Doctoral Dissertation

Computational Measures of Game Entertainment and Reward

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Abstract

The Game Refinement (GR) theory, widely employed for assessing the enjoyment and intricacy of games, is herein examined through the lens of audience perception. This theory delves into the cognitive information processes occurring in our minds while engaging in strategy games, in adherence to the principles set forth by Newton's laws. Its central focus is on the uncertainty surrounding game outcomes, employing abstraction to dissect game components and assess the progression of game information based on achievements and game duration.

The quantification of this progression leads to the measurement of informational acceleration, akin to Newton's second law ($F = ma$), wherein informational acceleration correlates with mental force and excitement. Changes in acceleration significantly impact game dynamics, especially in high-stress scenarios, and can be rectified using "jerk" values derived from acceleration readings. In this context, a comprehensive exploration of various physical factors influencing psychological experiences in gaming is conducted.

This study prominently emphasizes the intersection of physical and psychological elements by depicting the progression of game information within our minds. By establishing reasonable informational jerk values associated with the success rate and informational acceleration of the game process, the development of game refinement theory enhances the overall player experience.

The concept of "jerk", signifying a sudden change in acceleration, is a fascinating phenomenon experienced by humans and possesses practical applications in mechanical

and engineering domains, such as elevator rides. The paper forges a connection between the logistical model of game progression and "jerk" measurements, particularly in games with incomplete information, typified as AD values (representing addiction and the propensity for repeated gameplay).

To investigate the implications of AD values concerning GR metrics and its extension, the motion in mind model, several popular card games, including Wakeng, Doudizhu, Winner, Big Two, and Tien Len, are utilized. Self-playing simulations conducted by artificial intelligence (AI) agents validate these findings with empirical data, yielding valuable insights into game design and gameplay experiences.

Players of varying skill levels may encounter and endure diverse reward frequencies denoted as N and jerk values during the progression of game information. Thus, an exploration into the interplay between the player's performance level (k), reward frequency (N), and jerk values is undertaken.

The inherent game risk, often represented as "m" is a well-acknowledged factor. To overcome these challenges, players must skillfully devise strategies. Exceptionally skilled players can navigate uncertainties with ease, characterized by a game velocity represented as $\vec{v_0} = 1 - m$.

However, players possess differing abilities, resulting in varying levels of proficiency in dealing with uncertainties. Prior research employs "k" as a measure of performance, where higher values indicate lower performance. For imperfect players, the game velocity can be calculated as $\vec{v_k} = (1 - km)\vec{v_0} = (1 - km)(1 - m)$.

In the realm of reinforcement learning, "N" often signifies the "number of steps" or "time steps", representing the interval for acquiring rewards. In games with perfect information, players possess complete knowledge of moves, while imperfect information games entail partial insights or educated guesses. In such instances, "N" can symbolize the number of actions considered during decision-making, akin to the Monte Carlo Tree Search algorithm.

In this study, the Game Refinement (GR) theory is explored from the perspective

of the audience. The theory focuses on the cognitive processes that occur while playing strategy games, emphasizing the uncertainty of game outcomes and how it affects player experience. It introduces the concept of "jerk" to measure sudden changes in acceleration in games. The study investigates the relationship between "AD" values, representing addiction and repeat gameplay, and GR metrics in various card games. Additionally, it delves into the interplay of player performance (k), reward frequency (N), and unpredictability (AD) in gaming. This research provides valuable insights into game design and gameplay experiences.

Keyword: *Game refinement theory*;*Motion in mind*;*Perfect information game*;*Imperfect information game*;*Card game*;*Jerk*;*Addiction*;

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Chapter 1

Introduction

1.1 Chapter Introduction

Game Refinement Theory, originally proposed by Professor Hiroyuki Iida in 2004, is a widely recognized framework that utilizes the game progress model to analyze the uncertainty of game outcomes [4]. It introduces the concept of the "GR value" as a standard measure for assessing the entertainment value of games, with a desirable range typically falling within $GR \in [0.07, 0.08]$. Furthermore, this theory has found applications in diverse domains such as business evaluation [9], educational assessment [10], and even in examining the evolution of popular board games [4] and [11].

In recent years, Game Refinement Theory has evolved and expanded its scope to consider the analogy between in-game actions and physical movements, shedding light on the connection between these actions and players' psychological experiences. This research aims to investigate how different ranges of in-game movements can affect players' comfort and immersion. Achieving a state of comfort during gameplay is crucial because it can lead players to enter a state akin to "flow" where they are deeply engaged and motivated to continue playing and start the next round.

1.2 Problem Statement and Research Questions

In the world of gaming, players and their opponents come together in carefully orchestrated conflicts for the sake of entertainment, all governed by a set of rules and leading to quantifiable outcomes. Traditionally, these games have hinged on crucial elements such as difficulty levels and reward systems.

Drawing from previous research on game refinement theory, we have gained insight into the information science aspect of games. Games can be seen as a journey from a state of uncertainty to one of certainty, usually signaled by achieving a goal or determining a winner. The duration of the game, in terms of time, plays a pivotal role in shaping the gaming experience and the excitement surrounding the outcomes. Games that are excessively short or overly long tend to be seen as unreliable or tiresome.

This research delves into the realm of quantifiable measures, borrowing from the world of physical analogies, to understand the inner workings of our cognitive processes while engaging in distinct gaming scenarios. It seeks to gauge the level of uncertainty and unpredictability that players encounter, all while maintaining a broader perspective on the logic of these processes. The ultimate goal of this study is to create a formal model that can be applied to enhance the design and analysis of future games. A key contribution of this research lies in its transformation of temporal aspects, such as game duration, into contextual forms, such as search space and information uncertainty. These contextual forms are leveraged to characterize the psycho-physiological processes that underlie our interaction with information progression during gameplay.

Moving forward, the discussion explores the concept of "jerk" within the context of acceleration and its multifaceted applications as a parameter for comfort and safety in various domains. This segues into the examination of game refinement theory, with its central focus on the rate of change of information speed, known as the GR value, within games. This theory provides insights into how game progress unfolds and how players make choices, with a specific emphasis on understanding uncertainty and success rates.

The narrative then shifts its attention to card games, particularly those characterized by incomplete information. These games are known for their brevity, replayability, reliance on chance, and strategic depth. They have enjoyed a surge in popularity in the digital era, challenging players to navigate complex game states, remember their opponents' moves, and make probabilistic assessments.

The core objective of this study is to explore the addictive nature of these card games. Here, "addictive" refers to the intense enthusiasm and motivation that players exhibit when repeatedly engaging in these games, rather than indicating a psychological or substance-related addiction. Through simulations, the research investigates how different dynamics of game progress (GR) and unpredictability (AD) values impact player experiences. It also examines how varying levels of AI skill and player emotional responses, such as engagement and surprise, manifest in different rule scenarios. The ultimate aim is to identify potential metrics related to these emotional experiences, all grounded in the context of card games.

Furthermore, this study delves into a comparison of the psychological states and immersion levels of players with varying skill levels when confronted with games featuring different reward frequencies. It seeks to understand how reward frequency becomes a controllable or enriching factor as players refine their skills and explore various facets of gameplay. This comprehensive examination uncovers the interconnectedness of these elements, highlighting how they mutually influence and continually enhance the gaming experience.

Consequently, this thesis aims to address the following research questions:

- Apart from acceleration, what role do other physical quantities play during gameplay?
- How does motion in mind processing of movement function when players train or explore games of varying complexity?
- For players with different skill levels, how do players perceive the game when faced

with different reward frequencies?

1.3 Research Objectives

The second to fourth chapters are dedicated to addressing three specific objectives outlined in the introduction, providing comprehensive answers and explanations for the research questions posed, the research objectives are introduced, highlighting the need to expand the game refinement theory and investigate the role of various physical quantities in the gaming process.

Chapter 2 focuses on the first objective, aiming to extend the existing game refinement theory. The game process is developed, starting with the first derivative of acceleration, jerk. The expanded AD value is defined concerning the GR value. Concepts and calculation methods of AD are presented, along with an exploration of how speed, acceleration, and jerk influence the game progression and subsequently impact the player. Visualization techniques are employed to illustrate the game's progress.

The second objective is addressed in Chapter 3, which delves into the implications of jerk on the measure of game entertainment. The game information process is established, and solutions for each physical quantity are identified. Card games serve as a medium to study player abilities in the face of varying game complexities, observing changes in the AD value (jerk) and discerning its impact on the gaming experience. The conjecture that jerk measures unpredictability and surprise in a physical environment is explored and validated using card game data. Findings reveal that strong players exhibit smaller AD values for complex games, demonstrating adaptability and control over the gaming environment.

The third objective is explored in Chapter 4, focusing on the interplay between game design (specifically, reward frequency), player abilities, and AD (motion in mind). The relationship between individuals and the overall gaming experience is analyzed. Insightful correlations between game design elements, player capabilities, and the AD

value are uncovered, providing a comprehensive understanding of how these factors collectively contribute to the gaming experience.

In our study, we aim to delve into the impact of physical quantities beyond acceleration on the gameplay experience. We will investigate the cognitive processing of in-game movements and how they influence players' performance and engagement, especially in the context of games with varying levels of complexity. The research concludes by summarizing the key findings and contributions of each chapter, emphasizing the broader implications of the study for game refinement theory, player experience, and game design.

1.4 Thesis Structure

This thesis comprises six main chapters, given in the following structure.

- **Chapter 1: Introduction**

  The primary aim of this chapter is to provide an overarching view of the research, encompassing key definitions, the interrelationships among the core keywords within the study, and a concise historical context of the relevant domain. This chapter is instrumental in elucidating the central problem that the research endeavors to address. Additionally, the Introduction chapter encompasses the articulation of research questions, along with the objectives and significance of the study. Finally, this chapter concludes with an outline of the dissertation's structure.

- **Chapter 2: Related Works**

  This chapter reviews the theoretical background relevant to the present study and provides an overview of the latest research in the field. The first section of this chapter briefly introduces the subsequent literature review, followed by several sections that delve into necessarily related works, such as Game refinement theory, motion in mind, and the game models that will be employed in the subsequent analysis and so on.

- **Chapter 3: A Computational Game Experience Analysis via Game Refinement Theory.**

In this chapter, dynamic variation and process visualization are utilized through interactive games involving basketball and soccer to extend and elaborate upon the game refinement (GR) theory. The exploration begins with acceleration and extends to jerk, allowing players to not only experience forces (represented as attractiveness within the game) but also perceive changes in those forces (variations in gravitational attraction). Here, the focus is not solely on the mean value of acceleration but also on its fluctuations, emphasizing the importance of understanding both aspects.

- **Chapter 4: Implications of Jerk's on the Measure of Game's Entertainment: Discovering Potentially Addictive Games**

In this chapter, we primarily use card games as a medium to compare players of different skill levels horizontally and vertically. When players face games of varying complexities, they have the opportunity to enhance their skills or explore different strategies during training. We delve into the players' distinct perceptions throughout this process and examine the relationship between physical attributes in the information process and their psychological perceptions.

- **Chapter 5: The Impact of Performance Degree on Players: Exploring the Dynamics of Player Engagement and Enjoyment in Game Process**

In this chapter, we use player skill level as a baseline to explore games of varying difficulty levels. When facing both incomplete information games and complete information games, different games exhibit varying reward frequencies, leading to differences in player immersion and the addictive nature of repeated play. We examine the relationships between player skill level (k), reward frequency (N), and the degree of addiction (AD) experienced by players.

- **Chapter 6: Conclusion**

The last chapter is the conclusion of the dissertation. It concludes the whole dissertation relative to the main aim and objectives of the dissertation. Some potential future works are also outlined.

Chapter 2

Related Works

2.1 Chapter Introduction

This chapter reviews the theoretical background related to this research. It introduces the latest research results in this field, which include the influence of game refinement theory and its new perspective, the impact of jerk during the game process, motion in mind, the methods of data collection we used, and so on.

2.2 Game Refinement Theory and Motion in Mind Concept

The game progress model of game uncertainty is based on early work by [4]. It has been previously applied to measuring the design sophistication in domains of business [9], and education [10], and acts as a tool for exploring the evolution of popular board games [4] [11]. The GR values for most popular games are located in a reasonable zone of $GR \in [0.07, 0.08]$. From the player's viewpoint, the information on the game result is an increasing function of time (the number of moves in board games) t. Here, the information on the game result is defined as the amount of solved uncertainty (or information obtained) $x(t)$, as given by (2.1). The parameter n (where $1 \leq n \in N$) is the number of possible options and $x(0) = 0$ and $x(T) = 1$.

$$x'(t) = \frac{n}{t}\, x(t) \tag{2.1}$$

$x(T)$ stands for the normalized amount of solved uncertainty. Note that $0 \leq t \leq T$, $0 \leq x(t) \leq 1$. Equation (2.1) implies that the rate of increase in the solved information $x'(t)$ is proportional to $x(t)$ and inversely proportional to t. Solving (2.1), (2.2) is obtained.

$$x(t) = \left(\frac{t}{T}\right)^n \tag{2.2}$$

It is assumed that the solved information $x(t)$ is twice derivable at $t \in [0, T]$. The second derivative of (2.2) indicates the accelerated velocity of the solved uncertainty along the game progress, which is given by (2.3).

$$x''(t) = \frac{n\ (n-1)}{T^n}\ t^{n-2}\ |_{t=T} \quad = \frac{n\ (n-1)}{T^2} \tag{2.3}$$

Accelerated velocity implies the difference in the rate of acquired information during the game's progress. Then, the acceleration motion or free-fall motion in mind, a is given by (2.4). In the domain of board games, a is approximated as (2.5), where $B(n = \sqrt{B})$ and D stand for the average number of possible moves and game length, respectively.

$$x(t) = \frac{1}{2} a t^2 \tag{2.4}$$

$$a = \frac{n\ (n-1)}{T^2} \approx \frac{B}{D^2} \tag{2.5}$$

Figure 2-1 illustrates a model of move candidate selection based on skill and chance. This illustration shows that skillful players would consider a set of fewer plausible candidates (say b) among all possible moves (say B) to find a move to play, and that there is a core part of its original game with branching factor B. The core part is a stochastic game with a smaller branching factor, b since it is assumed that each among b candidates may be equally selected.

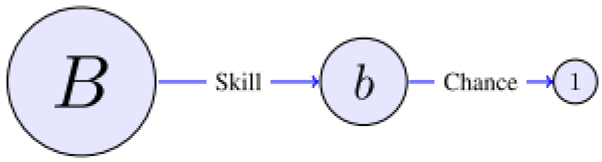

Figure 2-1: An illustration of move selection model based on skill and chance

2.2.1 Motion in mind model

In operant conditioning, a variable-ratio schedule is a schedule of reinforcement where a response is reinforced after an unpredictable number of responses [12] [13]. This type of schedule creates a steady, high rate of reaction. Stochastic games such as gambling and lottery games are typical examples of a reward based on a variable-ratio schedule. Mind sports [6] games such as chess and Go were also essentially stochastic games while applying the move selection model [1]. This situation implies that a reward of variable-ratio reinforcement schedule characterizes a game [14].

Therefore, the reward function of a game can be characterized by defining the reinforcement schedule's variable rate (denoted as $VR(N)$). Then, velocity v (win rate) and mass m (win hardness) of the motion in mind model are given by (4.11), where $0 \leq v \leq 1$ and $0 \leq m \leq 1$. As such, N was used to measure the frequency of getting rewards, where the player can get a reward in a total average of N steps [15]. Let v_0 be the reward function over various masses for the perfect player as given by (4.12), which corresponds to the objectivity of play. Note that there is a distinctive computation of the v for the board and scoring games [1]. The success rate is defined as $v = \frac{G}{T}$ for scoring games (such as basketball, soccer, etc.), where G and T are the average successful score and the total shoot attempts. Meanwhile, the success rate is defined as $v = \frac{B}{2D}$ in a board game (i.e., Chess, Shogi, etc.), where B is the average branching factor, and D is the average game length.

$$v = \frac{1}{N} \quad \text{and} \quad m = 1 - v, \quad \text{where} \;\; 1 \leq N \in \mathbb{R} \tag{2.6}$$

$$m + v_0 = 1, \quad \text{where} \;\; 0 \leq m \leq 1 \;\; \text{and} \;\; 0 \leq v_0 \leq 1 \tag{2.7}$$

The notion of energy conservation had been proposed by [16] as a potential measure of engagement, where the formulation of momentum in the game ($\vec{p_1}$) and potential energy in mind (E_p) are given by (4.13) and (4.14), respectively. Then, the momentum in mind ($\vec{p_2}$) can be derived based on the conservation of energy in mind, given by (4.15), which is associated with the measure of player's engagement, given by (4.16).

$$\vec{p_1} = mv \tag{2.8}$$

$$E_p = 2mv^2 \tag{2.9}$$

$$E_p = \vec{p_1} + \vec{p_2} \tag{2.10}$$

$$\vec{p_2} = E_p - \vec{p_1} = 2m^3 - 3m^2 + m \tag{2.11}$$

Applying (4.16) while assuming $\vec{p_2} = mv_2$, the subjective velocity v_2 is given by (4.17). Let $v_k(m)$ be a reward function over various m for a player with ability parameter k. Then, the relation is generalized as v_k using a parameter (say k where $0 \leq k \in \mathbb{R}$) that is the nature of the game under consideration, as shown in (4.18). The ability parameter k stands for players' strength in the competitive game context or error tolerance in the social or non-competitive context. For example, there is no error tolerance for the perfect player v_0. Note that objectivity and subjectivity perspective enables us to deepen the understanding of engagement and addictive mechanisms in games [16]. Thus, the objective velocity (v_0) and subjective velocity (v_k) were determined.

$$v_2 = 2m^2 - 3m + 1 = (1 - 2m)(1 - m) \tag{2.12}$$

$$v_k = (1 - km)\ v_0, \quad \text{where} \;\; 0 \leq k \in \mathbf{R} \tag{2.13}$$

The notion of potential energy in mind was originally discussed by [1] and its formula is given by (4.14). The notion of velocity is derived from the reinforcement schedule $VR(N)$ with frequency N, so we call objective reinforcement (E_0) for the potential energy in mind of the perfect player (v_0). Otherwise, we call subjective reinforcement (E_k) for the potential energy in the mind of other players (v_k). A game would produce its potential energy in the field of play (hence we call it potential energy of play) by which people would feel engagement or reinforcement.

In behavioral psychology, the term "reinforcement" refers to an enhancement of behavior. This term was used as a positive interpretation, i.e., greater reinforcement gives people a more substantial interest in staying in the event under consideration. In the game context, reinforcement depends on the player's ability. The potential energy of play (E_k) is given by, $E_k = 2mv_k^2$ which is denoted as *s*ubjective reinforcement. For the perfect player or game theoretical reward ($k = 0$), denoted as *o*bjective reinforcement E_0.

Figure 4-1 illustrates the objective and subjective reinforcement when $k = 3$. The reward function (v_k) represents a player's model or his/her sense of value. When assuming $k > 3$, $v_k < 0$ holds at $m = \frac{1}{3}$ where the objective reinforcement is maximized. This situation implies the learning context's most comfortable (peak of E_0). Therefore, it is highly expected to have $k \leq 3$. Furthermore, a board game like Go ($m = 0.42$) is still not yet solved; thus, $2.38 < k$ is expected to hold.

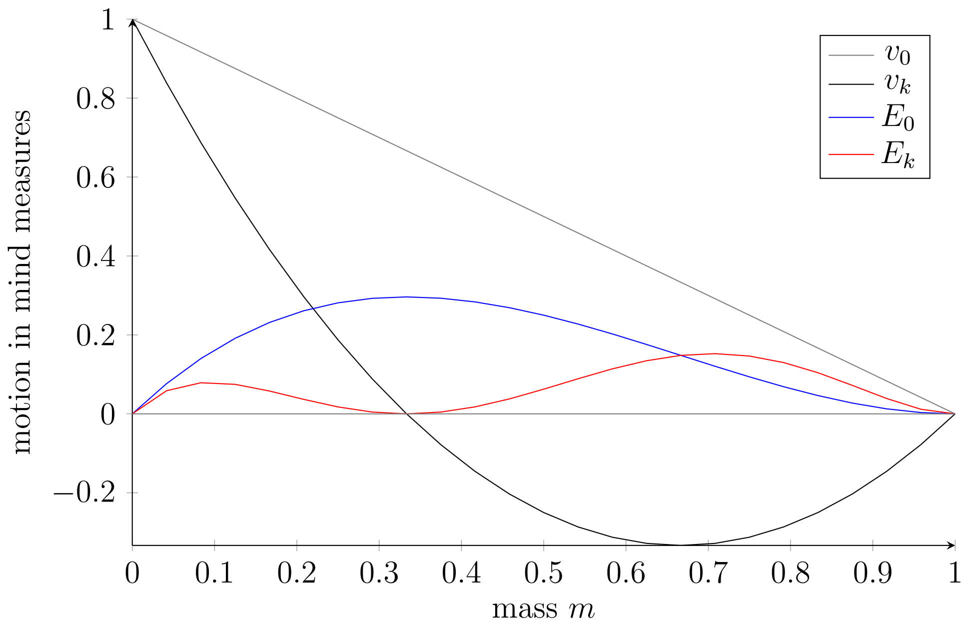

Figure 2-2: Objective and subjective reinforcement when $k = 3$

2.2.2 Jerk and comfort in mind

Two processes with the same GR-value at the end of the game information progress may have different instantaneous GR-value tendencies. For example, two basketball teams have the same successful shootings and total shot attempts. So their GR values are the same when the game is finished. However, each team felt different tendencies to get scores. The team with a stable scoring process is predictable, and vice versa. As such, each team's force was not only felt but also the change of the force. In physics, acceleration can be felt in motion and the feeling of jerk [17].

The third derivative of (2.2) indicates the change of accelerated velocity (or jerk [17]) of the solved uncertainty along the game progress [18], which is given by (2.14). Hence, the motion with constant jerk j is given by (2.15), where it is approximate in the domain of board games as (2.16).

$$x'''(t) = \frac{n(n-1)(n-2)}{T^n} t^{n-3} \mid_{t=T} = \frac{n(n-1)(n-2)}{T^3} \tag{2.14}$$

$$x(t) = \frac{1}{6} j t^3 \tag{2.15}$$

$$j = \frac{n(n-1)(n-2)}{T^3} \approx 3 \frac{B}{D^3} \tag{2.16}$$

Table 2.1 shows the measures of game refinement for board games. For sophisticated board games such as Chess, Shogi, and Go, it is assumed that there exists a reasonable zone for the acceleration (a) and jerk (j), which is between $0.07-0.08$, and $0.045-0.06$, respectively.

Table 2.1: Measures of game refinement for board games

	B	D	$\sqrt{a}$	$\sqrt[3]{j}$
Chess	35	80	0.074	0.059
Shogi	80	115	0.078	0.054
Go	250	208	0.076	0.044

We show, in Figure 2-3, an illustration of solved information over time, with our interpretation. The cross-point between the acceleration curve and jerk curve is the point where the maximum amount of achievement is greater than the discomfort (t_1), after t_1, the discomfort will be larger than achievement. The cross-point between velocity and jerk is the point where effort is greater than the discomfort (t_2), and the cross-point between velocity and acceleration is the point where effort is more excellent than achievement(t_3). The cross-point interval ensures a reasonable zone for game length [19].

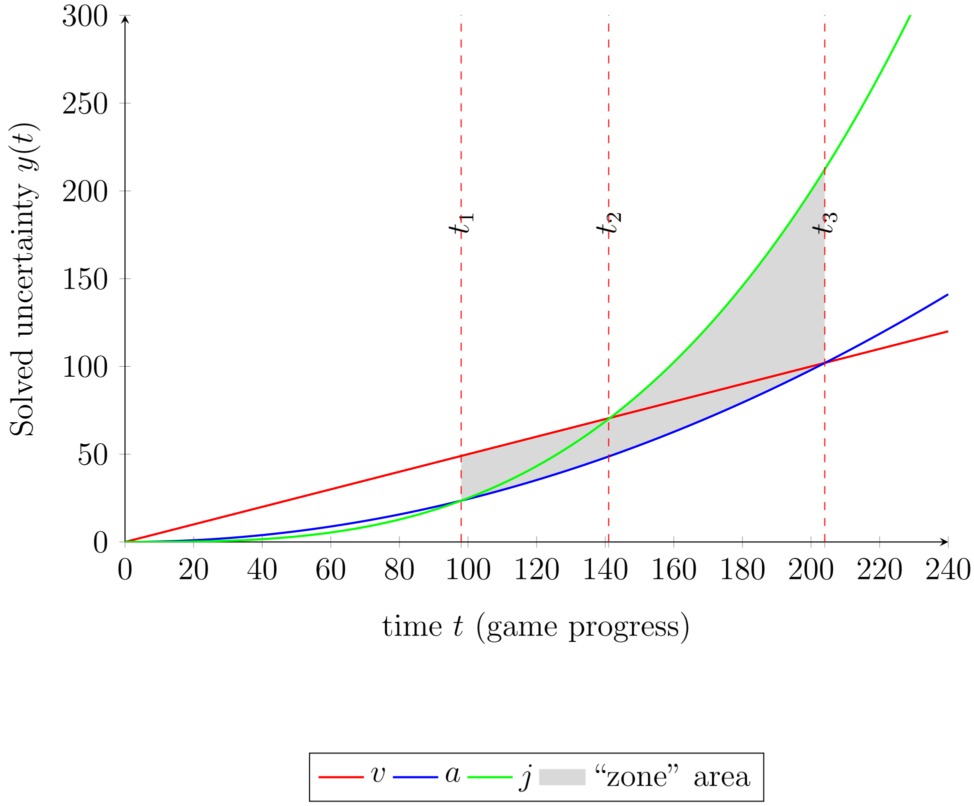

Figure 2-3: The cross point between the line with velocity v, curve with acceleration a and curve with jerk j. t_1; t_2 and t_3 represent the bound for effort, achievement, and discomfort, respectively.

2.3 Card Games as Testbed in This Study

Card games are a diverse category of recreational activities that utilize a standard deck of playing cards or specialized cards designed for a particular game. These games have been enjoyed for centuries and are popular worldwide due to their simplicity, strategic depth, and social nature. Here are some key points to understand about card games:

1. **Variety:** Card games encompass a wide range of options, from classic games like Poker, Bridge, and Solitaire to modern and innovative titles. They can be broadly categorized into several types, including trick-taking games, shedding games, and

matching games.

2. **Deck of Cards:** Most card games use a standard deck of 52 playing cards, which is divided into four suits: hearts, diamonds, clubs, and spades. Each suit contains thirteen ranks, from Ace to King. However, some card games may require additional decks or unique cards with specific designs.

3. **Objective:** The objectives of card games can vary significantly. Common objectives include achieving a certain hand or pattern of cards, accumulating points, or being the first to get rid of all cards. The specific goal depends on the rules of the game being played.

4. **Strategy:** Many card games involve a strategic element, where players make decisions based on the cards they hold and the actions of other players. Strategic thinking, planning, and adaptability are often key factors in winning card games.

5. **Popularity:** Card games have stood the test of time and remain popular across generations. They are played in homes, casinos, and online platforms, catering to a wide range of players.

6. **Rule Variations:** Card games often have multiple variations and house rules. Players may adapt the rules to suit their preferences or regional traditions.

7. **Educational Benefits:** Card games can help develop various cognitive skills such as memory, concentration, and strategic thinking. They are also used for educational purposes in mathematics and probability lessons.

Whether you're looking for a casual pastime or a competitive challenge, card games offer a rich and diverse world of entertainment for players of all ages and skill levels.

Shedding games are a category of card games with the primary objective of quickly getting rid of all the cards in hand. This is typically achieved by matching the cards played to previously played cards or by following specific card patterns. Players aim to empty their hands as rapidly as possible while adhering to the established rules. One of the most widely played and representative shedding card games in our research is "DouDizhu".

