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Computing Complete Sets by Narrowing

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In this thesis, we discuss how to solve equational unification problems automatically. Equational unification is the problem to compute equational unifiers of a given equation $s = t$ with respect to a given equational system $\mathcal{E}$. A complete set is a set of general solutions of an equation. We show examples for the equational problems and complete sets.

Examples Let us see the equation $x + y \approx \mathsf{s}(\mathsf{0})$ and the equational system $\mathcal{E}_0$

$$\mathsf{0} + x \approx x \qquad \mathsf{s}(x) + y \approx \mathsf{s}(x + y).$$

The set $\{\{x \mapsto \mathsf{s}(\mathsf{0}), y \mapsto \mathsf{0}\}, \{x \mapsto \mathsf{0}, y \mapsto \mathsf{s}(\mathsf{0})\}\}$ is the set of equational unifiers of $x + y \approx \mathsf{s}(\mathsf{0})$ with respect to the equational system $\mathcal{E}_0$ and this set is the complete set. This set corresponds to the set of general solutions of the equation $x+y = 1$ in arithmetic. We also consider equational systems related to functional programming. Let $\mathcal{F} = \{[\,]^{(0)}, \mathsf{0}^{(0)}, \mathsf{1}^{(0)}, \mathsf{isort}^{(1)}, :^{(2)}, \mathsf{insert}^{(2)}\}$. The function symbol : is infix and right-associative. Let us consider the equation $\mathsf{isort}(xs) \approx \mathsf{0} : \mathsf{0} : \mathsf{1} : [\,]$ and the equational system $\mathcal{E}_1$

$$\begin{aligned} \mathsf{insert}(x, [\,]) &\approx x : [\,] & \mathsf{isort}([\,]) &\approx [\,] \\ \mathsf{insert}(\mathsf{0}, xs) &\approx \mathsf{0} : xs & \mathsf{isort}(x : xs) &\approx \mathsf{insert}(x, \mathsf{isort}(xs)) \\ \mathsf{insert}(\mathsf{1}, x : xs) &\approx x : \mathsf{1} : xs. \end{aligned}$$

The set

$$\{\{xs \mapsto \mathsf{0} : \mathsf{0} : \mathsf{1} : [\,]\}, \{xs \mapsto \mathsf{1} : \mathsf{0} : \mathsf{0} : [\,]\}, \{xs \mapsto \mathsf{0} : \mathsf{1} : \mathsf{0} : [\,]\}\}$$

is the set of equational unifiers of $\mathsf{isort}(xs) \approx \mathsf{0} : \mathsf{0} : \mathsf{1} : [\,]$ with respect to the equational system $\mathcal{E}_1$ and this set is the complete set.

Contribution Computing complete sets has been studied in the equational unification theory [1, 4]. Narrowing [6] has been studied by Fay [2] and Hullot [5], and elements in a complete set can be computed by narrowing. However, narrowing cannot recognize that it has computed all elements in a complete set. In this thesis, combining narrowing and ununifiability analysis, we can compute a complete set. Ununifiablity problems can be converted to unreachability problems. Tree automata are used for unreachabilyty analysis [3]. In this thesis, we refine the tree automata technique for unreachability analysis.

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