Master's Thesis

LOSSY DISTRIBUTED MULTI-TERMINAL SOURCE CODING FOR END-TO-END COMMUNICATION SYSTEMS

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Abstract

The Gaussian CEO problem is a crucial joint-source coding (JSC) problem in distributed source coding (DSC) category. Although it has been studied for over two decades, researchers are still keen to make improvements. This research aims at solving the Gaussian CEO problem by combing Wyner-Ziv coding and convolutional lattice codes. The Wyner-Ziv coding scheme aims at compressing the source with the help of side information. In addition, the side information can be easily determined by using the cosets of the lattices. There are several classic lattices, such as the E_8 lattices and Barnes-Wall lattices, which are already known to researchers. However, such lattices have very strict structures, setting obstacles for researchers to use. Moreover, there always exists a huge gap between the Normalized Second Moment (error-correcting ability) and the theoretical bound. Therefore, convolutional code lattices are proposed, providing better Normalized Second Moment and flexibility for structures compared to classic lattices. However, there is not much research on combing them. Thus this work combines the Wyner-Ziv coding and convolutional code lattices to solve the Gaussian CEO problem. The simulation results show that convolutional lattice codes outperform classic lattice codes for the considered model.

Keywords: the Gaussian CEO problem, convolutional lattice code, rate-distortion function, Wyner-Ziv coding

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Chapter 1
Introduction

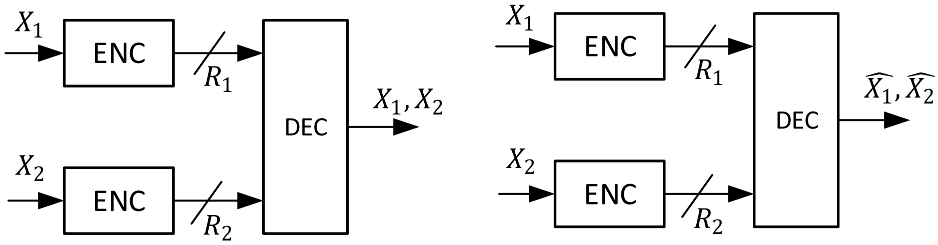
Wireless sensor networks (WSN) are network systems deployed in a large number of wireless sensors that monitor and record the physical and environmental conditions and then forward the collected data to a central location. WSNs rely on wireless connectivity and the spontaneous formation of networks so that data can be transmitted wirelessly. There are numerous WSN applications, and sometimes they have to be set in unpredictable environments to perform various tasks such as battlefield surveillance. Therefore, it is essential to design an efficient system for reliable transmission. The information gathered by different sensors is often correlated. Hence, exploiting correlated information is vital to reduce transmission power and decrease transmission rate. Distributed source coding (DSC) problem is one of the critical problems in information theory and communication. The DSC problem regards the compression of multiple correlated information sources that do not communicate with each other [1]. DSC is able to shift the computational complexity from the encoder side to the decoder side [2], which provides appropriate frameworks with complexity-constrained senders, such as sensor networks and video compression.

This chapter introduces the background of DSC problems and recent development results. The motivation of this research is introduced with a literature view of recent work. The outline of the thesis is shown at the end of this chapter.

1.1 Background

The core problem of DSC is to decide the tradeoff between the encoder rates and the accuracy of the recovered correlated sources. To solve this problem, it is critical to begin with rate-distortion analysis in information theory since it shows the criteria for how to design a communication system.

In general, the DSC problem can be divided into two categories: one is lossless DSC, and the other is lossy DSC. The difference between these two problems is that lossless DSC requires the distortion should be arbitrarily small while lossy DSC allows the distortion can be a certain value with a specified distortion measure function. The block diagram of the two problems is shown in Fig. 1 with two independent sources X_1 and X_2.



(a) Lossless case (b) Lossy case

Figure 1.1: Two cases of distributed source coding

1.1.1. Lossless Distributed Source Coding

In 1973, David Slepian and Jack Keil Wolf started the pioneering work by proposing the information-theoretical lossless compression bound on distributed compression of two correlated independent and identically distributed (i.i.d) sources X_1 and X_2 which are shown in Fig 1.1 (a) [3]. The region of the compression rate pair is determined for which two sources can be recovered with an arbitrarily small error probability. According to the Slepian-Wolf theorem, two correlated sources X_1 and X_2 can be losslessly recovered with joint decoding, as long as the compression rate of each source is larger than their conditional entropy for each and their joint entropy.

After the introduction of the Slepian-Wolf theorem, Thomas M. Cover extended to cases with more than two cases in 1975 [4]. Ahlwede and Korner focused on recovering one of the correlated sources. This problem was then referred to as source coding with side information[5]. Wyner independently studied the source coding with side information, where the joint decoder reconstructs two sources with side information[6]. Korner and Marton studied a specific problem where a primary source is the exclusive-OR (XOR) version of two helper information in [7]. Gel'fand and Pinsker [8] studied the rate region for reproducing an underlying source via corrupted observations of the source based on the assumption that the observations are conditionally independent. This problem is named as lossless CEO problem.

1.1.2. Lossy Distributed Source Coding

As shown in Fig 1.1 (b), another case of the DSC problem defines that the decoder needs to recover the sources at certain distortions with some functions, such as Hamming distortion.

Wyner and Ziv gave the solution for a source coding problem with side information only available at the decoder [9]. The rate-distortion function was induced, and the required transmission rate for this coding scheme is that the rate must be larger than the side information. The difference between the Wyner-Ziv coding problem and the Slepian-Wolf coding problem is that the knowledge of side information at the encoder does not reduce the rate. Afterwards, Wyner extended the Wyner-Ziv coding problem to non-discrete sources [10]. Henceforward, the Wyner-Ziv coding was intensively researched with applications to wireless video networks such as [11][12][13][14] and the wireless relaying network such as [15][16][17].

For the multi-terminal source coding problem, Berger and Tung first began their work by characterizing the bounds of the rate-distortion region [18], where correlated sources are encoded seperately and decoded together. Oohama derived the rate-distortion theory for multi-terminal source coding with correlated Gaussian sources and squared distortion afterward [19]. He gave a bound on the rate-distortion region and proved direct coding theorem using random coding arguments which first proposed by Berger. After that, Oohama [20] studied the multi-terminal Gaussian source coding problem, where multiple correlated Gaussian sources are encoded distributedly, one of which is the decoder's source of interest.

The chief executive officer (CEO) problem, which is a special case of multi-terminal source coding and has been attracting attention for two decades. T. Berger first gave the name of the CEO problem in [21]. The CEO problem can be given as: A CEO is interested in an underlying source that can not be observed directly. So, he hires several agents to observe the source and report to him from their point of view. Unfortunately, each agent suffers from unpleasant noise and then encodes the noisy source information independently under a sum rate R. Finally, the CEO gather all the encoded information and make an estimate of the underlying source as accurately as possible. In general, the CEO problem is to determine the tradeoff between the sum rate and the distortion. Berger *et al* studied the distortion function for the quadratic Gaussian CEO problem, where the source and observations are Gaussian distributed[22]. Oohama then derived an explicit form of the rate-distortion function for the quadratic Gaussian CEO problem by using the conditionally independent property[23].

1.2 Motivation

The CEO problem is still drawing attention nowadays, especially not only from the information theory community but also from industry, especially with the aim of creation of Internet-of-Things (IoT) applications, because IoT networks do not have to recover the observed information losslessly but may make accurate decisions based on observations. In this case, communications over multiple links should not necessarily be lossless, and the received data can be lossy so far as accurate decisions can be made. Nevertheless, in sensor networks, the observation in many cases is already noise-corrupted.

Numerous works have been proposed. For example, side information coding strategies based on Wyner-Ziv coding were proposed for a tree-structured sensor network [24]. A successive Wyner-Ziv coding strategy was applied to the quadratic Gaussian CEO problem to achieve the bound in the rate region [25]. A joint source-channel coding exploiting lattices was further considered for a Gaussian source with multiple sensors [27]. By using lattices, higher channel space dimensions were utilized to achieve the bound asymptotically. An application of successive Wyner-Ziv using Low-Density Generator-Matrix (LDGM) codes for binary quantization while Low-Density Parity-Check (LDPC) codes for syndrome generation indicate that the rate-distortion performance of the proposed scheme can approach the theoretical inner bound [27].