2.4 Perfect Information Game and Imperfect Information Game

2.4.1 Perfect information game

Perfect information games are a category of games where all players have complete knowledge of the game state at all times.

1. **Full Knowledge:** Players have access to all relevant information, including the current state of the game, the available options, and the history of moves made by all players.

2. **No Hidden Information:** There are no hidden cards, pieces, or elements that players are unaware of. Everything is open and visible to all participants.

3. **Examples:** Classic examples of perfect information games include Chess and Tic-Tac-Toe, where both players have complete knowledge of the board and all the pieces on it. In these games, success is determined solely by strategy and skill.

4. **Solved Games:** Some perfect information games have been fully solved, meaning that optimal strategies have been determined, and the outcome of the game is known with perfect play. Chess, for example, has not been fully solved, but certain endgames have been.

2.4.2 Imperfect information game

Imperfect information games, on the other hand, are games in which players do not have complete knowledge of the game state. In these games:

1. **Hidden Information:** Players lack complete information about the state of the game. This can include hidden cards in card games, hidden movements of opponents in board games, or uncertainty about the environment in video games.

2. **Bluffing and Uncertainty:** Imperfect information games often involve elements of bluffing, psychology, and decision-making under uncertainty. Players must make choices based on limited information and try to anticipate their opponent's actions.

3. **Examples:** Poker is a classic example of an imperfect information game. In Poker, players have hidden cards (hole cards), and they must use betting and psychological tactics to make the best possible decisions.

4. **Complexity:** Imperfect information games can be more complex and challenging due to the added element of uncertainty. Success in these games requires not only strategic thinking but also the ability to read and manipulate opponents.

5. **AI Challenges:** Creating artificial intelligence (AI) agents to excel in imperfect information games, like Poker, has been a significant challenge in the field of AI research.

In summary, perfect information games involve complete and open knowledge of the game state. In contrast, imperfect information games introduce elements of hidden information and uncertainty, requiring players to make decisions based on incomplete data and psychological tactics. Each type of game offers unique challenges and opportunities for strategy and skill development.

2.5 Reward Frequency in Reinforcement Learning

Reinforcement Learning is a subfield of machine learning where an agent learns to make decisions by interacting with an environment. The agent takes actions in the environment to maximize a cumulative reward signal over time.

Reward frequency is a critical aspect of reinforcement learning and refers to how often an agent receives feedback or rewards from the environment. It plays a significant role in shaping the learning process and affecting the agent's behavior.

2.5.1 Influence of reward frequency in gaming

In a gaming context, the concept of reward frequency takes on specific meanings depending on the type of game being played.

Scoring games

In scoring games, reward frequency is a measure of how many attempts or actions a player needs to make to successfully score points or achieve a desired outcome. For example: In basketball, reward frequency might represent the number of shots a player needs to take before successfully making a basket.

This type of game typically focuses on players achieving points by completing specific tasks, earning scores, or reaching particular objectives. The primary goal in scoring games is usually to accumulate as many points as possible, with the score often serving as a measure of a player's success. Classic examples of scoring games include brick-breaker games, shooting games, and racing games.

In these cases, a higher reward frequency may suggest that success is challenging to achieve, requiring multiple attempts, while a lower reward frequency indicates that success is easier to attain.

Choosing game

In board games, reward frequency refers to the number of available options or choices a player has when making decisions related to a specific location or position on the game board.

Choosing games emphasizes players making decisions during gameplay that impact the progress and outcomes of the game. Players often need to balance different options to select the most suitable strategy for achieving success in the game. These games may include strategy games, role-playing games (RPGs), and adventure games.

In these scenarios, a higher reward frequency means that the player has more strategic options and flexibility when making decisions related to a particular position on the game board. Conversely, a lower reward frequency implies limited choices and potentially more straightforward gameplay.

2.6 Chapter Conclusion

This chapter thoroughly examines several facets of gaming, game theory, card games, and reinforcement learning. These areas of study collectively contribute to our comprehension of the mechanics behind gameplay, the decision-making processes involved, and the profound impact of rewards on player behavior.

Our exploration commenced with an in-depth look at the diverse universe of card games. We highlighted their rich variety, the standard deck of cards used, and the overarching objectives that players strive to attain. Card games, whether they are classics or contemporary creations, offer a wellspring of entertainment, strategic thinking, and opportunities for social interaction that cater to players of all backgrounds.

Subsequently, we delved into the distinctions between perfect and imperfect information games. Perfect information games, exemplified by venerable classics like Chess and Tic-Tac-Toe, revolve around complete knowledge and transparent information, where success is primarily determined by strategic prowess and skill. In contrast, imperfect information games, such as Poker, introduce elements of hidden information and unpredictability, wherein psychology and decision-making become paramount.

Expanding our horizons even further, we ventured into the domain of reinforcement learning. In this subfield of machine learning, agents acquire the art of decision-making through iterative interactions with their environment, with the ultimate goal of maximizing cumulative rewards over time. The concept of reward frequency emerged as a pivotal factor that shapes the learning process and molds player behavior in a plethora of gaming contexts.

In summary, our comprehensive exploration of gaming, game theory, card games, and reinforcement learning casts light on the multifaceted landscape of decision-making, strategic thinking, and the role of rewards within the gaming world. These insights offer invaluable perspectives for both gaming enthusiasts and researchers in fields such as artificial intelligence and psychology. As our journey into the dynamics of games and decision-making continues, we gain a deeper appreciation for the intricacies and nuances that render gaming a captivating and ever-evolving realm.

Chapter 3

A Computational Game Experience Analysis via Game Refinement Theory

This chapter is based on the integration, update, and abridgment of the following publication:

- N. Gao, Y. Gao, Z. Zhang, M. N. A. Khalid, and H. Iida, "A computational game experience analysis via game refinement theory, Telematics and Informatics Reports", in Telematics and Informatics Reports, vol. 9, pp. 100039, 2023.

3.1 Chapter Introduction

Activity continuum was previously associated with the notion of spontaneity and freedom [20], providing the foundation of various game categorization and taxonomy [1, 21]. *Games* may be roughly described as a system in which one or more players engage in manufactured conflicts for the sake of amusement or enjoyment, which are regulated by established rules and result in a definite, quantifiable objective or outcome. The primary elements of the game category were traditionally key difficulties and gaming mechanisms [21].

"Huizinga" [22] described *play* as an important activity in creating civilizations and cultivating culture, which had evolved to eventually impact people's values [23]. A game would move from an uncertain to a certain state from the viewpoint of information sciences (when the goal was reached or the winner was determined). The size of the time (or rather, length of the play) played a role in describing the gaming experience and the outcome anticipation (too short or too lengthy were judged to be unreliable or tiresome, respectively) [1].

Players will have varied psychological sensations depending on the level of the game's information progression. When a player has the upper hand, they feel at ease since their risks are decreased, and they can anticipate winning. They will experience anxiousness as a result of being on the losing end. As a result, there should be a variety of conditions (e.g., success rate) to balance situations of advantage (relaxation) and disadvantage (anxiety), in which both the player and the opponent are engaged and comfortable (connected with the 'attractiveness' area). As a result, various success rates (speed or velocity) give an objective perspective and sense of the game throughout, which might be valuable and depicted based on our feelings.

Each game's progress is meant to give a range of utilitarian goals that indicate the amount of effort, success, and suffering (unpredictability) associated with the game's outcomes. Quantitative measures may be used to explain the utility of game information processes, and the mechanisms of game information processes can be discovered. The game refinement (GR) theory has been studied as a measure of complexity based on the "principle of seesaw" game, where the amount of option variants remained constant throughout the games [4]. Since then, GR has been used to determine the sophistication and magnitude of expected attractiveness in both games [11, 24, 25] and non-game contexts [9, 10]. According to this research, the second derivative of game information progress may be used to evaluate the attraction of $F = ma$ in our minds in the same manner that velocity can be used to estimate the game's pace.

The evolution of game information is analogically represented with physical quantities in this study, where the average acceleration (a derivative of the speed of solving uncertainty in games; associated with attractiveness) and jerk (a derivative of expected attractiveness in games; associated with unpredictability) can be measured. Basketball and soccer were chosen as the case study for such an analysis. Furthermore, the research looked at alternative representations of game information progress using game tree structure and value functions to simulate the most exciting and appealing situations during gameplay. By connecting to the well-known *Flow* theory, this visualization gave realistic estimations of the underlying play experience of the game-playing processes.

The quantities measured by physical analogies relevant to our mind's motions are explored concerning distinct process mechanisms connected with different playing circumstances. Furthermore, the degree of solved uncertainty and unpredictability is determined, as well as the extended perspective's logic. This research attempts to formalize the game-playing experience into a formal model that may be used to improve the design and analysis of future games. The study's primary contribution is the transformation of temporal forms (e.g., game duration) into contextual forms (e.g., search space and information uncertainty), in which the interaction of information progress is utilized to characterize psychophysiology processes.

The organization of this chapter is as follows. Section 3.2 introduces quite a little literature from different perspectives of game-playing experience measures, such as energy expenditure, perceived behavioral control, and social violence. Section 3.4 provides an overview of the GR theory and its recent developments. Section 3.5 provides a few example applications of the GR theory to scoring and choosing games. Section 3.6 expands the game refinement theory using physical quantities for two-sided dynamics, scoring dynamics, and game process visualization. Finally, Section 3.7 concludes the chapter.

3.2 Measurement of Play in Games

In this section, a brief review of the related works on player experiences that were observed in a game setting was outlined and described in Section 3.2.1. Then, a brief review of the previous works on player psychology was also described in the subsequent Section 3.3.

3.2.1 Player Experiences in Games

Activity continuum exists by extending play to games, and then into sports, characterized by decreasing spontaneity and freedom into regularization, orientation, habituation, and institutionalization (play-game-sport continuum) [20]. Such a continuum grounded the competitive activities into equal chances (agôn), unequal chances (alea), pretense (mimicry), and pursuit of giddiness (ilinx) as the classification system of games. Nevertheless, it is apparent that play, like games and sports, represents a specific kind of experience [20].

Several game-playing experience measures have been studied, such as energy expenditure, perceived behavioral control, and social violence. Based on the behavior of experienced "Dance Dance Revolution" (DDR) players and inexperienced players, it was found that different skillful players have different energy expenditures. Participants with greater playing experience can work at higher intensities, promoting greater energy expenditure [26]. Data collected using a structured questionnaire from 1584 university students in Malaysia with different backgrounds shows that perceived behavioral control has the most substantial influence on actual use. Other variables found to influence actual usage include perceived enjoyment, subjective norms, attitude, perceived enjoyment, and flow experience [27].

In the previous research about assessing the potential impact of playing video games, particularly violent games, the Game Engagement Questionnaire (GEQ) was developed using classical and Rasch rating scale models. The GEQ provides a psychometrically robust measure of levels of engagement specifically elicited while playing video games; thus, showing promise for future research examining risk and protective factors for negative game impact [28, 29]. Meanwhile, another study was conducted by [30] to potentially influence and evaluate quality of experience (QoE) in gaming service providers and provide expert evaluation in a passive viewing-and-listening scenario or interactive online/cloud gaming scenario; have roughly classified into three classes of influence factors: (a) human, (b) system, and (c) context. The author also highlighted that such QoE measurements would be invaluable in adjustable game input/output and mechanics, provide better realism, and quantify interaction degradation.

Some researchers investigate whether individual difference influences the characteristic experience of game playing [31]. The results suggest that physically aggressive people tend to engage in a more aggressive playing style after controlling the differences in gender and previous gaming experience. Implications of these findings and directions for future studies are discussed. Another study combines several 3-dimensional spatial technologies to enhance players' gaming experience [32], where the result showed that the play experience of different player levels and performance could be optimized, especially in games with 3-dimensional tasks.

From these perspectives, when engaged in any game experience, players' behavior (i.e., energy consumption) can measure skill level and violent tendencies. Thus, these studies showed the prospect of visualizing and measuring quite a few abstract factors during the game-playing experience. From another perspective, gravitational acceleration (like 4g and 5g) is essential in the context of the roller coaster ride to leave an impactful experience in people's hearts, and minds [1]. In essence, this study is interested in providing an alternative measure of game-playing experience by formalizing information progress of games via the game refinement theory to describe psycho-physiological processes. Recently, topics such as autonomous driving and gamification have become more and more popular. Vehicular automation involves the use of mechatronics, artificial intelligence, and multi-agent systems to assist the operator of a vehicle (car, aircraft, watercraft, or otherwise). These features and the vehicles employing them may be labeled as intelligent or smart vehicles or transportation. A vehicle using automation for difficult tasks, especially navigation, to ease but not entirely replace human input, may be referred to as semi-autonomous [33], whereas a vehicle relying solely on automation is called robotic or autonomous [34].

3.3 Player Psychology in Games

In this section, I discussed Player Psychology in Game, and how research by Johannes demonstrates the effectiveness of digital biomarkers in gaming for assessing social anxiety, with behaviors such as player accuracy and movement patterns indicating this condition. Additionally, Tng discovered that avatar identification is mediating in internet gaming disorder (IGD). Dang's analysis of temporal gaming patterns aimed to comprehend learner motivation, while Sevcenko revealed that in-game metrics can reliably forecast cognitive load. Furthermore, Harris established a connection between trait competitiveness and problematic online gaming.

A study conducted by [35] explored game-based digital bio-markers' effectiveness for social anxiety assessment, embedded in a gaming task, based on a player's interaction with and around a non-player character. The results of our study show that social anxiety manifests in several behaviors in-game, similar to avoidance behaviors from the physical world, useful as digital biomarkers. In particular, social anxiety was reflected in player accuracy and the movement paths (i.e., safety behaviors where the player stays farther away from non-player characters). Furthermore, this study presents the influence of customized avatars and camera perspectives on how much social anxiety is expressed in a gaming task. Meanwhile, [36] presents a study that examines the association of determinants, such as motivation of gaming and identity of in-game avatar, relative to the internet gaming disorder (IGD) among multiplayer online battle arena (MOBA) gamers. The study found the underlying determinants of IGD where avatar identification was found as a significant mediator, although such psychological motivations of gaming reflect unique meanings and consequences. For instance, youth must determine a more positive self-concept to strengthen greater self-acceptance and voluntary integration of the 'fatigue' system by game developers to discourage excessive time spent on online gaming indirectly.

Quitting and procrastination are evidence of exercising self-regulation in learners and provide the potential to understand their motivation by identifying and analyzing less desirable behaviors in learning. Such temporal patterns in gaming behaviors have been analyzed by [37] on an intelligent tutoring system in detecting motivation. The study's findings suggest that gaming behaviors indicate partial cognitive engagement and session-level influences on learning motivation. In addition, self-regulated behavior theories might infer temporal fluctuations in the motivations (such as ego-depletion and task-switching). Such information could be capitalized for responsive cognition and motivational dynamics. Another study conducted by [38] found that simple in-game metrics that describe the behavior and performance in an emergency simulation game can reliably assess and predict cognitive load based on a theory-driven approach. The result of the study indicates that increased difficulty causes significantly higher subjective ratings of cognitive load, accompanied by significantly poorer performance. Moreover, the early stages of gameplay are more informative and predictive for later gameplay outcomes than an aggregated score accumulated over a more extended time.

The development of problem video gaming was prominent in high-risk online games (i.e., massively multiplayer online first-person shooter games) where direct competition is the key motivator for participating in such a game. [39] had examined the relationship between three facets of trait competitiveness and problem online video gaming, where the study found that competitiveness motivates online gaming, suggesting that inherent dominant competitive tendencies may increase the risk of problematic participation in competition-centered online video games. Meanwhile, [40] conducted an exploratory analysis of the role of skill versus chance in determining outcomes in skill-based gaming machines (SGMs[1]) to assess and measures player understanding of gaming activities and erroneous beliefs of gambling. The study suggested that SGMs appeal to gamblers and new cohorts, especially younger players. At the same time, the use and understanding of SGMs are likely to change over time as players adapt to the presence of these new gambling activities and potentially lead to greater illusions of control in individuals, overconfidence in understanding how outcomes are determined, and greater gambling problem severity.

3.4 Game Refinement Theory and Its Development

In this section, the GR theory is revisited, where its original formulation from the game progress model of outcome uncertainty is described in Section 3.4.1. Then, a new perspective from the third derivative of the game progress model is introduced and described in Section 3.4.2.

3.4.1 Game Refinement Theory and Its Development

The game progress model of game uncertainty is based on early work by [4]. It has been previously applied to measure the design sophistication in domains of business [9], and education [10], and act as a tool for exploring the evolution of popular board games [4, 11]. The GR values for most popular games are located in a reasonable zone of $GR \in [0.07, 0.08]$. From the player's viewpoint, the information on the game result is an increasing function of time (the number of moves in board games) t. Here, the information on the game result is defined as the amount of solved uncertainty (or information obtained) $x(t)$, as given by (5.1). The parameter n (where $1 \leq n \in N$) is

[1]Gaming machine with a skill-element within the random mechanisms of electronic gaming machines (EGMs)

the number of feasible options and $x(0) = 0$ and $x(T) = 1$.

$$x'(t) = \frac{n}{t}\, x(t) \tag{3.1}$$

$x(T)$ stands for the normalized amount of solved uncertainty. Note that $0 \leq t \leq T$, $0 \leq x(t) \leq 1$. (5.1) implies that the rate of increase in the solved information $x'(t)$ is proportional to $x(t)$ and inverse proportional to t. Solving (5.1), (5.2) is obtained.

$$x(t) = \left(\frac{t}{T}\right)^{n} \tag{3.2}$$

It is assumed that the solved information $x(t)$ is twice derivable at $t \in [0, T]$. The second derivative of (5.2) indicates the accelerated velocity of the solved uncertainty along with the game progress, which is given by (5.3).

$$x''(t) = \frac{n\ (n-1)}{T^{n}}\ t^{n-2}\ |_{t=T} \quad = \frac{n\ (n-1)}{T^{2}} \tag{3.3}$$

Accelerated velocity refers to the difference in the rate at which information is gathered as the game progresses. Then (5.4)gives the acceleration motion or free-fall motion in mind, a. a is estimated in the domain of board games as (3.5), where B and D denote the average number of possible moves and game length, respectively.

$$x(t) = \frac{1}{2}at^{2} \tag{3.4}$$

$$a = \frac{n\ (n-1)}{T^{2}} \approx \frac{B}{D^{2}} \tag{3.5}$$

3.4.2 Jerk and Comfort in Mind

Two processes with the same GR value at the end of the game information progress may have different instantaneous GR value tendencies. For example, two basketball teams have the same total shots and total attempts. So their GR values are the same when the game is finished. However, each team felt different tendencies to get scores. The team with a stable scoring process is predictable, and vice versa. As such, force (uncertainty) was not only felt but also the change of the force (unpredictability). In physics, acceleration can be felt in motion and the feeling of jerk [17].

The change in accelerated velocity (or jerk [17])of the solved uncertainty along with the game progress [18], which is supplied by (3.6), is shown by the third derivative of (5.2). As a result, (3.7)gives the motion with continuous jerk j, which is approximated in the domain of board games by (3.8). In this study, j represents the change in informational acceleration (the magnitude of thrills, the tendency of getting attractiveness, and related to game addiction), which is linked to an addictive-like event due to the tendency of motivation retention [18]; thus, denoted as AD value.

$$x'''(t) = \frac{n(n-1)(n-2)}{T^n} t^{n-3} \mid_{t=T} = \frac{n(n-1)(n-2)}{T^3} \tag{3.6}$$

$$x(t) = \frac{1}{6} j t^3 \tag{3.7}$$

$$j = \frac{3a}{t} \approx 3 \frac{B}{D^3} \tag{3.8}$$

Table 3.1 shows the measures of game refinement for board games. For sophisticated board games such as Chess, Shogi, and Go, it is assumed that there exists a reasonable zone for the acceleration (a) and jerk (j), which is between $0.07-0.08$, and $0.045-0.06$, respectively. The cross point (Figure 3-1) between acceleration and jerk t_1 is the point where the maximum amount for achievement is greater than the discomfort (t_1). This condition is the first reasonable chance to stop the game; otherwise, the discomfort will be higher than achievement. Moreover, before the cross point between acceleration and jerk, players are considered beginners in the game, where lots of effort is vested in studying and training, and the achievement is much lower than the effort spent; thus, it is too short to stop the game.

Table 3.1: Measures of game refinement for popular board games, adopted from [1]

Board games	B	D	$\sqrt{a} = GR$	$\sqrt[3]{j} = AD$
Chess	35	80	0.074	0.059
Shogi	80	115	0.078	0.054
Go	250	208	0.076	0.044

B: Average branching factors; D: Average game length;
$\sqrt{a} = GR$: informational acceleration (thrill);
$\sqrt[3]{j} = AD$: informational jerk (surprise);

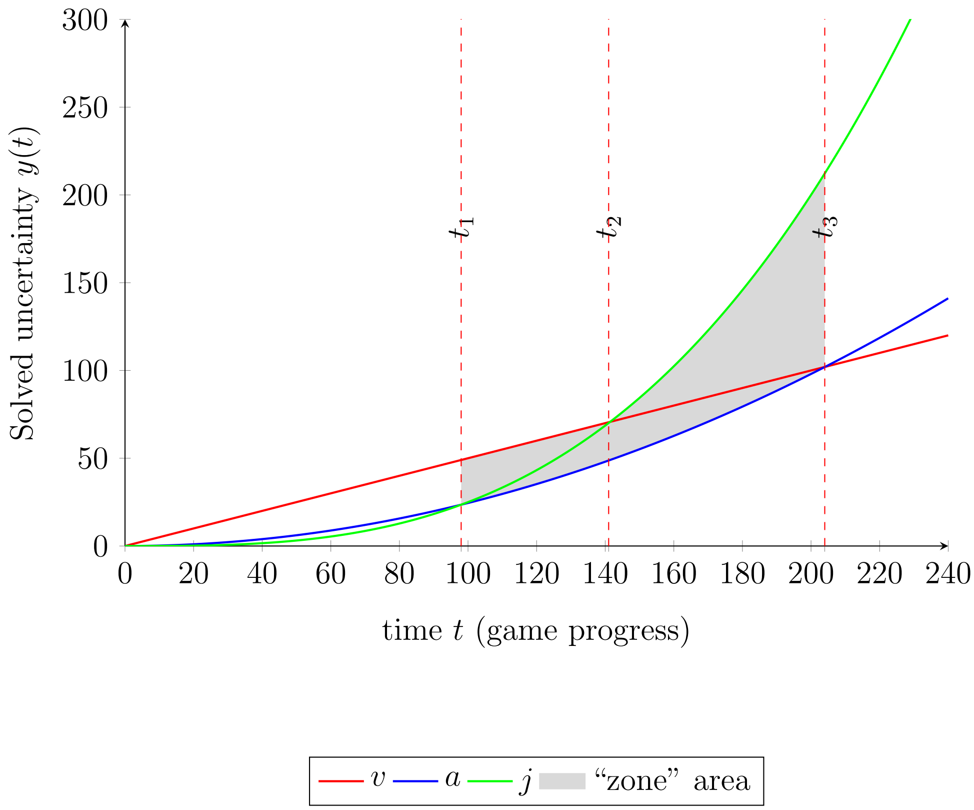

Figure 3-1: The cross point between the line with velocity v, curve with acceleration a and curve with jerk j. t_1; t_2 and t_3 represent the bound for effort, achievement, and discomfort, respectively.

However, after t_1, the discomfort or unpredictability becomes larger than achievement but still lower than effort, so t_1 is the first chance to stop the game. Otherwise, the discomfort will be higher than the achievement. The cross point between velocity and jerk (t_2) is where effort is greater than the discomfort or unpredictability, and it is the second reasonable chance to stop the game. After t_2, the discomfort is higher than the effort, inducing the feeling of discomfort and unpredictability. Furthermore, the cross point between velocity and acceleration is when the effort is greater than achievement (t_3), making it the last chance to stop the game.

Quite a few levels of risk in game-playing are the motives behind the desire and concentration to face the challenge, which is the point of entertainment. After t_3, the achievement will be higher than the effort, similar to the feeling of "reap without sowing." This situation is where the player had achieved mastery and possessed the ability to play the game expertly; thus, making the game bland. The interval of the cross points constitutes the reasonable zone for a reasonable game length. Moreover, players need enough steps to show their abilities to make the game fair enough; if game steps are too less, players do not have enough chance to show their skill, so game steps should be longer than t_1, So $t_1 \leq t \leq t_3$ can make sure game's fairness and entertainment.

A game is a constant task that invariably contains quite a few risks, where rewards are not expected at every step. As such, the best effort is devoted to playing the game. When the achievement is higher than the effort devoted, it makes us feel "reap without sowing" and "gain without pain." As such, effort with quite a few degrees of achievement makes the game progress a comfortable experience. However, ensuring a comfortable experience means that discomfort should be less than achievement and effort.

3.5 Gamified Experience From Metaphysical Perspectives

The reasonable GR (acceleration) and AD (jerk) measures of the game process of the sophisticated and popular board games such as Chess, Shogi, and Go have been previously established (Table 3.1). However, such measures were dependent on the temporal dimensions of game-playing, which makes generalizing the model of the play process challenging when dealing with problems with little to infinite time continuum. Therefore, considering the different dimensions of information progression in the game is essential.

The success rate in the game process is associated with the speed of what a player can gain, which is regarded as the physical domain's velocity. The total attempts (steps) are associated with the time or total costs in the physical domain, such as five minutes or steps. The total successful shoots in the game process are associated with the outcome gained, which is regarded as displacement in the physical domain. Hence, the correspondence of those physical quantities can be utilized to estimate the metaphysical game process in our minds. As total attempts and total shoots constitute the game process model, the physical model can express such progress while investigating the different interplay between various metaphysical properties. Hence, the analogical correspondence of the Newtonian dynamics and the game information progress is broadly determined as summarized in Table 3.2.

In this study, if the velocity of the process is directly proportional to the speed of getting a reward, the players invariably take n times for a single shoot. As such, the $v = \frac{1}{n}$, implies that there is no uncertainty during the process. The effort during the game process is described in (3.9). However, in the real world, the process is constantly subjected to uncertainty. Then, a continually changing velocity process is described as $v = at$, which produces (3.10).

Table 3.2: Contextual correspondence between game information progress, Newton dynamics, and their link

*	**Scoring games**	**Newton dynamics**	**Link**
$y(t)$	Total goals	Displacement	Total gain
t	Total shoot attempts	Time	Total cost
v	Total score rate	Velocity	Process speed

*: Mathematical notation; v: uncertainty solving rate; t: game length; $y(t)$: solved uncertainty;

$$y(t) = vt \tag{3.9}$$

$$y(t) = \frac{1}{2}at^2 \tag{3.10}$$

Furthermore, the process is also subjected to quite a few degrees of unpredictability. For example, when taking the elevator [41], if the acceleration is constant, the human body is immediately subjected to an upward thrust of $m(a+g)$ when the elevator starts, which causes discomfort to the passengers. Hence, the designers invariably set the acceleration to a gradient, giving the human body a period of adaptation, avoiding discomfort. By introducing $a = jt$ in the process, (3.11) is obtained.

$$y(t) = \frac{1}{6}jt^3 \tag{3.11}$$

The cross-linking between three domains (physics, games, and psychophysiology) was established by bridging the physics concepts to their psycho-physiological meaning, determined based on the cross point of various psycho-physiology quantities (Section 3.4.2). The 'velocity' is the speed to get the reward associated with the effort devoted. The 'acceleration' is the thrills of the obtained rewards associated with achievement due to changes in the effort. Finally, the 'jerk' is the degree of discomfort associated with unpredictability due to changes in the expected achievement. The analogy

Table 3.3: Contextual link between physics, games, and psychology

Physics	**Games**	**Psychology**
Velocity (v)	Solved uncertainty	Information gain (effort)
Acceleration (a)	Thrill magnitude	Reward gain (achievement)
Jerk (j)	Level of surprise	Unpredictability (discomfort)

of such links is given in Table 3.3.

