Title	折り畳み可能な単頂点展開図に関する研究
Author(s)	大内，康治
Citation	
Issue Date	2020-03-25
Type	Thesis or Dissertation
Text version	ETD
URL	http://hdl.handle.net/10119/16649
Rights	
Description	Supervisor:上原　隆平，先端科学技術研究科，博士

氏名	大内　康治		
学位の種類	博士(情報科学)		
学位記番号	博情第428号		
学位授与年月日	令和2年3月25日		
論文題目	Research on Flat-Foldable Single-Vertex Crease Patterns		
論文審査委員	主査　上原隆平	北陸先端科学技術大学院大学	教授
	金子峰雄	同	教授
	井口　寧	同	教授
	池田　心	同	准教授
	堀山貴史	北海道大学	教授

論文の内容の要旨

This paper aims to help origami designers by providing methods and knowledge related to a simple origami structure called *flat-foldable single-vertex crease pattern*. A *crease pattern* is the set of all given *creases*. A crease is a line on a sheet of paper, which can be labeled as "mountain" or "valley". Such labeling is called *mountain-valley assignment*, or MV assignment. *MV-assigned crease pattern* denotes a crease pattern with an MV assignment. A sheet of paper with an MV-assigned crease pattern is *flat-foldable* if it can be transformed from the completely unfolded state into the flat state that all creases are completely folded without penetration. In applications, a material is often desired to be flat-foldable in order to store the material in a compact room. A *single-vertex crease pattern* (SVCP for short) is a crease pattern whose all creases are incident to the center of the sheet of paper. A deep insight of SVCP must contribute to development of both basics and applications of origami because SVCPs are basic units that form an origami structure.

A decision problem whether a given crease pattern is flat-foldable or not was studied by Bern and Hayes in 1996. There are several theorems related to flat-foldable SVCP: for example, the Kawasaki Theorem, the Maekawa Theorem, and the Big-Little-Big Lemma. A *forcing set* is one of the promising properties in origami application. A forcing set is a subset of a given flat-foldable crease pattern C. If the creases in the forcing set are folded according to given MV assignment μ, the all other creases in C are also folded according to μ. In an application called self-folding origami, a thin material folds into an intended shape by rotating the planes around creases according to the label mountain or valley assigned on the creases. The cost of such an application can be reduced if it is enough to put actuators on a subset of creases. Such an optimization problem can be modeled as a *minimum forcing set problem*. The minimum forcing set problem supposes us to find a forcing set with the minimum number of creases. The input of this problem is a flat-foldable MV-assigned crease pattern (C,μ). Damian et al. proposed an algorithm for finding a minimum forcing set for arbitrary 1D origami in 2015. Ballinger et al. developed an algorithm for Miura-ori in 2015. The minimum forcing set for arbitrary 2D origami may be important in origami applications. However, there is no algorithm for such case so far.

In this paper, we propose an algorithm for finding a forcing set of flat-foldable MV-assigned SVCP, which might help us to construct an algorithm for arbitrary 2D origami. Our algorithm, which runs in $O(n^2)$ time where n is the number of given creases, is a variant of the algorithm by Damian et al. Furthermore, we show that the number of creases in the minimum forcing set for SVCP is $n/2$ or $n/2+1$. The proof for the size of minimum forcing set is by considering a situation that we repeatedly crimp consecutive creases forming a minimal angle with different assignments. Roughly speaking, such size is $n/2$ if the number of remaining creases after crimp repetition is two, and otherwise it is $n/2+1$.

