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An algebraic approach to logics over $\mathbf{FL_e}$

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Substructural logics are obtained from sequent calculus **LJ** for intuitionistic logic by deleting all or some structural rule (exchange, weakening, contraction). A basic substructural logic is called full Lambek calculus (**FL** which is obtained from **LJ** by deleting all structural rules. We call any extension of FL a substructural logic. A logic **FL** with exchange rule is called intuitionistic linear logic (which we denote FLe). Other examples of substructural logics are relevant logic, BCK-logic, and so on. Each logic mentioned above have been studied with own each motivations and interests. Recently, we can find that they are studied in the framework of substructural logics.

Early study of substructural logics is mainly done by using proof-theoretical methods. We can get many interesting results from cut-free sequent systems. But, proof-theoretical methods don't work well when cut-elimination theorem fails to hold for a given sequent system. So, we need other approaches to substructural logics. In this paper, we use algebraic methods to investigate properties for substructural logics. It is known that universal algebra is quite useful in getting general results on substructural logics.

Algebraic strucutres of substructural logics are residuated lattices, that can be defined as follows. An algebraic structure $\mathbf{A} = \langle A, \cap, \cup, \cdot, \backslash, /, 1\rangle$ is called a residuated lattice, if it satisfies the following conditions:

(1) $\langle A, \cap, \cup\rangle$ is a lattice,

(2) $\langle A, \cdot, 1\rangle$ is a monoid,

(3) for any $x, y, z \in A$, $x \cdot y \leq \Leftrightarrow y \leq x\backslash z \Leftrightarrow x \leq z/y$.

Operations $\backslash$ and $/$ are called *left* and *right* residuation, respectively. If we assume the commutativity of the monoid operation $\cdot$, then these two residuations become identical and hence the algebra $\mathbf{A}$ becomes a commutative residuated lattice. In this paper, we treat the logics over FLe. Therefore, we consider only commutative residuated lattices as algebraic structures.

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The purpose of this paper is to investigate properties of substructural logics by using algebraic methods. We study substructural logics with exchange rule. In other word, we study extensions of intuitionistic linear logic. We focus two topics. One is the finite model property for DFLe, DFLew, and DFLec. They are logics obtained from FLe, FLew, and FLec, respectively, by adding distributive law. We say that a logic has the *finite model property* if every formula that fails to hold in some model of the logic can be refuted also in a finite model of the logic.

We consider formulas without a logical connective *vee*. Then we can prove the following: For any $\vee$-free formula ϕ,

1. ϕ is provable in $\mathbf{FL_e} \Longleftrightarrow \phi$ is provable in $\mathbf{DFL_e}$
2. ϕ is provable in $\mathbf{FL_{ew}} \Longleftrightarrow \phi$ is provable in $\mathbf{DFL_{ew}}$
3. ϕ is provable in $\mathbf{FL_{ec}} \Longleftrightarrow \phi$ is provable in $\mathbf{DFL_{ec}}$

Using these facts, we can get the following:

1. If a $\vee$-free formula ϕ is not provable in $\mathbf{DFL_e}$, then there exists a finite $\mathbf{DFL_e}$-algebra such that ϕ does not hold.

2. If a $\vee$-free formula ϕ is not provable in $\mathbf{DFL_{ew}}$, then there exists a finite $\mathbf{DFL_{ew}}$-algebra such that ϕ does not hold.

3. If a $\vee$-free formula ϕ is not provable in $\mathbf{DFL_{ec}}$, then there exists a finite $\mathbf{DFL_{ec}}$-algebra such that ϕ does not hold.

Next we discuss Glivenko's theorem relative to logics over CFLe. To show this, we apply a closure operator. In 1929,V.Glivenko shows that classical propositional logic can be interpreted in intuitionistic propositional logic. More precisely,

> For any formula ϕ, ϕ is provable in classical propositional logic **CL** if and only if $\sim\sim \phi$ is provable in intuitionistic propositional logic **INT**.

We say that for logics **L** and **K**, Glivenko's theorem holds for **L** relative to **K**, whenever for any formula ϕ, $\sim\sim \phi$ is provable in **L** if and only if provable in **K**.

We show that the Glivenko's theorem hold between logics over CFLe and below CFLe by using closure operator defined following: 'h' is a *closure operator* if it satisfies follows,

(c1) $x \leq h(x)$,

(c2) $h(h(x)) \leq h(x)$,

(c3) if $x \leq y$ then $h(x) \leq h(y)$,

(c4) $h(x) \cdot h(y) \leq h(x \cdot y)$.

Let **K** be an involutive logic, then it can be represented, $\mathbf{K} = \mathbf{FL_e} + (\neg\neg\alpha \supset \alpha) + \{\beta_i\}_{i\in I}$ Now, we can construct G(**K**) as follows, $G(\mathbf{K}) = \mathbf{FL_e} + (\dagger 1) + (\dagger 2) + \{\neg\neg\beta_i\}_{i\in I}$. Then, we prove that G(**K**) is the minimum logic which holds Glivenko's theorem relative to**K**.