sections and phrases to create a sense of termination, by which sections and phrases are divided.

However, not all phrases can be divided into passages and motif. Instead,

[1] `en.wikipedia.org/wiki/Music`

different chords can distinguish more subtle emotional changes than phrases, since chords are created for emotional expressions and can make melody obtain a very diverse emotional tone and color [30], and changes of chords can show different emotion in the development of music work. Therefore, **chords** are used in this paper, as the smallest structure in a piece of music.

4.3.2 Harmonious Music

Music is a subject related to mathematics and physics. Thus, how to judge music with scientific methods remains curious.

Usually, while listening to a piece of music, the intuitive feeling of audiences is pleasant, or comfortable. According to this judgement, 'sound', especially simultaneous or successive sound, is categorized into **consonance** and **dissonance**. Basically, consonance, which is called harmonious sound in general, is closely connected with pleasure, relax, and acceptability. In contrast, dissonance can bring about nervous, unacceptability, such kind of negative emotion. No matter consonance or dissonance exists between more than two notes, therefore, consonance or dissonance is a crucial criteria in describing intervals or chords.

Although Hindemith mentioned that, there is no absolute boundaries between these two concepts, and the principles have changed through time [12], many scientific explanations with interdisciplinary studies are put forward. The definition of consonance varies in both physical and psychological considerations. [26] Pythagoras proposed the theory of frequency ratios, and mentioned that the simpler the ratio of the vibration frequencies of two notes are, the more consonant the intervals are. Helmholtz proposed voice relevance theory which is the more the overtones overlap, the more consonant they are. [11] Stumpf presented a theory of fusion and suggested that if untrained listeners are more likely to perceive two simultaneous sounds as the same one, these two are more consonant. [31] This theory is more of a qualitative explanation than a quantitative one though.

There is no definite reason why consonance can comfort listeners though, the distinguishment between consonance and dissonance has come up with a principle that while describing intervals, perfect consonance contains perfect fourths and perfect fifths, and imperfect consonance includes major thirds, minor sixths, minor thirds and major sixths, and others are dissonance. Similarly, chords can be also divided into consonant chord and dissonant chord. Only with consonant intervals, chords are called consonant chords. As long as there is dissonant intervals in the internal structures, they are dissonant chord. That is, except major triad and minor triad, other chords are all dissonant chord.

Progressing Direction.jpg

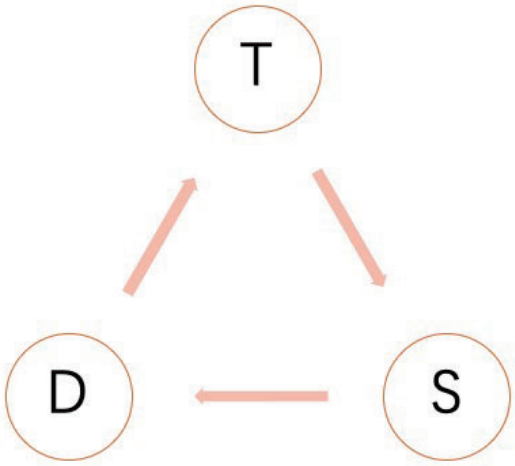

Figure 4.1: Chord Progressing Direction

Objectively, as the explanation of consonance and dissonance, the more the dissonance are, the more uncomfortable the music is. However, judging music is harmonious or not is not only relevant to academic definition, but also subjectively connected with knowledge of music, local culture, and even musical style of a specific period. For instance, consonance in classical music can be regarded as dissonant in atonal music. Even in the case of generally consonant music, dissonance tends to be significant points on pleasure, not unacceptable as defined. Since chords can express the unfolding of the whole melody, to study the separation of dissonant chords will confirm a harmonious range for music, and tell why some songs are popular.

4.3.3 Musical Structures

According to harmony theories [30], chords have a logical progression. In the most popular harmony theory – mode function theory, chords are divided by their functions, such as tonic tried (T), dominant triad (D), and subdominant triad (S). These three chords are called primary triad, identifying the mode in tonic music. Usually, primary triad has a definite progressing direction, which is shown in figure 4.1.

Among them, the tonic triad is the main support, because it can show the required stability of the structural termination and the completeness of the meaning in music.

