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Description	

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Geographical effects on the path length and the robustness in complex networks

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The short paths between any two nodes and the robustness of connectivity are advanced properties of scale-free (SF) networks; however, they may be affected by geographical constraints in realistic situations. We consider geographical networks with the SF structure based on planar triangulation for online routings, and suggest scaling relations between the average distance or number of hops on the optimal paths and the network size. We also show that the tolerance to random failures and attacks on hubs is weakened in geographical networks, and that even then it is possible for the extremely vulnerable ones to be improved by adding with the local exchange of links.

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I. INTRODUCTION

Complex networks have been studied with great interest inspired from physics to biology, computer science, and other fields, since the surprisingly common topological structure called a *small-world* (SW) and *scale-free* (SF) network has been found in many real systems [1–3]. Such structure is characterized by the SW property [2,3] that the average path as the hop count between any two nodes is as short as that in random graphs, and that the clustering coefficient, defined by the average ratio of the number of links connecting to its nearest neighbors of a node to the number of possible links between all these nearest neighbors, is as large as that in regular graphs. As to the scale-free property [1,2], the degree distribution follows a power law $P(k) \sim k^{-\gamma}$, $2 < \gamma < 3$; the fat-tail distribution consists of many nodes with low degrees and a few hubs with very high degrees. The heterogeneous SF network has good properties; economical and efficient communication by a small number of hops in a connected network with a few links [4] and robustness against failures [5]. Moreover, the restriction of link lengths has been observed, e.g., the internet at both router and autonomous system levels [6], road networks, and flight connections in a major airline [7]. Indeed, the distribution of link lengths was inversely proportional to the lengths [6] or exponentially decaying [8] in geographical space.

In this paper, we consider geographical SF network models for a number of research fields including urban planning, electric circuits, distributed robots, sensor networks, and communication networks. In particular, we discuss the dynamic configuration of planar networks for *ad hoc* communication not limited by wired connections. The planarity is important to avoid interference of the wireless beam, or to construct communication lines on the surface of the earth.

Geographical SF networks have attracted much attention from researchers recently. The interest includes not only the static topological structure [9] and the robustness [10] but also the dynamics on them, such as cascading breakdown [11], random walks [12], synchronization [13,14], magnetic models [15], competitive clusters [16], epidemic spreading [17], and immunization [18]. The geographical SF network models are categorized into three classes: the modulated Barabási-Albert (BA) model that has a penalty on the distance between connecting nodes [19,20], SF networks embedded in lattices [21,22], and space-filling networks [17,23,24]. A brief review of these models has been reported [8]. Unfortunately, except for the third class, crossing of links exists, and the positions of nodes are uniformly random or fixed on a lattice. On the other hand, as a typical model in the third class, a random Apollonian (RA) network [17,23] has some long-range links which cause dissipation of the beam power or the construction cost of links, although it is based on planar triangulation without crossing of links. Thus, to reduce the long-range links, we consider a modification of the RA network preserving the good properties of SF structure on a planar space. Moreover, we investigate the tolerance of the connectivity in geographical and nongeographical rewired networks. We emphasize the effects of geographical structures on the path length and the robustness.

The organization of this paper is as follows. In Sec. II we introduce dynamic configurations of networks based on planar triangulation for *ad hoc* communication, and propose a modified one to reduce long-range links. In Sec. III we numerically investigate both the paths of the shortest distance and the minimum hops between two nodes, and suggest scaling laws of these paths. In Sec. IV, we show the geographical effect on the robustness against random failures and attacks on hubs. Finally, in Sec. V we summarize these results and suggest further studies.

II. GEOGRAPHICAL NETWORK MODELS

A. Planar triangulation

Planar triangulation is a mathematical abstraction of sensor or *ad hoc* networks, in which the positions of nodes are temporarily fixed as service centers of backbone networks. Thus, the mobility of the nodes is out of our scope to sim-

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plify the discussion. In addition, the assumptions of different transmission ranges [25] and directed antennas [26] are also reasonable from the technological viewpoint. On such graphs, online routing algorithms [27] that guarantee delivery of messages using only local information about positions of the source, destination, and the adjacent nodes to a current node in the routing have been developed. To find a path through exploration is required in many cases, since knowledge about the environment in which routing takes place is not available beforehand, especially in *ad hoc* configurations with evolution of networks. In any case, since the optimal routing path depends on both topological and spatial structures of networks, as measures of the communication efficiency, the number of hops for transfer of a message and the path length measured by Euclidean distance are crucial.