By studying the literature on the state-of-the-art techniques for the CEO problem, most previous work focused on the binary CEO problem, even [26] proposed a practical encoding/decoding strategy by adopting lattice code, but only classical lattices like E8, Barnes-Wall lattices. It is of great importance to consider some new lattices like Low-Density lattices code (LDLC) and Convolutional Lattice code (CLC) which are more effective in error-correcting ability to achieve better performance.

What is more, IoT, the sensors' observations are often used to make an accurate decision rather than to achieve lossless reconstruction of the source information. According to this concept, many new applications are possible, such as autonomous car driving or factory automation. For the purpose of achieving greater signal to noise ratio (SNR) gain in the joint source-channel coding system, a new coding scheme needs to be designed. Lattice codes have been proven to work well for source and channel coding. Thus, this research focuses on multi-terminal lossy source-channel coding systems using lattices, which may contribute to future IoT design.

1.3 Outline of the Dissertation

This dissertation provides a practical encoding/decoding strategy for the quadratic Gaussian CEO problem.

In Chapter 2, the background knowledge required in this research is summarized. The basic concept of entropy and mutual information is first reviewed, followed by rate-distortion theory. Then, classical results in multi-terminal source coding, such as the Slepian-Wolf theorem and the Wyner-Ziv bound are included. After that, lattice codes, convolutional codes, and convolutional lattice codes based on construction A are briefly discussed.

In Chapter 3, the coding scheme design is introduced. Firstly, the Gaussian CEO problem is proposed, followed with the system model of two sensors. Then, the evaluation method CEO bound is given. Lattice Wyner-Ziv coding, which is the inspiration for the proposed model, is introduced. The model and block diagram are represented at the end of the chapter.

In Chapter 4, the method to decide the parameters is firstly introduced. Secondly, Simulation results under different scenarios are given.

In Chapter 5, the evaluation of the proposed system is briefly discussed. The first section is the strengths and weaknesses of the system. The second section is memories and the complexity of the convolutional lattices.

In Chapter 6, the conclusion and future work are discussed together.

Chapter 2

Preliminaries

In this chapter, the necessary background knowledge for this research is introduced. The concepts of entropy and mutual information are reviewed in Section 2.1. In Section 2.2, the rate-distortion theory is included. Then, some classical results in the DSC problem, such as the Slepian-Wolf theorem and the Wyner-Ziv problem, are reviewed in Section 2.3. Lattice codes are briefly discussed in Section 2.4. Finally, convolutional codes and convolutional lattice codes based on construction A are described in Section 2.5.

2.1 Entropy and Mutual Information

Information theory, entropy was first described by Shannon in [28], which is the measure of the uncertainty of information. Consider a random variable X taking i.i.d values from $\mathcal{X}$ with a probability function $P_X(x) = Pr\{X = x\}$, the entropy of X is given as follows:

$$H(X) = -\sum_{x\in\mathcal{X}} P_X(x)\, log\big(P_X(x)\big). \tag{1}$$

The logarithm is base 2, which corresponds to measuring entropy in bits.

The joint entropy $H(X, Y)$ of discrete random variables X and Y jointly distributed as $P_{XY}(x, y) = Pr\{X = x,\ Y = y\}$ is:

$$H(X,Y) = -\sum_{x\in\mathcal{X}} \sum_{y\in\mathcal{Y}} P_{XY}(x,y)\, log\big(p_{XY}(x,y)\big). \tag{2}$$

The conditional entropy of $H(Y|X)$ is the uncertainty of Y given that X is known. If X and Y are dependent, then knowledge of X can reduce the uncertainty of Y. Conditional entropy is one of the most important concepts in information theory. There are two types of conditional entropy, $H(Y|X = x)$ and $H(Y|X)$. The conditional entropy $H(Y|X)$ is given by:

$$H(Y|X) = \sum_{x\in\mathcal{X}} P_X(x) H(Y|X=x)$$
$$= -\sum_{x\in\mathcal{X}} P_X(x) \sum_{y\in\mathcal{Y}} P_{Y|X}(y|x) log\left(P_{Y|X}(y|x)\right)$$
$$= -\sum_{x\in\mathcal{X}} \sum_{y\in\mathcal{Y}} P_{XY}(x,y) log\left(P_{Y|X}(y|x)\right), \quad (3)$$

where $H(Y|X) = 0$ if and only if the exact state of Y can be entirely determined by X. Conversely, $H(Y|X) = H(Y)$ if and only if Y and X are independent.
The chain rule of entropy [29] for random variables X and Y:

$$H(X,Y) = H(X) + H(Y|X). \quad (4)$$

The proof is simple and basic in information theory. Therefore it is omitted.

Mutual information measures a quantity of the mutual dependence of two variables, mutual information $I(X;Y)$ is the reduction in the uncertainty of X by knowing Y. Consider random variables X and Y with a joint probability distribution function $P_{XY}(x,y)$ and marginal distributions $P_X(x)$ and $P_Y(y)$. Then $I(X;Y)$ is given by:

$$I(X;Y) = \sum_{x\in\mathcal{X}} \sum_{y\in\mathcal{Y}} P_{XY}(x,y) log\left(\frac{P_{XY}(x,y)}{P_X(x)P_Y(y)}\right), \quad (5)$$

which can also be expressed as:

$$I(X;Y) = H(X) - H(X|Y)$$
$$= H(Y) - H(Y|X). \quad (6)$$

2.2 Rate-Distortion theory

In lossless compression, for compression of a random vector X, the single-letter entropy $H(X)$ is a lower bound on the compression rate, that is $H(X) \leq R$. However, in lossy compression, rates less than $H(X)$ can be achieved. The reconstructed sequence is not the same as the original sequence. Regardless, the original and reconstructed sequences should be as similar as possible. Therefore, a way of measuring the similarity is introduced. There is a tradeoff between rate and distortion, and the tradeoff is the subject of rate-distortion theory.

2.2.1. Rate-Distortion Code

There is a source x with elements from χ^n:

$$x = (x_1, x_2, \dots, x_n). \tag{7}$$

A source encoder f maps x to a codeword, $f(x)$. There are $M = 2^{nR}$ codewords. A decoder g maps a codeword to a sequence $\hat{x}$, from an alphabet $\hat{\chi}^n$. Therefore, the encoding function f_n is:

$$f_n: \chi^n \rightarrow \{1,2, \dots, 2^{nR}\}, \tag{8}$$

and the decoding function g_n is:

$$g_n: \{1,2, .., 2^{nR}\} \rightarrow \hat{\chi}^n. \tag{9}$$

The set χ is called the source alphabet, and the set $\hat{\chi}$ is called the reconstruction alphabet.

A codebook C has M codewords of n symbols each. If $x \in \chi^n$ is a source sequence, then $f(x)$ is the message from the set $\{1,2, \dots, M\}$. If $m \in \{1,2, .., M\}$ is a message, then $g(m)$ is a reconstructed codeword, from $\hat{\chi}$. The codeword corresponding to x is $g(f(x)) \in C$. The encoding function should choose the message $f(x)$ so to minimize the distortion of the corresponding $g(f(x))$.

Since the codebook consists of M messages, the code rate is:

$$R = \frac{1}{n} log M. \tag{10}$$

2.2.2. Distortion Measure

We need a measurement of the difference,between two sequences and the measurement is called distortion. A distortion function is $d(x, \hat{x})$ for sequences. We want x and $\hat{x}$ to be as close as possible, and the distortion should be as small as possible.

