Abstract

We study the computational complexity of Matching-Match, which is a puzzle introduced by Daiso Publishing. The Matching-Match puzzle intended for ages 6 and up, so even children can easily understand it with simple problem settings. Moreover, we can play the Matching-Match puzzle just by looking at it input without reading the instructions. However, some instances of the Matching-Match puzzle are difficult and even adults can't solve them easily.

Study on the computational complexity of puzzles can explain the difficulty of the puzzles. Furthermore, the algorithms used to solve the puzzles can be applied to a variety of industries. While there have been studies of the computational complexity of various puzzles, the Matching-Match puzzle has not been studied. Although the Matching-Match puzzle's problem setting is very simple, it is slightly different from any other puzzle. The Matching-Match puzzle can be modeled as a variant of the graph coloring problem in a natural way. However, the corresponding graph coloring problem is new and there exists no previous work on it. Thus, it is unable to apply previous research's algorithms to the Matching-Match puzzle. The Matching-Match puzzle with quite restricted instance can be solved by using an algorithm that finds perfect matching. However, familiar matching algorithms cannot find feasible solutions almost all instances because they have more restrict than the ordinary perfect matchings. Therefore, we expect that algorithms for the Matching-Match puzzle can contribute to the design of matching algorithms.

The purpose of this research is to clarify the computational complexity of the Matching-Match puzzle. To achieve this, we conducted the following.

First, we formulate a generalized the Matching-Match puzzle. the Matching-Match puzzle's input is a problem card with a pattern drawn on it (target graph) and a set of sticks with colored ends (edge set). the Matching-Match puzzle's output is to arrange the edge set in the same pattern as the target graph. To investigate the computational complexity, we modeled the input and output of the Matching-Match puzzle as a variant of the graph coloring problem.

Second, we show the generalized the Matching-Match puzzle is NP-complete. We prove it by showing a polynomial-time reduction from the 3-Partition problem. We show that the same operation as the 3-Partition problem is required to obtain the solution by designing the instances of the Matching-Match puzzle (target graph and edge set). As a result, the Matching-Match puzzle is NP-complete even if we restrict the instance of target graph to a simple pattern such as paths.

Finally, we show the Matching-Match puzzle can be solved in polynomial time by restrict the instance of the problem. If try out all possible ways to arrange an edge set for target graph, execution time becomes exponential time. Therefore, we designed the algorithm to show, it can solve in polynomial time when appropriate restrict of target graph. If the instance of target graph is a star, a complete graph, a complete k-partite graph with a constant number of colors, or a spider with leg length 2 with a constant number of colors, which problem can be solved in polynomial time. According to these results, the radius of the graph seems to be related to the complexity of the Matching-Match puzzle.