

Title	ウォールズ・アンド・ウォリアーズ(Walls and Warriors)パズルの計算複雑性に関する研究
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The Walls & Warriors (W&W) puzzle is a two-dimensional grid-based logic puzzle for one player. The player must place wall segments to form a single closed loop that separates “Blue Warriors” (inside) from “Red Warriors” (outside). This thesis investigates the computational complexity of W&W through a rigorous graph-theoretic formulation, transforming the puzzle into a decision problem of finding a simple cycle under spatial separation constraints. Specifically, we model the game board as a graph where wall placements act as edge selections, requiring the loop to topologically isolate specific vertex sets based on their chromatic designation.

Our main contributions are twofold. First, we prove that the general W&W decision problem is **NP-complete**. Membership in NP is established by showing that a candidate solution can be verified in polynomial time. NP-hardness is demonstrated via polynomial-time reductions from two known NP-complete problems: Planar Tree-Residue Vertex-Breaking (TRVB) and Rectilinear Steiner Tree (RST). These reductions employ carefully designed gadgets that encode topological and geometric constraints into the puzzle’s loop-formation mechanics; the TRVB reduction captures the difficulty of navigating logical obstacles by mapping the connectivity states of graph vertices to local wall configurations that can either connect or disconnect adjacent regions. Conversely, the RST reduction addresses the complexity of geometric resource optimization, demonstrating that finding a valid loop enclosing specific points mirrors the minimum-length constraints inherent in Steiner trees.

Second, we identify a tractable case: when the grid height is bounded, the problem becomes **Fixed-Parameter Tractable (FPT)** with respect to the height. We develop a dynamic programming algorithm that processes the grid vertex-by-vertex. By maintaining a complex frontier state and scanning the grid in a top-to-bottom, left-to-right order, the problem can be solved efficiently on narrow grids. The state representation at each step involves a tuple (P, U, C, Q) , where P tracks the connected components of wall segments crossing the boundary (plugs), U indicates vertical connections, C is a bitmask dynamically tracking the inside/outside chromatic status of regions, and Q serves as a global flag to enforce the single-loop constraint. This precise state management allows the algorithm to achieve a time complexity of $O(W \cdot H \cdot 8^H)$. This confirms that W&W is solvable in linear time relative to width W for a fixed height H .

The research demonstrates that despite its deceptively simple ruleset, the W&W puzzle exhibits substantial computational complexity. Our findings

contribute to complexity theory for grid puzzles and offer insights into enforcing topological constraints.