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Research on the Minimum Moves of Rolling Cube Puzzles

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Rolling cube puzzles are a puzzle game that rolls the die which is placed on a board to complete a goal followed certain rules. A universal goal is to roll the die which is at the upper left corner of the board over all labeled cells of the board exactly once. When the die rolls over a labeled cell, the number on the top face of the die must be the same as the number of the cell on which it lies.

Rolling cube puzzles have a long history. Rolling cube puzzles were popularized by Martin Gardner in his mathematical games columns, which is published in *Scientific American*. In 2007, Kevin Buchin et al. researched the rolling cube puzzles and proved that solving rolling cube puzzles is NP-complete where rolling cube puzzles are defined by a six-sided die on a square grid board. However, there are several related puzzles and open problems posed in the paper by Kevin Buchin et al.

In this research, we focus on one of these open problems, that is, the problem to find the minimum moves of solving rolling cube puzzles when the initial state and the final state of the die are given. In order to get some properties of this problem, we refine this problem and introduce new motions of this problem. We only discuss the situation that the initial state of the die and the final state are given and the die can be rolled freely on a board without inaccessible cells. Then, we divide this problem into two cases. One is that the die can roll along a path, whose length is equal to Manhattan distance without any detours. The other is that the die can roll in all four directions on an unlimited board. By the experiment on the first case, we found the area, that is we can roll a die to find all the possible states on a cell only along a path of length equal to Manhattan distance. We also found the optimal number of the minimum moves to find all the possible states on a cell in the other area.

According to these results of the experiments, we give an algorithm for finding a general solution for the problem and analysis the complexity of this algorithm. In the algorithm, we divide the problem into four subproblems. The first one asks whether the path from the initial state to the final state exists or not. The second one asks if a die can reach to all the final states from the initial state along a path of length equal to the Manhattan distance or not. Thirdly, the algorithm determines if a die can reach the final state with a detour of length not greater than 2 addition to its Manhattan distance. The complexity of this part will take $O(4^{d(T)})$ time, where $d(T)$ is the depth of the BFS tree T, because we need to explore the four moving directions of the die. However, we know the upper bound of the maximum steps of a detour to find all the possible states on a cell. That means we only need to explore the area along the paths of the length of the Manhattan distance with the maximum detour. Furthermore, if the algorithm records all the states of the die after it has rolled and the cell of these states, the running time is drastically reduced in the manner of dynamic programming. Meanwhile, we verify that the new state does not duplicate in the table. After these optimizations, the complexity of this part will be reduced to $O(D_m^2 \times d(T))$ time, where D_m is the Manhattan distance between the initial cell and the goal cell. Finally, we only have the situation left, that is the minimum moves are equal to Manhattan distance plus 4. For this last subproblem, we transform the goal to find the corresponding state on the cell next to the initial cell. Follow these steps, we can get a general solution to find a shortest path between the starting point and the ending point when the initial state of the die and the final state are given and the die can be rolled freely on a board without inaccessible cells.