Title	折り畳み可能な単頂点展開図に関する研究
Author(s)	大内，康治
Citation	
Issue Date	2020-03-25
Type	Thesis or Dissertation
Text version	ETD
URL	http://hdl.handle.net/10119/16649
Rights	
Description	Supervisor:上原　隆平，先端科学技術研究科，博士

Abstract

This paper aims to help origami designers by providing methods and knowledge related to a simple origami structure called *flat-foldable single-vertex crease pattern*. A *crease pattern* is the set of all given *creases*. A crease is a line on a sheet of paper, which can be labeled as “mountain” or “valley”. Such labeling is called *mountain-valley assignment*, or MV assignment. *MV-assigned crease pattern* denotes a crease pattern with an MV assignment. A sheet of paper with an MV-assigned crease pattern is *flat-foldable* if it can be transformed from the completely unfolded state into the flat state that all creases are completely folded without penetration. In applications, a material is often desired to be flat-foldable in order to store the material in a compact room. A *single-vertex crease pattern* (SVCP for short) is a crease pattern whose all creases are incident to the center of the sheet of paper. A deep insight of SVCP must contribute to development of both basics and applications of origami because SVCPs are basic units that form an origami structure.

A decision problem whether a given crease pattern is flat-foldable or not was studied by Bern and Hayes in 1996. There are several theorems related to flat-foldable SVCP: for example, the Kawasaki Theorem, the Maekawa Theorem, and the Big-Little-Big Lemma. A *forcing set* is one of the promising properties in origami application. A forcing set is a subset of a given flat-foldable crease pattern C. If the creases in the forcing set are folded according to given MV assignment μ, the all other creases in C are also folded according to μ. In an application called self-folding origami, a thin material folds into an intended shape by rotating the planes around creases according to the label mountain or valley assigned on the creases. The cost of such an application can be reduced if it is enough to put actuators on a subset of creases. Such an optimization problem can be modeled as a *minimum forcing set problem*. The minimum forcing set problem supposes us to find a forcing set with the minimum number of creases. The input of this problem is a flat-foldable MV-assigned crease pattern (C, μ). Damian et al. proposed an algorithm for finding a minimum forcing set for arbitrary 1D origami in 2015. Ballinger et al. developed an algorithm for Miura-ori in 2015. The minimum forcing set for arbitrary 2D origami may be important in origami applications. However, there is no algorithm for such case so far.

In this paper, we propose an algorithm for finding a forcing set of flat-foldable MV-assigned SVCP, which might help us to construct an algorithm for arbitrary 2D origami. Our algorithm, which runs in $O(n^2)$ time where n is the number of given creases, is a variant of the algorithm by Damian et al. Furthermore, we show that the number of creases in the minimum forcing set for SVCP is $n/2$ or $n/2+1$. The proof for the size of minimum forcing set is by considering a situation that we repeatedly crimp consecutive creases forming a minimal angle with different assignments. Roughly speaking, such size is $n/2$ if the number of remaining creases after crimp repetition is two, and otherwise it is $n/2+1$.

It is also interesting to know how many flat-foldable MV-assigned crease patterns there are. In the case of SVCP, the tight upper and lower bounds on such count has been shown by Hull in 2003. However, enumeration of flat-foldable crease patterns has not been studied actively, although it is relative to counting. This paper tackles an efficient enumeration of flat-foldable MV-assigned SVCP. Such enumeration provides us concrete examples of MV-assigned SVCPs, which must be helpful to construct a new origami structure. In this enumeration, let a positive even number q be an input, and let the angle between two adjacent creases be a multiple of unit angle $(360/q)°$. Our algorithm reduces symmetrically duplicate patterns up to rotation and reflection. As far as the author knows, MV-assigned SVCP enumeration introducing the unit angle and reduction of symmetry in this paper is the first trial in the world. The author notes that the problem condition in this paper is different from that for the upper and lower bounds on counting by Hull. Our enumeration algorithm is composed of three phases: (1) enumerate crease patterns of at most q creases satisfying the Kawasaki Theorem; (2) enumerate MV assignments on the crease patterns obtained in (1) satisfying the Maekawa Theorem; (3) test flat foldability of the obtained MV-assigned SVCPs. The phase (1) can be done in parallel to (2) and (3) with master-worker model: the master process computes the phase (1); a worker process computes the phase (2) and (3) for an SVCP given by the master process. In experiment, our algorithm enumerates approximately 4.07×10^{13} flat-foldable MV-assigned SVCPs for $q = 40$ in 34 hours using a supercomputer.

This paper contributes to the development of origami by proposing algorithms for two problems: minimum forcing set for MV-assigned SVCP and enumeration of flat-foldable MV-assigned SVCPs with unit angle. The result of minimum forcing set for MV-assigned SVCP must help investigation of minimum forcing set for arbitrary 2D origami. The enumeration provides us examples of flat-foldable MV-assigned SVCPs and reveals that the number of flat-foldable SVCPs and that of flat-foldable MV-assigned SVCPs are numerous.
Keywords: computational origami, flat foldability, forcing set, enumeration