On the other hand, Delaunay triangulation (DT), which is the dual of a Voronoi diagram, is the optimal planar triangulation in some geometric criteria [28] with respect to the maximin angle and the minimax circumcircle of triangles on a two-dimensional space, and widely used in practical applications for facility location and computer graphics [29]. It is well known as a good property that the shortest path length between any two nodes on a Delaunay graph is of the same order as the direct Euclidean distance, since the ratio of the path length to the direct distance is bounded by a constant [30]. However, the average number of minimum hops on that graph is unknown. One of the fundamental techniques for equipping such properties is diagonal flipping. In a Delaunay triangulation, diagonal flips are globally applied to the triangles until the minimum angle is not increased by any exchange of diagonal links in a quadrilateral. Such a global process is unsuitable for *ad hoc* networks. In contrast, RA networks can be constructed by local procedures for subdivision of a randomly chosen triangle. Thus, we study the communication efficiency and the robustness in the typical network models based on planar triangulation: DT in computer science, RA networks in complex network science, and a modification to bridge them.

B. Delaunay-like SF network

We briefly explain the deterministic and random Apollonian networks. A random Apollonian network [17] is constructed from an initial triangulation of a polygon. Then, at each time step, a triangle is randomly chosen, and a new node is added inside the triangle and linked to its three nodes. We assume the new node is set at the barycenter of the chosen triangle. When all triangle faces are chosen for subdivision at each hierarchical level, it is called a deterministic Apollonian network [23,24]. We note that recursive graphs [31] divided from each q-clique subgraph by adding a new node and linking it to all the nodes of this subgraph include the case of triangulation from an initial tetrahedron at $q=3$. In addition, higher-dimensional Apollonian networks have been discussed [14,32,33]. The topological properties of power-law degree distributions with exponent nearly 3, large clustering coefficient, disassortative degree-degree correlation, and an average small number of minimum hops between any two nodes have been theoretically and numerically analyzed [17,23,24]. Although RA networks have the above advanced SF properties and the SW effect with a small diameter of the graph, some long-range links naturally appear near the boundary edges. To reduce the long-range links, we propose a modified model from RA networks. The main idea is based on a local strategy for connecting nodes at distances as short as possible by adding the diagonal flips in DT. The configuration procedures of the proposed network are as follows.

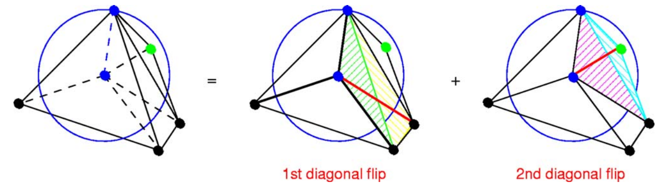

FIG. 1. (Color online) Linking procedures in a Delaunay-like SF network. On the middle and right, the (light green and cyan) long-range links are changed to the intersected shorter (dark red) ones in the shaded triangles by diagonal flips. The dashed lines are new links from the barycenter, and form five new triangles with contours given on the left (the two black solid lines are removed after the second diagonal flip).

(0) Set an initial planar triangulation on a space.

(1) At each time step, select a triangle at random and add a new node at the barycenter. Then, connect the new node to its three nodes temporarily. Moreover, by iteratively applying diagonal flips, connect it to the nearest node (or more than one of the neighbor nodes) within a radius defined by the distance between the new node and the nearest node of the chosen triangle. If there is no nearest node within the radius, this flipping is skipped; therefore the new node is connected to the three nodes.

(2) The above process is repeated until the required size N is reached.

We have two variations with one nearest node and all neighbors in the local circle. Note that these nodes are limited to the ones connected by applying iterative diagonal flips. We call our model RA+NN (one/all) which means a combination of the triangulation in the RA model and rewiring to one or all nearest neighbors as denoted in parentheses.

Figure 1 illustrates the linking procedures by iterative diagonal flips: in the quadrilateral of shaded triangles, the diagonal link is exchanged with the corresponding (dark red) link for maximizing the minimum angle. Clearly, our proposed model belongs in a class of the maximum planar network with fixed order like the RA models [17], because the numbers of nodes, links, and faces are invariant through the diagonal flips. Figure 2 shows the topological characteristic that our RA+NN (one) model has the intermediate structure between those of RA and DT models. The case of all neighbors is the same as that for one. We find that a heterogeneous structure with dense and sparse parts is constructed in the RA+NN (one/all) model. The dense part is more condensed in the generation process even with the random selection of a triangle. The assignment of many nodes to active areas is rational, since it can be covered by a sum of small regions within limited communication power. The dense-get-denser rule may correspond to the subdivision of a service area ac-

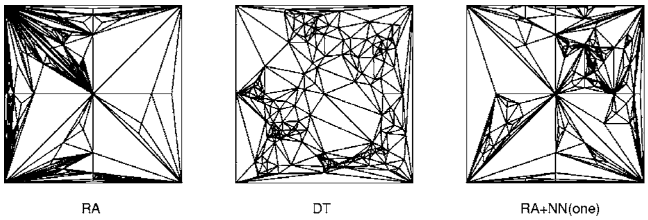

FIG. 2. Examples of random Apollonian (RA) network, Delaunay triangulation (DT), and our model (RA+NN), including the combination of random triangulation and diagonal flips to the nearest node. At each time step, a triangle is randomly chosen for the subdivision in each network; however, the triangulated structure is changed through rewiring with diagonal flips even from the same initial configuration.

