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Rule Removal for Confluence Analysis

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Aim: Confluence

Term rewriting is a computational model which uses directed equations as computation rules. This model has two important properties. One is termination which means that a result of computation is eventually obtained. The other property is confluence that guarantees uniqueness of computation results.

A term rewriting system is a set of a rewrite rules. For instance, consider the term rewriting system consisting of the three rules:

$$0 + x \to x \qquad \mathsf{s}(x) + y \to \mathsf{s}(x + y) \qquad x + y \to x + y$$

The term $(0 + \mathsf{s}(0)) + \mathsf{s}(0)$ is calculated as follows:

$$(0 + \mathsf{s}(0)) + \mathsf{s}(0) \to \mathsf{s}(0) + \mathsf{s}(0) \to \mathsf{s}(0 + \mathsf{s}(0)) \to \mathsf{s}(\mathsf{s}(0))$$

These calculation steps are called a rewrite sequence. Another rewrite sequence also exists:

$$(0 + \mathsf{s}(0)) + \mathsf{s}(0) \to 0 + (\mathsf{s}(0) + \mathsf{s}(0)) \to \mathsf{s}(0) + \mathsf{s}(0) \to \mathsf{s}(0 + \mathsf{s}(0)) \to \mathsf{s}(\mathsf{s}(0))$$

We got the same result $\mathsf{s}(\mathsf{s}(0))$ in the calculation of $(0 + \mathsf{s}(0)) + \mathsf{s}(0)$. This property is *confluence* and how to prove or disprove it automatically is the theme of this thesis.

Approach: Compositional Confluence Criteria and Rule Removal

Recently, Shintani and Hirokawa [1] proposed compositional confluence criteria for confluence analysis. This method ensures that, given a rewriting system and its subsystem, confluence of the subsystem implies the original TRS. Since such a subsystem can be analyzed by any other (compositional) confluence criterion, compositional confluence criteria can be applied to subsystems successively. This method enables us to decompose a rewriting system into its subsystem for showing confluence of the original one. They also developed another method that is a rule removal [1] which ensures equivalence of confluence of a given TRS and its subsystem and we can prove confluence of given TRS by analyzing its subsystem as well as compositional confluence criteria while preserving confluence. By using rule removal, we can search a subsystem strongly and efficiently. This rule removal is based on compositional confluence criteria and actually it consists of a compositional confluence criterion and a sufficient condition which guarantees the reverse direction meaning that confluence of given a TRS implies its subsystem. In this thesis, we present a criterion ensures the reverse direction of rule removal based on persistency. Furthermore, we need to revisit the automation technique for rule removal by Shintani and Hirokawa. Whereas they aim to automate compositional confluence criteria, we aim to automate rule removal criteria. To this end, we will recast their automation method for rule removal.

Illustration

With an example, we demonstrate our rule removal approach. Consider the TRS $\mathcal{R}$:

$$1\colon\ \mathsf{s}(\mathsf{p}(x)) \to \mathsf{p}(\mathsf{s}(x)) \qquad 2\colon\ \mathsf{p}(\mathsf{s}(x)) \to x \qquad 3\colon\ \infty \to \mathsf{s}(\infty)$$

We show the confluence of $\mathcal{R}$ by using the rule removal criteria based on parallel critical pair system and rule labeling.

1. The TRS $\mathcal{R}$ admits two parallel critical peaks and the corresponding critical pairs join:

Let $\mathcal{S} = \{3\}$. The parallel critical pair system $\mathsf{PCPS}(\mathcal{R}, \mathcal{S})$ consists of the four rules:

$$\mathsf{s}(\mathsf{p}(\mathsf{s}(x))) \to \mathsf{s}(x) \qquad \mathsf{p}(\mathsf{s}(\mathsf{p}(x))) \to \mathsf{p}(\mathsf{p}(\mathsf{s}(x)))$$
$$\mathsf{s}(\mathsf{p}(\mathsf{s}(x))) \to \mathsf{p}(\mathsf{s}(\mathsf{s}(x))) \qquad \mathsf{p}(\mathsf{s}(\mathsf{p}(x))) \to \mathsf{p}(x)$$

By taking the linear polynomial interpretation $\mathcal{A}$ on $\mathbb{N}$ with

$$\mathsf{s}_{\mathcal{A}}(n) = 2n \qquad \mathsf{p}_{\mathcal{A}}(n) = n + 1 \qquad \infty_{\mathcal{A}} = 0$$

the inclusions $\mathsf{PCPS}(\mathcal{R}, \mathcal{S}) \subseteq >_{\mathcal{A}}$ and $\mathcal{R} \subseteq \geqslant_{\mathcal{A}}$ hold. Thus, $\mathsf{PCPS}(\mathcal{R}, \mathcal{S})/\mathcal{R}$ is terminating. We define many sorted signature Σ as follows:

$$\mathsf{s} : A \to A \qquad \infty : A$$

Since $\mathcal{R}\restriction_{\Sigma} = \{3\} = \mathcal{S}$, we obtain $\mathcal{R}\restriction_{\Sigma} \subseteq \to_{\mathcal{S}}^{*}$. Therefore, by rule removal based on parallel critical pair system the TRSs $\mathcal{R}$ and $\mathcal{S}$ are equi-confluent.

2. As $\mathcal{S}$ admits no parallel critical peaks, the rule removal criterion based on rule labeling proves the equi-confluence of $\mathcal{S}$ and $\varnothing$.

3. The empty TRS $\varnothing$ is trivially confluent.

Hence the original TRS $\mathcal{R}$ is confluent. In our automation technique, the suitable subsystem $\mathcal{S}$ and $\varnothing$ are found by solving SMT constraints. This correctness follows from our persistency result.

References

[1] K. Shintani and N. Hirokawa. Compositional confluence criteria. *Logical Methods in Computer Science*, 20:6:1–6:28, 2024.