3.5.1 Basketball

A quintessential two-sided time-limited sports game is basketball. The shot attempts and successful shots, determine the game information progress by promoting a similar idea from board games to sports games. In the previous research, the analysis of game information progress focuses on the data of one team. However, focusing on the entire basketball process, the game information progress needs to be created by the attempts and goals of the two teams.

If the game process progresses at a constant speed, then the game process is certain, which is given as in (3.12) where G and T stand for the average number of goals and shoot attempts, respectively. In the domain of sports games, if the process is uncertain, then the informational acceleration is given as in (3.13). If the process is unpredictable, the informational jerk is given as in (3.14). Solving $a = \frac{2G}{T^2}$ and $j = \frac{6G}{T^3}$, then $\sqrt{a} = 0.073$ and $\sqrt[3]{j} = 0.046$ were obtained, respectively.

$$y(t) = \frac{G}{T}t \tag{3.12}$$

$$y(t) = \frac{1}{2}at^2 \tag{3.13}$$

$$y(t) = \frac{1}{6}jt^3 \tag{3.14}$$

This situation implies that if acceleration is reasonable ($GR \in [0.07, 0.08]$), the game is enjoyed with an appropriate degree of uncertainty and thrills. This condition corresponds with the attractiveness "felt" during the game process, which is related to the measure of force ($F = ma$). Meanwhile, jerk is a part of the underlying components that are essential in our daily life, where it generates phenomena such as vibrato in music and the elevator system with progressive acceleration. Quite a few times, slow and gentle change of force is better perceived when the amount of jerk is reasonable ($AD \in [0.045, 0.06]$). A game with tolerable perturbation from unpredictability and discomfort during the game process would remain enjoyable. It can be observed from Table 3.4 that the GR (acceleration) and the AD (jerk) values of the basketball are 0.073 and 0.046, respectively. Hence, basketball has a reasonable amount of thrills and unpredictability, which makes the game characterized as a sophisticated scoring game.

Table 3.4: Quintessential two-sided time-limited shooting game - basketball (adopted from [2] and basketball reference website*)

	$G^\dagger$	$T^\ddagger$	GR	AD
Basketball	72.76	164.02	0.073	0.046

†: Successful goals; ‡: Total shots; GR: Informational acceleration; AD: Informational jerk;
*: https://www.basketball-reference.com/leagues/NBA_stats_per_game.html;

The correspondence between physical quantities in the game and psychological feelings enables the visual representation of such processes to be mapped to its effort, achievement, and unpredictability, as given in Figure 3-2. The velocity of the game is linear based on effort. With the change in velocity, its acceleration describes the game's achievement levels. With the change of acceleration, unpredictability can be expressed through the linear jerk. Thus, the game progress satisfies the relationships of $unpredictability \leq achievement \leq effort$. This relationship provides an important justification that shows a clear separation between different progressions relative to the number of actions that can be made in the game setting, which corresponds to the expected psychological feelings of a player.

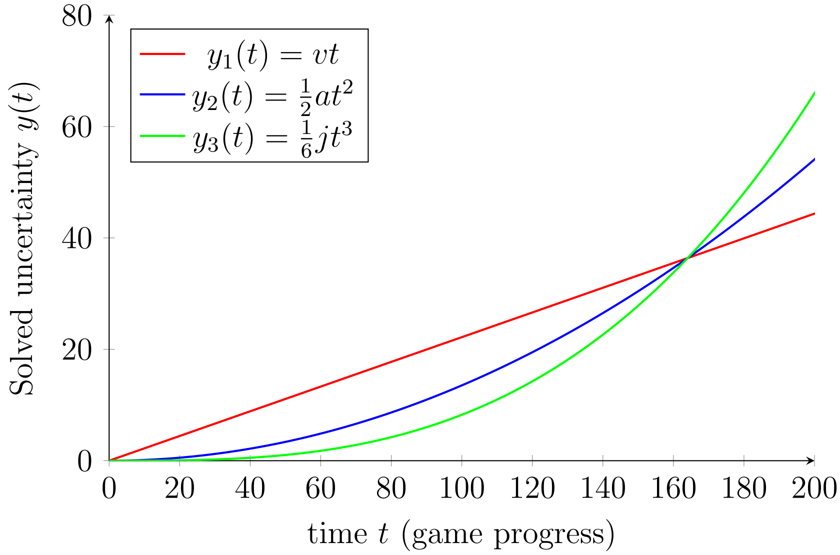

Figure 3-2: The cross point between the curves of the velocity v, acceleration a, and jerk j, where such a cross point describes the comfortable moment of the basketball game. After the cross point, it can be observed that with enough training and skill, achieving rewards becomes easy. However, the feeling of discomfort will also be higher due to boredom and insufficient challenge.

3.5.2 Soccer board

Quite a few times, the difficulty for every game is different, where there exist quite a few games with low points and successful score rates being much less than $\frac{1}{2}$, e.g., soccer [42]. In soccer, it is hard to get a goal, and the game becomes very exciting when a team gets a single goal. The average number of goals is 2.64 per match, and the promising shots (shots on target) are 7.86 [43]. As such, the previous study has calculated the game refinement measure for soccer [3], given as in Table 3.5.

However, quite a few countries have quite a few board games to play soccer, such as “Paper Soccer” in Poland [44] and “Philosopher’s Football” in the USA [45]. Thus, it is reasonable to apply the board game model to soccer based on such a context. A quintessential soccer board game called “Zuqiuqi” in Chinese and “soccer board” in

Table 3.5: Measures of game refinement for soccer game according to [3]

Goals	**Shots on target**	**Total shoots**	GR
2.64	7.86	22	0.073

English [46], is given in Figure 3-3.

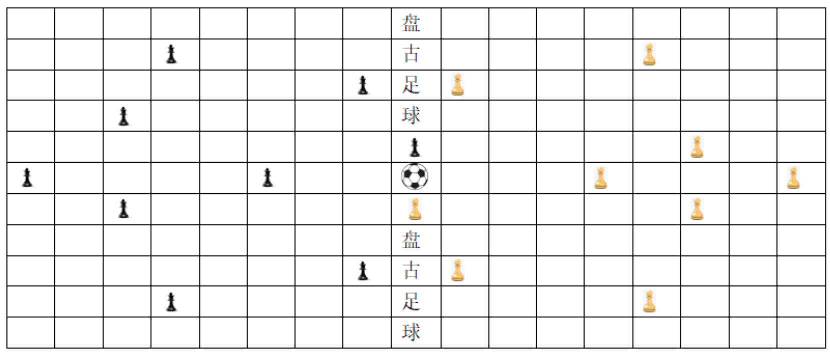

Figure 3-3: Using board game to play soccer

The average feasible options (or feasible time) (B) to get a single goal is $\frac{22}{2.64} = 8.33$. Meanwhile, the average promising options (average feasible time over the shots on target) B_1 for soccer is $\frac{22}{7.86} = 2.8$ (Table 3.6). Hence, the promising options B_1 are approximate as the root feasible options $\sqrt{B}$ ($\sqrt{B} = B_1$, $\sqrt{8.33} \approx 2.8$).

Table 3.6: Links between board game to soccer, where D is total shots, B is average feasible options, B_1 is average promising options (i.e., n is assumed as ideal options $n = \sqrt{B_1}$), GR is the informational acceleration, and AD is the informational jerk.

D	B	B_1	n	GR	AD
22	8.33	2.8	1.67	0.076	0.09

In the soccer case, the best physical strength and the ideal state of a player correspond to the strongest AI in the board game cases, we can see the promising options are approximated to the root square of feasible locations($\sqrt{B} = B_1$), so the ideal value of $n = \sqrt{B_1} = 1.67$ is considered. As such, an ideal state for the players is 1.67 times for one shoot. By associating with soccer board games, a new expression of acceleration and jerk for soccer was obtained and given as in (3.15) and (3.16), respectively.

$$a = \frac{n\ (n-1)}{T^2} \approx \frac{B_1}{D^2} \tag{3.15}$$

$$j = \frac{n(n-1)(n-2)}{T^3} \approx 3\frac{B_1}{D^3} \tag{3.16}$$

Every game has a different success rate where its velocity differs. Consequently, a different design of unpredictability to balance the success rate and retain players' interest and loyalty is required. According to the soccer board results, players cannot quickly and frequently achieve the reward because the velocity is too low. Although the informational acceleration falls in the zone values ($\sqrt{a} \in [0.07, 0.08]$), it does not sufficiently describe soccer's attractiveness. Therefore, having a higher jerk value increases the level of unpredictability, which offsets the rewarding difficulty in soccer.

If the velocity (successful score rate) is small, a high jerk value helps to generate an unpredictable (or surprise) experience, which improves the entertainment outlook. While rewards are not easily obtained, the game can still be enjoyed (watched). On the other hand, if the successful score rate is too high and the process is perceived as too easy, a low jerk value would stabilize players' enjoyment and comfort.

3.6 Physics and Psychophysiology Processes in Games

In this section, the application of physics quantities relative to its psychophysiology expectation based on *flow* theory, in the context of games, is investigated. In Section 3.6.1, the dynamics of an adversarial setting (two players/teams) are explored based on the interaction of velocity, uncertainty, and forces. Then, the relationship between scoring dynamics in the game and flow theory in psychology is established (Section 3.6.2). Finally, the interplay between the various physical quantities leading to the conception of the motions in mind is discussed in Section 3.6.3.

3.6.1 Interaction Dynamic

Generally, the game process can be expressed using a game tree structure. For example, Figure 3-4 depicts the game process of the players in such a way that the first player spends three attempts on the first shoot, two attempts for the second shoot, and a single attempt for the third one. Then, the player's corresponding success rates (velocity) in every state can be determined, where the acceleration and jerk can be derived to express different game-playing processes at each moment.

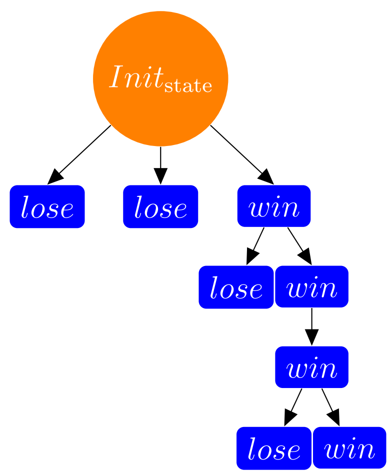

Figure 3-4: Using a game tree model to visualize the scoring process

The successful score rate at any moment is the velocity of the game to obtain the reward. For two side players (or teams), p_1 and p_2 denote the first player (or team) and second player, respectively, and their velocity (or successful score rate) is v_1 and v_2, respectively. Therefore, it can be conjectured that if $v_1 = v_2 = \frac{1}{2}$, the level of attractiveness felt by both parties would be maximized. This situation reflects the best scoring effort given by both sides to gain a reward (or score), where both sides have an equal chance.

From another perspective, the scoring process can be regarded as a quintessential binary variable, where the player's success rate and loss rate have a distribution of $[v, 1-v]$. In the context of information theory, entropy can be used to measure the uncertainty in the random variables in each state [47]. Then, the uncertainty of state based on such distribution is given as (3.17), where the uncertainty is maximum ($H = 1$) when $v = \frac{1}{2}$. Therefore, if both the success rate and loss rate are $\frac{1}{2}$, then the uncertainty is highest. This situation describes that the game's outcome is uncertain (the highest uncertainty), which again implies that both sides have an equal chance.

$$H = -v\log(v) - (1-v)\log(1-v) \tag{3.17}$$

Solving the cross point between (3.9) and (3.10), then the information acceleration expression is obtained as $a = \frac{2v}{t}$. Then, v can be aggregated based on each player by considering the interactions between two players (teams), in which the resulting expression is given as (3.18). In the physics domain, there are two quintessential interactive forces: universal gravitational force and electromagnetic force, which are expressed as $F_u = G\frac{M_1M_2}{R^2}$ [48] and $F_e = k\frac{q_1q_2}{R^2}$ [49], respectively. Here M and q are the weight of two masses and the quantity of electric charge, respectively. For the attractiveness in the game process, these are the interactive forces in mind. Relative to the interactive forces, the constant G and k in the game-playing context are associated with players' ability and the successful score rates of players (given by v_1 and v_2). Hence, the analogy of such interactive force in the game is given by (3.19).

$$a_1 = \frac{2v_1}{t}, \qquad a_2 = \frac{2(1-v_1)}{t} \tag{3.18}$$

$$F = Ka_1a_2 = K\frac{4v_1(1-v_1)}{t^2} \tag{3.19}$$

Based on the premise of $v_1 + v_2 = 1$ in two-sided game, only when $v_1 = v_2 = \frac{1}{2}$, the force between each other becomes highest, which is $\frac{K}{t^2}$. If one side is much stronger than another side, their abilities and success rate are either $\frac{1}{3}$ or $\frac{2}{3}$, respectively, while their interactive force will be less than $\frac{K}{t^2}$ (or $\frac{8K}{9t^2}$). The notion of mass is associated with the weights of the objects (how heavy it is to hold the object) and the difficulty of changing its state (difficulty of putting stress on an object). Mass in the game context is prevalent based on the latter, which refers to the difficulty of moving. This situation corresponds to the challenge of the game (a measure of risk rate). Hence, $m_1 = 1 - v_1$ and $m_2 = 1 - v_2$ refers to the mass of first and second players (or team), respectively. The analogy of universal gravity in the game context can be described as given by (3.20). When the risk rates of both sides approach $\frac{1}{2}$, the universal gravity in the game context is highest.

$$F_u = G\frac{m_1 m_2}{R^2} = G\frac{(1 - v_1)(1 - v_2)}{R^2} = G\frac{v_2(1 - v_2)}{R^2} \tag{3.20}$$

In essence, interaction dynamics in games can be analyzed when it is assumed that each side has similar abilities, implying successful score or risk rate is both approaching $\frac{1}{2}$. In such a condition, the interactive attractiveness between the two players becomes the highest. It is easy to understand that the winning possibilities will also be the same when the ability is well-matched with both sides in a contest, where the interactive force in our minds is highest when both sides tried their best, they will feel indulged like flow in a non-game context.

3.6.2 Game Playing Experience and Flow Theory

It was found that in two-sided games, when both players (or teams) abilities are similar, the successful score rates are approximately balanced, and the interactive attractiveness between two-sided players becomes the highest. However, what happens when the successful score rate is not $\frac{1}{2}$? Based on such regards, differences between the two sides are best described in conjunction with the Flow theory. Such condition is bridged via the measures of scores and their differences to describe conditions in a game-playing context (Table 3.7).

Table 3.7: The Correspondence between Game Context and Non-Game Context using S (Scores)

Non-Game Context	**Game Context**	**Notation**
Skill	Player's Ability	S
Challenge	Opponent's Ability	Opponent's S
Δ(Skill-Challenge)	The Distance between Abilities	ΔS

S: Scores; $\Delta(S)$: Score Gap between 2 sides;

The Flow theory is a distinguished psychological theory that explains the feelings of people acting on quite a few things, fully immersing or engaged, based on differences between two dimensions: skills and challenges (Figure 3-5). The standard of measurement is the $\Delta(skill - challenge)$ [50]. When $\Delta(skill - challenge) = 0$, both skill and challenge are high, where the experience that happened is defined as the "flow." Meanwhile, $\Delta(skill - challenge) > 0$ is where the experience people feel is related to control and relaxation. Meanwhile, the $\Delta(skill - challenge) < 0$ is related to arousal and anxiety. Nevertheless, what are the challenges and skills in the game context?

If the skills are the abilities a player obtains, it can be linked to the success rate in the game, and its opponent's ability is the challenge they need to overcome. Hence, the risk rate is the expected challenge in the game context.

Different success rates are utilized to estimate the affected processes during the game relative to the Flow theory. Since the scoring process is a dynamic mechanism during the game progression, the velocity (v), acceleration (a), and jerk (j) can be visualized. The sine function is used to describe the scoring process (v) starting from 0:0 (Figure 3-6). Then, the derivative of v is given by the cosine function to describe the a that is associated with attractiveness. Subsequently, the derivative of a is the minus sine function representing j, associated with the unpredictability or discomfort in the game process. Consistent with previous research, [18] when the scores are similar, the absolute value of acceleration is the highest (the greatest attraction).

Similarly, Figure 3-7 depicted the player scores to measure their abilities, and the

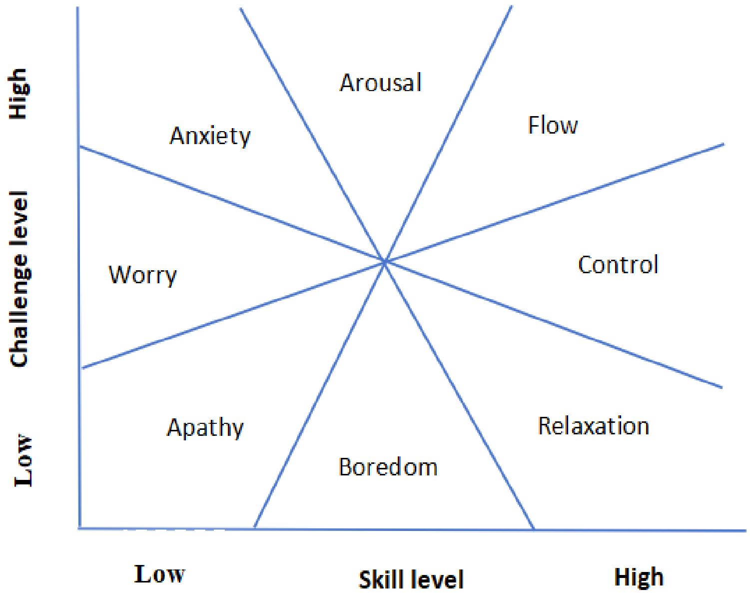

Figure 3-5: *Challenge vs. Skill, illustrating the "flow" region* Source: English Wikipedia. `https://en.wikipedia.org/wiki/File:Challenge_vs_skill.jpg`

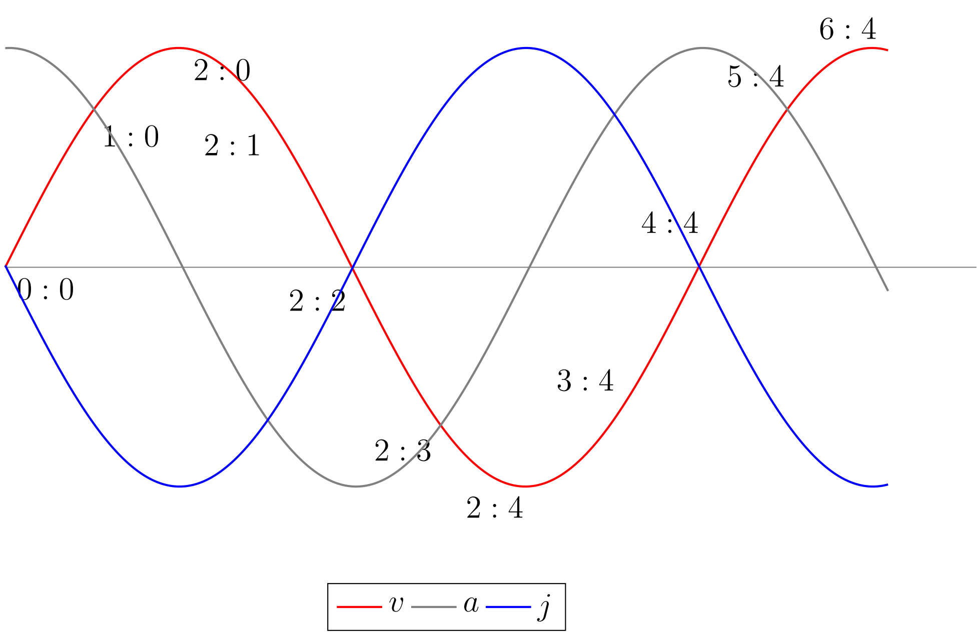

Figure 3-6: The description of game process using Δ scores

opponent's scores correspond to the challenge the player needs to face. In the game-playing process, when a player and his opponent's abilities (scores) are similar and strong, they will be fully concentrated and attentive to the game because such a moment is the most uncertain, and both players can expect the greatest excitement. The most stable (predictable) situation means that players can expect the most comfort and control. GR theory explains that when two side teams' $\Delta scores = 0$, the absolute value of acceleration is the highest because this condition is the most uncertain. In such a setting, every feasible thing could happen, since similar skills were expected from both sides.

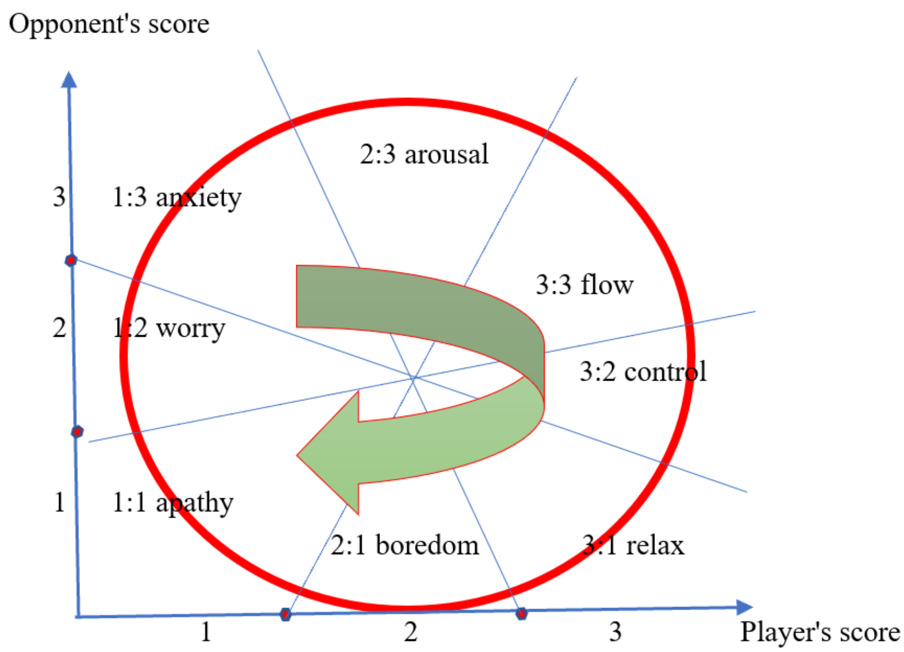

Figure 3-7: The description of Flow theory using the game-playing process to associate the context of the expected experience when playing, based on self's (ability) and opponent's (challenge) score

In addition, when the success rates are comparable, interactive attractiveness becomes the highest, implying a balance between ability and challenge in the game context ($\Delta[skill - challenge] = 0$). As such, players will experience the "flow" zone, which has the highest absolute value of acceleration (the greatest thrills). As a result, the game outcome becomes unpredictable. At the same time, the $j \to 0$ because it is hard to change the highest uncertainty; players will not feel discomfort and unpredictability. In essence, players will potentially feel both thrill and anxiety, blended or in parallel, for the mere possibility of a situation change. Besides the $\Delta scores = 0$ condition, different ability gaps correspond to psycho-physiological experiences of various non-game contexts.

3.6.3 Conceptual Basis of Motion in Mind

Different game-playing mechanisms were revealed in each state based on v, a, and j during their game processes. Since, $a = \frac{2G}{T^2}$ and $GR = \sqrt{a}$, then the GR value would approach the sophisticated zone as the game progressed. In physics, mass is the factor determining the difficulty of changing the state of motion when an object is stressed. Thus, mass is a physical quantity that describes the inertia of an object, which describes the "heaviness" and the difficulty of holding an object [51]. In games, the risk factor is the reason that determines the difficulty of progression and; thus, the difficulty of obtaining scores. Therefore, assuming the mass as the risk rate given by $m = 1 - v$, the analogical translation between physics, games, and psycho-physiological context can be determined, which is given in Table 3.8.

Table 3.8: Analogical translation between motion in minds, its game-playing implications, and its psycho-physiological context

Motion	**Implication**	**psycho-physiological**
v	Solved uncertainty	Information gain (effort)
m	Risk uncertainty	Uncertainty gain (challenge)
$\vec{p_1}$	Continuity tendency	Concentration
$\vec{E_p}$	Information expectation	Expectation
$\vec{p_2}$	$\vec{p_1}$ and $\vec{E_p}$ gap	"knowledge" gap

v; velocity; $m = 1 - v$: mass; $\vec{p_1}$: game's objective momentum; $\vec{E_p}$: game's potential energy; $\vec{p_2}$: game's subjective momentum;

Concerning the relative motions in the mind, the analogy of momentum in mind ($\vec{p}$)

is adopted, which is given as (3.21). To identify the peak point, the first derivative of $\vec{p}$ is $\vec{p}' = 1 - 2v = 0$, where $v = \frac{1}{2}$, then $\vec{p} = \frac{1}{4}$ holds. Initially, the probabilities of win and loss are both $\frac{1}{2}$, so the velocity is $\frac{1}{2}$ and the risk rate $m = 1 - v = \frac{1}{2}$. In this situation, the momentum $\vec{p}$ is highest.

$$\vec{p} = m \cdot v = (1 - v) \cdot v \tag{3.21}$$

In physics concept, momentum $\vec{p}$ is the difficulty of stopping moving objects and the tendency of moving objects to keep moving. Also, momentum is an instantaneous variable, and it measures the momentary trend based on the current velocity. Concerning the game process, momentum in mind $\vec{p}$ describes players' tendency to maintain the game's state and continue to focus on the game. It can be seen that their success rate is the same, and their tendency to keep playing the game is highest. Hence, the $\vec{p}$ measures the players' tendency to keep playing the game in different velocity situations [16].

Potential energy ($\vec{E_p}$) is the energy stored in a specific location. It is related to the expectation in the specific state. In different winning rate states, there are different energy and different expectations in our minds. The potential energy $\vec{E_p} = 2mv^2$ in the game is defined as the amount of the required game information a player needs in the game process [1]. It explains the expectation of the player in finishing the game. As a game progresses, the game's potential energy reflects the amount of anticipation the game may give to the player (degree of win comfort). More importantly, this helps the game designer determine the game sophistication's appropriate stability, given as (3.22).

$$\vec{E_p} = 2mv^2 \tag{3.22}$$

Relative to the process in our mind, momentum $\vec{p_1} = mv$ is the momentum of the game's motion, which is followed by the momentum of mind's motion ($\vec{p_2} = \vec{E_p} - \vec{p_1}$), where $\vec{p_2}$ is adopted as the measurement of the magnitude of engagement. In the context of psychophysiology, the gap between concentration and expectation can be related to the epistemic rationality of determining the "knowledge gap," in which the former is related to the truth system (actual/observed information) and the latter is related to the belief system (predicted/expected information) from the classical theory of knowledge [16, 52].

$$\vec{p_2} = 2mv^2 - mv \tag{3.23}$$

When both sides of the players' success rate are similar, ΔS approaches 0, and momentum $\vec{p}$ increases. This situation implies that the difficulty of stopping the game is high, and the tendency to keep playing the game is high. Otherwise, when ΔS increases, momentum $\vec{p}$ approaches 0 because the game's uncertainty is reduced, and the disadvantaged side already had no chance to win the game. Such game processes are illustrated as in Figure 3-8.