For discrete random variables, including binary random variables, an important distortion function is Hamming distortion [29]:

$$d(x,\hat{x}) = \frac{1}{n}\sum_{i=1}^{n} d(x_i,\widehat{x_i}), \tag{11}$$

where

$$d(x_i,\widehat{x_i}) = \begin{cases} 0 & if\ x_i = \widehat{x_i} \\ 1 & if\ x_i \neq \widehat{x_i} \end{cases}. \tag{12}$$

For continus random variables, widely used distortion function is Quadratic distortion [23]:

$$d(x,\hat{x}) = \frac{1}{n}\sum_{i=1}^{n} d(x_i,\widehat{x_i}), \tag{13}$$

where

$$d(x_i,\widehat{x_i}) = E(x_i - \widehat{x_i})^2. \tag{14}$$

2.2.3. Rate-Distortion Theorem

Expected distortion is the distortion averaged over all source sequences. The expected distortion for a $(2^{nR}, n)$ code is defined as:

$$D = E\left[d\left(X, g\big(f(X)\big)\right)\right]$$
$$= \sum_{x\in\chi^n} p_X(x)\, d\left(x, g\big(f(x)\big)\right). \tag{15}$$

The optimized tradeoff between rate and distortion is characterized by the rate-distortion function. It is expected that as the allowed distortion D increases, the code rate R will decrease. Therefore, the information-rate distortion function $R(D)$ for a source X is given by [29]:

$$R(D) = \min_{p_{\widehat{X}|X}(\hat{x}|x):\, E\left[d\left(X,\widehat{X}\right)\right] \leq D} I\left(X;\widehat{X}\right), \tag{16}$$

where the minimum is taken over all $p_{\widehat{X|X}}(\hat{x}|x)$ that satisfy $Ed(X,\hat{X}) \leq D$ where,

$$E[d(X,\hat{X})] = \sum_{x\in\chi}\sum_{\hat{x}\in\hat{\chi}} p_{X,\hat{X}}(x,\hat{x})d(x,\hat{x}). \tag{17}$$

The goal of rate-distortion theory is to find good $p_{\widehat{X|X}}(\hat{x}|x)$. The source distribution $p_X(x)$ is fixed, this is equivalent to finding good $p_{\widehat{X|X}}(\hat{x}|x)$. A good code C can satisfy $p_{\widehat{X|X}}(\hat{x}|x)$. With those definitions, we can now give the rate-distortion theorem for an i.i.d source X with distribution $p_X(x)$ and distortion function $d(\cdot,\cdot)$ is equal to the information rate-distortion function. That is [29]:

$$R(D) = R_I(D) \tag{18}$$

$$R(D) = \min_{p_{\widehat{X|X}}(\hat{x}|x):E[d(X,\hat{X})]\leq D} I(X;\hat{X}). \tag{19}$$

The rate-distortion function $R(D)$ is given by the definition that the infimum of rates R such that (R,D) is in the rate-distortion region of the source of a given distortion D, which concerns the construction of codes. The information rate-distortion function $R_I(D)$, given by (16)(17), concerns minimization of mutual information.

2.2.4. Rate-Distortion for Gaussian Sources

Consider a Gaussian source with $X \sim N(0,\sigma^2)$. The rate-distortion function $R(D)$ for lossy compression of a Gaussian source X with squared-error distortion D is:

$$R(D) = \begin{cases} \frac{1}{2}log\frac{\sigma^2}{D} & 0 \leq D \leq \sigma^2 \\ 0 & D > \sigma^2 \end{cases}. \tag{20}$$

The proof is in [29].

2.3 Theorems in Distributed Source Coding

Fig. 1(a) shows a lossless source coding problem where the decoder aims to reconstruct two correlated sources without any loss. Consider two sources X_1 and X_2 with n symbols X_1^n and X_2^n. Each encoder assigns an index from the set $\{1,2,\dots,2^{nR_i}\}$ for the sequence X_i^n and transmits to the decoder, $i = 1,2$. After receiving the indices from the encoder, the joint decoder assigns an estimate of $(\hat{X}_1^n, \hat{X}_2^n)$ or an estimate of error. The probability of error is defined as:

$$P_e^{(n)} = Pr\{(\hat{X}_1^n, \hat{X}_2^n) \neq (X_1^n, X_2^n)\}. \tag{21}$$

A rate pair (R_1, R_2) is admissible if there exist codes $(2^{nR_1}, 2^{nR_2})$ such that $P_e^{(n)} \to 0$ with $n \to \infty$. Slepian and Wolf [3] characterized the admissible rate region, which is a closure of a set of admissible rate pairs.

The Slepian-Wolf rate-region is given by:

$$\mathcal{R}^{SW} = \left\{ \begin{array}{c} (R_1, R_2): R_1 \geq H(X_1|X_2) \\ R_2 \geq H(X_2|X_1) \\ R_1 + R_2 \geq H(X_1, X_2) \end{array} \right\}. \tag{22}$$

Two sources X_1 and X_2 are i.i.d sources determined by a joint distribution $P_{X_1X_2}(x_1, x_2)$. The rate region is determined by the joint entropy of X_1 and X_2 and conditional entropy of each.

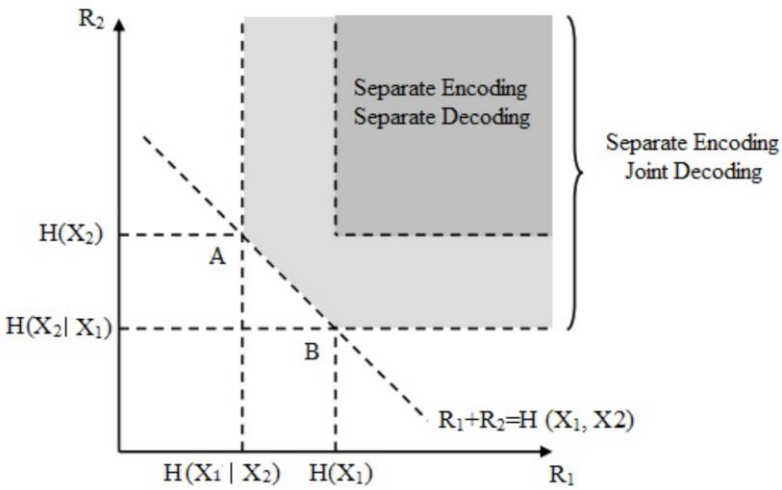


Figure 2.1 : Slepian-Wolf rate region for lossless distributed source coding of two sources

Figure 1.2 shows the admissible rate region of compressing two sources X_1 and X_2 by applying the Slepian-Wolf theorem. From the figure, it can be seen that the total compression rate is significantly decreased compared to the independent case.

The generalized Slepian-Wolf theorem [4] is aimed to achieve lossless compression of L correlated sources $\{X_1, X_2, \cdots, X_L\}$, the source rate R_i satisfies the following conditions:

$$\sum_{i\in\mathcal{L}} R_i \geq H\left(X_{\mathcal{L}}|X_{\mathcal{L}^c}\right) \quad \mathcal{L} \in \{1,2,\dots,L\}, \tag{23}$$

where $\mathcal{L}^C = \{1,2,\dots,L\}\backslash\mathcal{L}$ represents the complementary set of $\mathcal{L}$ and $X_{\mathcal{L}} = \{X_i | i \in \mathcal{L}\}$.

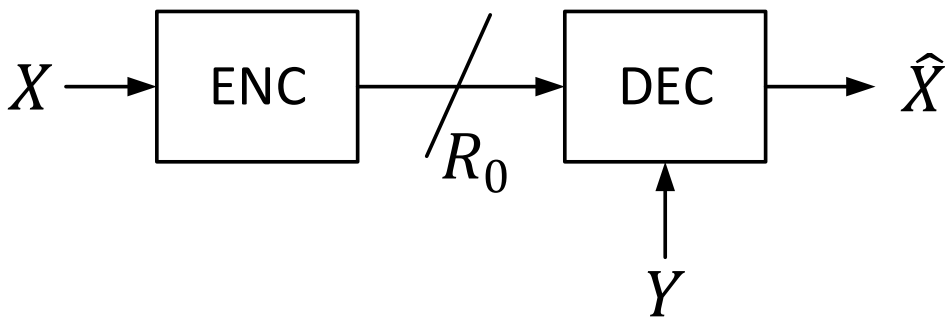


Figure 2.2 : The Wyner-Ziv source coding problem

Figure 1.3 shows the source coding problem that Wyner and Ziv studied in [6]. The problem depicts that there is a pair of dependent i.i.d sources (X,Y), the encoder assigns an index from the set $\{1,2,\dots,2^{nR_0}\}$ for the source sequence X^n, the decoder estimates the source $\hat{X}^n$ using the side information provided by Y and the received index from the encoder. Wyner-Ziv derived the rate-distortion function $R^{WZ}(D)$. Therefore, the Wyner-Ziv bound is given that a pair of sources (X,Y) which generates i.i.d sequnces, and there exists a Z forms Markov chain $Z \to X \to Y$. The decoder reconstructs $\hat{X}$ based on Z and Y. The rate R is achievable if $R \geq R^{WZ}(D)$, where $R^{WZ}(D)$ is defined as:

$$R^{WZ}(D) = inf[I(X;Z|Y)]. \tag{24}$$

The D is distortion measure $D = E\left[d\left(X,\hat{X}\right)\right]$ and $inf[\cdot]$ is the infimum of a subset.

