A research on rep-octahedron

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A replicative net of polyhedron Q is a polygon that serves as a net for Q and can be subdivided into nets of (small copies of) Q. The number of small nets is called order of the replicative net. If they have the same surface area, the replicative net is called regular, and if they are congruent, it is called uniform. When Q is a cube, which is called rep-cube. The notion is well investigated as a rep-cube in the past few years. Additionally, there is preliminary research for the case where Q is a regular tetrahedron. However, studies extending this notion to other polyhedra have not been undertaken.

We investigate the case that Q is a regular octahedron, which is called rep-octahedron, as another application of the notion of a replicative net. We note that the classic platonic solids consist of five regular polyhedra, namely, a regular tetrahedron, a cube, a regular octahedron, a regular dodecahedron, and a regular icosahedron. Especially, it is known that a cube and a regular octahedron are dual and the numbers of distinct ways of edge unfolding are the same, which is equal to eleven. The purpose of this study is to investigate whether the properties already known for the rep-cube are also applicable to a rep-octahedron or not. It clarifies the similarities and differences between a pair of dual regular polyhedra.

First, we explored regular or uniform rep-octahedra of order k to exist. Then we found regular rep-octahedra of order k when k is 3,4,7,9,12,16 or 64 and uniform rep-octahedra of order k when k is 3,4,9,12 or 16 by actually designing them. These rep-octahedra were mainly figured out by hand, although it was done using a free puzzle solver called "Burrtools".

Next, we investigated whether there is a way to generate infinitely many non-isomorphic regular rep-octahedra. We found a way to generate a regular rep-octahedron of order $k = 64i^2$ for any integer i, which yields infinitely many non-isomorphic regular rep-octahedra.

Third, we proved to establish the necessary conditions for the existence of a regular rep-octahedron. As a result, a regular rep-octahedron of order k does not exist when k is not an element of the set $\{x \mid \exists a \in Z, , \exists b \in Z, a > 0, b \geq 0, x = a^2 + b^2 \ + ab\} = \{1, 3, 4, 7, 9, 12, 13, 16, \ldots\}$.

Finally, we analyzed the uniform rep-octahedrons of smaller orders based on the eleven types of edge unfolding using computational experiments. In the computational experiments, the integer programming solver SCIP 8.0.3 was used. From the computational experiments, we showed that no uniform rep-octahedron of order 7 exists. There is a dual relationship between eleven edge unfoldings of a regular octahedron and ones of a cube, however,

no interrelation was observed in the results of this study when focusing on replicative nets.