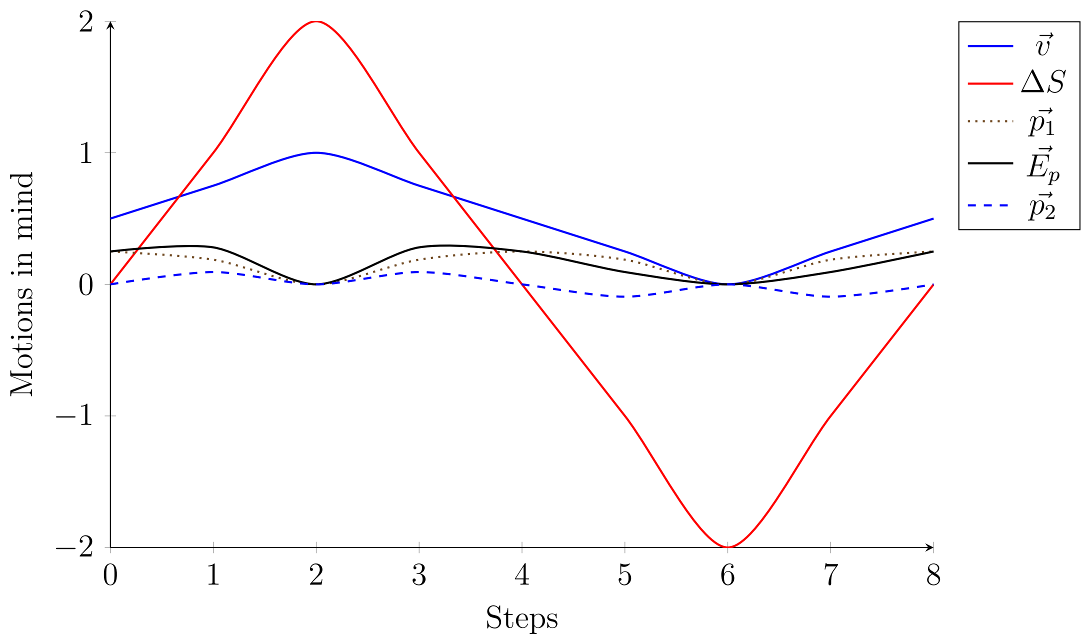

Figure 3-8: The analysis of motions in mind based on dynamical scores gap

If ΔS approaches 0 or 1 when they evenly match each other, the player's expectation is high, and if the player comes to the disadvantaged side, and ΔS approaches -1, lessen the player's expectation. When the situation became apparent, ΔS is 2 or -2; expectation reduced to 0 because the winner is almost determined, so any work cannot reverse the tide. To this end, $\vec{p_2}$ represents the gap between expectation and concentration; when the score difference is 1 or -1, the situation at this time is a situation that can quickly change. A player quintessentially wants to anticipate the reverse of such a situation, making the magnitude of engagement the highest.

Based on the conceptual basis of the motion in mind and flow theory, the professional basketball games' dynamic process is analyzed. The process of obtaining in-game scores can be associated with the player's abilities. Meanwhile, the abilities of their opponent (opponent's scores) can be regarded as a challenge. Hence, the instantaneous psychophysiology measures, expected from the interaction dynamics of the two sides (player and their opponent) in the game-playing, can be visualized. For this purpose, actual data of NBA games between two teams were adopted, where the game between the Milwaukee Bucks (ranked second) versus the New York Knicks (Ranked 26th) was analyzed for their interaction dynamics.

The analysis of the game match between the two teams was summarized in Table 3.9. Their score S is less than ten at the beginning of the process, and their score gap (ΔS) is smaller than one. Such a situation makes the prospect of the game unclear since the score gap is not apparent; thus, associated with apathy. However, as the game progresses, $S > 10$ while ΔS were still smaller than one, implying that the game is trapped in a stalemate and white-hot stage; in this situation, the potential to reverse the situation could happen in time. Flow conditions could occur due to the need to be attentive and indulged in such a moment.

As the game progresses further, their score gap gradually becomes larger ($2 \leq |\Delta S| \leq 4$); when their scores are less than 10, it means that it is also the beginning of the process, the advantageous team would experience boredom while the other team

Table 3.9: Dynamical emotions and the corresponding description

Emotions	**Game Score**	**Skill:Challenge Ratio**
Apathy	$\|\Delta S\| \leq 1,\ S \leq 10$	Low:Low
Flow	$\|\Delta S\| \leq 1,\ S > 10$	High:High
Boredom	$2 \leq \|\Delta S\| \leq 4$, ΔS is positive, $S \leq 10$	Medium:Low
Worry	$2 \leq \|\Delta S\| \leq 4$, ΔS is negative, $S \leq 10$	Low:Medium
Control	$2 \leq \|\Delta S\| \leq 4$, ΔS is positive, $S > 10$	High:Medium
Arousal	$2 \leq \|\Delta S\| \leq 4$, ΔS is negative, $S > 10$	Medium:High
Anxiety	$\|\Delta S\| \geq 5$, ΔS is negative	Low:High
Relax	$\|\Delta S\| \geq 5$, ΔS is positive	High:Low

experience worried. As the game progresses gradually, this condition implies that the advantageous team understands this process and experiences the notion of control, and the other team experiences arousal. Also, when two side teams' $\Delta S > 5$, the condition becomes too difficult to reverse, in which the team in an advantageous position would undergo a relaxing experience while the other team in a disadvantaged position would experience anxiety. Such condition can be validated from the observation made from the recorded game recap of the final matches between Milwaukee Bucks versus the New York Knicks (i.e., Game Recap: Bucks 112, Knicks 97, see `https://www.youtube.com/watch?v=8VduEjEsNO8`; Game Recap: Bucks 112, Knicks 100, see `https://www.youtube.com/watch?v=sKra9m6YqUo`; and Game Recap: Knicks 113, Bucks 98, see `https://www.youtube.com/watch?v=2dlearLKJEk`).

The listing of such dynamic processes for the winner's side is depicted in Figure 3-9 to consider the composition of such experiences. It can be observed that the advantageous (or stronger) team would experience mainly relaxation and control during the game progression due to their skill being higher than the challenge they faced (opponent's ability). Therefore, considering these interactions and dynamics of the game-playing experience would be beneficial in designing a personalized entertainment, specialized monitoring, or training program (e.g., medical rehabilitation) and leveraging the flow experience in a specific activity or task.

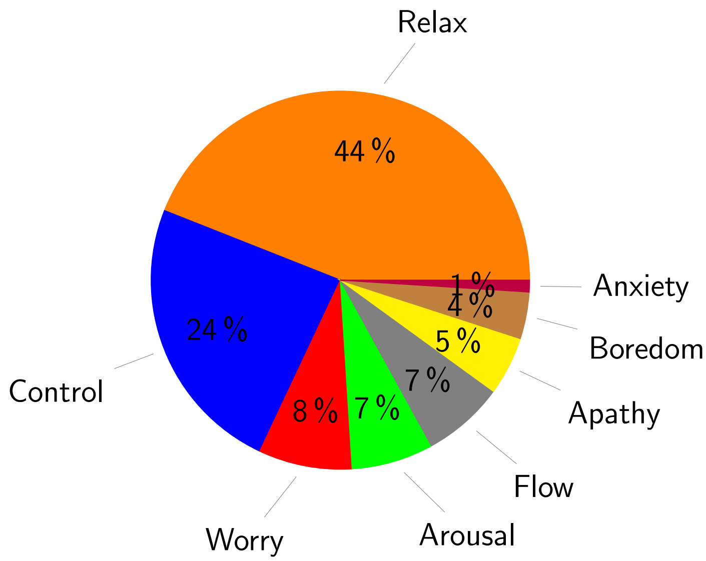

Figure 3-9: An example of the dynamic interactions and game-playing experience of one game process based on its association with the Flow theory

3.6.4 Limitations and Future Works

The limitations of the study are as follows. The proposed model of interaction dynamic and game-playing experience involves mapping the game processes as psycho-physiological measures. However, such a measure would be a reliable measure when the game-playing scores of the game are obtained by actual human players (such as the NBA score data). However, this measure would not provide a psycho-physiological measure of players when the data is obtained using AI agents and requires further justification of human players in those contexts.

In addition, validating the collective emotion experienced by the players in a team-based setting remains a challenging feat in the research community. Moreover, justification was needed from the theoretical studies in psychology, empirical evidence on the influence of games on human perceptions, the importance of personality characteristics, and the oscillating psychological states in game-playing. Those areas would be fertile grounds for future prospective studies to verify and validate this study.

Future works can also investigate the influences of information dynamics, such as jerk value, on different player strengths, varying game complexity and rules, and differences between simple classical board games and sophisticated modern video games. Another aspect of potential future study involves developing a framework of player-driven game-playing experience by manipulating surprising events via such information dynamic (e.g., jerk value).

3.7 Chapter Conclusion

Our research linked game progression to emotions through game refinement theory and a physical model, in games, effort, achievement, and unpredictability are aligned with velocity, acceleration, and jerk. Higher jerk values in games increase surprise and make players addictive and repetitive.

The construction of a game information model is rooted in a game tree, striving for a comprehensive visualization and quantitative representation of intricate game processes. Concurrently, it integrates pertinent theories such as game refinement theory and flow theory, intending to thoroughly assess the entertainment value and complexity of games.

Psychological states in the game's progression were elucidated by linking scoring situations with the emotional states outlined in flow theory. This connection was revealed through the lens of game refinement theory and the analogy of a physical model. Various game elements were correlated with the physical model to ascertain the mind's psychological conditions, where appropriate effort, achievement, and unpredictability corresponded to the physical quantities of velocity, acceleration, and jerk, respectively.

The dynamics of diverse associated psychological states during gameplay were established, showcasing how sophisticated games leverage jerk value to heighten difficulty, leading to players experiencing sudden rewards that enhance the element of surprise. Additionally, players were anticipated to contend with interactive forces, illustrating that when players' abilities are similar, the expected "attractiveness" peaks, aligning with the flow zone. Conversely, when players' success score rates differ, distinct psychological aspects are associated with varying measures of game progression. A higher success score rate than the opponent is linked with relaxation, while a lower score rate is associated with anxiety.

It was found that momentum ($\vec{p}$) is essential to measure the tendency of playing the game during the progress when their scores are different. When linked to the game process, $\vec{p}$ implies the tendency to retain focus. When ΔS becomes lessened, the game tends to be continued (peak $\vec{p}$ value). Otherwise, the game tends to stop due to gap differences between the losing and winning teams (high ΔS).

Essentially, a novel perspective on game refinement theory emerged through the introduction of the jerk value, offering a quantitative framework for game design and mechanistic measures of the player's anticipated psychology. This viewpoint facilitates a more player-driven experience by pinpointing pertinent jerk values concerning success rate and informational acceleration. Consequently, maintaining a stable game progression yields a comfortable and appealing gaming experience, while incorporating relevant surprise and discomfort enhances the motivational and attractive aspects of a more challenging game progression. Motions in mind for design, ensuring a player-centric experience balancing skill and challenge, integrating physics, psychology, and design for immersive gaming.

Chapter 4

Implications of Jerk's On The Measure of Game's Entertainment: Discovering Potentially Addictive Games

This chapter is based on the integration, update, and abridgment of the following publication:

- N. Gao, H. Chang, Z. Zhang, M. N. A. Khalid and H. Iida, "Implications of Jerk's On the Measure of Game's Entertainment: Discovering Potentially Addictive Games", in IEEE Access, vol. 10, pp. 134362-134376, 2022.

4.1 Chapter Introduction

The time derivative of acceleration, termed jerk (or jolt), represents a physical phenomenon that is represented through a sudden change of acceleration (expressed in m/s^3), which humans can feel. It has been widely used as a standard (dis)comfort parameter (e.g., roads, rails, vehicles, trains, elevators, amusement rides) and was applied in manufacturing, engineering, sports science, safety science, and more [53][54]. Other aspects of jerk were studied, focusing on minimizing the wear-and-tear effect of jerk-induced operations of machinery [55] and its impact on the structural-dynamic applications (i.e., bio-mechanics and robotics) [56]. Nevertheless, jerk application from the domain of games was limited since its measurements were typically focused on physical and external implications in games (c.f. [17][57]).

Research on game refinement (GR) theory focused on exploring the dimensional implications of the game information progress [58][59][6], where the balance between certainty and uncertainty in reaching a goal is considered the rate of change of information speed (or acceleration), denoted as GR value. Therefore, game progress and move selection models were proposed from the game context using the game refinement theory and its new perspective, called the motion in mind model [1]. This model allowed the uncertainty of the game progress to be analogously measured relative to a physical measure associated with the success rate (v).

Subsequently, considering its first (acceleration, a) and second (jerk, j) derivatives were associated with speculative psychology, such as engagement and unpredictability, respectively. Therefore, a jerk in games is interchangeably regarded as an informational jerk, leading to the association of unpredictability (or surprise), which is denoted as a AD value [60]. When comparing different games, AD value changes could provide insights between classical and sophisticated games. As such, such a measure could establish the concept of unpredictability and stability concerning jerk dimensions in games. The higher the AD value of the game, the more unpredictable and full of surprises, and vice versa.

The card game is a typical imperfect information game. Players can only see their cards and the cards that have already been played. Therefore, they need to learn all the information about the game process. People indulge in this kind of game (incomplete information game) since it can be characterized as short, repeatable rounds (replayability), chance-based, and strategy-oriented, making it among the most entertaining games among adults and teenagers. Coupled with the digital platform and low-cost mobile devices, the popularity of multiplayer online card games has been increasing where its market value of about $4 billion in 2019 [61], and has positively grown its market share in recent years [62]. With a unique blend of recreational and intellectual entertainment, card games extensively test the ability of the player to navigate massive state space, recall other players' moves, probabilistically estimate the missing information induced from incomplete and asymmetric knowledge, and possibly even be addictive. Therefore, this study adopted card games as the test environment suited for the interest of this research.

There are many types of popular card games in China, and among the most popular were the Doudizhu[25], 24 points (also called as 24 puzzle) [63], and Wakeng [64]. They all have various playing styles and are widely accepted by different target audiences. Wakeng and Doudizhu were two games that share rule similarities but differ in gameplay. This gameplay includes one based on single cards, pair cards, triple cards, quadruple cards, chain cards, and various extensions and variations. In addition, it is only related to the number of points and has nothing to do with the suit. Wakeng's gameplay rules were conservative and straightforward, while Doudizhu, the succession version of Wakeng, was more varied, changeable, and sophisticated.

This study investigates the impact of the rule design of Wakeng, Doudizhu, and other similar card games. As these card games were prevalent, it is unclear why these kinds of incomplete information games were associated with being addictive. The *addictive* term used here refers to the game being repeatedly played and/or preferred from the behavioral standpoint, which is not referring to a psychological or substance-related addiction; instead, a "healthy excessive enthusiasms" that relate to motivation and from a non-pathological perspective [65]. As such, *addictive* term used in this study refers to excessive enthusiasm and motivation to repeat and continue the behavior, which statistically becomes a popular choice by most. As such, simulations were conducted to imitate difficult situations faced by players beyond their ability, a condition that matches player ability, and a case that improves player's intuitions to explore more ways to increase the game's playability; each represented by the different dynamics of GR and AD values. Several computational experiments were adopted to achieve such conditions. First, this study compares the game against other related card games using the game refinement theory and its new perspective. Second, different AI levels were adopted to simulate different skill levels of players, to understand the stability and controllability of the game (horizontal view). Third, players' affective modality (such as engagement and surprise) regarding the gameplay with different rules was also investigated (vertical perspective). Finally, the potential measures of such affective modality were identified based on the card games.

4.2 Previous work

The terminology of the "jerk" had been extensively provided by [57], which defined a standard term widely used; the first derivative of acceleration. However, interestingly, there were alternative terms, such as "acceleration onset rate", and "jerk-dot", that refer to the same concept that describes the fourth derivative of position. Therefore, for consistency and generality, the term "jerk" is used throughout this study to describe the fourth position derivation relative to its alternative uses.

The jerk had been used to measure, control, or generate smooth trajectories, motion profiles (horizontal or vertical or lateral or rolling), ride (dis)comfort criteria or parameters, human kinesiology and sport science, and shock spectrum analysis [57]. The advancement of unmanned machinery (i.e., robotic excavators) has led to the investigation of optimizing jerk for the joint trajectory planning problem of hydraulic robotics excavators. The study found that stable and smooth motion with accurate trajectory can be achieved while mitigating equipment damage (structural and wear) and prolonging its service life [66][67]. Furthermore, understanding and studies on jerk had been increasing for earthquake engineering had been conducted for the improvement of the structural dynamics such as structural control, isolation systems, and dampers are applied in mitigating jerk [68].

Previous research also focuses on the influence of acceleration and higher derivatives of motion relative to the classical mechanics of rigid bodies [17] and acoustics [69][70]; often associated with stability and vibration. However, such research works focus on the physical phenomenon where the influence of jerk had been investigated from the physical activities and other bodies of implications or systems. Adopting the analogy of classical mechanics and motions, the information in a game can be modeled as the canonical representation of location, where information can undergo displacement (progress), velocity (success rate), and acceleration (attractiveness). Then, the law of motion in the game is formalized by considering the derivatives of the game progress model, where the link between game refinement theory and flow theory is determined; leading to a more natural yet pleasurable human experience based on the sophisticated game's underlying mechanism [1].

Based on human experience modeling in the game, scarce works had been focused on it, but rather on developing artificial intelligence (AI) agents or systems that could efficiently maximize the winning rate in the card game. For instance, the Big Two AI (Big 2 AI) framework had been proposed by [71] to predict multiple movements of multiple opponents and determine the card set with a high chance to win. The known played-card information and unknown holding-card information are explored to analyze the superiority of card sets. In addition, the weight values for different card sets are customized based on the analyzed card superiority and game-playing data. Furthermore, historical data teaches frequent playing patterns with different game features. Experimental results show that Big2AI outperforms existing AI and can achieve the highest win rate (if a winning chance exists) or the least losing points (if there is no chance to win) against computer and human players.

From the perspective of incomplete information games, [72] proposes a complete game framework for Doudizhu, fully considering the confrontation and cooperation in the Doudizhu game, considering a multirole modeling-based card-playing framework that includes role modeling, card-carrying, and decision-making strategies. Role modeling learns different roles and behaviors by using a convolutional neural network. Cards carrying can calculate reasonable rules using an evaluation algorithm. Finally, decision-making is for implementing different card strategies for different player roles while discussing some key factors affecting them. Experimental results showed that this card-playing framework makes playing decisions like human beings, learn, collaborate, and reason when facing incomplete information game problems. Meanwhile, the complexity and attractiveness of Doudizhu as a multiplayer incomplete information game were investigated by analyzing the combinations of player numbers, player roles, and scoring systems [25]. AI players of different levels were constructed for self-game simulation. Game refinement measures were used to evaluate and identify the best settings for the Doudizhu game and clarify its popularity.

Previous works focused on taking the physical measure of "jerk" in smoothing and comfort of physical operations. Meanwhile, such a measure is still a nascent area in the development of games [60], where it was essentially dominated by the development of AI. Although game concepts have been adopted in various problem domains, understanding the nature of game playing's underlying mechanism has been limited. This study investigates such a mechanism by adopting a logistic model of game outcome uncertainty, where a measure of game refinement has been derived. It was applied to the popular board and sports games where the implications of its sophisticated design had been investigated [2],[58][59][6]. It also has defined the harmonic balance between skill and chance, where theoretical analysis using empirical data confirms the effectiveness of the proposed model.

4.3 Analysis of Card Games

Some of the prominent characteristics of defining card games were their suites, the number of players, starting conditions, outplay mechanisms, and card decks. A suit is one of the groups into which the cards in a deck are classified as playing cards [73][74]. Typically, each card contains one of many pips (symbols) indicating which suit it belongs to (♣: clubs; ♦: diamonds; ♥: hearts; ♠: spades); the suit may also be denoted by the color written on the card. Except for face cards, the rank of each card is determined by the number of pips on it. There is no order between the suits unless established in the rules of a given card game (for instance, in Tien Len, the ranking of cards based on numerical and suit: 2♠, A♥, A♦, A♣, A♠, K♥), so ranking reveals which cards within a suit are better, higher, or more valuable than others [74]. There is exactly one card of each rank in each suit in a single deck. In addition, a deck may contain one-of-a-kind cards that do not belong to any suit, known as jokers.

General characteristics of the considered card games were outlined and compared in Table 4.1. The studied card games were considered climbing games, where the player can simply pass if they cannot or do not wish to beat the previous play [74]. Among the five-card games considered, only two were suits irrelevant. Note that the characteristics outlined were the typical conditions or rules associated with the game characteristics. In addition, these card games were suited for evaluating this study due to the unique characteristics associated with the card games. Firstly, card games were incomplete information games, where some information about the game is hidden throughout the game-playing progression. Secondly, card games involve some magnitude of the affective modalities (engagement and surprise), where the change of information throughout the game progression impacts the play experience. Therefore, the variants of card games were adopted where the impact of the rule design and their combinations with the skill level of the players were explored.

Table 4.1: Comparison of card games considered in this study

Characteristic	Wakeng	Doudizhu	Winner	Big Two	Tien Len
Suit Types	N/A	N/A	Suits	Poker Hands	Reverse
Typical Player Numbers	Three	Three	Four	Four	Four
Standard Card Deck	54	54	52	53	52
Initial Cards	16	17	13	13	13
Start Condition	4♥	Role Bidding†	Trick card⋆	Lowest card (3♦)	Lowest card (3♠)

N/A: Suit irrelevant card games, where players can play the game with all the suits erased from the cards; ⋆: trick card determined by the dealer, placed face up in the pile of cards he/she deals; †: role bidding involving bidding for the landlord or peasant-based on self-declared stakes (low to high stakes, 1–3); *Poker hands:* Each hand belongs to a category determined by the patterns formed by its cards [75]. A hand in a higher-ranking category consistently ranks higher than a hand in a lower-ranking category. A hand is ranked within its category using the ranks of its cards. Individual cards are ranked from highest to lowest: A, K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3, and 2. Suits are not ranked, so hands that differ by suit are of equal rank [76].

4.3.1 Suits irrelevant card games: Wakeng and Doudizhu

Wakeng

The meaning of Wakeng is "dig the pit" there may be treasures and gold in the pit, but there also may exist a crisis in the hole, so sometimes Wakeng implies "trap" in Chinese. Usually, the game is played by three players, and everyone keeps 16 cards first. Meanwhile, four cards are hidden in the "pit", and the priority player can decide to dig or not; those four cards may be a bonus but sometimes a punishment for the digger. It is a card game popular in Northwest China and a basic classical poker game. The player who keeps "4 of Hearts" has the priority to be the digger to get four more hidden cards and start the first round.

Moreover, if the "4 of Hearts" was hidden, the priority player changed to be the player who keeps "5 of Hearts". And so on. When the priority player does not want

Table 4.2: The card types of Wakeng

Card Type	**Description**	**Examples**	
		LS	**HL**
single	one card	4	3
pair	2 same cards	44	33
triple	3 same cards	444	333
quadruple	4 same cards	4444	3333
chain	$\geq$3 single*	456	4567890JQK
paired†	$\geq$3 pairs*	445566	4567890JQK*2
triple†	$\geq$3 triples*	444555666	4567890JQK*3
quadruple†	$\geq$3 quadruples*	444455556666	4567890JQK*4

†: chain; *: consecutive; **LS**: lowest & short rank;
HL: highest & longest rank;

the hidden card, other players can choose to dig the four more hidden cards. After the hidden cards are decided to keep by someone, the rest of the players will become a team automatically and against the digger, and the player who plays all the cards out first wins. There are only 8 card types, as Table 4.2 shows, and every player can play according to these 8 card types. Also, the next player only can play the cards with the same type and more robust than the card of the previous hand.

There are 52 cards in total (two jokers are not included). Except for the four cards covered in the pit, the remaining 48 cards are equally divided among each player, 16 cards each. The player who holds the “4 of hearts” decides whether the four hidden cards at the pit are required. If not, the priority is transferred to the next player. If no players want it, then the priority holder has to pick it. When none of the three players hold the card “4 of hearts”, the priority will be transferred to the player who holds the card “5 of hearts”. So, the player who holds the card “5 of hearts” will prioritize getting four hidden cards or not. This process could repeat until the card “8 of hearts”.

At the same time, the person holding the card “4 of hearts” first plays the card; he has the priority to play and set the card type. After that, he can play any of the

Table 4.3: The card types of Doudizhu

Card Type	**Description**	**Examples**	
		LS	**HL**
single	one card	3	JOKER
straight	≥5 single*	34567	34567890JQKA
pair	2 same	33	22
wings	≥3 pairs*	334455	34567890JQKA*2
	3 same	333	222
triplet	3 same + a kiker	3334	222A
	3 same + pair kiker	33344	222AA
plane	≥2 triplets	333444	QQQKKKAAA88990022
bomb	4 of a kind	3333	2222
rocket	JOKER and joker		

*: consecutive; **LS**: lowest & short rank; **HL**: highest & longest rank;

available card types, and the next player needs to draw the corresponding card type, which must be stronger than the previous player's card. The process repeats until it cannot be continued, constituting one round. If a player's card cannot be defeated by any other player and the round is stopped, then this player has the priority to play cards and can play any of the eight card types to start a new round. He and his team will win if someone plays out all the cards in his hand.

DouDiZhu

DouDiZhu also called 'Fighting Landlord,' 'Landlords,' or '2 against 1,' is China's most popular computer card game. In 2017, the download amount of DouDiZhu network clients reached 1.13 billion [77]. Although the categories of cards are more diverse, the categories of cards of the Doudizhu game, as given in Table 4.3, and its rules are more sophisticated but still similar [78]. As a result, Doudizhu is more popular and has a broader audience than Wakeng.

The classic DouDiZhu game features three players, two of whom, referred to as "the peasants", must collaborate against another, referred to as "the landlord". The game usually lasts between one and three minutes long [77][78]. This circumstance allowed individuals to play the game anytime and from any location. If the peasants' hands are played first, they win; otherwise, the landlord wins. Profits and losses are shared among the peasants in the game, whereas the landlord carries himself alone, indicating that the game is a zero-sum game satisfying Nash Equilibrium. As in most card games, the starting hand of DouDiZhu has the most considerable influence on the game's outcome. The rules of DouDiZhu are simple, but the two most essential parts of winning the game demand strategies and expertise.

4.3.2 Suits relevant card games: Big Two, Winner, and Tien Len

Winner

The Winner shared similarities to the DouDiZhu. The main differences between Winner and classic DouDiZhu are found in two aspects. Firstly, the number of players where the Winner can play at least two players, usually not more than eight, using two decks of cards when there are more than four players. Secondly, each player of the Winner fights on their own. The Winner plays out all the hands, while other players continue until there is just one player, and they score by sequence ranks to play out cards. Thirdly, hands in Winner have a different weight of the suit, which ranked from biggest to lowest ($\spadesuit > \heartsuit > \clubsuit > \diamondsuit$). Suit discrimination gives the Winner many card combination possibilities [79].

Big Two

Big two (also known as deuces, Capsa, pusoy dos, dai di, and various other names) is a card game of Chinese origin. It is similar to the games of the Winner, Daifugō, President, Crazy Eights, Cheat, and other shedding games [80]. The game is very popular in East Asia and Southeast Asia. It is played casually and as a gambling game, usually with two to four players, where the entire deck is dealt out in either case (or sometimes with only 13 cards per player if there are fewer than four players). The game involves using poker hands, where the hand belongs to a category determined by the patterns formed by its cards [75]. The game's objective is to be the first to get rid of all the cards in hand by constantly one-upping opponents with cards played singly or in certain poker combinations (pair cards, three cards, or five cards) [81].

Tien Len

Tien Len, also known as Vietnamese cards, Thirteen, Poison, or Killer 13, is a shedding-type card game popular in Vietnam [82]. It is derived from the Chinese card game Winner, which uses a specially printed deck of cards, and Big Two. Considered the national card game of Vietnam, the game is intended and best for four players. For the first hand, the dealer is picked randomly; for subsequent hands, the loser of the previous hand deals. Cards ranked from high to low: 2 A K Q J 10 9 8 7 6 5 4 3. Within the numerical ranking, suits rank (from high to low): Hearts ♥ – Diamonds ♦ – Clubs ♣ – Spades ♠. So for example (from higher to lower): 2♠ A♥ A♦ A♣ A♠ K♥. Thus, 2♥ is the highest card and 3♠ the lowest. Players, in turn, discard a single or combination of cards to a central face-up pile. The objective is to avoid being the last player to hold any cards.

