A Rewriting Game Theory Analysis of a Dynamic Router-Layer

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1 Introduction

We are doing various decision-makings in daily life. Our decision-making influence other people's decision-makings and vice versa. Therefore, in the social life, our decision-making is interdepent. We imagine the daily life as a game, people as a player and each player(agent) has its own strategies(decision-makings) to behave in the game. Game theory is the theory to study this decision-making game. Game theory was formulated by von Neumann and Morgenstern as a general theory of rational behavior. The theory aims to give insight into how strategies are formed and euilibrium points arise whent agents interact. Equilibrium points are situation where all agents are happy, especially, the non-cooperative game of the equilibrium point are called Nash equilibrium point. Game theory is nowadays used in the fields of Economics, Biology, Law, Networking and so on, however, game theory is mainly used in the field of Economics. Because the outcome of Economics' game is always based on the real number, and the outcome of classic game thoery is the real-valued preference. Therefore Economics and Game theory are going well. We are interested in dynamic

game. Because, we want to see where equilibrium points arise in the dynamic game and in what situations. Network game has a lot of dynamic aspects such as the router-layer of the P2P communication. However, the classic game theory is inappropriate to model the dynamic aspect of the network game, because of real-valued preference, instead, we aim to use abstract game and rewriting game theory, as proposed recently by LeRoux, Lescanne, and Vestergaard to analyze the dynamic network game.
In this thesis, we proposed the dynamic router-layer of network game, and we use the rewriting game thoery to apply the game to analyze the 2routers and 2clients' network game of equilibrium points. And, we also used the LEDA and Mathematica as a tool to compute the equilibrium point automatically.

2 Rewriting Game Theory

A rewriting game theory is proposed by LeRoux, Lescanne, and Vestergarrd. It is a new approach to study Nash Equilibria especially in simultaneous games. The rewriting game theory consists of abstract games. Simultaneous game in classic game theory needs outcomes as number in order to use probability theory that Nash proposed. But, equilibria in abstract game do not use probability theory and therefore, rewriting equilibria are discrete and dynamic in nature. The rewriting equilibrium point is the terminal strongly connected components(SCC). We use this theory to analyze the dynamic equilibria in our router-layer of network game.

3 LEDA & Mathematica

We use the LEDA & Mathematica as a tool to compute the equilibrium point automatically for us. The concept of why we need the tools is, the game with more players and strategies will be very complicate. Therefore, we use tools to apply our game. The equilibrium point in the rewriting game theory is the terminal SCC. The terminal SCC can be found in the acyclic graph. LEDA and Mathematica have a lot of implemented algorithm to compute the graphs. So, by rewriting game theory we compute

our game as a graph and we are implemented the method for computing the rewriting equilibria by Mathematica and LEDA.

4 Router-layer of network game

Network game has many dynamic aspects of uses. We modeled the nework game with the dynamic aspects are called router-layer of network game. We are defined the general model of router-layer game with the usage proflies and usage graph especially, in the 2routers and 2clients game. Finally we use the Mathematica to apply to our model and we analyze the result of its. The results of the computations includes in the appendix.

5 Conclusion

In the 2by2 network game, what the equilibrium point says is, if the usage graph with only one usage profile, the graph can not be shrunken so that the pure Nash equilibrium point always exists. And if the usage graph with two or more usage graph, then the graph can be always shrunken, so that the rewriting equilibrium point always exists. We can conclude, the calssic game theory without Nahs probability theory can not apply to the dynamic network game. And, the classic game can not express the cyclic strategies(SCC) well, however, the rewriting game thoery treats SCC as one equilibrium called rewriting quilibria. So we are able to understand the equilibrium point easier as a situation. See also appendix.

6 Related Work

In classic game theory the network model consists of m agents and a source and a sink computer connected through n wires and using Nash equilibria, Papadimitriou and others have established a O(ln n / ln ln n) upper bound between router and idealized regulated network traffic. Also the study the probability of Nash theory to analyze the latency.

7 Future Work

We have studied the general 2by2 network game usage profiles with the usage graph, however, the specific usage profiles might have a lot of interesting cases to analyze (such as two or more terminal sccs exsits in the same usage graph, the equilibrium point does not exist and so on). The computation might take time, however, to study ..by.. network game is important, we might be able to find out a good example that takes palce in the real world of the performance.