Reconfiguration Problems of Shifting Tokens on Graphs

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Reconfiguration problems are computational problems that explore connectivity between two feasible solutions or arrangements of a given problem. The general questions that are considered in reconfiguration problems are: can any arrangement be reconfigured to any other (connectivity); what is the worst-case number of steps required (diameter); and what is the complexity of computing the minimum number of steps required to get from one given configuration to another given configuration (distance).

The famous 15-puzzle inspired the token reconfiguration problem which is a reconfiguration problem on a graph with initial and target arrangments of tokens. Different versions of token reconfiguration problems on various graph structures have been studied widely in computer science. Token Swapping Problem was introduced by Yamanaka et al. . It was shown that 2-Colored Token Swapping can be solved in polynomial time and 3-Colored Token Swapping is NP-complete even for planar bipartite graphs of maximum degree 3. It also showed that c-Colored Token Swapping is $O(n^{c+2})$-time solvable for graphs of maximum degree at most 2, and c-Colored Token Swapping is fixed-parameter tractable for complete graphs if c is the parameter. In Sequentially Swapping Colored Tokens on graphs, inapproximability of the Sequential Token Swapping problem was demonstrated and the positive results for trees, complete graphs, and cycles were presented.

Token Shifting Problem was introduced by Sai et al. and it was shown that the Labeled Token Shifting Problem is solvable in polynomial time on a large class of graphs while solving the k-Colored Token Shifting Problem in the minimum number of moves is NP-hard, even for $k = 2$.

In this thesis, we investigate a new variation of a token reconfiguration problem on graphs using the cyclic shift operation. A colored or labeled token is placed on each vertex of a given graph, and a "move" consists of choosing a cycle in the graph and shifting tokens by one position along its edges. Given a target arrangement of tokens on the graph, our goal is to find the shortest sequence of moves that will re-arrange the tokens as in the target arrangement. The novelty of our model is that tokens are allowed to shift along any cycle in the graph, as opposed to a given subset of its cycles. We focus our investigation on graph classes with high connectivity so as to reflect the potential applications in areas such as logistics and telecommunications.

We first present the efficient algorithms for solving the token shifting problem on special graph classes and then give the hardness result of the problem.

- We give linear-time algorithms for the shortest shift sequence for both the 2-Colored and the Labeled Token Shifting Problem for complete graphs.

- We also show that the shortest shift sequence for the Labeled Token Shifting Problem on standard barbell graphs, and then on generalized barbell graphs with more than one connecting edge can be constructed in O(n) time.

- We then describe the procedure of solving the 2-Colored Token Shifting Problem for block graphs in $O(n^2)$ time.

- Finally, we prove that, in the 2-Colored Token Shifting Problem, the shortest sequence of moves is NP-hard to approximate within a factor of $1/2+\varepsilon$, even for planar graphs with a maximum degree of 4 by reduction from the NP-Complete problem of Hamiltonian Cycle on Grid Graph.