4.4 Proposed Computational Models

4.4.1 Game progress model

The proposed logistic model of game uncertainty is based on the early work by [4]. From the player's viewpoint, the information on the game result is an increasing function of time (the number of moves in board games) t. Here, the information on the game result is defined as the amount of solved uncertainty (or information obtained) $x(t)$, as given by (4.1). The parameter n (where $1 \leq n \in N$) is the number of possible options and $x(0) = 0$ and $x(T) = 1$. $x(T)$ is the normalized amount of solved uncertainty, where $0 \leq t \leq T$ and $0 \leq x(t) \leq 1$. Equation 4.1 implied that the rate of increase in the solved information $x'(t)$ is proportional to $x(t)$ and inversely proportional to t. Solving (4.1), (4.2) is obtained.

$$x'(t) = \frac{n}{t}\, x(t) \tag{4.1}$$

$$x(t) = \left(\frac{t}{T}\right)^{n} \tag{4.2}$$

It is assumed that the solved information $x(t)$ is twice derivable at $t \in [0, T]$. The second derivative of (4.2) indicates the accelerated velocity of the solved uncertainty along with the game progress, given by (4.3). The acceleration of velocity implies the difference in the rate of acquired information during the game's progress. The motion with constant acceleration a or free-fall motion in mind is given by (4.4). In the domain of board games, the acceleration a is approximated as (4.5), where B and D represent the average number of possible moves and game length, respectively.

$$x''(t) = \frac{n\ (n-1)}{T^n}\ t^{n-2}\ |_{t=T} \quad = \frac{n\ (n-1)}{T^2} \tag{4.3}$$

$$x(t) = \frac{1}{2}at^2 \tag{4.4}$$

$$a = \frac{n\ (n-1)}{T^2} \approx \frac{B}{D^2} \tag{4.5}$$

Two processes with the same GR-value at the end of the game information progress may have different instantaneous GR-value tendencies. For example, two basketball teams have the same successful shootings and total shot attempts. So their GR values are the same when the game is finished. However, each team felt different tendencies to get scores. A team with a stable scoring process is predictable, and vice versa. As such, each team's force was not only felt but also the change of the force. In physics, acceleration can be felt in motion, and the feeling of jerk [17].

The third derivative of (4.2) indicates the change of accelerated velocity (or jerk [17]) of the solved uncertainty along the game progress [18], which is given by (4.6). Because acceleration is always associated with the uncertainty of the game process, we can measure the attractiveness $F = ma$ concerning Newton's second law, which measures the acceleration process of attractiveness. Then, the first derivative of acceleration, called "jerk", would be associated with the measures of the tendency of getting attractiveness (or unpredictability), which could be speculated with the addiction mechanism. Hence, the motion with constant jerk j is given by (4.7), where it is approximate in the domain of board games as (4.8).

$$x'''(t) = \frac{n(n-1)(n-2)}{T^n} t^{n-3} \mid_{t=T} = \frac{n(n-1)(n-2)}{T^3} \tag{4.6}$$

$$x(t) = \frac{1}{6} j t^3 \tag{4.7}$$

$$j = \frac{n(n-1)(n-2)}{T^3} \approx 3\frac{B}{D^3} \tag{4.8}$$

Table 4.4 shows the measures of game refinement for board games. For sophisticated board games such as Chess, Shogi, and Go, it is assumed that there exists a reasonable zone for the acceleration (a) and jerk (j), which is between $0.07-0.08$, and $0.045-0.06$, respectively. Given the relations described by (4.9) and (4.10), then Table 4.4 can be obtained, which shows the reasonable acceleration and jerk zone of games that are derivative of popular, sophisticated board games.

$$GR = \sqrt{a} = \frac{\sqrt{B}}{D} \tag{4.9}$$

$$AD = \sqrt[3]{j} = \frac{\sqrt[3]{3B}}{D} \tag{4.10}$$

Table 4.4: Measures of game refinement for board games

	B	**D**	$\sqrt{a}$	$\sqrt[3]{j}$
Chess	35	80	0.074	0.059
Shogi	80	115	0.078	0.054
Go	250	208	0.076	0.044

4.4.2 Motion in mind model

In operant conditioning, a variable-ratio schedule is a schedule of reinforcement where a response is reinforced after an unpredictable number of responses [12, 13]. This type of schedule creates a steady, high rate of reaction. Stochastic games such as gambling and lottery games are typical examples of a reward based on a variable-ratio schedule. Mind sports games such as chess and Go were also essentially stochastic games while applying the move selection model [1]. This situation implies that a reward of variable-ratio reinforcement schedule characterizes a game [14].

Therefore, the reward function of a game can be characterized by defining the reinforcement schedule's variable rate (denoted as $VR(N)$). Then, velocity v (win rate) and mass m (win hardness) of the motion in mind model are given by (4.11), where $0 \leq v \leq 1$ and $0 \leq m \leq 1$. As such, N was used to measure the frequency of getting rewards, where the player can get a reward in a total average of N steps [15]. Let v_0 be the reward function over various masses for the perfect player as given by (4.12), which corresponds to the objectivity of play. Note that there is a distinctive computation of the v for the board and scoring games [1]. The success rate is defined as $v = \frac{G}{T}$ for scoring games (such as basketball, soccer, etc.), where G and T are the average successful score and the total scores. Meanwhile, the success rate is defined as $v = \frac{B}{2D}$ in a board game (i.e., Chess, Shogi, etc.), where B is the average branching factor, and D is the average game length.

$$v = \frac{1}{N} \quad \text{and} \quad m = 1 - v, \quad \text{where} \;\; 1 \leq N \in \mathbb{R} \tag{4.11}$$

$$m + v_0 = 1, \quad \text{where} \;\; 0 \leq m \leq 1 \;\; \text{and} \;\; 0 \leq v_0 \leq 1 \tag{4.12}$$

The concept of energy conservation was introduced by [16] as a prospective metric for engagement. The formulation of momentum in the game ($\vec{p_1}$) and potential energy in the mind (E_p) is provided by Equations (4.13) and (4.14), respectively. Subsequently, the momentum in the mind ($\vec{p_2}$) can be deduced through the conservation of energy in the mind, as expressed in Equation (4.15). This derived momentum in the mind is closely linked to the quantification of a player's engagement, as defined by Equation (4.16).

$$\vec{p_1} = mv \tag{4.13}$$

$$E_p = 2mv^2 \tag{4.14}$$

$$E_p = \vec{p_1} + \vec{p_2} \tag{4.15}$$

$$\vec{p_2} = E_p - \vec{p_1} = 2m^3 - 3m^2 + m \tag{4.16}$$

Applying (4.16) while assuming $\vec{p_2} = mv_2$, the subjective velocity v_2 is given by (4.17). Let $v_k(m)$ be a reward function over various m for a player with ability parameter k. Then, the relation is generalized as v_k using a parameter (say k where $0 \leq k \in \mathbb{R}$) that is the nature of the game under consideration, as shown in (4.18). The ability parameter

k stands for players' strength in the competitive game context or error tolerance in the social or non-competitive context. For example, there is no error tolerance for the perfect player v_0. Note that objectivity and subjectivity perspective enables us to deepen the understanding of engagement and addictive mechanisms in games [16]. Thus, the objective velocity (v_0) and subjective velocity (v_k) were determined.

$$v_2 = 2m^2 - 3m + 1 = (1 - 2m)(1 - m) \tag{4.17}$$

$$v_k = (1 - km)\ v_0, \quad \text{where} \;\; 0 \leq k \in \mathbf{R} \tag{4.18}$$

The notion of potential energy in mind was originally discussed by [1] and its formula is given by (4.14). The notion of velocity is derived from the reinforcement schedule $VR(N)$ with frequency N, so we call objective reinforcement (E_0) for the potential energy in mind of the perfect player (v_0). Otherwise, we call subjective reinforcement (E_k) for the potential energy in the mind of other players (v_k). A game would produce its potential energy in the field of play (hence we call it potential energy of play) by which people would feel engagement or reinforcement.

In behavioral psychology, the term "reinforcement" refers to an enhancement of behavior. This term was used as a positive interpretation, i.e., greater reinforcement gives people a more substantial interest in staying in the event under consideration. In the game context, reinforcement depends on the player's ability. The potential energy of play (E_k) is given by, $E_k = 2mv_k^2$ which is denoted as $s$ubjective reinforcement. For the perfect player or game theoretical reward ($k = 0$), denoted as $o$bjective reinforcement E_0. Figure 4-1 illustrates the objective and subjective reinforcement when $k = 3$. The reward function (v_k) represents a player's model or his/her sense of value. When assuming $k > 3$, $v_k < 0$ holds at $m = \frac{1}{3}$ where the objective reinforcement is maximized. This situation implies the learning context's most comfortable (peak of E_0). Therefore, it is highly expected to have $k \leq 3$. Furthermore, Go ($m = 0.42$) is still not yet solved;

thus, $2.38 < k$ is expected to hold.

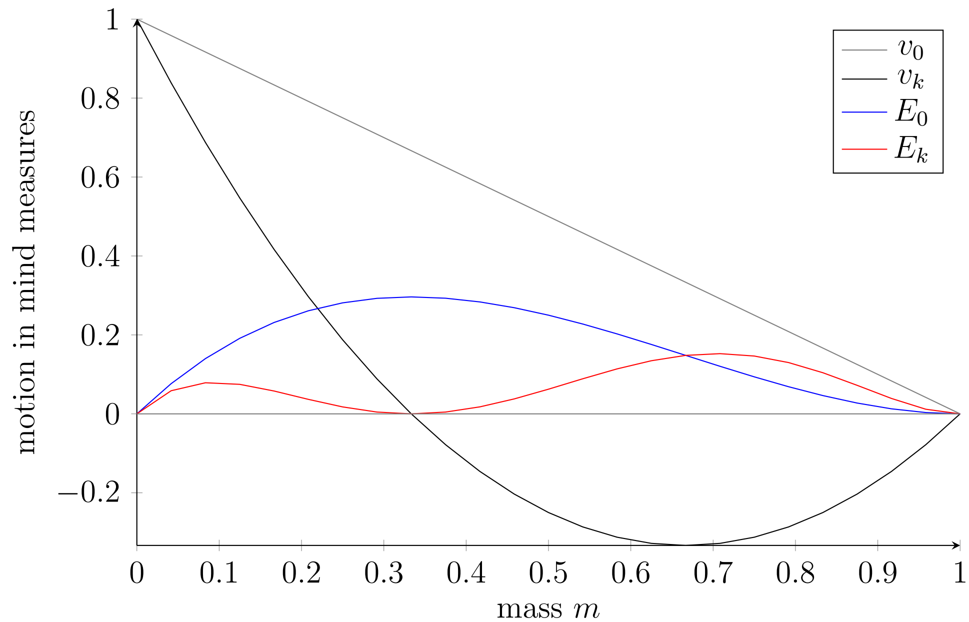

Figure 4-1: Objective and subjective reinforcement when $k = 3$

4.4.3 Experimental design

The experiment is based on several logical settings to mimic the different experiences of players. Different logical rules were set by listing all possible actions, and a pruning algorithm was utilized to find the optimal solution based on the different weights (Table 4.5). The first experiment focuses on fixing the game but is played with different AI levels (horizontal perspective): weak, fair, and strong. As such, analysis of players with different skill levels was simulated.

Subsequently, the second experiment was conducted on the same player (fixed AI level) who played games with varying levels of complexity (vertical perspective). Such analyses were simulated from the different rules represented by the increasing complexity of the card games, from classical ones to more sophisticated games (i.e., between Wakeng and Doudizhu).

Algorithm 1 Card Game Algorithm

```
pass_count_l ← 0
pass_count_p ← 0
is_end ← false
while not is_end do
    if is_played then
        last_played ← 0
    else
        pass_count_leq1
    end if
    for i ← 0 to 2 do
        check_end(card_groups[i])
        player_action_policy(card_group, i + 1, last_player, his, valid_moves)
        if is_played then
            last_played ← 0 ELSE
            pass_count_l1
        end if
    end for
    current_player ← id
    opponent ← 3 − current_player {Find the opponent player}
    if player_id == last_player then
        valid_cards, length ← list_greater_cards(, card_group)
    else
        if check_weight(his[−1][0], his[−1][1]) ≤ thresholdandlength > 1 then
            valid_cards, length ← list_greater_cards(his[−1][1], card_group)
        end if
    end if
    played_cards ← play_rules(valid_cards)
    remove_cards(valid_cards, card_group)
    if length == 1 then
        return  false
    end if
end while=0
```

Table 4.5: The experiment design of Wakeng and Doudizhu

AI level	Description
Weak	The priority players play the longest weakest cards first, and other players play their cards as much as possible based on the cards of the previous player.
Fair	Besides the basic logic of weak AI, two players other than the digger cooperate. If one of them plays a card with a large difference in weight to defeat the digger's card, he chooses not to defeat his teammate.
Strong	Add a new restriction to the previous logic, play any cards but can not cause two more solo cards. If a player can defeat the previous player, but he played this hand, it will cause two more solo cards, then he will give up playing cards.

4.5 Computational Results

4.5.1 Result analysis of DouDiZhu

The classic Doudizhu is the most popular version in China, played by three players. Valid cards include a single card, pair card, straight card, triple card, double-wing card, plane card, bomb card, rocket card, and 300 cards, which kickers can play. DouDiZhu game has a unique score system. The basic score is set at 3 points in this study, doubling the score when the player played a bomb or rocket [78]. For example, the landlord obtains a total of 24 points while the peasant loses 12 points if there are three bombs; thus, the landlord wins the game. The conducted simulation considers a reasonable level of DouDiZhu AI, which corresponds to the average level of a human player (Table 4.6).

The result of the conducted simulation considers a reasonable level of DouDiZhu AI, which corresponds to the average level of a human player. As the $GR = 0.0715$, it is located in the sophistication zone ($GR \in [0.07, 0.08]$). So, most people could enjoy the

Table 4.6: Measures of game refinement for classical DouDiZhu

B	**D**	**GR**	**AD**
8.197	40.032	0.0715	0.0726

B: average possible options;
D: total steps;

Doudizhu game. Moreover, the $AD = 0.0726$ is higher than the reasonable zone value ($AD \in [0.045, 0.060]$). This situation indicates that the Doudizhu game contains lots of unpredictability and instability. Because of Doudizhu's vibrant and diverse rules, players often feel surprised and unpredictable. Subsequently, the results of different levels of AI with different settings were analyzed (Table 4.7).

Table 4.7: Results of different levels of AI with different DouDiZhu game settings

Setting†	**AI level**	**B**	**D**	**GR**	**AD**
(3, 0, 3)(18, 18, 18)	fair	7.344	45.418	0.060	0.062
	strong	6.657	53.212	0.048	0.051
(3, 1, 2)(20, 17, 17)	weak	7.667	38.486	0.072	0.074
	fair	8.198	40.032	0.072	0.073
	strong	6.613	53.212	0.048	0.051
(3, 1, 2)(18, 18, 18)	weak	7.232	38.964	0.069	0.072
	fair	7.644	40.696	0.068	0.070
	strong	6.021	53.075	0.046	0.049
(3, 1, 2)(22, 16, 16)	weak	9.312	37.605	0.081	0.081
	fair	10.02	39.070	0.081	0.080
	strong	8.037	50.929	0.056	0.057
(3, 1, 2)(24, 15, 15)	weak	13.438	36.520	0.100	0.094
	fair	14.373	37.810	0.100	0.093
	strong	11.761	48.026	0.071	0.068

†: the first parenthesis represents the player's configuration (number of players, landlord number, peasant number); the second parenthesis represents the respective player's initial hands (landlord, peasant 1, peasant 2); B: average possible options; D: total steps;

Two observations can be made from the analysis of the result of the sophisticated card game Doudizhu (illustrated as Figure 4-2). Firstly, weak players' ability is insufficient to adapt to the game's difficulty. At this time, the GR value is higher than the reasonable range. As a result, the game becomes difficult for the player, where the AD value is higher than the reasonable range, which makes the game too varied and unpredictable. After learning and training, players can master and enjoy the game, and the GR value and AD value become lower and close to the reasonable zone. Secondly, if the game is not balanced and fair enough for the players (for example, if the difference in the number of cards is too large), the uncertainty and unpredictability of the game become too high. For players, the AD value has a more significant impact at this time, which they will indulge in surprises.

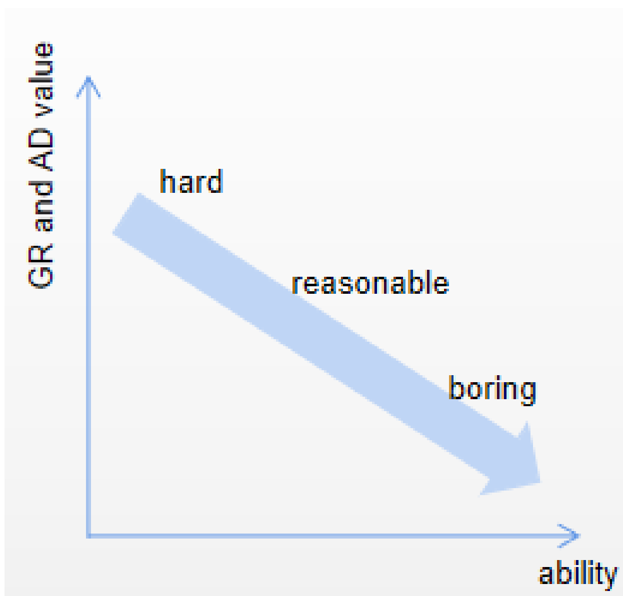

Figure 4-2: The tendency of GR and AD based on the ability level of sophisticated card games

4.5.2 Result analysis of Wakeng

Wakeng was explored and analyzed using different levels of AI to simulate different players with varying skill levels (Table 4.8). Based on the table, it can be observed that the GR value increases as the player's skill increases (canonical to players of weak skill to strong skill). This condition implies that when players explore the variation of the classical gameplay and develop more ways to play, they will feel more attracted to playing the game.

Table 4.8: The analysis of Wakeng based on different level AI based on the setting of (3, 1, 2)(20,16,16)

AI level	**B**	**D**	**GR**	**AD**
weak	4.10	49.72	0.041	0.046
fair	4.69	54.15	0.040	0.045
strong	4.12	37.48	0.054	0.062

B: average possible options; D: total steps;

However, classical games do not have much variety where the rules are relatively fixed and straightforward, making the unpredictability very low ($AD \in [0.045, 0.060]$). Nevertheless, players can play more cards after learning and training, improving the games' unpredictability (as indicated by the increasing GR and AD values). However, the change between the different AI levels was insignificant, implying that even though mastery of the game is achieved, players' entertainment value and controllability will not change significantly.

4.5.3 Result analysis of suits-relevant card games

It can be observed that the stability and controllability measured by the AD value based on different AI levels and levels of sophistication between Doudizhu and Wakeng were related to the points of cards; suits are irrelevant. Subsequently, similar suit-relevant card games, such as Winner, Big Two, and Tien Len, were explored further. From the data analysis of Table 4.9, a similar observation can be found from these suit-relevant card games, verifying the previous findings. For players with a high skill (simulated by strong AI), or the game was advanced and sophisticated enough, the GR and AD values increased and reached a reasonable zone leading to a higher entertainment experience and higher unpredictability.

Table 4.9: The analysis of several card games based on different level AI and setting

Games	**AI level**	**B**	**D**	**GR**	**AD**
Big two	weak	3.22	63.29	0.028	0.034
	fair	3.33	65.67	0.028	0.033
	strong	3.68	44.71	0.043	0.050
Winner	weak	3.43	61.10	0.030	0.036
	fair	4.54	64.10	0.033	0.037
	strong	3.88	42.27	0.047	0.055
Tien Le	weak	3.95	56.94	0.035	0.040
	fair	5.09	56.93	0.040	0.044
	strong	3.72	43.70	0.044	0.051

The influence of suits usually demands the game to be played by more players. Typically, the number of cards in each player's hand will be reduced, slightly impacting the balance it has on the gameplay. Collected data showed that GR and AD values increase to the reasonable zone when the player becomes stronger. For the classical games, the GR and AD values of strong players are higher than the weak, implying players need to train to explore more gameplay to make the relatively bland game process more entertaining (Figure 4-3).

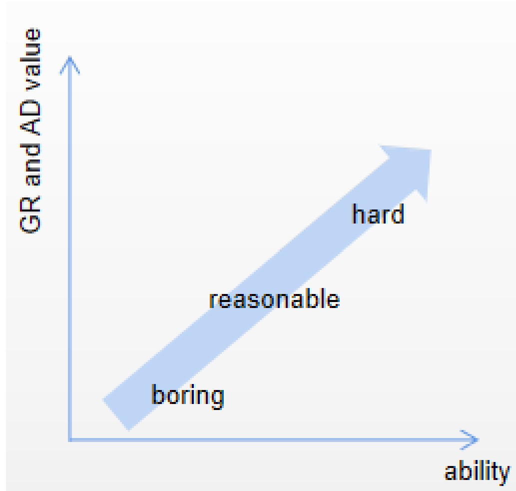

Figure 4-3: The tendency of GR and AD based on the ability level of classical card games

4.5.4 Result analysis of fixed AI levels

Based on the standard game settings, Doudizhu of parameters of (3, 1, 2)(20, 17, 17) and Wakeng of parameters of (3, 1, 2)(20, 16, 16) have the AD value between 0.051 to 0.074 and 0.04 to 0.045, respectively. It can be observed that the AD value of Wakeng is lower than Doudizhu because its gameplay was conservative, with fewer options, and thus much more stable than Doudizhu. Therefore, the rules of Wakeng are more predictable, associated with the low value of AD. This finding is aligned with previous studies that associate low AD values with simple and plain games [18]. Thus, the relationship between player level and game complexity, based on GR and AD values, was described in Table 4.10.

The GR and AD values decrease when players become stronger for high-complexity games like Doudizhu. When the player starts to play, the ability is not enough to adapt to the game's difficulty. Currently, the GR and AD values are higher than the reasonable range. The game is challenging and unpredictable for the player. However, players can master and enjoy the game after learning and training. For simple games, The GR and AD values increase when players become stronger. When the player starts

Table 4.10: Closeness to reasonable zone of GR and AD given by different levels of players of different game complexity

Player Level	**Classical**	**Advanced**
Weak	$<$	$>$
Normal	$\approx$	$\approx$
Skillful	$=$	$=$

$<$: less than reasonable zone of GR/AD;
$>$: more than reasonable zone of GR/AD;
$\approx$: approximately close to reasonable zone of GR/AD;
$=$: within the reasonable zone of GR/AD;

to play, they will feel simple and predictable. Both GR and AD are smaller than a reasonable range. However, different gameplay can be developed and explored after training, so the GR and AD values will slowly increase, and players will slowly build interest and unexpectedness.

Table 4.11: The entertainment aspects of different GR and AD value expression

GR & AD	**Description**
$<$ zone	skill $>$ sophistication, too easy, need to continue to explore the evolution in the game rules to make the game more interesting
$=$ zone	Perfect match
$>$ zone	skill $<$ sophistication, too difficult, need to improve their skills to master the game

Table 4.11 summarizes the expression of different GR and AD values from the game entertainment perspectives. If the game is challenging for the player, the player keeps training on the process, where they can become proficient and skillful, improving their skills. The process is slow, yet mastering the game would gradually match the game, and the player can feel the joy of conquering. On the other hand, if the game is easy for the player, the player keeps developing and exploring other gameplay, which they find variable ways to play. The process includes exploring more game possibilities to make

the process simpler, more exciting, and more variable. As such, GR and AD values tendencies relative to the training time influence the game's difficulty and mastery (Figure 4-4).

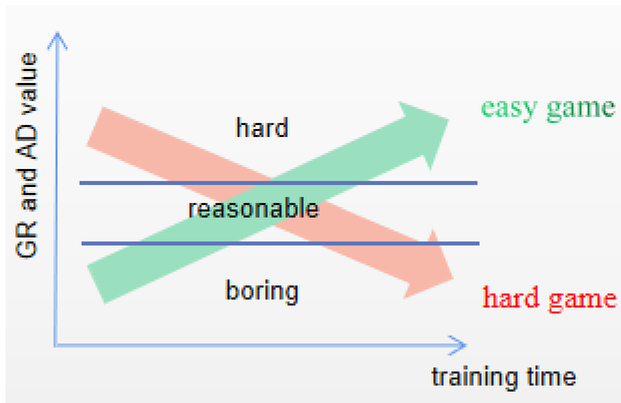

Figure 4-4: The tendency of GR and AD value of different complexity game

4.6 Discussion

4.6.1 Comparison on different game complexities

The suit-relevant and suit-irrelevant card games were further explored from the perspective of the motion in mind model, in comparison to the game progress model (GR and AD values), in which the results were presented in Table 4.12 and Table 4.13. For suit-irrelevant games, the velocity of Doudizhu is higher than Wakeng, And for suit-relevant games, the velocity of Tien Len is higher than Big Two. This condition showed that the speed of a sophisticated game is higher than a classical and simple game. Then, the momentum of the game's motion ($\vec{p_1}$) described the players' attention and concentration and the tendency to keep playing games. It can be observed that the momentum ($\vec{p_1}$) of Doudizhu is higher than Wakeng, while Tien Len is higher than Big Two. These situations implied that players tend to continue the sophisticated and advanced games and have a greater tendency to play advanced games.

Table 4.13: The comparison of suit-relevant card games based on a standard setting

Setting	AI level	B	D	GR	AD	v	m	$\vec{p_1}$	$\vec{E_p}$	$\vec{p_2}$	a_2
Tien Len [(4, 2, 2)(13, 13, 13, 13)]	strong	3.721	43.700	0.044	0.051	0.043	0.957	0.041	0.003	-0.037	0.039
	fair	5.090	56.930	0.040	0.044	0.045	0.955	0.043	0.004	-0.039	0.041
	weak	3.950	56.931	0.035	0.040	0.035	0.965	0.033	0.002	-0.031	0.032
Big two[(4, 2, 2)(13, 13, 13, 13)]	strong	3.680	44.705	0.043	0.050	0.041	0.959	0.039	0.003	-0.036	0.038
	fair	3.330	65.670	0.028	0.033	0.025	0.975	0.025	0.001	-0.023	0.024
	weak	3.220	63.290	0.028	0.034	0.025	0.975	0.025	0.001	-0.024	0.024

Table 4.12: The comparison of suit irrelevant based on a standard setting

Setting	**AI level**	**B**	**D**	**GR**	**AD**	v	m	$\vec{p_1}$	$\vec{E_p}$	$\vec{p_2}$	a_2
DDZ [(3, 1, 2)(20, 17, 17)]	strong	6.61	53.21	0.048	0.051	0.062	0.938	0.058	0.004	-0.055	0.751
	fair	8.20	40.03	0.072	0.073	0.102	0.898	0.092	0.009	-0.082	0.590
	weak	7.67	38.49	0.072	0.074	0.100	0.900	0.090	0.009	-0.081	0.602
WK [(3, 1, 2)(20, 16, 16)]	strong	4.12	37.48	0.054	0.062	0.055	0.945	0.052	0.006	-0.046	0.049
	fair	4.69	54.15	0.040	0.045	0.043	0.957	0.041	0.004	-0.038	0.040
	weak	4.10	49.72	0.041	0.046	0.041	0.959	0.040	0.003	-0.036	0.038

Also, the potential energy ($\vec{E_p}$) of Doudizhu is higher than Wakeng, and Tien Len is higher than Big Two. These conditions imply that the player's expectation of finishing the advanced game is higher than a classical game [83]. It can be observed that Doudizhu and Tien Len have higher AD values and higher ($\vec{E_p}$) values. This likely can be speculated as the unpredictability is great, and the players keep the expectation high. Moreover, advanced games' absolute value of $\vec{p_2}$ is higher than a classical one, which means it has more engagement to continue. The result (Table 4.12 and Table 4.13) also revealed that advanced games have high velocity, AD value, $\vec{p_1}$, $\vec{E_p}$, and $\vec{p_2}$ compared to the classical games. This condition implies that the sophisticated game has more unpredictability and expectation, thus, better engagement. Relative to the AD value, such a condition can also be induced, which means that higher unpredictability and surprise can make us feel more concentrated, engaged in continuing the game, and have a greater expectation of the game process.