2.4 Lattices

Lattices serve as a bridge from the high dimension of Shannon's theory to that of digital communication techniques [30]. Good lattices can form effective structures for various coding problems. Lattices for quantization and modulation always draw the attention of communication engineers and information theorists. Lattices have been proved that they are suitable for both lossy compression and noise immunity, also known as source coding and channel coding respectively. Hence, it is crucial to review lattices in this part.

2.4.1. Fundamental Characteristics of Lattices

An n-dimensional lattice Λ is a discrete additive subgroup of $\mathbb{R}^n$. Since lattice Λ is a Euclidean space, this space can be spanned by a set of basis vectors $g_1, g_2, \dots, g_n$, where,

$$g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} \tag{25}$$

is a column vector representing a point in $\mathbb{R}^n$.

If there is a lattice point x, it can be formed as a linear combination of the basis vectors scaled by $b_i \in \mathbb{Z}$.

$$x = g_1 b_1 + g_2 b_2 + \cdots + g_n b_n. \tag{26}$$

The matrix form is:

$$x = \boldsymbol{G} b, \tag{27}$$

where G is an n-by-n generator matrix of lattice Λ.

For a lattice Λ, a fundamental region $\mathcal{V} \in \mathbb{R}^n$ is a region that, if each lattice point in Λ shifts in this region, will cover the whole real space $\mathbb{R}^n$. This can be expressed as:

$$\mathbb{R}^n = \cup_{x \in \Lambda} \mathcal{V} + x \text{ and} \tag{28}$$

$$\{\mathcal{V} + x\} \cap \{\mathcal{V} + y\} = \emptyset. \tag{29}$$

For any $x \neq y$. Any point $y \in \mathbb{R}^n$ is in one fundamental region. The volume of any fundamental region is constant, which is denoted as $V(\Lambda)$:

$$V(\Lambda) = |det(\boldsymbol{G})|. \tag{30}$$

The Voronoi region is an important fundamental region, the Voronoi region for $x \in \Lambda$ is a region that, if there is a point x, covers all points with Euclidean distances to x less than that to any other lattices.

Lattices can also be scaled. If there is a lattice Λ and $k \in \mathbb{R}$ with $k \neq 0$, $k\Lambda$ is a scaled lattice. If Λ has a generator matrix G, then $k\Lambda$ has a generator matrix $k\boldsymbol{G}$. For $k > 0$, the volume of $k\Lambda$ is:

$$V(k\Lambda) = |det(k\boldsymbol{G})| = k^n|det(\boldsymbol{G})|. \tag{31}$$

Lattice cosets are sets of points such that the difference vectors between every pair of points belong to the lattice, defined as:

$$\Lambda_x = \boldsymbol{x} + \Lambda = \{\boldsymbol{x} + \lambda : \lambda \in \Lambda\}. \tag{32}$$

2.4.2. Lattice Quantization

Lattice quantization is to find the lattice point $\boldsymbol{x} \in \Lambda$ which is closest to an arbitrary $y \in \mathbb{R}^n$. If there is a lattice point $\boldsymbol{x}$ which is closest to $\boldsymbol{y}$, the definition of lattice quantization is:

$$\boldsymbol{x} = \arg\min_{\lambda \in \Lambda} ||\boldsymbol{y} - \lambda||^2. \tag{33}$$

Also written as:

$$\boldsymbol{x} = Q_\Lambda(\boldsymbol{y}). \tag{34}$$

Quantization in scaled lattices is simple. The point closest to $\boldsymbol{y} \in \mathbb{R}^n$ in a scaled lattice $k\Lambda$ for any real number k is:

$$Q_{k\Lambda}(\boldsymbol{y}) = kQ_\Lambda\left(\frac{1}{k}\boldsymbol{y}\right). \tag{35}$$

Quantization in a lattice coset is also simple. The closest point to $\boldsymbol{y} \in \mathbb{R}^n$ in a lattice coset $\Lambda' = \Lambda + s$ of a lattice Λ is:

$$\boldsymbol{z} = Q_{\Lambda'}(\boldsymbol{y}) = Q_{\Lambda}(\boldsymbol{y} - s) + s. \tag{36}$$

Lattice modulo function is to find the difference between $\boldsymbol{y} \in \mathbb{R}^n$ and the quantization of $\boldsymbol{y}$, which is usually written as $\boldsymbol{y}$ mod Λ:

$$\boldsymbol{y}\, mod\, \Lambda = \boldsymbol{y} - Q_{\Lambda}(\boldsymbol{y}). \tag{37}$$

In order to measure the quantization capability for a lattice, the normalized second moment is an important tool to measure the quantization error and be used for designing lattice codes.

The normalized second moment, also known as $G(n)$, which is used to measure the average quantization error by the mean squared error per dimension, defined as:

$$G_n(\Lambda) = \frac{1}{nV(\Lambda)^{\frac{2}{n}+1}} \int_{\mathcal{V}} \|\boldsymbol{t}\|^2 d\boldsymbol{t}, \tag{38}$$

$\boldsymbol{t}$ represents the quantization error $\boldsymbol{t} = \boldsymbol{y} - Q_{\Lambda}(\boldsymbol{y})$.

The shaping gain measures the improvement in Normalized Second Moment (NSM) to scalar quantization. It is defined as:

$$\gamma_S(\Lambda) = 10log_{10} \frac{1}{12G_n(\Lambda)}\ dB. \tag{39}$$

The shaping gain is usually expressed in dB. As the lattice dimension increases, the shaping gain also increases, the asymptotic value of $\frac{\pi e}{6}$, which is 1.53 dB.

2.4.3. Nested Lattice Codes

A nested lattice code is a lattice code where the shaping region is the fundamental region of some other lattice. Therefore, there are two lattices. The coding lattice Λ_C provides the lattice code with error-correcting ability, and the shaping lattice Λ_S provides the lattice code with shaping ability. Nested lattice codes require Λ_S is a sublattice of Λ_C. The definition can be given as follows:

Λ_s and Λ_C are two lattices with $\Lambda_S \subseteq \Lambda_C$. $\mathcal{F}$ is the fundamental region for Λ_S. Then:

$$\mathcal{C} = \Lambda_C \cap \mathcal{F} \tag{40}$$

is a nested lattice code.

The rate of a lattice code with M codewords is $R = \frac{1}{n} log_2 M$. For a nested lattice code, the size of the codebook is $V(\Lambda_S)/V(\Lambda_C)$, the code rate is:

$$R = \frac{1}{n} log \frac{V(\Lambda_S)}{V(\Lambda_C)} = \frac{1}{n} log \frac{|det(G_S)|}{|det(G_C)|}. \tag{41}$$

If the shaping lattice is a scaled coding lattice, such kind of nested lattice code is called self-similar lattice code. If $\Lambda_S = K\Lambda_C$, where K is a positive integer. The code rate R is:

$$R = \frac{1}{n} log \frac{det(K\Lambda_C)}{det(\Lambda_C)} = log_2 K. \tag{42}$$

2.5 Convolutional Lattice Code based on Construction A

Convolutional lattice code (CLC) proposed by Erez and ten Brink to be used for vector quantization for dirty-paper coding problems [31] has recently attracted attention from academics and industries. Several works proved that employing convolution lattice code for shaping brings many advantages, such as good shaping gain, flexibility for dimensions, and straightforwardness for applying the Viterbi algorithm which is optimal. Therefore, CLC should be considered for solving DSC problems.

