An Order-sorted Logic with the many nagations in the Intuitionistic logic

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In this paper, we consider the system of the order-sorted logic that includes multiple negations in the intuitionistic logic which represents human knowledge representation.

The order sort logic is able to represent the hierarchy of the concept, called the sort hierarchy. If we consider the logic fomula is the formalized human thinking, the sort hierarchy is very important for us logically to deal with the natural language lexicon, because the human beings understand the meaning of an lexical item relative to another lexical item in the sort hierarchy. For example, when we understand the meaning of the 'white', the meaning is decided by the meaning of 'color' which is more general than 'white' in the sort hierarchy. As the basis, we can express that the meaning of white is represented in the color of reflecting all of the visible rays. Hence, the order sort logic is very useful tool for the problems of how to represent the knowledge and how to deal with the meaning of the natural language lexicon logically. Furthermore, the inference rules of the logic system which based on the deduction was defined. Therefore, the order-sorted logic is widely used for the representation of knowledge in the

discipline of Artificial Intelligence. Some representations of the negations are introduced in the order-sorted logic by Kaneiwa, Tojo[2002]. That is the strong negation, the weak negation and the negation of the lexical opposition.

The negation is very important factorin logics, because the negation is the factor causes the logical contradiction. The examples of such sentences are "I do not say that I do not like it", "I do not hate it though not like", and "This is insufficient". There are cases of the sentential negation('- not -'), the negation of lexicalized polar contraries('like','hate') and the negation by an affix('insufficient'). In this paper, the sentential negation are called the weak negation, the negation of lexicalized polar contraries is called the negation of the lexical opposition and the negation by an affix is called the strong negation.

Now, we consider one example about the weak negation. If we represent 'like' for the proposition 'A', we can express that example sentence to '$\neg\neg A$' for the formula. But, the formula '$\neg\neg A$' logically corresponds to 'A' in the classical logic because of the rule of elimination of the double negation. However, When we use a sentence in the double negation, we want to express the case in the positive, but we want to negate the negative of the sentence. Therefore, we are not able to represent the human mind in the classical logic, and we do not success the correct representation. Therefore, if a certain proposition exists, we can decide true or false about the proposition in any situation on the classical logic. But, the human beings cannot decide true or false for the entire proposition. Hence, the representation of knowledge in the classical logic is not adequate way to represent our usage of natural language.

However, that order-sorted logic is based on the system of negation in the classical logic. Therefore, the present order-sorted logic cannot lead to adequate representation for handling human thought. Hence, in this paper, we reconsider the system of negation in the intuitionistic logic and expand the representation.

The semantics of the intuitionistic logic is constructed by multiple possible worlds. That semantics model is called Kripke model. Therefore, the intuitionistic logic can represent the state of the knowledge of the human mind.

In according to that reason, the interpretation is different from the classical logic. The truth of a proposition corresponds to the proposition is correct in the classical logic. But, we use another way of the interpretation which is known as the proof interpretation in the intuitionistic logic. The truth of a proposition in the proof interpretation corresponds to the fact that we have the way to prove that the proposition, otherwise the case is false. Then, a negative proposition in the proof interpretation implies that if we assume the proposition is true then we can get the proof which yields to the contradiction. Hence, since we cannot have a proof for a proposition, the situation exists that neither the positive proposition nor the negative proposition are proved .

In classical logic, there is the principle of the excluded middle '$A \vee \neg A$'. The principle is not tautology in the intuitionistic logic. Hence, this principle is interpreted into "we can have the concrete proof that A is true, otherwise if we assume that A is true then we can get the concrete proof which yields to the contradiction". Therefore, in the intuitionistic logic we can not decide the truth of formula until we get the proof A or $\neg A$. In the other word, we cannot always decide wether we say "Yes" or "No".

Thus, It is to use intuitionistic logic that we depend on only the thing which we know in the state. Hence, this way is along with the human thought. That is the main reason why we use the system of negation of the intuitionistic logic in this paper.

In this manner, when we introduce the system of negation of the intuitionistic logic into the order-sorted logic, naturally that causes a transition in the logic system. Particularly, the weak negation sort which represents the weak negation (expressed by using $\neg$ operator) is directly affected by that. First, the double negation sort did not exist in the conventional order-sorted logic. Furthermore, as the rule of the double negation elimination dose not exist, the double weak negation sort is also introduced. Therefore the order sorted logic in this paper has the single weak negation sort and the double weak negation sort. Consequently we can express it in the order-sorted logic, for example, that purple is

not red, nevertheless we do not say that purple is not red. This is formalized as follows:$\neg(purple \supset red) \wedge (purple \supset \neg\neg red)$

The inference rule of Hilbert style is introduced into the order-sorted logic on the present order-sorted logic. But, Hilbert style is incompatible with the implementation on the computer system. Therefore, we introduc the sequent calculus which is compatible with the machine, and implement the system into the computer.