A summary of motion in mind measures for different games was given in Table 4.14. It can be observed that the more advanced and complex the game, the higher the velocity. The velocity of DouDiZhu is the highest, and the Big Two is the lowest, which means that the speed of information progression in a sophisticated game is higher than in a classical game, where the information contained in each step requires multiple options of consideration. Correspondingly, the higher the difficulty and complexity of the game, the lower the mass, and the easier it is to progress. The higher the momentum, the higher the tendency to continue, and the higher the potential energy, the more expectation the players have.

Table 4.14: Summary of motions in mind measures of different games

Card games	**Rank**	m	v	$\vec{p_1}$	$\vec{Ep}$	N
DoudDiZhu	1	0.898	0.010	0.092	0.0187	9.8
Wakeng	2	0.954	0.046	0.044	0.0040	21.7
Tien Len	3	0.959	0.041	0.039	0.0030	24.4
Winner	4	0.964	0.035	0.034	0.0024	28.2
Big Two	5	0.970	0.030	0.029	0.0020	33.3

Moreover, in card games, if $N > 20$, the game is too random, and players feel too easy, forcing players to improve their ability to explore more gameplay to reduce the impact of a surprise and make the process more enjoyable. The GR and AD values would increase to the reasonable zone. In contrast, if $N < 15$, the game is too static, where limited N is required to acquire the skill to distinguish the optimal choice. Such conditions challenge players to become more skillful in adapting to the game rules and feel enjoyable; thus, the GR and AD values decrease to the reasonable zone.

Some games have perfect information (such as Chess, tic-tac-toe, Checkers, Go, and so on), where each player can see all the pieces on the board at all times [84]. For these kinds of games, the game length (denoted as D) is enormous, while the optional moves/positions to be made (denoted as B) at each step are small, resulting in a low-risk rate with a long game duration and a minimal number of rounds. Therefore, players will be more cautious because they know all the current and possible future information to determine the winning state. Moreover, given enough rounds (or unlimited resources), the game mimics the skill demonstrated by the strong player, where only a limited number of rounds is enough to distinguish the winners.

However, some games are characterized by their aspect of the play hidden from opponents, such as the cards in poker and bridge, which were typical examples of games with imperfect information [85]. For card games, the D is small, and B is high, making the risk rate high, short game duration, and a considerable number of rounds. This condition implies that in imperfect information games, players are more based on luck (or likelihood) for each hand because most information is hidden. Furthermore, each round does not have enough game steps to demonstrate the skill to identify winners, which demands more rounds to determine the real winner. So for imperfect information games, players need many rounds to distinguish winners.

4.6.2 GR and AD relative to addictive situation

The impact of GR and AD values were further explored where the cross point of $GR = AD$ is determined, which could be related to the balance between entertainment/excitement (associated with the gravity of play) and surprise/fluctuation (related to the jerk of play). If $GR = AD$, people should feel exceedingly engaged and addicted. This condition is caused by not only the player feeling entertained due to the GR sophisticated zone, but the high AD value is associated with the feeling of surprise and unpredictability. As such, the play is likely to be repeated to maintain fairness, with speculation of winning the next round.

Revisiting (4.9) and (4.10), solving $GR = AD$ would have $B = 9$. Then, considering $GR = AD$ from the perspective of reward frequency N would involve resolution involving (4.11); thus, leading to (4.19) which results in $N = 9$. When $N = 9$, there is a cross point between GR and AD, where the velocity of the game is $v = \frac{1}{9}$ and $m = \frac{8}{9} = 0.89$ (Figure 4-5). At the same time, when $B = 9$ and $GR \in [0.07, 0.08]$, then $D \approx 40$. That is the reason $(B, D) = (9, 40)$ would be the candidates for an addictive game. This condition would explain why the setting of Doudizhu of $(B, D) = (8, 40)$ is the most popular and addictive game for common players among many similar card games.

$$AD = \sqrt[3]{\left(\frac{3}{2} \cdot \frac{B^2}{D^4} \cdot \frac{2D}{B}\right)} = \sqrt[3]{\left(\frac{3}{2} \cdot GR^4 \cdot N\right)} \tag{4.19}$$

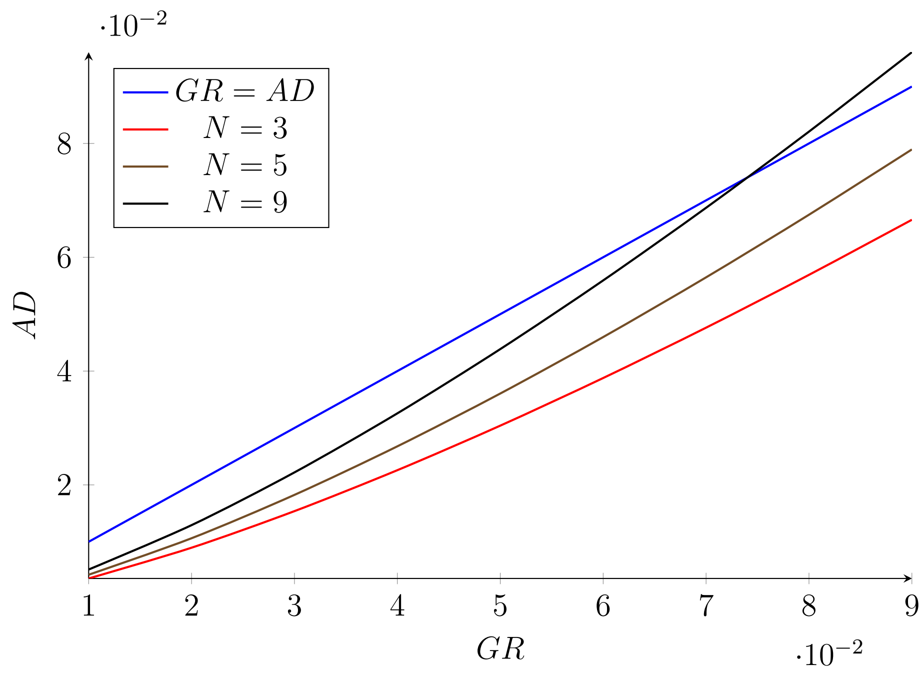

Figure 4-5: The relations between GR and AD

Based on the given N and GR, solving them can help in discerning the reasonable game length (D), given by the (4.20). It can be observed that the D is shorter (Figure 4.6.2) when AD and N get higher (Figure 4.6.2). This situation implies that for a fast-paced game, the rate of getting the reward is shorter, high predictability, full of surprise, and encourages players to play the game repeatedly, thus, making it addictive.

$$D = \frac{2v}{a} = \frac{2}{N \cdot GR^2} \tag{4.20}$$

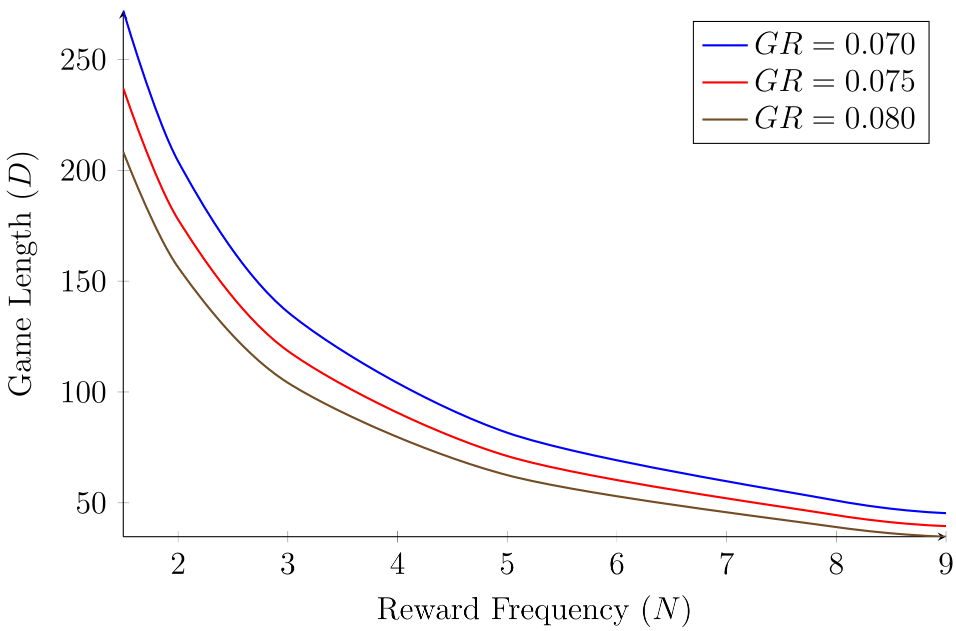

Figure 4-6: The relations between reward frequency (N) and game length (D)

Based on results and previous studies, the GR value could provide the balance of the game length; if it is too short, the game is unfair, and players do not have enough steps to show their ability. Otherwise, it is too boring. The cross point between velocity and acceleration shows the balance between fairness and entertainment. In that sense, the winner can be determined within reasonable steps. Meanwhile, based on the value of AD, it provides the balance of the game rounds. The higher jerk value means unpredictability; it causes shorter steps (the game process relies on chance, and players have insufficient steps to show skill in the stochastic game). Otherwise, the game steps are longer. This situation can be defined based on $y_1 = vt$, $y_2 = \frac{1}{2}at^2$, and $y_3 = \frac{1}{6}jt^3$ where the relation AD and GR were given by $AD^3 = j$ and $GR^2 = a$. When

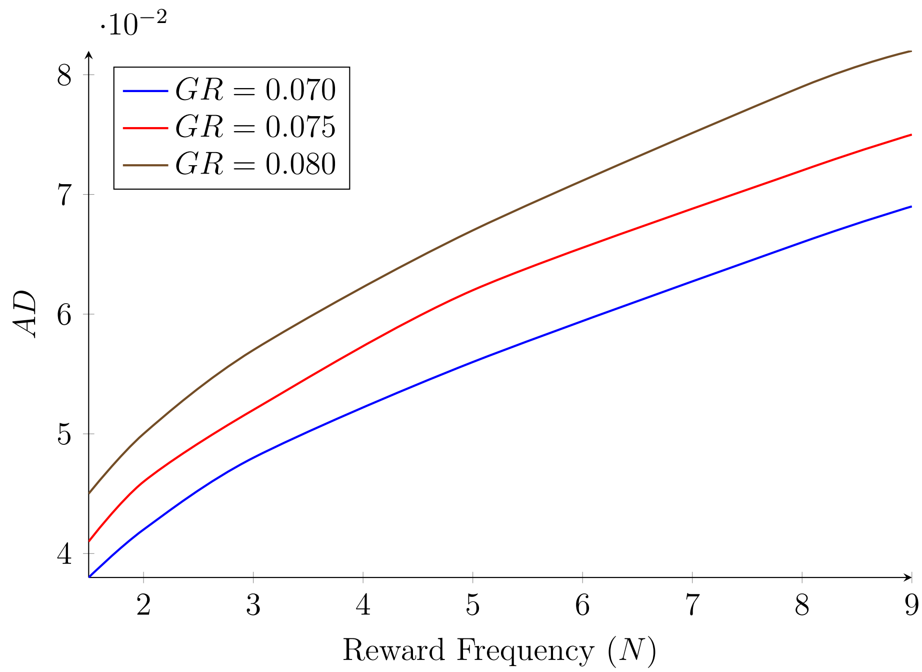

Figure 4-7: The relations between reward frequency (N) and AD

AD=GR, the cross point between acceleration and jerk can balance the unpredictability and entertainment; in that sense, the winner can be determined with good rounds (Figure 4-8).

In addition, the reward frequency (N) differences between perfect and imperfect information games are observed, where the N of the former is lower than the latter. This condition implies that imperfect information games have random choices compared to perfect information games, so the entire process requires careful choosing. For example, if the game is stochastic, there is a high possibility of surprises, the game steps are too short, and the jerk value is high. This situation can be found in a card game, where many rounds were played to balance the impact of surprises. On the other hand, if the game is too static, while the player can fully demonstrate the ability, longer steps are needed to complete the game. This situation causes the jerk value to be low (like a board game), where the winner can be distinguished under limited rounds.

A low jerk value can balance the deterministic and stochastic of the game features.

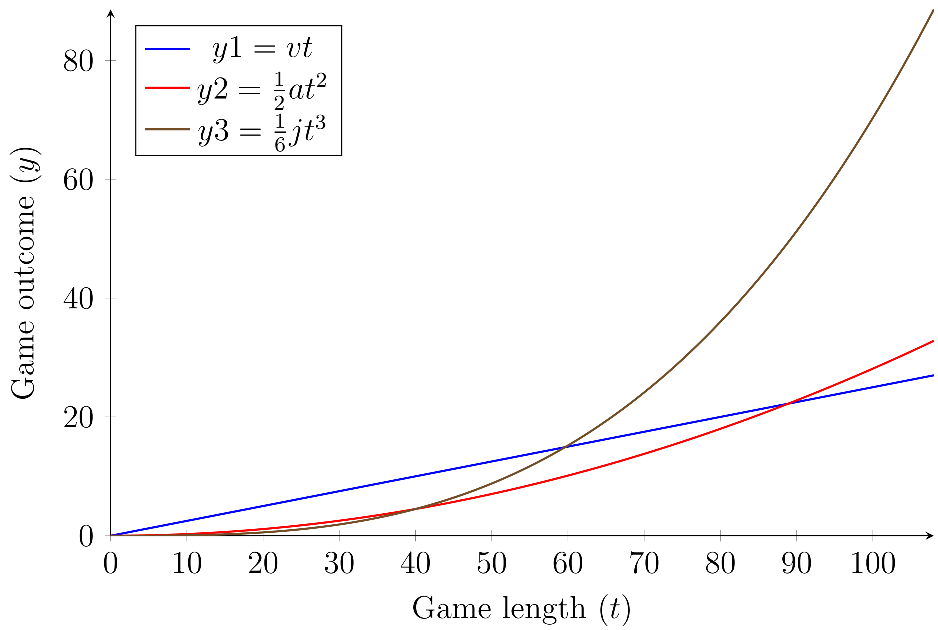

Figure 4-8: The crosspoint between fairness (y), reinforcement (v), entertainment (GR), and unpredictability (AD)

Otherwise, a high jerk value would be attractive to the players where the steps are limited, full of surprises, and engaging. The data from card games could suggest an equivalence relation between game refinement theory and motion in mind to determine the nature of addictive games. Given the condition of $GR \in [0.07, 0.08]$ (zone values) and $AD = GR$, it is where the $N = 9$ was obtained, which suggests the nature of strongly addictive games when $m = 0.9$ ($N = 9$) under the assumption that $k = 2.5$. As such, Table 4.15highlights the possible corresponding popular games.

An extension of the game progress model leads to the four possible major principles of play. These principles were defined relative to the measures of the game progress model, based on both GR and AD values, which were given in Table 4.16. GR value implication was derived from the principles of (1) to (3). The game length should not be too small/large; otherwise, unfair/boring. It was found that addictive entertainment is derived from all principles. Games that satisfy principles (3) and (4) would be exceedingly engaged and should be repeatedly played to meet fairness relative to principles

Table 4.15: Possible corresponding games

Game type	**N**	v	m	AD	t	*Games*
Perfect information	1.5	$^2/_3$	$^1/_3$	0.041	237	Go
	2	$^1/_2$	$^1/_2$	0.046	178	Basketball
	3	$^1/_3$	$^2/_3$	0.052	119	Shogi
	5	$^1/_5$	$^4/_5$	0.062	71	Chess
Imperfect information	8	$^1/_8$	$^7/_8$	0.072	44	Mahjong
	9	$^1/_9$	$^8/_9$	0.075	40	Doudizhu

(1) and (2). A certain type of game with a larger mass (for instance, $m > 0.9$) can be implemented on a computer and thus can be easily played repeatedly.

Table 4.16: Possible principle of game element

Game Element	**In-game Description**	**In-game Implication**
Game length (t)	Cost of getting reward	Fairness
Velocity (v)	Frequency of getting reward	Reinforce of play
Acceleration (a)	Uncertainty of getting reward	Attractiveness of play
Jerk (j)	Unpredictability of getting reward	Surprise of play

4.7 Chapter Conclusion

This study explored the implications of motion in mind measure, in particular jerk value, to measure a game's entertainment aspects and identify what constitutes an addictive game. Several card games with different complexities and rule designs were investigated and computationally analyzed using varying AI levels to represent different skill levels of the player. By simulating the performance of players with different skill levels in card games of various difficulty levels, we can conduct both horizontal and vertical comparisons to delve into the interactive relationship between players and games. This allows us to explore the balance between player abilities and game complexity. The differences between the two dimensions of games were observed: (1) deterministic and stochastic games; and (2) simple and sophisticated games.

For sophisticated games, such as board games, the GR value is 0.07-0.08 and the AD value is 0.045-0.06, ensuring entertainment and predictability (stability, controllability). For games with imperfect information, such as poker and mahjong, the AD value is similar to the GR value. The AD value is higher than that of the perfect information games, which means that the game is highly unpredictable and there are surprises in the process. The balance between the AD value and the GR value makes the game addictive. The process is unparalleled and players prefer to play repetitively.

For deterministic games (i.e., board games), the possible options (N) are small; they are deterministic and stable, ranging from $1.5 \leq N \leq 5$, where the GR and AD value are located in their sophistication zones ($GR \in [0.07, 0.08]$ and $AD \in [0.045, 0.06]$) and $GR > AD$. Longer steps make the process more deterministic and less surprising. For stochastic games (i.e., card games), the possible options (N) are large, ranging from $5 \leq N \leq 10$, but limited in terms of game length while being more unpredictable and full of surprise ($AD > GR$), while solving $GR = AD$ allowed the N value to be determined. When $N = 9$, there is a cross point between GR and AD curves (see Figure 4-5), where $B = 9$ and $D = 40$. This condition is the most addictive and entertaining case, which explains the reason for card game popularity and addiction (for instance, the Doudizhu game setting of $(B, D) = (8, 40)$).

Analysis of the AD value relative to the GR value showed that it could measure the stability and predictability of game players in complete and incomplete information games. Analyzing GR and AD provides insights into player entertainment and unpredictability. Complex games start above zone values, posing initial challenges, while simpler games begin below, becoming more engaging over time. For instance, high-complexity games (like Doudizhu) typically have GR and AD values higher than the zone values, implying that the game difficulty and unpredictability were not easily mastered by the player by just having a better ability. Nevertheless, players can become more skillful in mastering and enjoying the game after learning and training. Meanwhile, for simpler games (like Wakeng, Big Two, Winner, and Tien Len), the player could quickly get bored since the GR and AD values are smaller than the zone values. However, different gameplay can be developed after exploring and training, so the GR and AD values will slowly increase, and players will feel interested and unexpected.

Furthermore, the value of the potential energy ($\vec{E_p}$), the momentum in the game ($\vec{p_1}$), and in mind ($\vec{p_2}$) of a complex game is higher than the simple game, implying a high level of player's tendency, expectation, and engagement to play sophisticated games. This condition can also be related to the AD value, where higher unpredictability can be observed. While GR and AD focused on balancing the game sophistication corresponding to the game length and winning rate, while relative to the $\vec{E_p}$, $\vec{p_1}$, and $\vec{p_2}$, provided the indication of the game stability based on the player's tendency and expectation. Therefore, the principle of play can be implied, providing insights into the game's elementary components.

Nevertheless, the study has several limitations. Firstly, the collected data were based on the established assumption that the difference between the levels of AI indicates the player's skill level. The gap between amateur-level and expert-level players may be huge in some game types and insignificant in others; thus, caution is advised, and the findings are interpreted with a grain of salt. Secondly, the findings were the application of the proposed computational model in the domain of card games. The implication in complex real-time continuous games (such as video games) is currently unknown, which is open for future studies. Finally, the study's theoretical basis was based on the culmination of works built on analogical relations between classical physics and mechanical game-playing schemes. Games based purely on sensors or gestures, psycho-physiological measures or signals, and another mode of interfaces (augmented or virtual reality technology) would be an exciting area to explore in the future.

Chapter 5

The Impact of Performance Degree on Players: Exploring the Dynamics of Player Engagement and Enjoyment in Game Process

5.1 Chapter Introduction

In the gaming domain, it is common to acknowledge the presence of inherent risks, represented by the variable "m". Players are entrusted with the challenge of formulating strategic solutions to overcome these obstacles. Proficient players who have mastered the game can effortlessly progress, resulting in a game progression ideal velocity denoted as $\vec{v_0} = 1 - m$ [86].

However, the gaming landscape reveals a diverse range of player abilities, leading to varying competencies in managing uncertainties and advancing in the game. Prior research introduces the variable "k" to evaluate player performance, with higher values indicating diminished performance. For players with imperfections, the game progression velocity can be computed as $\vec{v_k} = (1-km)\vec{v_0} = (1-km)(1-m)$. In this regard, we will discuss the comparison between perfect players and imperfect players in the field of motion in mind during the game, and explore how the psychological changes evolve as players continue to improve and become proficient players.

Performance level "k" is important on the player side, and reward frequency "N" also plays a big role on the game side. In the context of reinforcement learning, "N" often signifies the "number of steps" or "time steps" representing the timeframe for earning rewards. In games with perfect information, players enjoy complete access to moves influencing the game's outcome or the count of potential positions that can be eliminated. Conversely, in games involving imperfect information, players may possess only partial insights into other players or engage in informed speculation about strategies and inclinations. In such scenarios, "N" may denote the number of options or actions considered during decision-making. For instance, in the Monte Carlo Tree Search algorithm, "N" frequently signifies the number of simulations or playouts conducted from a specific game state, so the velocity also can be described as $\vec{v_0} = \frac{1}{N}$.

As mentioned earlier, the concept of "AD" emphasizes the inherent unpredictability of games. This study aims to delve into the intricate relationship between the performance level represented by "k" and the reward frequency represented by "N", as well as the unpredictability manifested by "AD" in the gaming domain. These three fundamental elements are closely intertwined, forming a cohesive whole that provides valuable insights for analyzing both the player's and the game's perspectives. This offers valuable perspectives for game designers, optimizers, product managers, and others involved in the gaming industry.

5.2 Related Works

This section begins by examining the disparity between perfect players and imperfect players from the standpoint of physical quantities such as speed, acceleration, and potential energy. Additionally, it investigates the conditions under which non-perfect players can emulate sensations akin to those in an ideal state. Furthermore, it explores the relationship between the speed of the game and the optimal number of turns.

5.2.1 The meaning of playing performance

In different player performances, discernible distinctions emerge in the velocities of game progression. As illustrated in Figure 5-1, players with lower performance levels demonstrate a diminished capacity to overcome uncertainties, even when faced with an equivalent risk rate, as compared to their higher-performing counterparts[1].

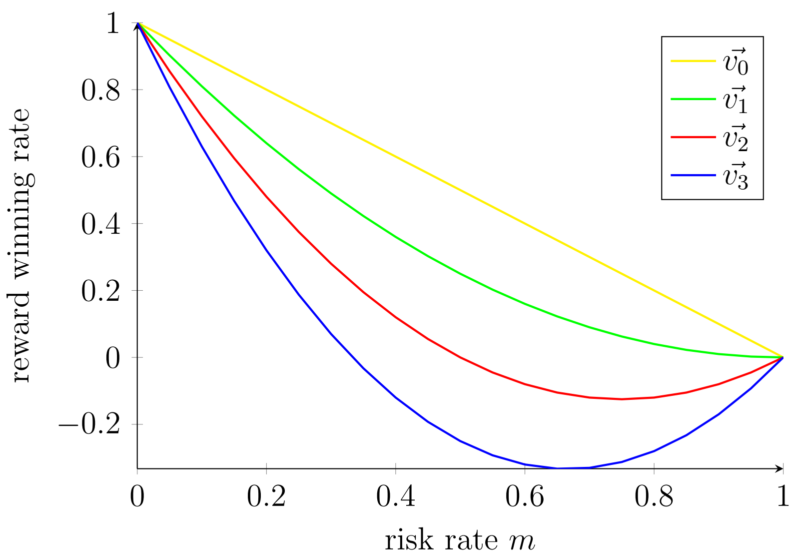

Figure 5-1: Game progression velocity as a function of risk rate m

Upon solving the inequality $v_k = (1 - km)(1 - m) \geq 0$, we determine the critical threshold for the admissible risk rate, corresponding to a player's given performance level. This threshold is precisely defined as $m_x = \frac{1}{k}$.

This clarifies that a player's velocity maintains a positive trajectory within a range spanning from 0 to $\frac{1}{k}$, as long as the risk rate remains within this defined limit. Nevertheless, if the risk rate surpasses this critical threshold, the capacity to resolve uncertainties diminishes significantly. Players may then encounter stagnation in their game progress when employing strategies based on their performance level.

5.2.2 The acceleration a_k of different kinds of players

From the previous discussion, we have the derivation of v_k.

$$\vec{v_k} = (1 - km)\vec{v_0} \tag{5.1}$$

Correspondingly, we can also derive the acceleration a_k

$$a_k = \vec{v_k}' = (1 - km)\vec{v_0}' = (1 - km)a_0 \tag{5.2}$$

The acceleration a_k consistently stays below a_0, and proficient players may experience greater entertainment than less skilled players.

5.2.3 The balance of perfect player and imperfect player from the perspective of potential energy

"Subjective expectation", E_0 can be translated as"personal expectation[87]" or "individual expectation". This term is typically used to indicate an individual's subjective estimation or anticipation of a certain outcome or event, rather than being based on objective data or probabilities.

"Objective expectation", E_k can be translated as "factual expectation". This term usually denotes the expected value based on objective facts, probabilities, or data, without consideration of subjective viewpoints [16]. By solving the intersection between perfect and imperfect players, E_0 represents the objective expectation (Ideal reinforcement), while E_k denotes the subjective expectation (Players side).

$$E_0 = E_k \tag{5.3}$$

So we have

$$2m{v_0}^2 = 2m{v_k}^2 \tag{5.4}$$

In this scenario, both perfect and imperfect players participate on equal footing, implying that regular players can share the same motivation and expectations. This inclusivity ensures the game's appeal to a broad audience of ordinary players. Solving (5.4), we have

$$k = \frac{2}{m} \tag{5.5}$$

As m always falls within the range of $m = [0 - 1]$, if k is greater than 2, an intersection point exists between E_0 and E_k.

Assume $k = 3$ as depicted in Figure 5-2. In high-risk games, such as classical board games (i.e., $m = \frac{2}{3}$), the skill level of ordinary individuals is farther from that of masters ($k = 3$). However, ordinary players have the advantage of extended thinking time, making this skill gap acceptable.