2.5.1. Construction A

Assume that there is an (n, k, d) binary code C, which maps k information bits into binary codewords of length n. Construction A is a method for generating lattices by lifting C to the Euclidean space [30]. For example, for a modulo-2 lattice, the set of all integer vectors whose modulo-2 results belong to code C forms a lattice. As is shown below:

$$\Lambda_C = \{x \in \mathbb{Z}^n : x \ mod\ 2 \in C\}. \quad (43)$$

Equivalently,

$$\Lambda_C = 2\mathbb{Z}^n + C. \quad (44)$$

The volume of binary construction A lattice is:

$$V(\Lambda_C) = |\mathbb{Z}^n/\Lambda_C| = \frac{2^n}{M}, \quad (45)$$

where M is the size of code C. Also, if C is generated by a full-rank $n \times k$ matrix, then $M = 2^k$, the volume is:

$$V(\Lambda_C) = 2^{n-k}. \quad (46)$$

The minimum distance between any two points in Λ_C is:

$$d_{min}(\Lambda_C) = min\{2, \sqrt{d}\}, \quad (47)$$

where d is the minimum Hamming distance of code C.

2.5.2. Convolutional Code

Convolutional codes were first introduced by Elias in 1955 [32]. Different from block codes, convolutional codes contain memories in the encoder, an (n, k, m) convolutional code can be implemented with a k inputs, n outputs with m input memories. For example, there is a $(3,1,3)$ binary convolution codes, the encoder is shown in Figure 2.3:

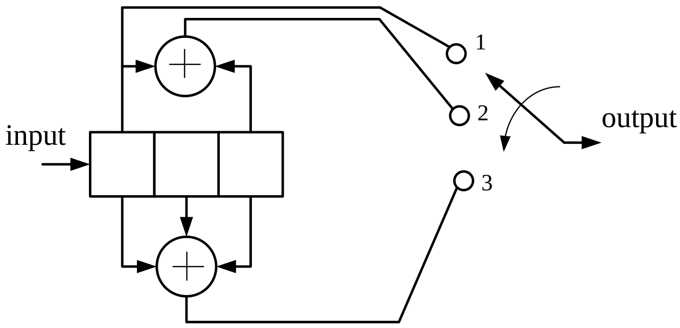


Figure 2.3 : $(3,1,3)$ binary convolutional encoder

For a general case of an (n, k, m) code, the generator matrix is formed by $\frac{n}{k}$ generator polynomials, which are:

$$\begin{aligned} \boldsymbol{g}^{(0)}(D) &= g_0^{(0)} + g_1^{(0)}D + \cdots + g_m^{(0)}D^m \\ \boldsymbol{g}^{(1)}(D) &= g_0^{(1)} + g_1^{(1)}D + \cdots + g_m^{(1)}D^m \\ &\vdots \\ \boldsymbol{g}^{\left(\frac{n}{k}-1\right)}(D) &= g_0^{\left(\frac{n}{k}-1\right)} + g_1^{\left(\frac{n}{k}-1\right)}D + \cdots + g_m^{\left(\frac{n}{k}-1\right)}D^m \end{aligned} \quad , \tag{48}$$

where $g_i^{(j)}$ can be interpreted as the encoder transfer function relating input i to output j, D is the delay operator, m is defined as the number of the shift registers. The generator matrix of a rate $\frac{k}{n}$ convolutional code $\boldsymbol{G}(D) = \left[g^{(0)}(D)g^{(1)}(D) \dots g^{\left(\frac{n}{k}-1\right)}(D)\right]^t$ is equivalent to $\boldsymbol{G}(D) = G_0 + G_1D + \cdots + G_mD^m$, where $\boldsymbol{G}_i = \left[g_i^{(0)}g_i^{(1)} \dots g_i^{\left(\frac{n}{k}-1\right)}\right]^t$, for $i = 0,1, \dots, m$. $\boldsymbol{G}_i$ is the submatrix of the generator matrix, which has $\frac{n}{k}$ rows and 1 column for each submatrix.

Therefore, the convolutional code generator matrix is defined as:

$$
\boldsymbol{G} = \begin{bmatrix} \boldsymbol{G}_0 & & \\ \boldsymbol{G}_1 & \boldsymbol{G}_0 & \\ \vdots & \vdots & \ddots \\ \boldsymbol{G}_m & \boldsymbol{G}_{m-1} & \\ & \boldsymbol{G}_m & \ddots \\ & & \ddots \end{bmatrix}. \tag{49}
$$

The generator matrix $\boldsymbol{G}$ is an $n \times k$ matrix.

2.5.3. Convolutional Lattice Code

Convolutional lattice code based on construction A is obtained by applying convolutional code to construction A. The method is straightforward, first the convolutional code generator matrix $\boldsymbol{G}$ is transformed to a canonical form which is:

$$
\boldsymbol{G} = [\boldsymbol{I}_k | \boldsymbol{P}^t]^t, \tag{50}
$$

where $\boldsymbol{I}_k$ is the $k \times k$ identity matrix, $\boldsymbol{P}$ is an $(n-k) \times k$ matrix. The information vector w is encoded as $c = \boldsymbol{G}w$, concatenated with $\boldsymbol{P}w$. Therefore, the generator matrix of convolutional lattice code based on construction A is defined as:

$$
\boldsymbol{G}_{\Lambda_C} = \begin{bmatrix} \boldsymbol{I}_k & 0 \\ \boldsymbol{P} & 2\boldsymbol{I}_{n-k} \end{bmatrix}. \tag{51}
$$

$\boldsymbol{G}_{\Lambda_C}$ is an $n \times n$ generator matrix for Λ_C.

Chapter 3

Coding Scheme Design

In this chapter, the model for the Gaussian CEO problem is proposed. The Gaussian CEO problem with two sensors and the evaluation method are described in 3.1. Then, the Wyner-Ziv coding problem is discussed in 3.2, which is widely used in lossy DSC problems. Finally, the proposed model based on Wyner-Ziv coding strategy is given in 3.3.

3.1 Problem Statement

In this section, the Gaussian CEO problem [20] is discussed. As is introduced in Chapter 1, the Gaussian CEO problem estimates a Gaussian source using multiple agents who can only provide a corrupt version of the source under Gaussian noise. The target of the CEO problem is to determine the tradeoff between the total rate R and the distortion D. The Gaussian CEO problem with two agents (sensors), which is a basic case of the problem, is mainly investigated.

3.1.1. Gaussian CEO Problem with Two Sensors

The system model of estimating a Gaussian source through two sensors with joint-source coding is shown in Figure 3.1.

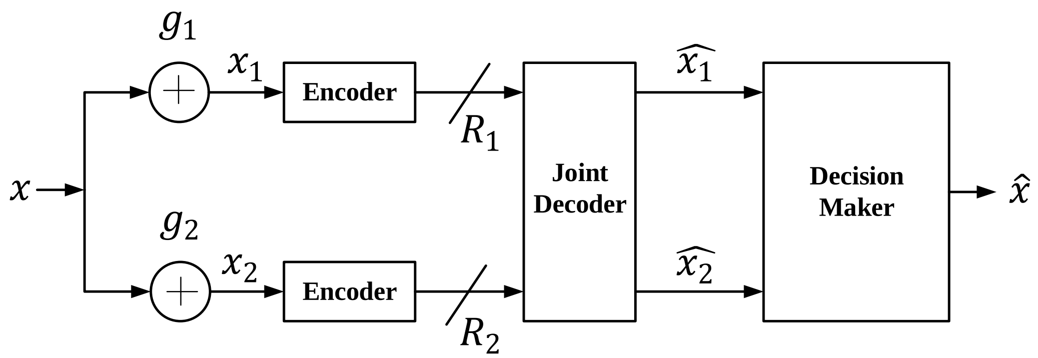


Figure 3.1 : The system model of estimating a Gaussian source through two sensors with joint-source coding

In Figure 3.1, x is an i.i.d Gaussian source, which produces a sequence $\boldsymbol{x} = \{x(t)\}_{t=1}^{n}$. Source $x(t) \sim N(0, \sigma_x^2)$ is observed by two sensors and forwarded to the encoders. Due to the corruption during the observation, the sequences acquired by sensors contain observation errors. For $i = 1,2$, the corrupted version of x is called $\boldsymbol{x}_i = \{x_i(t)\}_{t=1}^{n}$, which can be expressed as:

$$x_i(t) = x(t) + g_i(t) \tag{52}$$

where $\boldsymbol{g}_i = \{g_i(t)\}_{t=1}^{n}$ for $i = 1,2$ are independent additive white Gaussian noise, each $g_i(t)$ obeys an identical distribution with mean 0 and variance σ_o^2. Although $\boldsymbol{x}_i$ contain errors, the sensors still forward the erroneous sequences to the two encoders, which is also known as lossy forwarding [33]. $\boldsymbol{x}_i$ are seperately lossy encoded, which based on rate-distortion function $R_i(D_i)$ for $i = 1,2$. The Joint-source coding (JSC) decoder then performs JSC coding to estimate $\boldsymbol{x}_i$, the estimation results are denoted as $\widehat{\boldsymbol{x}_i} = \left\{\widehat{x_i(t)}\right\}_{t=1}^{n}$. Finally, the decision maker reconstructs the source x with an estimation denoted as $\widehat{\boldsymbol{x}} = \left\{\widehat{x(t)}\right\}_{t=1}^{n}$. The decision rule is decided by:

$$\widehat{\boldsymbol{x}} = F_d(\widehat{x_1}, \widehat{x_2}), \tag{53}$$

where function $F_d(\cdot)$ is designed based on the variance of the source σ_x^2 and the variance of the observation error σ_o^2.

