Title	Dynamic Logic Epistemic Logic
Author(s)	,
Citation	
Issue Date	2006-03
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/1967
Rights	
Description	Supervisor: , ,

1 Introduction

Modal logic is based on classical logic, and is obtained by adding various modal operators to it. Modal logic has been studied in order to handle ordinary reasoning that we cannot explain enough in propositional classical logic.

As for modal logic, a systematic study was begun by Aristotle, and it is C.I. Lewis and C.H. Langford that arranged it from a modern point of view. Necessity and possibility were studied in the early days mainly, but logic of Knowledge and logic of belief to have an important role by a theory of an agent attracting attention are performed recently.

In this study, we aim for building the logical system which integrated Dynamic Logic and Epistemic Logic which can describe the change of knowledge with an action. Therefore in this study as a case study we really use the logical system which we introduced axioms and inference rules of two logic into after having thought about how we should have formalized it, and analyze The Aces and Eights game.

2 Dynamic Logic and Epistemic Logic

Dynamic Logic is the logical system introduced modal operators to express a state after the execution of every program into. In this study, we use Propositional Dynamic Logic(PDL) which is basic Dynamic Logic. In addition, we think as an action of a person in substitution for the execution of a program.

Epistemic Logic is a kind of the modal logic, and it is the logical system introduced as the modal operator "knowing" "believing". A study of such a knowledge is used for a problem of a distributed system of a computer effectively.

3   Analysis of The Aces and Eights game

The Aces and Eights game is the simple game that includes some complicated reasoning about knowledge. We distribute six pieces to three player A, B, C by two pieces from eight pieces of cards comprising four pieces of *Ace* of cards and four pieces of *Eight* and we do it and put two pieces of the remainder on the back. Of course three people are informed of that these cards are six pieces beforehand. It is not allowed for all players to watch contents of a card, but contents of a card are seen to two other players. Three players are going to decide what one's card is by turns then and watch two other players again. A player must raise his/her hand if the player is not decided what one's card is. And other two people know that the player raised his/her hand at the time.

   We introduce a way of progress and perform the commentary next.
At first A raise his/her hand after A watched other two people. B raises his/her hand in the same way next, too. C watches two people of A and B, and C know A having two pieces with *Ace* and B having two pieces with *Eight*, that all two of them raise their hand more. Can C know what one's card is then? Actually, one's card is by *Ace* and *Eight* one piece, and C can know that.

   If C has two pieces of *Eight*, A should know that oneself has two pieces of *Ace* because A watch four pieces of *Eight* when A watched B and C. In addition, B should know that oneself has two pieces of *Eight* if C has two pieces of *Ace* likewise. However, two people raised their hand. Therefore, C can reason that oneself has by *Ace* and *Eight* one piece.

   We define relations of formulae and propositions, and actions.

   • A_p : A has two pieces of *Ace*
   • A_q : A has two pieces of *Eight*
   • A_r : A has by *Ace* and *Eight* one piece
   In addition, for player B, C, We define B_p, C_q equally.

   • α : A watches B and C
   • β : B watches A and C
   • γ : C watches A and B

With the system added the following axioms and inference rules to in the logical system LK, we analyzed this game.

$$K_i\varphi \to \varphi \qquad \frac{\Gamma \to \psi}{K_i\Gamma \to K_i\psi}$$

$$[\alpha;\beta]\varphi \to [\alpha][\beta]\varphi \qquad [\alpha][\beta]\varphi \to [\alpha;\beta]\varphi \qquad \frac{\Gamma \to \psi}{[\alpha]\Gamma \to [\alpha]\psi}$$

Furthermore, we add the following axioms peculiar to this game to it.

$$X_p, Y_p \to Z_q \qquad X_q, Y_q \to Z_p \qquad [\tau]K_X Y_i \to K_X Y_i \quad \text{etc.}$$
$$(X, Y, Z(\in \{A, B, C\}) \text{ are different each other.}$$
$$i \in \{p, q, r\} \quad \tau \text{ is an arbitrary action.})$$

As a result of having analyzed it in consideration of only knowledge, It was as follows.

$$K_C(\neg K_A A_p \wedge \neg K_B B_q), K_C(A_p \wedge B_q) \rightarrow K_C C_r$$

As a result of having analyzed it in consideration of knowledge and actions, It was as follows.

$$K_C([\alpha]\neg K_A A_p \wedge [\alpha;\beta]\neg K_B B_q), K_C(A_p \wedge B_q) \rightarrow K_C C_r$$

Among two analysis, the latter can express the order of actions.

4   Conclusion

In this study we analyzed The Aces and Eights game by two methods when we used the system which considered knowledge and actions when we used the system which considered only knowledge to express the reasoning that players performed in this game formally. In addition, it seemed that we could easily understand progress of this game if we read a commentary of a game, but we understood that we needed various suppositions when we really formalized it.