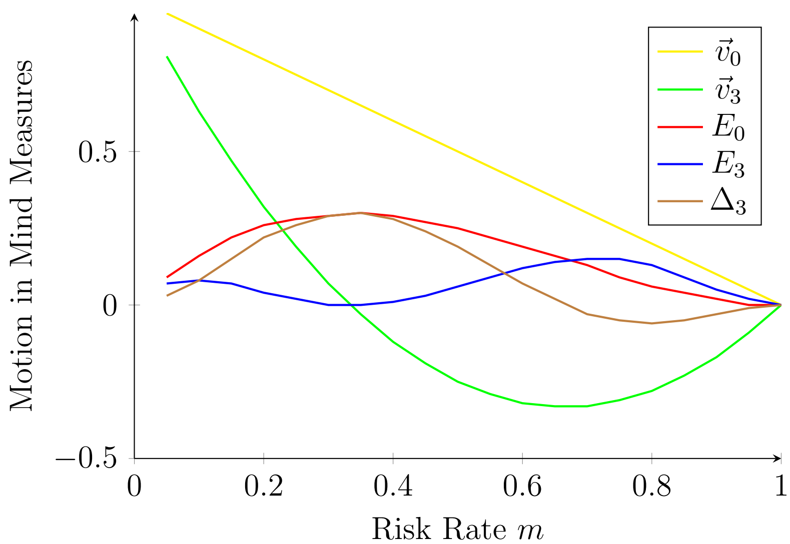

Figure 5-2: Motion in Mind Measures for $k = 3$

When the risk rate hovers around $m = \frac{1}{3}$, player concentration is at its peak. As the risk rate increases beyond $m = \frac{1}{3}$, the disparity between E_0 and E_k diminishes, contributing to a heightened sense of comfort among players. This trend persists until $m = \frac{2}{3}$, where, E_0 aligns with E_k, resulting in maximum player engagement. Hence, $m = \frac{2}{3}$ is the most suitable and reasonable risk rate for ordinary players.

If the risk rate surpasses $m = \frac{2}{3}$, the challenges become too formidable for both original and perfect players, leading to decreased engagement. However, even at this point, actual players may still exhibit higher engagement levels compared to perfect players.

5.2.4 Exploring optimal rounds for distinguishing real strength

In prior research, game outcomes have been traditionally characterized using average velocity as a representation of the effort required. The equation for the game outcome in an ideal case y_0 ($v = 1$) is expressed as follows:

$$y_0 = t \tag{5.6}$$

For other cases with a speed of $v = \frac{1}{N}$, the equation becomes:

$$y_1 = vt = \frac{1}{N}t \tag{5.7}$$

However, since the speed of resolving uncertainty is not constant, considering average acceleration as a measure of the game outcome becomes relevant. Acceleration, in this context, is relative to attraction, and mental attraction is closely linked with achievement. The equation for y_2 is presented as follows:

$$y_2 = \frac{1}{2}a_0t^2 \quad \text{where} \quad a_0 = GR^2 \tag{5.8}$$

Here, a_0 is given by $a_0 \approx \frac{B}{D^2}$, hence $GR = \frac{\sqrt{B}}{D}$, in the domain of board games with the average branching factor B and game length D [4].

Subsequently, we can determine the intersection point (say t_{02}) between ideal effort and achievement by solving $y_0 = y_2$, leading to:

$$t_{02} = \frac{1}{2}{a_0}^2{t_{02}}^2 \tag{5.9}$$

The initial sensible option for terminating the game is $t = t_{02}$, representing the minimum game duration. If this duration is exceeded, the achievement becomes excessively high, the attainment comes too effortlessly, the game appears too simple and smooth, and players may lose motivation to overcome challenges, leading the game into a dull and unengaging stage.

$$t_{02} = \frac{2}{a_0}\ (= T) \tag{5.10}$$

In this phase, the resolved uncertainty associated with y_{02} can be quantified. If players decide to conclude the game at this point, it is reasonable for a single round to determine the winner, ensuring a relatively fair outcome. However, if the objective is to identify the genuinely stronger side, a single round may not suffice. Repeating multiple rounds becomes necessary to mitigate the influence of randomness and establish a more accurate assessment of skill.

$$y_{02} = T \tag{5.11}$$

At the same time, we can solve the cross point (say t_{12}) between y_1 and y_2.

$$t_{12} = \frac{2v}{a_0} = \frac{T}{N} \tag{5.12}$$

Hence, y_{12} can be described as,

$$y_{12} = \frac{1}{N}t_{12} = \frac{T}{N^2} \tag{5.13}$$

Therefore, multiple rounds are required to encompass the entire spectrum of uncertainty and effectively discern the genuinely stronger side.

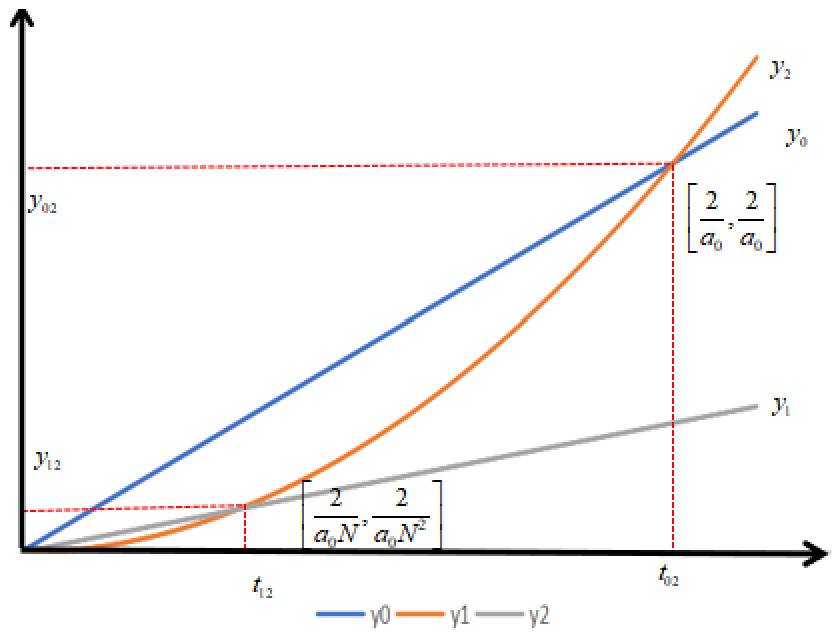

Figure 5-3: Total and each round solved game uncertainty

$$\text{Rounds} = \frac{y_{02}}{y_{12}} = N^2 \tag{5.14}$$

Corroborating with real-life observations, in the case of perfect information games where the process is openly transparent and there are fewer available options, coupled with a lower frequency of rewards N, the number of rounds tends to be lower. Simultaneously, each round takes longer than in imperfect information games; every move in each round requires careful consideration, making each round relatively "stable" and "fair".

For imperfect information games, the game process is unpredictable, necessitating the determination of actions amidst numerous possibilities. The frequency of rewards is higher, leading to an increased number of rounds. However, each round's duration is often shorter in such cases, enabling a swift transition to the next round. Repeated iterations balance the potential "randomness" and "bias" in each round. These surprises contribute to an addictive, almost irresistible feeling, akin to an addictive experience.

5.3 Performance Degree (k): In-Depth Analysis

In this section, our primary focus is on exploring the connection between playing performance (k) and the suitable complexity of the game. We delve into the specific evolution of "motion in mind" as players enhance their skills through training and exploration.

5.3.1 The risk rate m and performance degree k

There exists a negative correlation between the risk rate m and the performance degree k. In games with higher stochastic characteristics, featuring a high-risk rate and shorter game steps, chance assumes a more significant role. This leads to a reduced gap between ordinary and expert players, resulting in a lower value of k. Conversely, in games with more statistical characteristics, marked by a low-risk rate and a greater emphasis on skill over chance (as indicated by a higher GR), the gap between ordinary and expert players widens, yielding a higher value of k.

In most common games, a balance is struck between stochastic and statistical elements, with the risk rate typically falling within the range of $\frac{1}{3}$ to $\frac{2}{3}$. For classic games, a player's performance level can be quantified using a k value, which typically ranges from 3 to 6.

$$m_x = \frac{1}{k} \tag{5.15}$$

5.3.2 The explanation of the correspondence system of m and the maximum acceptable performance degree k

Certainly, here are some general guidelines for the correspondence between the risk rate m and the maximum acceptable performance degree k.

High-risk games ($m > \frac{2}{3}$): In high-risk games where the risk rate m exceeds $\frac{2}{3}$, the recommended maximum acceptable player performance level k typically falls within the range of 1 to 1.5. These games have a significant

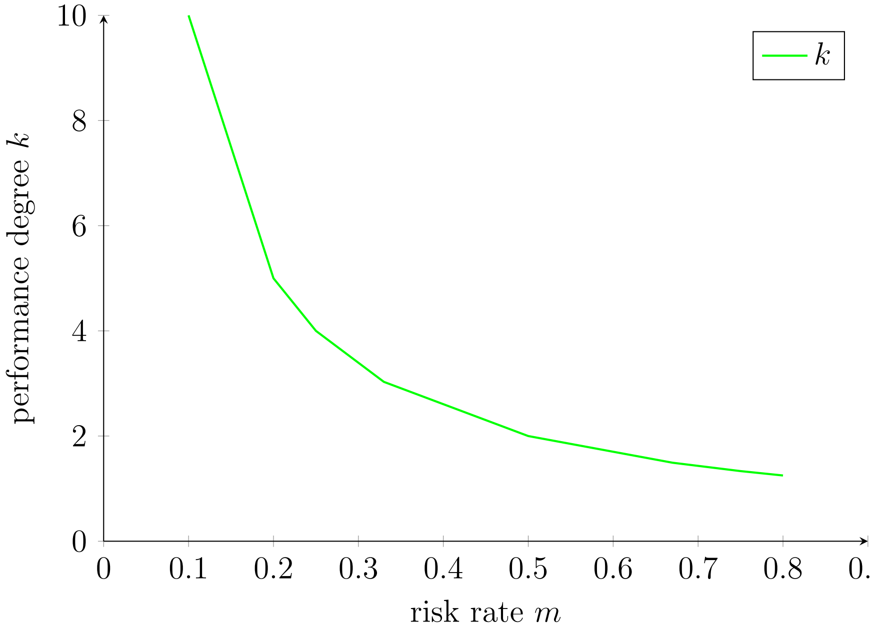

Figure 5-4: Possible relation between performance degree and risk rate

element of luck, so players need to have a higher level of skill to navigate the elevated risk and strive for GR cater control over the game.

Balanced-risk games ($\frac{1}{3} \leq m \leq \frac{2}{3}$): Games with a balanced risk rate typically see a maximum acceptable player performance level k ranging from 1.5 to 3. These games strike a harmonious balance between skill and luck, allowing both casual and professional players to participate in a fair and challenging gaming experience.

Low-risk games ($m < \frac{1}{3}$): For low-risk games with a risk rate m less than $\frac{1}{3}$, it's advisable to have a maximum acceptable player performance level k in the range of 3 to 10. These games primarily emphasize skill, and luck plays a diminished role in determining the game's outcome.

It is important to keep in mind that these guidelines are flexible, and variations may arise based on the specific game and player preferences. Moreover, individual players may possess varying skill levels and luck, influencing their perception of the maximum acceptable playing performance level.

Table 5.1: Relations between m and k

m	N	k	Explanation
0.1	1.11	10	Typically, the game is designed for young children and is relatively easy, making it accessible even to those with limited skills.
0.2	1.25	5	Geared towards elementary school students, this game is relatively easy, allowing children with varying abilities to enjoy it.
0.3	1.43	3.3	Targeted at older elementary school students, as they mature and learn, the skill gap among students narrows.
0.4-0.6	1.7-2.5	1.7-2.5	The intended players are typical adults, and the risk rate in most two-player games is around $\frac{1}{2}$. When both players have similar abilities, and the game is fair, the win rate should be roughly equal, offering enjoyment to a broad range of players. However, the scope of players who can enjoy the game is limited.
0.7-0.9	3.3-10	1.1-1.4	High-risk games primarily rely on randomness rather than skill. To relish these games, strong skills are necessary, further restricting the range of player levels.

Players with lower gaming skills often lean towards games with lower difficulty levels (higher reward frequency). In such scenarios, the jerk value of the game is reduced, resulting in a more predictable and straightforward gaming experience. These players enjoy a higher rate of success and derive GR cater satisfaction from their achievements. Conversely, players with advanced gaming skills tend to favor games with higher difficulty levels (lower reward frequency). Although these games are associated with higher jerk values, they also offer GR cater challenges. Players with high gaming skills relish the process of identifying optimal choices from a plethora of options and experience an enhanced sense of accomplishment.

In the case of random games, characterized by larger values of N and increased randomness and unpredictability, gameplay tends to be more intense. However, highly skilled players often opt for games with elevated difficulty levels. On the other hand, statistical games, featuring smaller N values and GR cater certainty, typically offer less intense gameplay. These games find diverse applications in fields such as business and education, where statistical analysis and decision models are frequently utilized.

5.3.3 Comparison of motion in mind measures based on performance degree k

In prior studies examining the connection between motion and mindset, it was discovered that the concept of energy, denoted as E_p, can serve as a metric for measuring a player's expectation [88]. Notably, players with different skill levels exhibit distinct energy levels, and when k surpasses 2, a crossover point emerges between perfect and imperfect players. At this juncture, imperfect players can harbor similar expectations to those of perfect players.

$$E_p = 2mv^2 \tag{5.16}$$

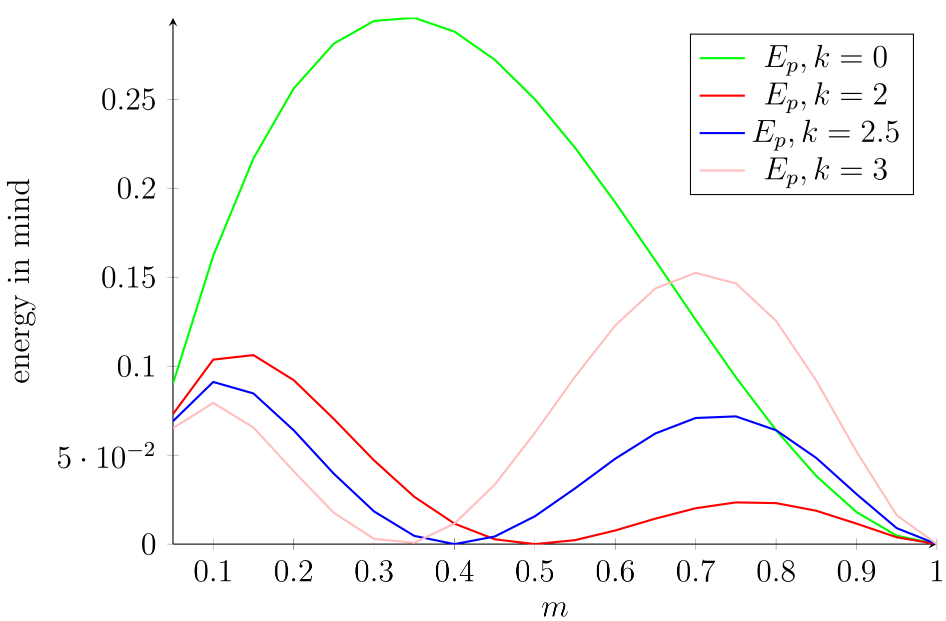

Figure 5-5: The comparison of energy in mind based on the player performance level k

In the gaming domain, when $k = 3$ and $m = \frac{2}{3}$, an intriguing equilibrium arises with

$E_k = E_0$, delicately balanced. Exploring the evolution of players transitioning from the introductory stage when $k = 3$ to the realm of heightened skill, we unveil a fascinating phenomenon: at $k = 2.5$ and $m = \frac{4}{5}$, once again, the convergence of $E_k = E_0$ unfolds. This prompts profound contemplation. From $k = 3$ to $k = 2.5$, as a player's skill advances, the crossover point shifts to the right. This suggests that players with higher skill levels will achieve the same expectations but at a higher risk rate. It implies that skillful players can confront more substantial challenges.

This signifies that for players endowed with relatively refined expertise, the allure of more challenging games seems to strike a chord akin to the expectations harbored by perfect players. As players transition from $k = 3$ to an elevated tier of skill, their performance ascends. As k is attenuated to 2.5, while the risk factor escalates to $m = \frac{4}{5}$, the anticipated outcomes of the game are reminiscent of those of an ideal player. In essence, even when confronted with more formidable challenges, adept players manage to evoke the anticipation reminiscent of an idyllic scenario. This intimates that for skillful players, engagement with games of heightened complexity can elicit expectations on par with the archetype of the perfect player.

Building upon prior research findings, it has been established that when $k = 3$, there exists a corresponding value of $m = \frac{2}{3}$, resulting in $E_0 = E_k$. This convergence point signifies that players can experience expectations similar to those of perfect players.

Moreover, in the domain of incomplete-information games like card games [5], our investigation reveals that when N falls within the range of 8 to 9 and then m spans from $\frac{7}{8}$ to $\frac{8}{9}$, with k ranging between 2.25 and 2.3, the equilibrium condition $GR = AD$ is attained. This balance point suggests that players' enjoyment and addiction levels are in equilibrium. Consequently, in this chapter, we will delve into the physical and psychological changes in our minds as players undergo continuous training, observing the shift in k from 3 to 2.

In addition to potential energy E_p [89], we utilized the momentum concept to assess the concentration level among players. It becomes apparent that momentum varies for

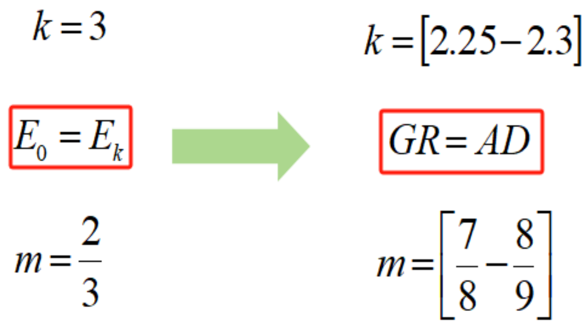

Figure 5-6: Motion in mind measure compared based on the performance degree k transition from 3 to 2

players with different performance levels. In instances where performance is subpar, the corresponding momentum diminishes. This suggests that players with weaker skills display reduced focus and concentration on the game.

$$p_1 = mv \tag{5.17}$$

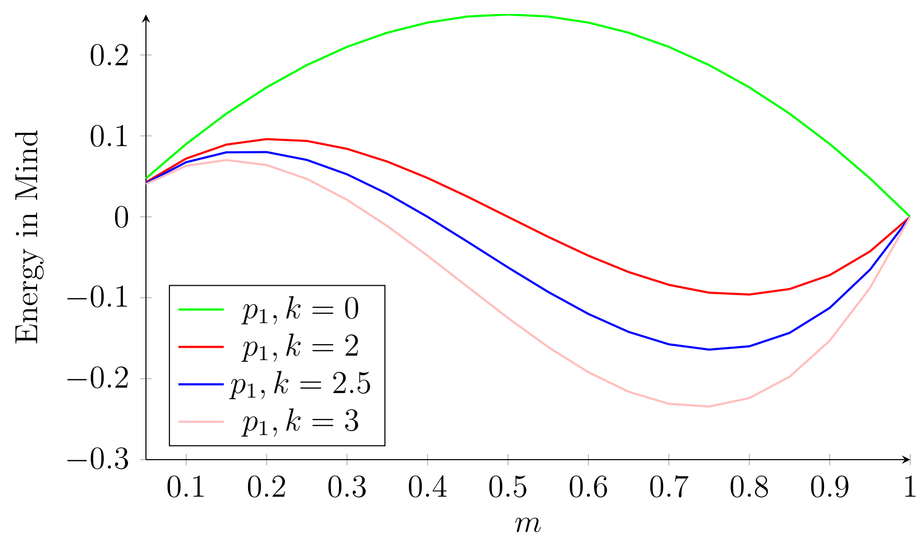

Figure 5-7: Comparison of momentum in mind based on the player performance level k

The subjective momentum represents the engagement of players, and the subjective momentum of players with different performances is also different, but performance level k and subjective momentum p_2 are not linear relationships.

$$p_2 = E_p - p_1 \tag{5.18}$$

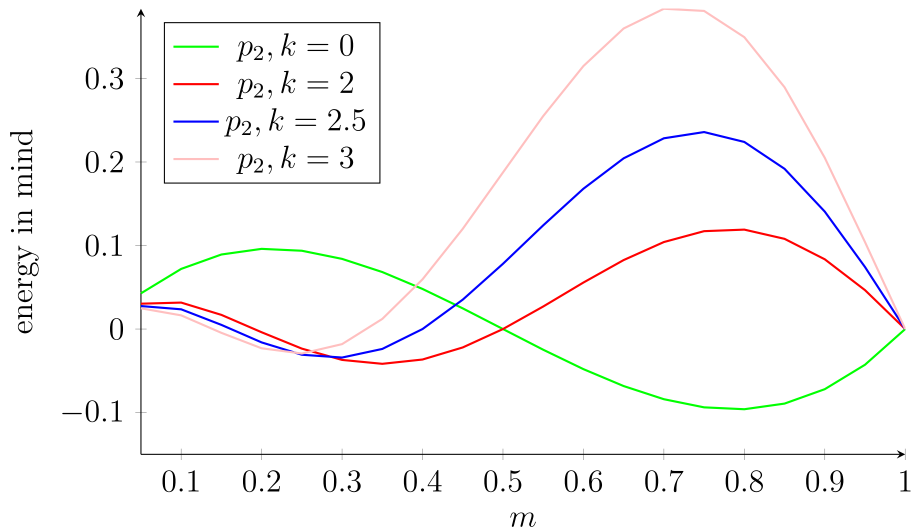

Figure 5-8: The comparison of subjective momentum in mind based on the player performance level k

It is evident that through continuous training and performance enhancement, transitioning from $k = 3$ to $k = 2$, stronger players can overcome the bottleneck at a lower risk rate, showcasing their readiness to face challenges and achieve higher levels of participation than perfect players. Additionally, we observe a progressive increase in the maximum value of p_2. For players with higher skill levels, both the peak and average values of p_2 are elevated. Essentially, this signifies that players with greater capabilities experience higher average and peak engagement levels, setting them apart from those with weaker skills.

This trend underscores the intricate interplay between players' skill levels, perceived risk, and subjective momentum. It reveals how these factors collectively shape the degree of players' immersion and commitment to the gaming experience, shedding light on the nuanced dynamics of player engagement. We have explored the intricate relationship among player competence, risk rate, and subjective momentum, offering valuable insights into understanding player engagement across diverse contexts. This contributes to the enhancement of game design and user experience, enabling us to better cater to the needs and expectations of various types of players.

5.4 Dynamic Interaction between Performance Level k, Reward Frequency N, and AD

This section delves into the interplay among three vital elements: k, N, and AD, from both player and game perspectives.

The symbiotic relationship between player performance and game rules is evident, as they mutually complement and influence each other, fostering a dynamic interplay. Importantly, players of varying skill levels can align their expertise (k) with a game's unpredictability (AD) or the number of choices (N) through dedicated training and exploration. This adaptability empowers players to tailor their skills and strategies, fostering a personalized gaming experience that achieves a harmonious balance between enjoyment and challenge.

5.4.1 Performance level k and reward frequency N

Building on our earlier discussion, novice players are typically inclined towards simpler games characterized by higher speed of getting rewards, resulting in a more predictable and gratifying gaming experience (GR). On the flip side, proficient players gravitate towards demanding games with lower velocity, relishing the complexity and the sense of achievement they provide.

In random games with larger values of N and higher unpredictability, gameplay reaches heightened intensity, attracting highly skilled players. Conversely, statistical games with smaller N values and increased predictability are less intense and are frequently employed in business and educational settings for statistical analysis and decision-making, showcasing their applicability beyond the realm of gaming.

$$k = \frac{1}{m} = \frac{N}{N-1} \tag{5.19}$$

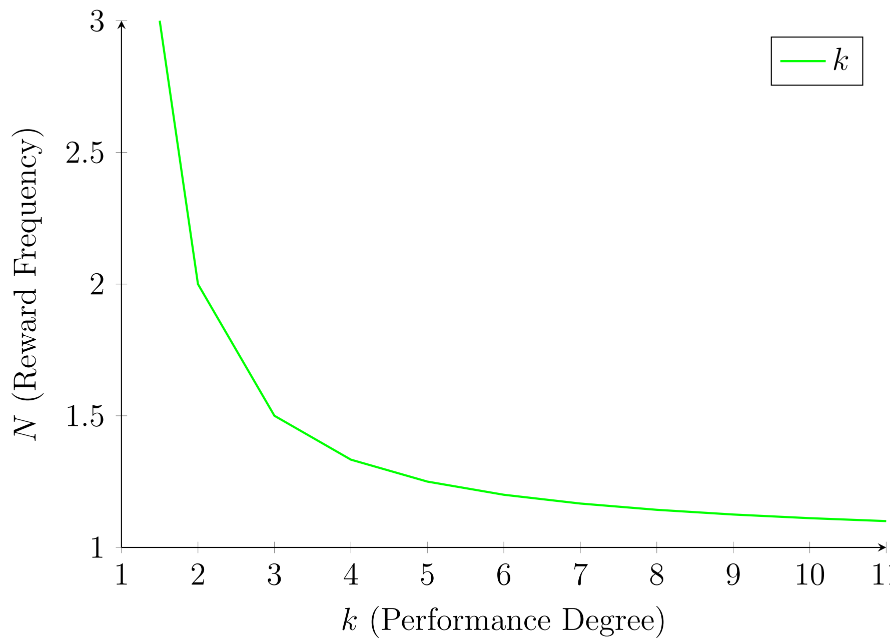

Figure 5-9: Performance Level k and Reward Frequency N

5.4.2 Performance level k and AD

Building upon prior research on AD through card game analysis [5], it becomes evident that in simpler games, where the player's skill surpasses the game's difficulty, the AD value starts small, and players continually explore ways to enhance the game's enjoyment. Conversely, in complex games, where the player's skill falls short of coping with the game's difficulty, the AD value is excessively high, rendering the game uncontrollable. In such cases, players persistently engage in training to improve themselves and enhance their skills to bring the AD value within a reasonable range.

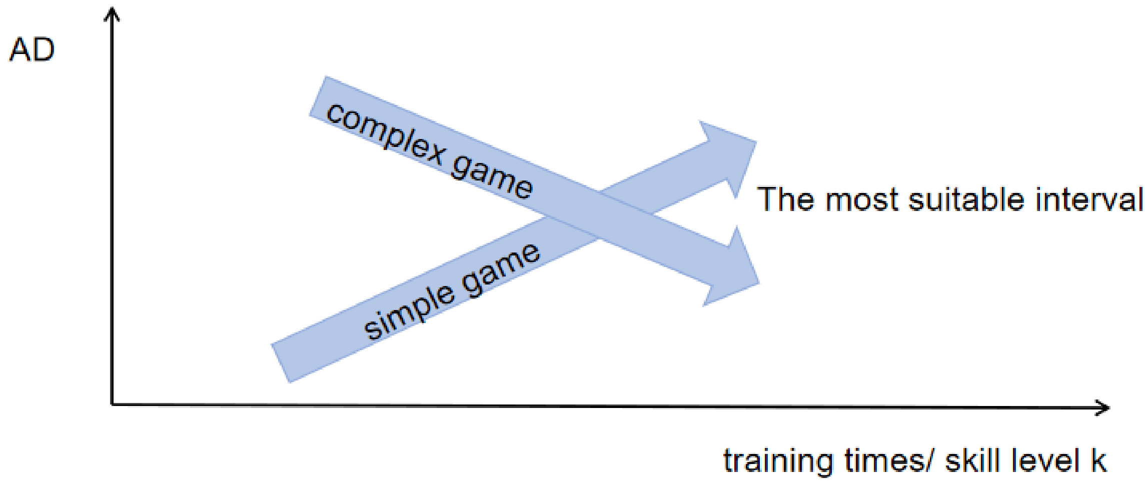

Figure 5-10: The relations between k and AD

Players with varying levels of skill can indeed achieve a better match between their skill level (k) and the level of unpredictability (AD) or the number of choices (N) in the game through continuous training and exploration. By refining their skills and adapting their strategies, players can tailor their gaming experience to a level that is both enjoyable and appropriately challenging for their abilities.