3.1.2. Evaluation Method

Quadratic distortion function (13) (14) is applied to measure the distortion. For evaluating the proposed system, the Gaussian CEO bound [20] is given as:
For every $D > 0$

$$R^{CEO}(D) = \frac{\sigma_o^2}{2\sigma_x^2}\left[\frac{\sigma_x^2}{D} - 1\right]^{+} + \frac{1}{2}log^{+}\left(\frac{\sigma_x^2}{D}\right), \tag{54}$$

where $[a]^{+} = max\{0, a\}$ and $log^{+}a = [loga]^{+}$.

3.2 Lattice Wyner-Ziv Coding

For the coding strategy, it is not difficult that Wyner-Ziv coding strategy to be considered, which exploits the correlation of the two estimations of the source. Moreover, under the quadratic-Gaussian model there is no loss in rate-distortion performance for the side information only available at the decoder.

For the code selection, as is introduced in Chapter 2, lattices are defined directly in the Euclidean space, they do not require mapping, which saves complexity. Besides, it is easy to design the codebook by applying lattices since the rate can be changed by changing the volume of the lattice.

Therefore, in this part, the lattice Wyner-Ziv coding is briefly introduced since it may inspire the idea of how to design the system for the CEO problem.

Assuming that there is a source X and Y, which Y is known to the decoder as the side information. $Z = X - Y$ is called the innovation component, $Z \sim N(0, \sigma_Z^2)$. According to the Wyner-Ziv rate-distortion function,

$$R_{WZ}(D) = \frac{1}{2} log\left(\frac{\sigma_{X|Y}^2}{D}\right) = \frac{1}{2} log\left(\frac{\sigma_Z^2}{D}\right), \tag{55}$$

where $\sigma_{X|Y}^2$ represents the conditional variance of X is given Y.

The idea is to transmit innovation Z without wasting bits on Y which is already known to the decoder. Therefore, the encoder initially quantizes the source X to lattice points λ in a lattice Λ_1, Then sends the relative cosets λ/Λ_2 to the decoder. The rate is $log|\Lambda_2/\Lambda_1|$ bits. The decoder knows that λ is around Y, at a distance determined by d_{min} of lattice Λ_2 and innovation Z. If the lattice Λ_2 is sparse enough, the λ would be the only member in the cosets within the distances to Y. Finally, the decoder can reconstruct X by decoding λ.

The coding scheme is based on a nested lattice pair $\Lambda_1 \subset \Lambda_2$, which generates a codebook $C = \{\boldsymbol{v}_1, \dots, \boldsymbol{v}_m\}$. Given a source $\boldsymbol{x} = \boldsymbol{y} + \boldsymbol{z}$, the encoding operation can be expressed as:

$$\boldsymbol{v} = \left[Q_{\Lambda_1}\boldsymbol{x}\right] mod\ \Lambda_2, \tag{56}$$

and the decoding operation is:

$$\hat{\boldsymbol{z}} = (\boldsymbol{v} - \boldsymbol{y})\ mod\ \Lambda_2. \tag{57}$$

$$\hat{\boldsymbol{x}} = \boldsymbol{y} + \hat{\boldsymbol{z}}. \tag{58}$$

3.3 Proposed Model

The model for the CEO problem with two sensors is proposed in this part. As is introduced in 3.1, $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are correlated. Thus it is reasonable to transform the CEO problem into a special case of the Wyner-Ziv coding problem. Since $\boldsymbol{x}_i \sim N(0, \sigma_x^2 + \sigma_o^2)$ for $i = 1,2$, the innovation component is $\boldsymbol{z} = \boldsymbol{x}_1 - \boldsymbol{x}_2$ with mean 0 and variance σ_o^2. Hence, it is straightforward to generate a codebook of z, which is similar to the Wyner-Ziv coding.

The block diagram of the coding scheme is shown below:

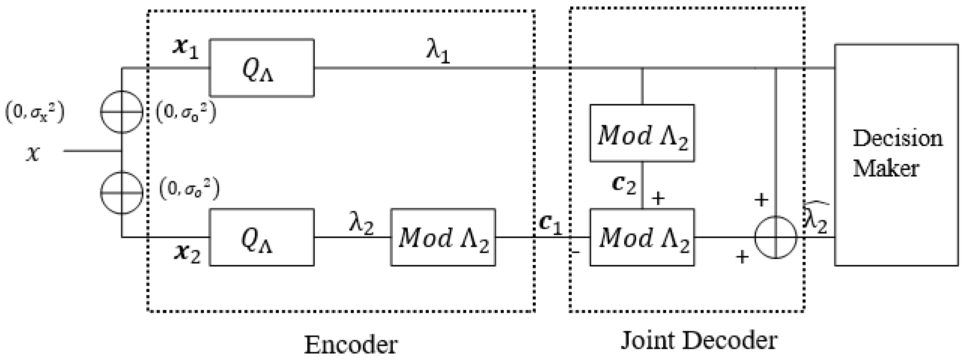


Figure 3.1 : The block diagram of the coding scheme

where $\Lambda = \alpha\Lambda$, $\Lambda_2 = K\Lambda$.

First, encoder 1 and encoder 2 quantize the source vector $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ to points λ_1 and λ_2 in a fine lattice $\alpha\Lambda$, where α is a scaling factor.

Encoder 1 transmits λ_1, encoder 2 sends relative coset $\boldsymbol{c}_2$ of λ_2 to the joint decoder. $\boldsymbol{c}_2$ is defined as:

$$\boldsymbol{c}_2 = [\lambda_2]\ mod\ \alpha K\Lambda, \tag{59}$$

where K is a shaping factor, which is a positive integer.

The joint decoder receives λ_1 and $\boldsymbol{c}_2$. The decoder needs to recover λ_2 by using $\boldsymbol{c}_2$ and λ_1. The decoder first finds the coset $\boldsymbol{c}_1$ of λ_1:

$$\boldsymbol{c}_1 = [\lambda_1]\ mod\ \alpha K\Lambda, \tag{60}$$

then the decoder can recover λ_2:

$$\widehat{\lambda_2} = \lambda_1 + [\,\boldsymbol{c}_2 - \boldsymbol{c}_1]\ mod\ \alpha K \Lambda \tag{61}$$

Finally, an estimation of x can be made by the decision maker:

$$\hat{x} = F_d\left(\lambda_1, \widehat{\lambda_2}\right). \tag{62}$$

The parameter settings and simulation results will be introduced in Chapter 4.

Chapter 4

Numerical Results

In this chapter, the parameter settings and simulation results are introduced. Parameter settings are discussed in section 4.1. Simulation results and analysis are shown in section 4.2.

4.1 Parameter Settings

As is introduced in Chapter 3, scaling factor α is to match the source to the fine lattice resolution. In order to avoid failing to reconstruct the source, it is necessary to make lattice points λ_1 and λ_2 not so far from each other. In other words, the source variance should match the volume of the lattice.