Progressing Model.jpg

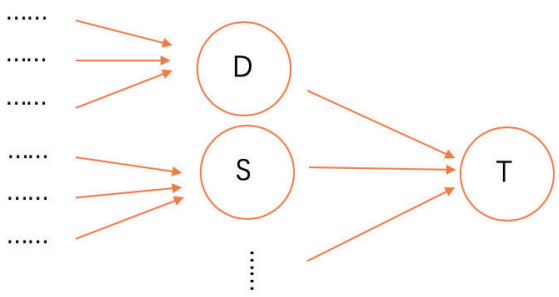

Figure 4.2: Chord Progressing Model

On the basis of the primary triad, other kinds of chords are added to enrich the emotional color. Composers also change the forms of chords to keep movement of melody and rhythm without changing chord positions, to avoid harmonic chaos caused by too much chord changes. But the additional chords and changes are unnecessary, which means chord progression can be flexible during the process. On the other hand, the tonic triad at the structural end never change in mainstream music for its stability.

Thus, the logical progression of chords can be presented as figure4.2.

Primary chords are originally consonant, yet changing forms can lead to dissonance. Additional chords also can be dissonant. Therefore, let D and C be the number of dissonant chord and total number of structure units, here we refer to the number of chords. At the end the chord progress $x(t)$ will be a linear function of time t with $0 \leq t \leq L$ and $0 \leq x(t) \leq K$, in Eq. (4.1).

$$x(t) = \frac{D}{C}\, t \tag{4.1}$$

However, during the music progression, listeners don't know the next chord. Thus, progress given by Eq. (4.1) is not accurate. In reality, the progress is reasonably assumed to be exponential. Hence, the realistic model of chord progress is given by Eq. (4.2).

$$x(t) = D(\frac{t}{C})^n \tag{4.2}$$

Here we can confirm the progress of chord is also like game information progress. $n \in N$ stands for a parameter which is given based on the task's

progress patterns or hardness of completing the task. Larger n corresponds to more hardness of the goal achievement. According to relative calculation, the equation becomes

$$x''(C) = \frac{Dn(n-1)}{C^n} t^{n-2} = \frac{D}{C^2} n(n-1)$$

Similarly, we get the refinement value $GR = \frac{\sqrt{D}}{C}$.

4.4 Data Collection and Analysis

In this section, music samples are selected as objectives to find their crucial factors that are effected by their harmony degree.

4.4.1 Chord Analysis

In this study, Chordify [2] is used to analysis chords in songs. Chordify started from the audio signal of a song. The spectrum diagram gives us better understanding of music content of audio. Then neural network is used to analyse chords. The developers input large numbers of samples to train it and the finished web-page is built.

Chordify takes links from YouTube, Deezer, Soundcloud or private collections from listeners. While inputting a song, Chordify will recognize audio signal automatically and show the chords in a simple player, as figure 4.3

Then I analyzed chords by Chordify for each song and calculate the average value for certain categories, and the result is shown in the next section.

4.4.2 Acceleration of Music

Firstly, Billboard rankings are chosen as the popular songs objectives. Billboard is the most authoritative list in America, held by Billboard-Hollywood Reporter Media Group. It was an entertainment magazine created in 1894, and started the music ranking in 1940s. Therefore, it can shows the development trend of popular music for over 50 years.

Here, I selected Billboard year end top ten songs from every ten years, and calculate the game refinement value for each song. Among them, I also calculated the ratio m for dissonance number out of total chords, in eq. (4.3)

