Representation of Agent's Knowledge with Logic of Awareness

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An agent refers to a system that autonomously infers under their knowledge or belief. To analyze a realistic agent's reasoning, it matters to focus on available knowledge instead of holding knowledge. For example, if we could activate inference on all holding knowledge, such as $1 + 1 = 2$, then we could calculate 6^6 in our brain. However, our actual capacity to activate inference is restricted to available knowledge. The aim of this study is to build a logical system that can describe each agent's inference in situations where multiple agents consider the opponents' knowledge and reasoning by using their available knowledge.

Epistemic logic has been used in a logical description of inference concerning knowledge or belief. The semantics of modal logic is given by possible worlds semantics called Kripke semantics. Kripke semantics is composed of a tuple $\langle W, R_i, V \rangle$, where W is a set of possible worlds which represent possibility; R_i is a binary relation on W that labeled by i; V is a valuation, which is a function that takes a proposition and returns a set of possible worlds. It is supposed that we denote by φ an arbitrary logical formula and read $K_i\varphi$ as 'an agent i knows φ,' and may write anonymously $K\varphi$. When there are some accessible worlds on R whose different truth value for φ, he/she does not know whether or not φ is true, that is $\neg K\varphi$. On the contrary, if all accessible worlds on R have the same truth value for φ, he/she knows φ, that is $K\varphi$.

In epistemic logic, an ideal inference capacity to activate inference on all holding knowledge, as I touched on at the beginning, is called logical omniscience. In ordinary logic, we employ *Modus Ponens* (MP) for logical inference that is from φ and $\varphi \to \psi$, we conclude ψ. When one knows φ and $\varphi \to \psi$, i.e., $K\varphi$ and $K(\varphi \to \psi)$, respectively, $K\psi$ would necessarily be inferred in his/her knowledge if we adopt the axiom K: $K(\varphi \to \psi) \to (K\varphi \to K\psi)$. However, such exhaustive reasoning is unrealistic for human model.

To avoid logical omniscience, logic of awareness has been explored mainly in the field of philosophy, computer science, and economics. This logic is a kind of epistemic logic incorporating the notion of awareness, and the notion has been given the interpretation such as 'he/she pays attention to things' or 'things are under his/her conscious.

Awareness logic is an early study in the field and proposed components that represent agents' state of awareness called an *awareness set* and incorporated it into epistemic logic. This logic distinguishes the knowledge that

the agents cannot use for their reasoning, called *implicit knowledge*, from that they can, called *explicit knowledge*. The former, implicit knowledge, represents unaware information. The idea is to classify knowledge into implicit or explicit knowledge according to whether an agent is aware of the proposition.

However, we argue that awareness should also affect the distinction of possible worlds in addition to propositions. When an agent is unaware of a certain proposition, he/she must not also be aware of the distinction between two possible worlds whose different valuations. This indistinction plays an important role when agents make inferences about other agents' knowledge states, as shown in our illustrative example in Chapter 3. Since in the previous study, awareness only concerns the propositions, it is not possible to fully handle knowledge and reasoning in such situations.

Therefore, in this paper, our study develops this idea, and we propose a logic $\mathcal{ALP}$ incorporating an idea that being aware of propositions itself affects an agent's width of reasoning. Specifically, we introduce course partitions of a set of possible worlds reflecting an agent's width of reasoning. We introduce the axiom system **ALP** based on **S5** and prove its soundness and completeness. Finally, we outline an idea to incorporate some epistemic actions with dynamic operators that change the state of awareness.

Our logic can work as a foundation in an analysis of game situations or formalization of interaction between each agent by communication. For example, in game theory, players make their own decisions by guessing other players' reasoning. It is based on specific decision criteria, such as the best strategy to a dominant strategy. Whether player b is aware of actions that player a can take affects b's inference to find an equilibrium. In this sense, our logic is supposed to be useful in its application to game theory.