Consider quantizing a single source using a nested lattice $\alpha N\Lambda/\alpha\Lambda$ with rate $R = log\ N$. The source is fixed with variance σ^2. Given a rate R and probability P_e that the source is not in the Voronoi region of $\alpha N\Lambda$, N and α should be found.

Let S be an n-ball of radius r. The probability that Gaussian noise is inside the sphere is,

$$I_n = \int_S f(x)\, dx, \tag{63}$$

where

$$f(x) = \frac{1}{\left(\sqrt{2\pi}\sigma\right)^n} exp\left(\frac{-\|x\|^2}{2\sigma^2}\right) \tag{64}$$

and is given by [34],

$$I_n = I_{n-2} - e^{-z}\frac{z^{\frac{n}{2}-1}}{\left(\frac{n}{2}-1\right)!} \tag{65}$$

where $z = \frac{r^2}{2}$ and

$$I_2 = 1 - e^{-z} \tag{66}$$

So,

$$P_e = e^{-z}\left(1 + \frac{z}{1!} + \frac{z^2}{2!} + \cdots + \frac{z^{\frac{n}{2}-1}}{\left(\frac{n}{2}-1\right)!}\right) \tag{67}$$

where $z = \frac{r^2}{2\sigma^2}$.

Let $\rho^*(\sigma^2)$ be the value of r such that $P_e = P_{target}$. Let ρ_Λ be the packing radius of Λ, $\rho_{\alpha N\Lambda}$ be the scaled packing radius of $\alpha N\Lambda$. Then:

$$\rho_{\alpha N\Lambda} = \alpha N \rho_\Lambda. \tag{68}$$

When $P_e = P_{target}$,

$$\rho^* = \alpha N \rho_\Lambda, \tag{69}$$

$$\alpha = \frac{1}{N}\frac{\rho^*}{\rho_\Lambda} \tag{70}$$

Therefore, scaling factor α depends on rate $logN$ and P_{target}.

Consider designing a coding scheme to test the rate-distortion performance. The source variance σ_{src}^2 and the observation variance σ_{obs}^2 are fixed. Let $R_2 = logK$ to transmit the difference between the two cosets $\boldsymbol{c}_2 - \boldsymbol{c}_1$ and $R_1 = logK + logM$ to transmit the lattice code $\alpha\Lambda$. The procedures are shown below:

1. Make a prelimary choice for R_1 and work with 2^{R_1}.

2. Find α to satisfy the target error probability P_{target}.

$$\alpha = \frac{1}{N}\frac{\rho^*(\sigma_{src}^2)}{\rho_\Lambda}. \tag{71}$$

where $N = 2^{R_1}$.

3. Find K to satisfy the target error probability P_{target}. (α is already fixed).

$$K = \left\lceil \frac{1}{\alpha}\frac{\rho^*(\sigma_{obs}^2)}{\rho_\Lambda} \right\rceil = \left\lceil 2^{R_1}\frac{\rho^*(\sigma_{obs}^2)}{\rho^*(\sigma_{src}^2)} \right\rceil. \tag{72}$$

4. Find M by using R_1 and K,

$$M = \left\lceil \frac{2^{R_1}}{K} \right\rceil. \tag{73}$$

5. Find value for R_1 using $R_1 = logM + logK$.

4.2 Simulation Results

Several well-known low-dimensional lattices, such as E_8 lattice, Barnes-Wall lattice, and Leech lattice have been well-studied and proved to provide good shaping gains. What is more, they all have optimal quantization algorithms [35][36].

Besides, convolutional lattices also have been proved to provide good shaping gains, flexibility for dimensions, and straightforwardness for applying the Viterbi algorithm, which is already discussed in Chapter 2.

The quantization algorithm of convolutional lattice code (CLC) is given to show how to apply the Viterbi algorithm [37] to it.

Algorithm: Quantizing convolutional lattice code

Input: noisy input $\boldsymbol{y}$, Viterbi decoder $Vdec(\cdot)$,
convolutional encoder $Enc(\cdot)$.
Output: convolutional lattice point $\hat{\boldsymbol{x}}$ nearest to $\boldsymbol{y}$
Compute the following:
$\boldsymbol{y}' = |mod_2(\boldsymbol{y} + 1) - 1|$;
$\boldsymbol{b} = Vdec(\boldsymbol{y}')$;
$\hat{\boldsymbol{c}} = Enc(\boldsymbol{b})$;
$\boldsymbol{y}'' = \frac{\boldsymbol{y} - \hat{\boldsymbol{c}}}{2}$;
$\hat{\boldsymbol{z}} = \lfloor \boldsymbol{y}'' \rceil$;
$\hat{\boldsymbol{x}} = \hat{\boldsymbol{c}} + 2\hat{\boldsymbol{z}}$;

Table 4.1: Quantization algorithm of CLC

Simulations are implemented on MATLAB by applying Barnes-Wall lattice code and convolutional lattice code. Barnes-Wall lattices have 16 dimensions, while the dimension of CLC is set to 1152 with a 1/2 rate. The CEO makes a final decision as $\hat{x} = \frac{\widehat{\lambda_1}+\widehat{\lambda_2}}{2}$ because the two sensors suffer the same observation error.

Given the source variance $\sigma_{src}^2 = 1$, the variance of observation error $\sigma_{obs}^2 = 0.01$, the error target $P_{target} = 10^{-4}$. figure 4.1 shows the simulation result:

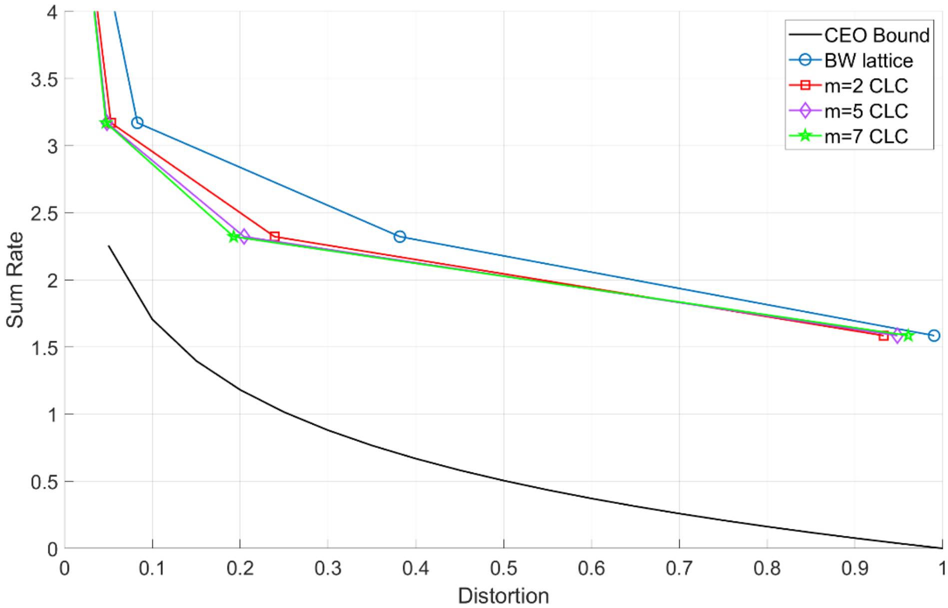


Figure 4.1 : Rate-distortion function of BW lattice and m=2, 5, 7 CLC when $\sigma_{src}^2 = 1$, $\sigma_{obs}^2 = 0.01$

Fix the source variance $\sigma_{src}^2 = 1$, set the variance of observation error $\sigma_{obs}^2 = 0.05$, the error target $P_{target} = 10^{-4}$. figure 4.2 shows the simulation result.