$$m = \frac{D}{C} \tag{4.3}$$

[2] `https://chordify.net`

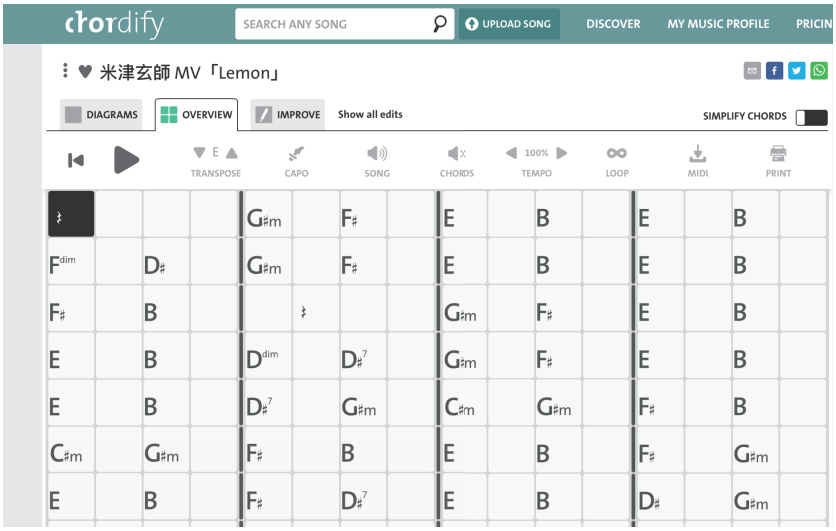

Figure 4.3: Chordify Screenshot

The original result of 90 songs is shown in Table 4.1.

Table 4.1: Billboard Year End Ranking from 1940 to 2019

Year	Songs	C	D	m	GR
2019	OTR	88	44	0.5	0.075377836
	Sun	28	20	0.714285714	0.159719141
	WM	112	98	0.875	0.088388348
	BG	28	0	0	0
	Wow	31	23	0.741935484	0.154704243
	Hap	100	25	0.25	0.05
	7R	52	39	0.75	0.120096115
	Tal	124	88	0.709677419	0.075651867
	SM	56	37	0.660714286	0.108620759
	Suk	171	72	0.421052632	0.049621529
2010	TiT	128	38	0.296875	0.048159484
	NYN	69	30	0.434782609	0.079380081
	HSS	114	14	0.122807018	0.032821556
	CG	217	52	0.239631336	0.033230887
	OMG	99	3	0.03030303	0.017495463
	Airp	98	34	0.346938776	0.059499509
	LTWYL	91	48	0.527472527	0.076134101
	BR	120	0	0	0
	Dyn	170	44	0.258823529	0.039019115
	BYH	145	80	0.551724138	0.061684634
2000	Bre	120	16	0.133333333	0.033333333
	Smo	164	56	0.341463415	0.045629968
	MM	62	8	0.129032258	0.045619792
	IWK	161	122	0.757763975	0.068604727
	EYW	151	93	0.61589404	0.063865237
	SMN	130	50	0.384615385	0.054392829
	INILY	147	42	0.285714286	0.044086671
	Ama	63	50	0.793650794	0.112239172
	Ben	89	35	0.393258427	0.066472807
	HWME	110	10	0.090909091	0.028747979

1990	HO	69	27	0.391304348	0.075306557
	IMHBL	83	9	0.108433735	0.036144578
	NC2U	91	36	0.395604396	0.065934066
	Poi	101	68	0.673267327	0.081645656
	Vog	52	51	0.980769231	0.137335162
	VOL	69	31	0.449275362	0.080692237
	ADIP	115	67	0.582608696	0.071176981
	HoO	28	13	0.464285714	0.128769688
	COL	83	32	0.385542169	0.06815487
	BOG	80	0	0	0
1980	CM	89	16	0.179775281	0.04494382
	ABOTW	54	8	0.148148148	0.05237828
	Mag	114	45	0.394736842	0.058843894
	RWY	95	60	0.631578947	0.081536491
	DTTMOMT	96	11	0.114583333	0.034548175
	CLTCL	97	18	0.18556701	0.043738564
	CU	85	73	0.858823529	0.100517691
	Fun	53	18	0.339622642	0.080049824
	ISRARTM	107	18	0.168224299	0.039650848
	TR	49	0	0	0
1970	BOTW	141	114	0.808510638	0.075723959
	CTY	73	35	0.479452055	0.081042189
	AW	9	2	0.222222222	0.15713484
	RKFOMH	74	56	0.756756757	0.101125875
	War	83	83	1	0.10976426
	ANMHE	188	111	0.590425532	0.056040711
	IBT	113	44	0.389380531	0.058701324
	GR	68	8	0.117647059	0.041594517
	LIB	121	1	0.008264463	0.008264463
	BOG	70	0	0	0
1960	TFASP	127	88	0.692913386	0.073864815
	HHTG	40	40	1	0.158113883
	CC	67	0	0	0
	RB	53	40	0.754716981	0.119331232
	TA	48	6	0.125	0.051031036
	IS	73	20	0.273972603	0.061262136
	INON	54	6	0.111111111	0.045360921
	HM	50	28	0.56	0.105830052
	SOY	44	3	0.068181818	0.039364791
	ThT	37	0	0	0
1950	GI	75	45	0.6	0.089442719
	ML	50	50	1	0.141421356
	TMT	60	24	0.4	0.081649658
	SaS	79	59	0.746835443	0.097229693
	SiM	97	68	0.701030928	0.085012487
	MMM	98	32	0.326530612	0.057723003
	TMT2	40	18	0.45	0.106066017
	CSSB	97	44	0.453608247	0.068384016
	HL	52	27	0.519230769	0.099926008
	IIF	64	26	0.40625	0.07967218
1940	WC	108	62	0.574074074	0.07290748
	TCS	104	101	0.971153846	0.096633419
	GBTC	101	71	0.702970297	0.083427225
	TTAT	83	69	0.831325301	0.100079806
	Sta	56	32	0.571428571	0.101015254
	SOAS	81	58	0.716049383	0.09402189
	YAHTOYL	78	55	0.705128205	0.095079468
	BWBB	51	13	0.254901961	0.070697084
	CCC	98	23	0.234693878	0.048937056
	PD	70	65	0.928571429	0.115175111

