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The Finite Embeddability Property for Some Modal Algebras

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If it is possible to determine with a mechanical method whether a given well-formed formula φ is a theorem of a logic L, the logic L is said to be *decidable*. This problem, called *decision problem* (of L), has been one of the many concerns for logicians since Hilbert's *Entscheidungsproblem*.

In this thesis we (re)prove decidability of modal and substructural logics. *Modal logics* are propositional logics enriched with unary connectives such as $\Box$ or $\Diamond$. These dates back to Aristotle and have philosophical origin. Today many applications are found in linguistics, computer science and more. On the other hand *substructural logics* are those obtained by deleting structural rules from a Gentzen formulation of intuitionistic logics. They subsume various kinds of logics such as relevant logic and linear logic and allow a systematic investigation of them.

In general a given logic corresponds to a class of algebras. This makes it possible for us to talk about properties of classes of algebras when we want to show some property of logics. The subject of our thesis, *the finite embeddability property(FEP)*, is an example. The finite embeddability property of a class $\mathcal{K}$ of algebras implies decidability of its *universal theory*, which directly relates to decidability of logic associated with $\mathcal{K}$. Our aim is to prove the FEP and get decidability of classes of algebras, or equivalently logics.

The well-known proof strategy for decidability of modal logics is to prove *the finite model property(FMP)*. A major path to that end is to use the construction called filtration, which requires Kripke semantics for the logic under consideration. This is no obstacle when we consider only modal logic, for which Kripke semantics were originally designed. But when we try to apply the method to substructural logics, it simply fails us because we cannot, in general, define Kripke semantics for them. Thus model-theoretical proof of decidability was not known in substructural logics, while of course standard cut elimination argument has done the job.

In 2002, Blok and van Alten's paper in *Algebra Universalis* introduced a generic construction to prove the FEP. This blazed a trail for model theoretical decidability proofs of the substructural logics. A class $\mathcal{K}$ of algebras is said to have the FEP

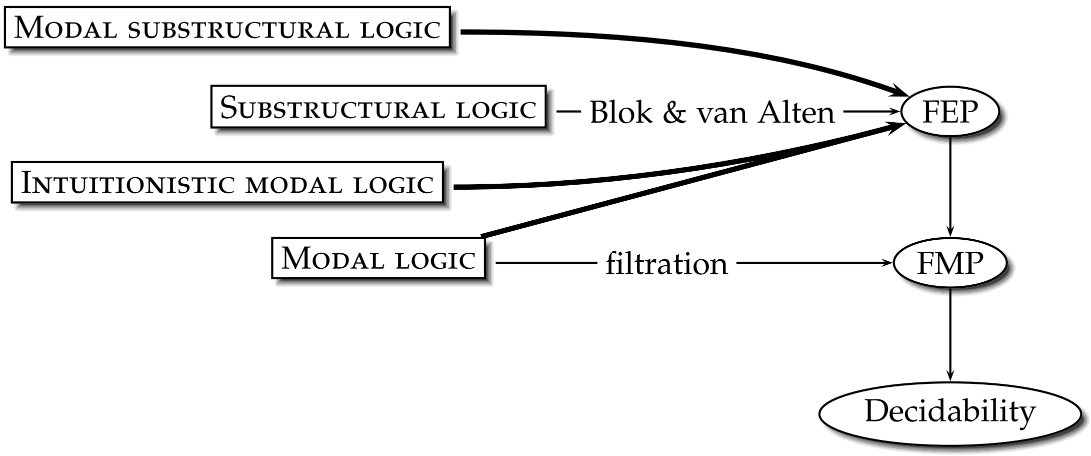

Figure 1. a road map to decidability

if every finite partial subalgebra of a member of $\mathcal{K}$ can be embedded into a finite member of $\mathcal{K}$. This is a stronger version of the finite model property in the sense that the decidability implied by the FEP is more general than that implied by the FMP.

Most of the FEP result so far is on the classes of algebras without modality. Our concern is the FEP of those classes of algebras which have a unary modal operator. Related results and ours(bold arrows) can be illustrated as in figure 1.

In this thesis we first show that the classes corresponding to normal modal logics, called *normal modal algebras*. The fundamental result is:

Theorem *The class of all normal modal algebras has the FEP.*

Further we prove the FEP for KT-, KTB-, S4-, S5-algebras in a similar fashion.

Our proof of this theorem mimics that of Jónssn-Tarski's theorem: every normal modal algebra can be embedded into the complex algebra of its ultrafilter frame. An ultrafilter frame of an algebra is Kripke frame built upon the set of all ultrafilters of the algebra. Instead we use a more "localized" version of ultrafilter, which we call a saturated set. This is in fact an algebraic version of the consistent sets used by Schütte's method of proving the FMP. Thus we have a finite Kripke frame and then embed the original algebra into the complex algebra(powerset algebra with modality appropriately defined). We can apply this method to the class of intuitionistic and intuitionistic modal algebras. The basic result is:

Theorem *The classes of all intuitionistic algebras and all intuitionistic modal algebras have the FEP.*

Then we consider a modal substructural logic which was introduced by Ono(2005). Roughly speaking this is FL_{ew} with an S4-like modality. Then we define modal residuated lattice as the algebraic semantics for it. We show that the modality can be naturally dealt with by Blok-van Alten's proof of the FEP for residuated lattices. Thus we have:

Theorem *The class of all modal residuated lattices has the FEP.*

Future work: First we need a more systematic study of relationship between Schütte's method and the FEP. Second it remains open whether some modification of the method used here will provide the FEP for residuated lattices with weaker modalities.