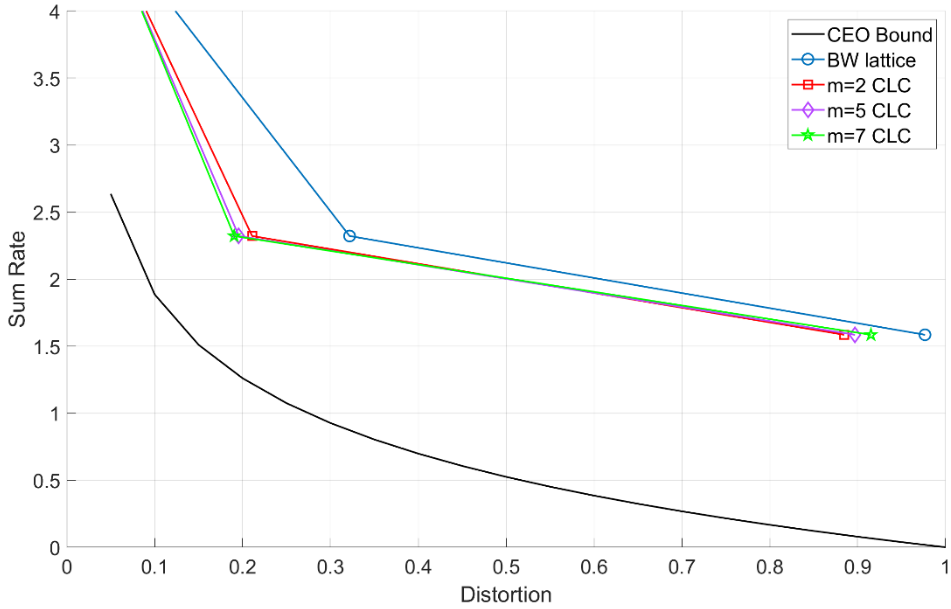


Figure 4.2 : Rate-distortion function of BW lattice and m=2, 5, 7 CLC when $\sigma_{src}^2 = 1$, $\sigma_{obs}^2 = 0.05$

Fix the source variance $\sigma_{src}^2 = 1$, set the variance of observation error $\sigma_{obs}^2 = 1$, the error target $P_{target} = 10^{-4}$. figure 4.3 shows the simulation result:

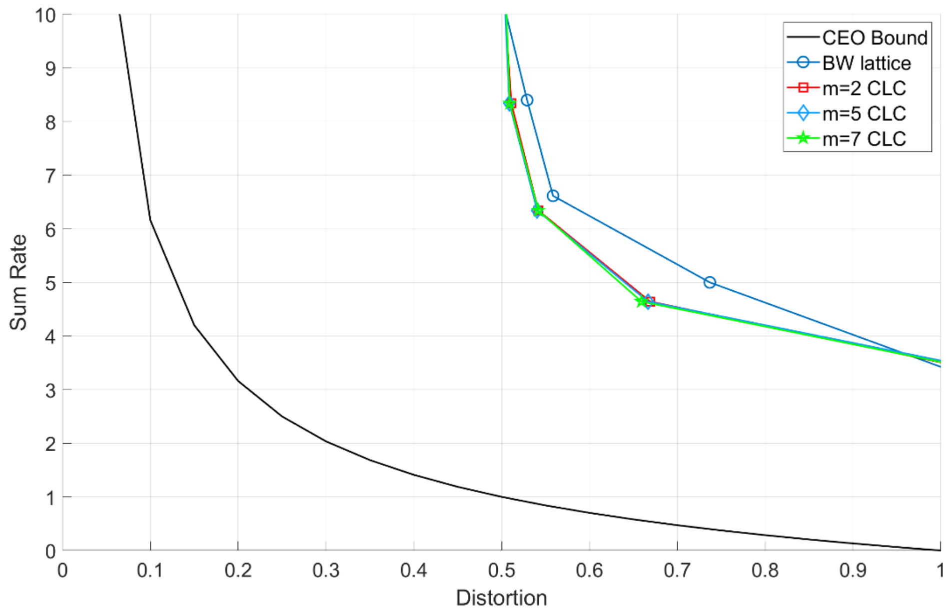


Figure 4.3 : Rate-distortion function of BW lattice and m=2, 5, 7 CLC when $\sigma_{src}^2 = 1$, $\sigma_{obs}^2 = 1$

From Figure 4.1, Figure 4.2, and Figure 4.3, it can be seen that there is a gap between the Gaussian CEO bound and the proposed system. The CEO bound assumes that an infinite number of sensors are used, while the proposed system only uses two sensors. However, the proposed system still performs well when the observation error is small. Besides, The convolutional lattice codes outperform the BW lattice in general. Moreover, convolutional lattice codes with high memories perform better than those with low memories. In addition, when the observation error becomes very large, the distance between the curve of the proposed system and the CEO bound becomes very far, which shows that this two-sensor system may not be able to deal with such cases, more sensors should be considered.

Chapter 5
Conclusion and Future Work

In this chapter, the evaluation of the proposed coding system is discussed. Strengths and weaknesses are discussed in 5.1. The relationship between memories and complexity is introduced in 5.2.

5.1 Strengths and Weaknesses of the Proposed System

As is shown in Figure 4.1, Figure 4.2, and Figure 4.3, the system performs well under the variance of observation error $\sigma_{obs}^2 = 0.01$ and $\sigma_{obs}^2 = 0.05$ with fixed $\sigma_{src}^2 = 1$. Significantly, the convolutional lattices with only 2 memories outperform the Barnes-Wall lattices, which means the complexity of the quantization algorithm is not so high. Besides, the block length is variable for different practical use. In addition, the parameter settings are based on the probability of error. Therefore, the system can satisfy different requirements by changing the parameters.

As for the weaknesses, it is obvious that the two-user system is only a primary case of the multi-terminal communication system. Considering the real communication system, the observation error and channel noise are very large. Normally hundreds, thousands, even millions of sensors are applied. The coding system is much more complicated than the two-user case.

5.2 Memories and Complexity

According to Figure 4.1, Figure 4.2, and Figure 4.3, the performance becomes better as the memory of the convolutional code increases. Therefore, the normalized second moment of convolutional lattice code with different memories $\boldsymbol{m}$ are shown below [38]:

m	$g^{(0)}$	$g^{(1)}$	$\gamma_{NSM}(dB)$
2	7	5	0.9734
3	17	13	1.0622
4	31	23	1.1233
5	75	57	1.1814
6	165	127	1.2251
7	357	251	1.2574
BW lattice			0.86

Table 5.1: Half-rate convolutional code generator polynomials for different m based on best-found convolutional lattices for NSM

Table 5.1 gives the asymptotic shaping gains based on different memories, which shows the reason why the convolutional lattices achieve better performance with m increases.

However, the complexity comes from the Viterbi algorithm, which means the Viterbi decoder has 2^m states and basically the complexity of viterbi decoding is 2^m. In addition, the quantization algorithm based on construction A requires 5 steps to lift the binary codeword to a lattice point and inverse. Therefore, the complexity of the whole quantization is $5 + 2^m$.

5.3 Conclusion

This research aims to solve the Gaussian CEO problem, which is one of the most important and popular problems in lossy DSC. The CEO problem describes recovering an underlying source with the help of several observations of the source which are corrupted by the noise. In this research, a system model of estimating a Gaussian source through two sensors with joint-source coding is proposed, combining the Wyner-Ziv coding scheme and convolutional lattice code. The Wyner-Ziv coding scheme exploits the side information in the research, which represents the subtraction of the cosets in this research. By applying the Wyner-Ziv coding scheme to the CEO problem, the source being transmitted can be further compressed. Besides, applying the convolutional lattice codes based on construction A provides more shaping gain, which outperforms the classic lattices such as E_8 lattice, Barnes-Wall lattice, and Leech lattice according to the simulation results. However, this research only focuses on the two-sensor system, a basic CEO problem model. It is believed that the system can be improved by utilizing more sensors.

5.4 Future Work

There are several directions for extending this research.

1) Extending the two-sensor model to the multiple-sensor model. As is discussed already, with the variance of the observation error increases, there is a large gap between the two-sensor system and CEO bound in the rate-distortion bound. By utilizing more sensors, more side information can be used to compress the source.

2) Considering introducing the channel and the noise. This research does not consider introducing the channel and the noise for simplicity. Different channels such as additive white Gaussian noise (AWGN) channel, binary symmetrical channel (BSC), and rayleigh fading channel can be considered in future work.

3) Applying successive Wyner-Ziv coding scheme. The successive Wyner-Ziv coding scheme [39], also known as quantization splitting, is a generalization of the source splitting technique and the rate splitting technique in channel coding. [27] shows that quantization splitting can be achieved via successive Wyner-Ziv coding, and the results show that the rate-distortion performance can approach the theoretical bound based on BSC. Therefore, applying successive Wyner-Ziv coding also deserves further study.

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