From this table, the average length and dissonant chord can be calculated as table4.2.

Table 4.2: Average dissonant chord separation of 1940 to 2019

Years	C	D	m	GR
2019	79.0	44.6	0.565	0.085
2010	125.1	34.3	0.274	0.047
2000	119.7	48.2	0.403	0.058
1990	77.1	33.4	0.433	0.075
1980	83.9	26.7	0.318	0.062
1970	94.0	45.4	0.483	0.072
1960	59.3	23.1	0.390	0.081
1950	71.9	39.3	0.552	0.088
1940	83.0	54.9	0.661	0.089
Average	88.11	38.88	0.4412	0.0708

Table 4.2 tells that, high ranking popular music has an average acceleration value of 0.0708, which is the same degree with sophisticated games. Thus, popular music can be assumed have a common factor in acceleration. Since consonance and dissonance is mainly considered in the mathematical model of music progression, the acceleration can be defined as **harmony value** in music, same with the game refinement value in game theories. Also, harmonious music can bring about similar feeling with sophisticated games, say appropriate excitement during the process.

Therefore, a conjecture can be drawn.

Conjecture 4 *Popular music with common factor in acceleration value (0.07 - 0.08) is harmonious music. It can bring about both appropriate excitement and comfort.*

Also, we can consider the other parameter m. It is defined as the ratio of dissonant chord separated in music. Dissonance means discomfort theoretically, yet certain ratio of dissonant chord can enrich the emotional color in music. Here the average m is resulted in 0.4412, close to half of the total number. It shows that appropriate ratio of dissonance can bring fairness.

On the other hand, regard of both GR value and m ratio, the intersection zone can be shown in figure 4.4.

The average GR value and ratio m of popular music is also within the intersection "Zone" area. This area represents the proper length of total

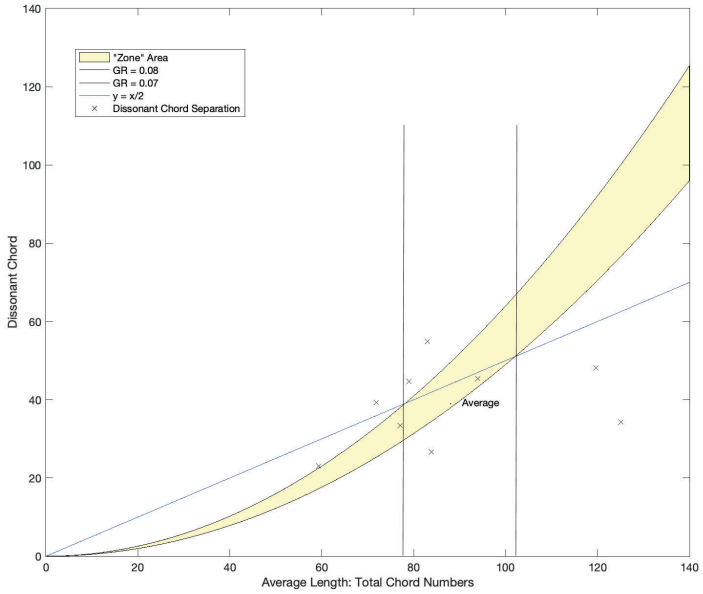

Figure 4.4: Game refinement values of existing studied games belongs to a "Zone" value. GR = 0.07 and 0.08 is the upper and lower bound. Here we consider m= 0.5, for the fairest condition.

progression. With the same dissonant chord number, music lasting longer time means a lower dissonance ratio and lower acceleration. It is considered boring. While too shorter time music contains too many dissonances. This will lead to too much flexibility and sometimes means noisy.