It is also interesting to know how many flat-foldable MV-assigned crease patterns there are. In the case of SVCP, the tight upper and lower bounds on such count has been shown by Hull in 2003. However, enumeration of flat-foldable crease patterns has not been studied actively, although it is relative to counting. This paper tackles an efficient enumeration of flat-foldable MV-assigned SVCP. Such enumeration provides us concrete examples of MV-assigned SVCPs, which must be helpful to construct a new origami structure. In this enumeration, let a positive even number q be an input, and let the angle between two adjacent creases be a multiple of unit angle $(360/q)°$. Our algorithm reduces symmetrically duplicate patterns up to rotation and reflection. As far as the author knows, MV-assigned SVCP enumeration introducing the unit angle and reduction of symmetry in this paper is the first trial in the world. The author notes that the problem condition in this paper is different from that for the upper and lower bounds on counting by Hull. Our enumeration algorithm is composed of three phases: (1) enumerate crease patterns of at most q creases satisfying the Kawasaki Theorem; (2) enumerate MV assignments on the crease patterns obtained in (1) satisfying the Maekawa Theorem; (3) test flat foldability of the obtained MV-assigned SVCPs. The phase (1) can be done in parallel to (2) and (3) with master-worker model: the master process computes the phase (1); a worker process computes the phase (2) and (3) for an SVCP given by the master process. In experiment, our algorithm enumerates approximately 4.07×10^{13} flat-foldable MV-assigned SVCPs for $q = 40$ in 34 hours using a supercomputer.

This paper contributes to the development of origami by proposing algorithms for two problems: minimum forcing set for MV-assigned SVCP and enumeration of flat-foldable MV-assigned SVCPs with unit angle. The result of minimum forcing set for MV-assigned SVCP must help investigation of minimum forcing set for arbitrary 2D origami. The enumeration provides us examples of flat-foldable MV-assigned SVCPs and reveals that the number of flat-foldable SVCPs and that of flat-foldable MV-assigned SVCPs are numerous.

Keywords: computational origami, flat foldability, forcing set, enumeration

論文審査の結果の要旨

本研究テーマは計算幾何学の中でも比較的新しい計算折り紙についての研究である．具体的には，紙の中心から放射状に折り線が与えられたモデルについての研究を行った．研究成果は主に二つあり，一つ目は強制集合と呼ばれる概念の研究であり，もう一つは折りたたみの数え上げに関する研究である．

まず強制集合について概観する．折り紙では，与えられた折り目に山折りと谷折りを割り当てるが，このとき一般に，一部の折り目に山折りと谷折りを割り当ててしまえば，残りの折り目の山谷は強制的に決まってしまう．こうした折り目の集合の大きさは，折り目に依存してきまる．本研究では，紙の中心から放射状に折り線が n 本出ているとき，この強制集合の大きさが n/2 か n/2+1 のどちらかになることを示し，さらに具体的な強制集合を求める効率の良いアルゴリズムを示した．強制集合については，これまで 1 次元の場合（つまり直線を折る場合）と，三浦折りと呼ばれる限定的なパターンについてしか知られていなかった．

次に折りたたみの数え上げについて概観する．本テーマでは，中心から n 本等間隔で放射状に出ている折り線に，山折り，谷折り，さらには「折らない」という選択肢を与えて，

平坦に折りたたみ可能なパターンを全列挙するというテーマである．これは実際には「1次元バージョンですべて折る」という最も単純な場合ですら「切手折り問題」と呼ばれる難問であり，離散数学の分野では長い研究の歴史がある．本研究の場合は，川崎定理，前川定理と呼ばれる折り紙の分野では有名な定理と，いくつかの組合せ論の結果を用いて対称性を排除し，最後はスーパーコンピュータですべて効率良く列挙するアルゴリズムを開発した．その結果，n=40までの数え上げに成功した．

これらの研究成果は，計算折り紙分野においては，基礎研究を発展させる新規の結果であり，オリジナリティが高い．情報科学の観点から言えば，前者は非自明な数学的特徴づけと，効率的な線形時間アルゴリズムを与えており，後者は様々な組合せ論の結果を用いて高度なアルゴリズムを構築し，それを実際にスーパーコンピュータに実装し，並列化も駆使して，指数時間アルゴリズムであるにも関わらず，十分大きな値の n に対して，現実的な時間で解を与えた意義は大きい．また特に日本では近年，列挙アルゴリズムの研究開発が活発に行われているが，その文脈においても，大きな貢献である．以上，本論文は，計算折り紙の単頂点のパターンについて，理論的にも工学的にも新たな研究成果を示したものであり，学術的に貢献するところが大きい．よって博士（情報科学）の学位論文として十分価値あるものと認めた．