5.4.3 Reward frequency N and AD

In reinforcement learning, N often represents the "number of steps" or "time steps" indicating that a reward unit is obtained from every N step.

From a gaming perspective, N can be interpreted as the reward frequency, considering game expertise and motion in mind. The $G-T$(Goal-Total steps) model implies receiving a reward once every N operation. For example, in basketball, we may make N attempts before successfully scoring a basket, and each successful basket results in a reward [6].

Table 5.2: The relationship between N and AD in the sport games domain

Games	N	D	AD
basketball	2.25	178	0.046
soccer	8.33	22.0	0.090

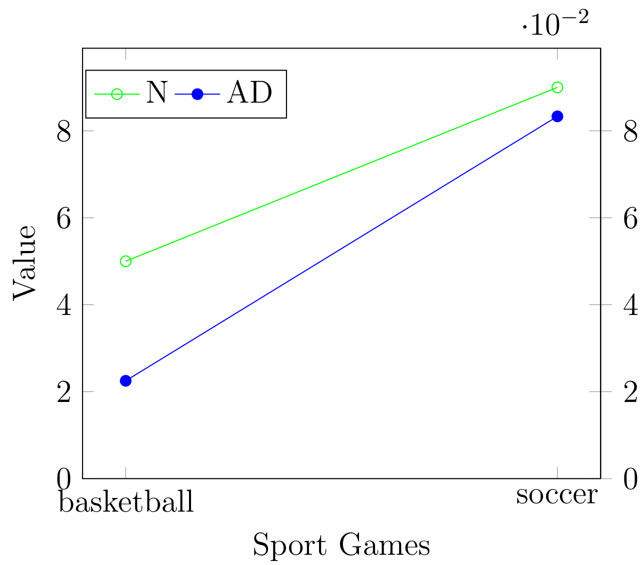

Figure 5-11: The relationship between N and AD in the sports domain

It is evident that in sports games [6], there is a positive correlation between the number of options (N) and the level of unpredictability (AD). In other words, as the number of available options increases, the uncertainty and unpredictability of the game also increase. This is because more options imply more variables and possibilities, making it more challenging to predict the outcome of the game.

In the $B - D$(Branch factor - Depth) model, it is inferred that we select the best option from N choices. This concept is similar to a game tree, where we consider N possible positions and make our final decision. For example, in a chess game, we may evaluate N potential moves and select the one with the highest expected outcome as our final decision [6].

Table 5.3: The relationship between N and AD in the board games domain

Games	$N(B)$	D	AD
Go	1.66	237.0	0.041
shogi	2.88	119.0	0.052
chess	4.57	71.0	0.062

Similarly, for board games [90], there is also a positive correlation between the number of options (N) and the level of unpredictability (AD). In other words, as the number of available options increases in a board game, the level of unpredictability also tends to increase. This is because a higher number of options for GR introduces increased complexity and potential outcomes, making it more challenging to anticipate the final result of the game.

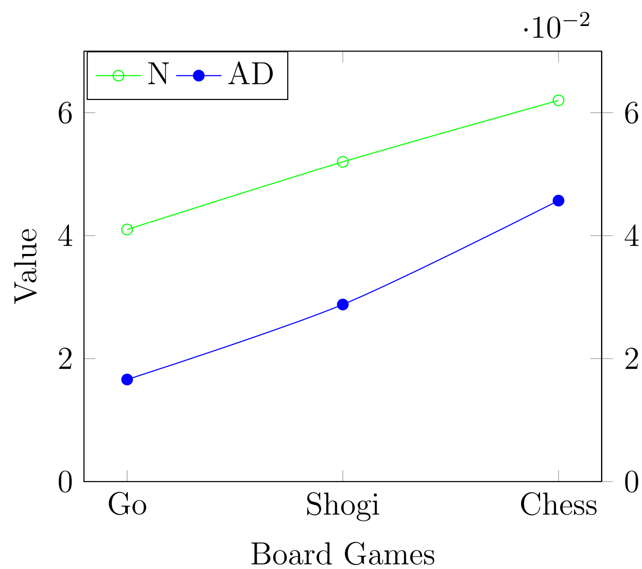

Figure 5-12: The relationship between N and AD in the board games domain

In general, for perfect information games, where all game information is available to players, the more options there are to choose from, the higher the level of unpredictability. This is because we need to narrow down the range of possibilities from numerous available options to make the best decision.

In the case of imperfect information games like card games [5], where factors such as suits, rules, and the game state are not entirely known, and there is significant uncertainty, your assumption aligns with the concept of a game tree. In this context, N can be considered similar to the concept of branching.

In a game tree model, N would represent the number of possible actions or moves available at a given game state. Each action or move leads to a different branch in the game tree, representing a different possible outcome or game state. By exploring the branches and considering N different options, the player can make informed decisions based on the available information and their strategic understanding of the game.

Considering the uncertainty and imperfect information in card games, the player can use the available information to assess the potential outcomes associated with each branch and make decisions accordingly. The N branches represent the different choices or actions available to the player at a particular game state, and the player would aim to select the most favorable or strategic option based on their analysis of the game tree.

It's worth noting that the implementation and specific details of modeling card games using game trees or similar approaches can vary depending on the specific game and the chosen algorithm or framework. However, your hypothesis of N being analogous to the concept of branching in a game tree model aligns well with the context of imperfect information games like card games.

Table 5.4: The relationship between N and AD in the card games domain

Games	$N(B)$	D	AD
Big Two	3.33	65	0.033
Winner	4.54	64	0.037
WK	4.69	54	0.045
Tien	5.09	57	0.044
DDZ	8.20	40	0.073

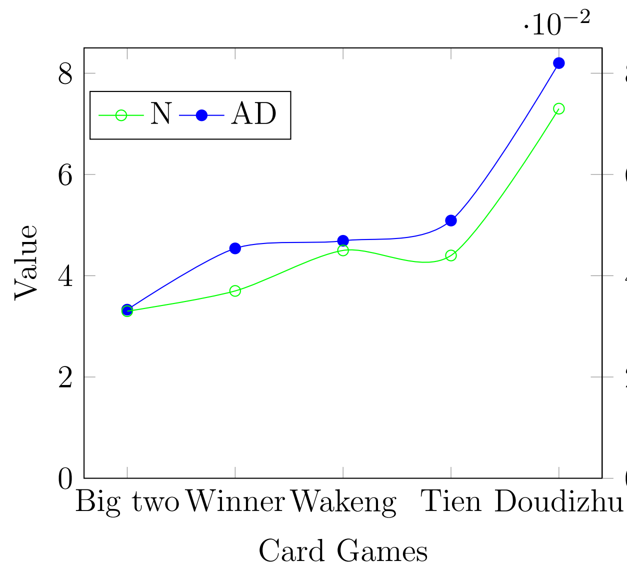

Figure 5-13: The relationship between N and AD in the card games domain

Additionally, in the realm of card games[5], it's noteworthy that there exists a positive correlation between the number of options (N) and the degree of unpredictability (AD). This implies that as the number of options (N) increases in card games, so does the level of unpredictability, adding an intriguing layer of complexity to the gaming experience.

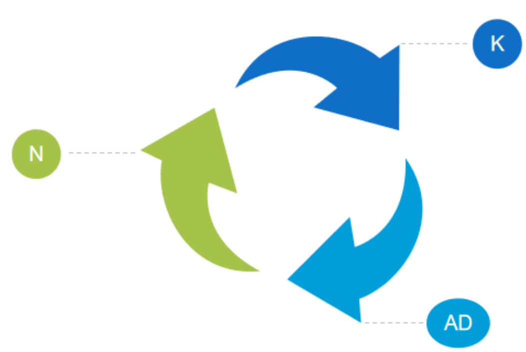

Figure 5-14: The relations between N, k and AD

The three important elements(k, N, and AD) on the player side and the game side are related in pairs.

Table 5.5: Correlation between player performance and game characteristics

Variables		Relationship
k	N	We observe a clear correlation between players' skill levels (k) and game characteristics, particularly in game selection and experience. Novice players (with higher k) tend to prefer simpler games with higher speed (resulting in lower reward frequency N) for a more predictable and enjoyable gaming experience. In contrast, experienced players gravitate towards more challenging games with lower reward frequencies, appreciating the complexity and sense of achievement they offer. Thus, k and N exhibit a negative relationship.
k	AD	Furthermore, our analysis of card games indicates that as players train or explore different complex games, their performance level (k) tends to converge towards that of an ideal player, leading to a decrease or increase in the average deviation (AD) within a reasonable range.
N	AD	Additionally, upon examining various game types, we discovered a positive correlation between reward frequency (N) and average deviation (AD).

In random games with higher N values and AD cater to unpredictability, the gaming experience becomes more intense, attracting highly skilled players. Conversely, statistical games with smaller N values and higher predictability are utilized in business and educational contexts for statistical analysis and decision-making.

Player performance and game rules complement each other, promote each other, continuously iterate, and influence each other. Crucially, players with varying skill levels can align their expertise (k) with a game's unpredictability (AD) or the number of choices (N) through training and exploration. Adapting their skills and strategies enables players to customize their gaming experience, striking a suitable balance between enjoyment and challenge.

5.4.4 The difference between perfect information games and imperfect information games

In general, when comparing perfect information games [91] with imperfect information games, the latter typically present a greater number of choices and more intricate scenarios. Perfect information games are renowned for their explicit information-sharing characteristics, where players possess complete and clear information throughout the game. In contrast, imperfect information games involve a higher degree of uncertainty and concealed information, requiring players to make decisions in the absence of perfect information.

In imperfect information games [92], players need to consider the potential actions of others and hidden information while also making decisions based on probabilities and conjectures. In such scenarios, there may exist a broader range of strategies and variations, as different hidden information and varying opponent behaviors can impact a player's optimal decision-making process. Compared to perfect information games, the features of imperfect information games contribute to a more diverse strategic landscape and a more complex decision-making process, as players must balance risks and benefits in the face of incomplete information

Table 5.6: Comparison between Perfect Information Games and Imperfect Information Games

Type	Perfect Information Games	Imperfect Information Games
Game Characteristics	Players have perfect information about the game state and all past player actions.	Players may lack perfect information, leading to uncertainty and hidden information.
Examples	Chess, Go, many strategic board games	Card games, games with hidden information
Player Decisions	Based on perfect information for calculations and decisions	Based on limited information, probabilities, strategies, and possible opponent actions
Information Impact	Every move in the game can be calculated and predicted.	Requires decisions based on speculation and possible opponent actions
Possible choices (N)	Prone to fewer choices and complexities	Prone to more choices and complexities
Unpredictability (AD)	Lower	Higher

5.5 The influence of ratio ϕ (GR/AD)

In previous studies, we investigated the balance between game refinement (GR) and addictiveness (AD), recognizing that a harmonious interplay between these factors results in an engaging player experience characterized by entertainment, repetition, and immersion in the game's surprises and delights. This chapter delves into the implications of imbalanced GR and AD values, examining how different ratios convey unique meanings and influence the overall gaming experience.

Here, we employ the ratio of GR (Game Refinement) value to AD (Addictiveness) value, denoted as ϕ, as a metric for assessing the relationship between entertainment value and the level of addictiveness. Through this analysis, we explore the impact of enjoyment and addiction (repeatability) across various game genres. This approach offers insights into the underlying motivations guiding game designers' decisions and facilitates a discussion on the varying importance of entertainment and habituation in game design.

Table 5.7: Measures of game refinement for board games [4]

Games	B	D	GR	AD	ϕ
Chess	35	80	0.074	0.059	1.25
Shogi	80	115	0.078	0.054	1.44
Go	250	208	0.076	0.044	1.73

In the realm of board games, the ϕ value typically resides in the [1.25-1.75] range. This range suggests that board games offer an enjoyable and pleasurable experience, coupled with a degree of stability and statistical influence. Engaging in board games demands a certain level of strategic thinking and decision-making.

Table 5.8: Comparison of Card Games using GR and AD values [5]

Games	B	D	GR	AD	ϕ
DoudDiZhu	8.2	40.03	0.072	0.073	0.99
Wakeng	4.69	54.15	0.040	0.045	0.89
TienLen	5.09	56.93	0.040	0.044	0.91
Winner	4.54	64.1	0.033	0.037	0.89
BigTwo	3.33	65.67	0.028	0.033	0.85

In the realm of card games, the ϕ value typically falls within the [0.85-1] range, signifying that card games tend to display more unpredictability than board games and are marked by a higher level of instability. Engaging in card games not only necessitates skillful play but also demands the ability to adapt to unforeseen situations.

Table 5.9: Comparison of Basketball and Soccer using GR and AD values [6]

Games	B	D	GR	AD	ϕ
Basketball	72.76	164.02	0.073	0.046	1.59
Soccer	2.8	22	0.076	0.09	0.84

In the realm of sports games, the ϕ value commonly falls within the approximate range of [0.85-1.5]. This range underscores the importance of entertainment impact over unpredictability, suggesting that the enjoyment derived from sports games is a key factor. The implication is that entertainment value plays a more pivotal role than unpredictability in sports games. These games often showcase higher statistical measures, reflecting a greater reliance on skills rather than chance-based elements, as indicated by the GR parameter.

The concept of gamification has gained widespread popularity in recent years and is applied across various facets of life, including education [93] and business [94]. In the following sections, we will delve into typical applications of gamification in both educational and business contexts.

Table 5.10: Comparison of Hotels using GR and AD values [7]

Hotel	B	D	GR	AD	ϕ
IHG	3	75	0.023	0.028	0.833
Hilton	5	60	0.037	0.041	0.907
Starwood	3	50	0.035	0.042	0.833
Marriott	6	75	0.033	0.035	0.935
Hyatt	5	60	0.037	0.041	0.907

This table compares different hotel brands based on the ratio of GR values to AD (Addictive Dynamics) values, denoted as ϕ. This ratio assesses the relationship between entertainment value and addictiveness level, providing insights into the appeal and gaming aspects of various brands.

Similar to card games, when the ϕ value is less than 1, it signifies that entertainment value and addictiveness are significant. Due to the AD value surpassing the GR value, repetition plays a crucial role at this point. Under the influence of gamification concepts, there is a tendency for players to consistently choose this hotel, fostering loyalty and habituation.

However, what sets it apart is that in this scenario, both the GR and AD values are relatively small. This implies that the amusement and addictive qualities of commercial activities are relatively low. They merely apply gamification concepts, lacking the equivalent enjoyment and thrill players might experience in traditional games.

Table 5.11: Comparison of Languages in Duolingo using GR and AD values [8]

Language	B	D	GR	AD	ϕ
English	55.619	291.958	0.026	0.019	1.35
Spanish	64.142	319.571	0.025	0.018	1.39
French	72.222	346.333	0.025	0.017	1.41
German	89	381.25	0.025	0.017	1.47
Portuguese	68	379	0.022	0.016	1.40
Italian	66	385	0.021	0.015	1.39

In the domain of educational games, the ϕ value typically hovers around 1.5. This indicates that entertainment value holds a significant role within the realm of educational gaming, catering to the importance of the GR parameter. The overall gameplay tends to be more stable and transparent, characterized by a clear and open process.

However, what distinguishes it in this scenario is that both the GR and AD values are relatively small. This suggests that the entertainment and addictive elements of educational games are somewhat modest. While these games often incorporate gamification concepts, unlike traditional games, players tend to perceive platforms such as Duolingo primarily as learning tools rather than entertainment platforms.

Table 5.12: Game types compared using ϕ values

Game Types	GR	ϕ	Impact	Explanation
Business Games	0.033	0.8-0.9	Repetitive (Builds Loyalty)	Habit-forming games often found in daily activities
Card Games	0.072	0.85-1.0	Enjoyable and Addictive (Create Harmonize)	Games associated with randomness and exhibit addictive tendencies
Sports Games	0.074	0.85-1.6	Enjoyable and Exercise (Promotes Health)	Games connected to physical activities and fitness
Educational Games	0.024	1.35-1.5	Learning (Encourages Improvement)	Games designed for learning and expected skill enhancement
Board Games	0.076	1.25-1.75	Enjoyable and Stable (Cultivates Competition)	Games involving competition with stable dynamics

Different values of ϕ represent the boundaries between games and various activities. Through observation, we can discern that when ϕ is larger and both GR values and AD values are higher, as seen in board games, it marks the intersection between games and competition. Conversely, when ϕ is smaller and both GR values and AD values are lower, as evident in business games, it signifies the boundary between games and habits.

In the case of games that combine elements of competition, such as board games, they exhibit higher ϕ values along with elevated GR and AD values. This indicates that these games not only offer entertainment and enjoyment but also possess a certain

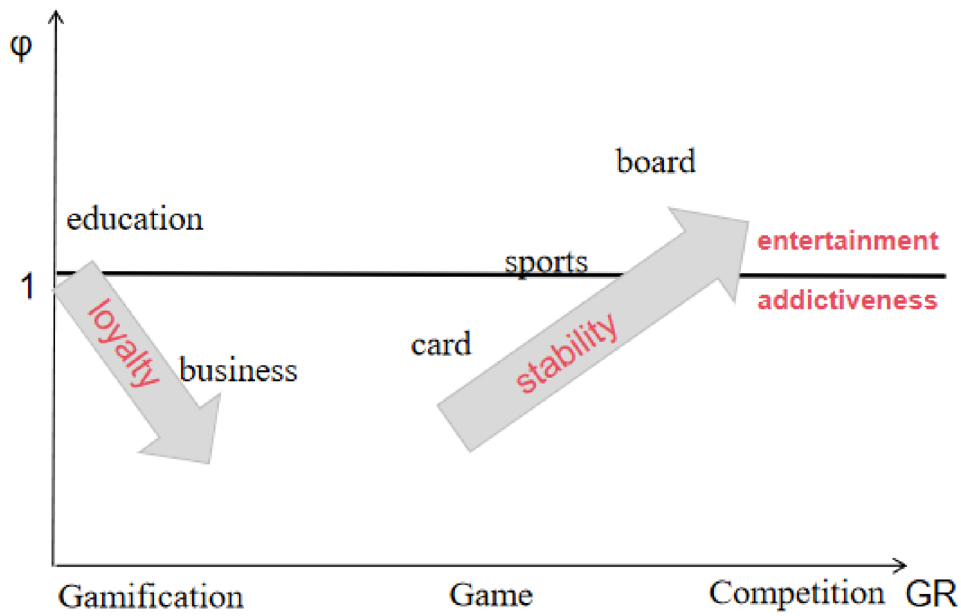

Figure 5-15: Gamification, Game and Competition

level of addictiveness, possibly tied to the competitive nature and outcomes. These games ignite players' competitive spirit, urging them to strive for better performance, all while maintaining their entertainment value.

On the other hand, examples of business games showcase lower ϕ values, accompanied by lower GR and AD values. This highlights that these games emphasize cultivating habits and daily activities rather than pursuing high entertainment or addictiveness. While these games may incorporate gamification elements to engage users in specific business activities like purchases or earning points, players generally view them as routine tasks rather than traditional games.

In gamification scenarios, a lower ϕ value indicates a greater emphasis on the AD value. In contrast to entertainment, designers aim for the development of habits, encouraging repetition and fostering increased loyalty.

In game-to-competition scenarios, an increase in the phi value suggests a greater significance of the GR value. This implies that the game becomes progressively more entertaining, standard and stable, less surprising, and unpredictable. The overall process, blending entertainment and stability, aids in identifying winners and showcasing strength.

In conclusion, ϕ values serve as a significant metric for gauging game characteristics, aiding us in better understanding the distinctions in terms of entertainment, addictiveness, and impact among different game types, as well as the varying positions they hold in players' perspectives.

5.6 Chapter Conclusion

In this section, we primarily explored the relationship between player skill level (k), reward frequency (N), and unpredictability (AD).

Firstly, in terms of player skill level (k), it's evident that players at different skill levels have varying speeds in handling game uncertainties, with less-skilled players solving uncertainties at a slower pace than highly-skilled players. Consequently, the rate of information processing in the game also differs, with less-skilled players experiencing a lower acceleration than highly-skilled players. This implies that highly skilled players derive more enjoyment from the game than average players. Moreover, we analyzed player performance when they can achieve similar expectations as perfect players, denoted as $k = 3$. Nevertheless, the equilibrium point, where GR equals AD, is approximately at $k = 2.3$. As a result, we examined how various motions in mind change as players transition from $k = 3$ to $k = 2$, and how they enhance their performance to attain a heightened gaming perception.

To ensure that player speed is not catering than 0, there exists a negative correlation between player skill level (k) and game risk rate (m). In games with high randomness, high-risk rates, and shorter game steps, the role of chance is more prominent, resulting in a smaller gap between average players and professional players, with lower values of k. Conversely, in games with stronger statistical elements, lower risk rates, and GR catering to emphasize skill rather than chance, the gap between average players and professional players is larger, leading to higher values of k.

This suggests that less experienced players select games with higher reward frequencies and simplicity, aiming for a more predictable and satisfying gaming experience. Conversely, skilled players prefer games with lower reward frequencies to cater to challenges, appreciating the complexity and sense of accomplishment they provide. The relationship between k and N displays a negative correlation.

Expanding from the analysis of card games, it's apparent that in simpler games, players' skills surpass the game's difficulty, resulting in initially low AD values, as players continuously explore methods to enhance the enjoyment of the game. Conversely, in more complex games, players' skills may not match the game's difficulty, leading to excessively high AD values, making the game unmanageable. In such cases, players need persistent training to improve themselves and their skills, maintaining AD values within a reasonable range. Players with different skill levels can indeed achieve a better match between their skill level (k) and the level of unpredictability (AD) or the number of choices (N) in the game through continuous training and exploration. By refining their skills and adapting their strategies, players can customize their gaming experience to strike a balance between enjoyment and an appropriate level of challenge. Simultaneously, game designers and promoters can gather user data, and tailor and promote games that align with their preferences, achieving a mutually beneficial two-way matching for a win-win outcome.

Additionally, our analysis encompassed various types of games, including card games, business games, educational games, and board games, to explore the relationship between N and AD, which also exhibited a positive correlation.

Moreover, we expanded our analysis beyond AD to include the ratio ϕ of GR to AD, offering additional insights into diverse game types and their respective boundaries. Various ϕ values represent the distinctions between games and various activities. Through observation, we noted that when ϕ is larger and both GR and AD values are higher, as seen in board games, it signifies the intersection between games and competition. In contrast, when ϕ is smaller and both GR and AD values are lower, as observed in business games, it denotes the boundary between games and habits. Using board games as an example, which integrate competitive elements, they displayed higher ϕ values along with increased GR and AD values. This indicates that these games not only offer entertainment and enjoyment but also possess a certain level of addictiveness, possibly related to their competitive nature and outcomes. These games ignite players' competitive spirit, motivating them to strive for better performance while maintaining their entertainment value. On the other hand, examples of business games showcased lower ϕ values, accompanied by lower GR and AD values. This highlights that these games emphasize cultivating habits and daily activities rather than pursuing high entertainment or addictiveness. While these games may incorporate gamification elements to engage users in specific business activities like purchases or earning points, players generally view them as routine tasks rather than traditional games.

In conclusion, ϕ values are a significant metric for assessing game characteristics, assisting game developers and optimizers in adjusting and improving games to cater to different player preferences and levels.

Chapter 6

Conclusion

This research has significantly advanced our understanding of game design and player behavior through a comprehensive investigation into various aspects of gaming. The examination of three critical areas in this study has yielded valuable insights capable of guiding game designers and developers. Ultimately, these findings contribute to the creation of more captivating and well-balanced game experiences for players.

Our principal contribution revolves around the creation of an extensive game information model based on game trees. We innovatively introduced the concept of "jerk", rooted in game refinement theory, and explored the integration of flow theory within the gaming context. Introducing the Δscore, effectively bridging the divide between skill and challenge outside a game setting, added depth to our exploration. By incorporating dynamical velocity, acceleration, and jerk, we probed the intricate links between motion and emotions, particularly concerning flow theory. This model not only provides a comprehensive visualization of intricate game processes but also quantifies them. By synthesizing established theories such as game refinement theory and flow theory, our model emerges as a valuable instrument for evaluating games, taking into account both their entertainment value and complexity. This comprehensive approach provides developers with the insights necessary to make informed decisions during the design and optimization of games, ultimately elevating the overall gaming experience.

After introducing the concept of jerk and the game dynamics model, our expanded study explored the motion in mind measure, particularly the jerk value, to assess game entertainment and addiction potential. We conducted computational analyses on various card games using AI to represent different player skill levels and game complexities.

By simulating players with varying skills in card games of different difficulties, we performed horizontal and vertical comparisons, investigating the interaction between players and games. This exploration aimed to find a balance between player abilities and game complexity in two dimensions: weak vs strong players and plain vs sophisticated games. For deterministic games, like board games, options (N) are limited ($1.5 \leq N \leq 5$), leading to deterministic and stable gameplay with $GR > AD$. Longer steps increase determinism. Stochastic games, such as card games, offer more options ($5 \leq N \leq 10$), resulting in unpredictable and surprising gameplay with $AD > GR$. Solving $GR = AD$ identifies the most addictive and entertaining scenario, occurring at $N = 9$ (e.g., Doudizhu with parameters $B = 9$ and $D = 40$).Analyzing the relations between AD and GR reveals the ability to measure player stability and predictability in games. High-complexity games challenge players, while simpler games may become more engaging after exploration and training. Additionally, we observed that complex games have higher potential energy (E_p), game momentum (p_1), and mental momentum (p_2) values, indicating greater player engagement, expectations, and unpredictability. This also correlates with higher AD values. These insights provide a deeper understanding of game dynamics.

Finally, we explored the relationship between player skill level (k), reward frequency (N), and unpredictability (AD), from the player side and game side including reward frequency and reward surprises.

Firstly, varying player skill levels (k) impact the handling of game uncertainties. Less-skilled players solve uncertainties more slowly than highly-skilled ones, resulting in slower information processing and less enjoyment for the former. Analyzing player performance at k=3, we found the equilibrium point of $GR = AD$ around k=2.3.

We examined how player transitions from k=3 to k=2 affect their performance and gaming experience enhancement. A negative correlation exists between player skill level (k) and game risk rate (m). Higher randomness, risk rates, and shorter game steps diminish the gap between average and professional players, yielding lower k values. Conversely, games emphasizing skill over chance lead to larger k values, indicating a preference for greater challenges.Less experienced players gravitate towards games with higher reward frequencies and simplicity for predictable enjoyment. Skilled players favor lower reward frequencies, relishing complex challenges. The k-N relationship displays a negative correlation. Analysis revealed that in simpler games, player skills surpass game difficulty, resulting in initially low AD values. In more complex games, skills may not match game difficulty, resulting in excessively high AD values. Continuous training and skill improvement help maintain AD values in a reasonable range, allowing players to balance enjoyment and challenge.

We expanded our analysis to various game types, finding a positive correlation between N and AD. Additionally, we introduced the GR-AD ratio (ϕ) to distinguish between games and other activities. Larger ϕ values, as seen in competitive board games, indicate increased addictiveness due to competition. Smaller ϕ values, as observed in business games, suggest a focus on routine activities over entertainment. In summary, ϕ values are crucial for evaluating game characteristics and assisting developers in catering to diverse player preferences and levels.

In conclusion, our comprehensive exploration has shed light on the intricate nuances of player psychology and gameplay dynamics. The integration of Flow theory, game refinement theory, and physical analogies has offered valuable insights into how psychological states evolve during gameplay. Moreover, our examination of various game dimensions, ranging from complexity to unpredictability, has deepened our understanding of what makes games engaging and even addictive. This holistic perspective equips us with a richer understanding of player experiences in diverse gaming scenarios, paving the way for more player-centric and enjoyable game design in the future.

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