Thus, conjecture can be drawn as below.

Conjecture 5 *Harmonious music with a dissonance ratio close to 0.5 is* ***charm music****. It contains proper length with lingering charm to listeners.*

4.4.3 Music Genre Classification using the method of Dissonant Chords Separation

In this section I selected songs from different music genre, mainly according to Japanese mainstream ranking classification. That is **Japanese popular music** (marked as 'JP Music'), **Out-of-Japan music** (marked as 'OJ Music'), **Enka**, and **Rock**. I chose the list of year 2019 and collected the data from 10 songs each genre (8 songs for Rock). The result is shown as table 4.3, among it, C and D is the average value of each genre.

Table 4.3 suggests that, there is an obvious difference in length, dissonant chord numbers, dissonance ratio and GR value. Thus, it can be confirmed that dissonant chord separation can tell difference between genres.

To combined the average value of each genre with the "Zone" value mentioned above, a classification can be shown in Figure 4.5.

The size of circle for each genre is decided by the standard deviation of the GR value. The bigger the circle is, the more diversity the genre has. Therefore, we can see clearly from the figure that, Japanese music is more similar to each other, while overseas music is more abundant.

Here, I raise 4 conjectures of the classification for music genre, according to the result.

Conjecture 6 *OJ music and Enka are more scientific in harmony and charm than JP music and Rock, because the former ones get closer to "Zone" area, which means they have a proper length and dissonant chord ratio.*

Conjecture 7 *JP music are probably considered simple in most cases. It may be due to the development of idol culture in Japan. Since idol is created with a neighbourhood image, their sound may not be hard to understand.*

Conjecture 8 *JP music and Rock in Japan are similar from harmonious and charm point of view, the difference between these two genres may refer to other parameters such as types of musical instruments or the way of singing.*

Table 4.3: Average dissonant chord separation of different genre

Genre	Songs	C	D	m	GR
JP Music	JWRD	132	22	0.166666667	0.035533453
	KSNI	138	30	0.217391304	0.03969004
	Sus	139	41	0.294964029	0.046065642
	Bra	136	42	0.308823529	0.047652505
	Lig	131	42	0.320610687	0.049471303
	YAM	127	60	0.472440945	0.060991864
	KH	200	66	0.33	0.040620192
	Kyu	204	77	0.37745098	0.043014531
	12345	158	79	0.5	0.056254395
	SO	181	115	0.635359116	0.059247543
	Average of JP Music	154.6	54.7	0.371	0.049
OJ Music	OTR	88	44	0.5	0.075377836
	Sun	28	20	0.714285714	0.159719141
	WM	112	98	0.875	0.088388348
	BG	28	0	0	0
	Wow	31	23	0.741935484	0.154704243
	Hap	100	25	0.25	0.05
	7R	52	39	0.75	0.120096115
	Tal	124	88	0.709677419	0.075651867
	SM	56	37	0.660714286	0.108620759
	Suk	171	72	0.421052632	0.049621529
	Average of OJ Music	79	44.6	0.565	0.085
Enka	KSGRT	87	9	0.103448276	0.034482759
	JHB	120	22	0.183333333	0.039086798
	KSR	97	79	0.81443299	0.09163087
	YNF	60	5	0.083333333	0.0372678
	BKSK	111	49	0.441441441	0.063063063
	OK	79	32	0.405063291	0.07160575
	YRK	77	70	0.909090909	0.108657146
	KNT	83	2	0.024096386	0.017038718
	ONJ	104	1	0.009615385	0.009615385
	Average of Enka	86.2	29.6	0.312	0.060
Rock	Unl	117	42	0.358974359	0.055390946
	GRK	111	3	0.027027027	0.015604061
	HRRY	146	40	0.273972603	0.043318872
	KKNU	189	95	0.502645503	0.05157034
	PJ	190	42	0.221052632	0.034109162
	DKA	106	48	0.452830189	0.065360408
	PRRS	133	88	0.661654135	0.070532568
	Average of Rock	141.7	51.1	0.361	0.050

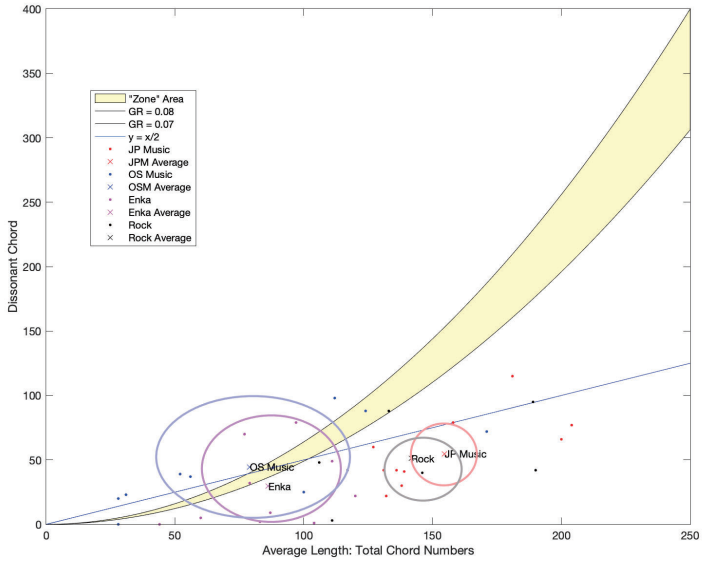

Figure 4.5: Music Genre Separation

Conjecture 9 *OJ music and Enka are unexpectedly similar. Yet Enka is more single. Enka is the transition between traditional Japanese music and popular music, thus, OJ music can be considered have a certain degree of professionalism without the influence by idol culture. Therefore, the gap between popular music and classical music may be smaller than expected.*

4.5 Chapter Conclusion

The progression model of music is built based on musical theory with consideration of consonant chord and dissonant chord. Then results of chord analysis given by Chordify shows a certain trend of dissonant chords separation. The general acceleration which is defined as harmony value is 0.0708, belonging to "Zone" area. This result gives the proof that music is like game from harmonious point of view and music with acceleration in "Zone" value can bring about appropriate excite and comfort. Besides, harmonious music with a dissonance ratio close to 0.5 can be considered charming, because the length is proper to give listeners a feeling of lingering charm. In addition, this method is also used to classify music genre. Four different music genres are distinguished according to Japanese mainstream classification, which are Japanese popular music, Out-of-Japan music, Enka and Rock, and they behave differently in this study. Among them, Out-of-Japan

music and Enka are more scientific in harmony and charm, with a proper length and dissonance ratio, than other two genres. Japanese popular music may be considered simple in most cases for its lengthiness, because of idol culture in Japan. Also, Japanese popular music and Rock are similar from harmonious point of view. They may differ from other parameters such as musical instrumentals or ways of singing. Out-of-Japan music and Enka are unexpectedly similar to each other. Thus, the gap between popular music and classic music may be smaller than considered.

Chapter 5

Conclusion and Discussion

The result of Mafia game and music shows that, popular activities share common factor with traditional games such as Chess, Shogi in board game and Tennis, Soccer in sports game. The acceleration of comfortable popular activities also belongs to "Zone" area (0.07 - 0.08). Thus, it can be proved that the movement of human mind is similar in different kinds of entertainment.

During the research, I'm focusing on the progression analysis. As progress model is decided by outcome and game length, in Mafia game, that is the number of killed players and the average turns, while in music, it becomes the average dissonant chords number and total chords number. Also dissonance ratio is calculated in the case of music. This ratio gives an idea that the existing popular music is mainly in a right length.

The result of acceleration and ratio can be also used in classification. Different settings in games or genres in music have different performances, thus, the result of "Zone" area is used to define comfortable players number setting in Mafia game, and to tell differences between music genres. It is proved that, the exiting version of each activity is suitable in a certain degree.

Although two objectives with totally different characteristics are concerned in this research, there are some limitations. Besides mafia game, other party games with communication can also be taken to consideration. While popular music is mainly considered in this research to analyze music acceleration, since they are more distinct in structure. Yet there is still a lack of data in classical music. With the limitation of Chordify, the chord, or we can call it harmony, in classical music may have other analysis methods. In the future, with more research on all music types, a more complete music map can be drawn.

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