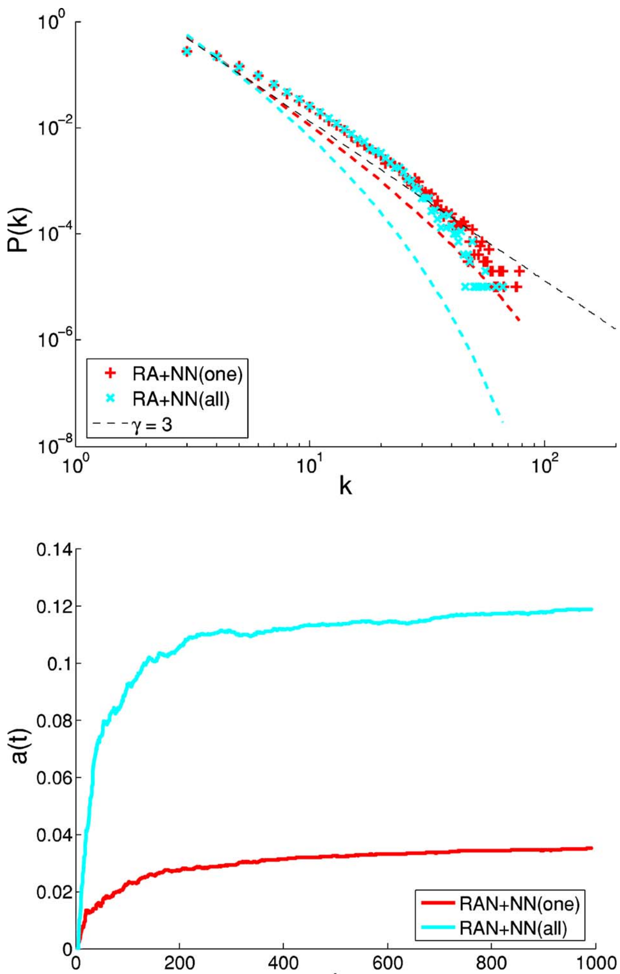

FIG. 3. (Color online) Numerical estimation of the power-law degree distribution with exponential cutoff. (a) Degree distribution $P(k)$. The red and (more curved) cyan dashed lines show the approximations in the form $k^{-\gamma}\exp(-ak)$ with $\gamma=(N_{\triangle}+N)/N \approx 2.994$, $a=0.03$ for RA+NN (one), and $a=0.12$ for RA+NN (all). The black dashed line indicates the slope of 3. The cyan line for RA+NN (all) deviates slightly because of ignoring the complex configuration procedures in the approximate analysis. (b) The cumulative rate $a(t)$ of diagonal flips as a function of time step t.

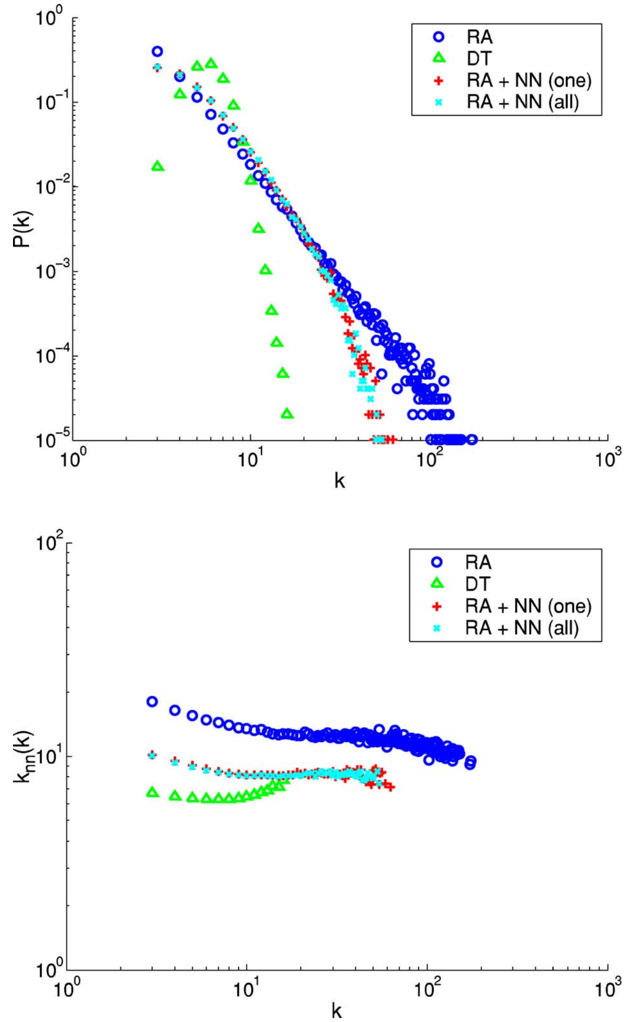

FIG. 4. (Color online) Distributions of (a) degree $P(k)$ and (b) degree-degree correlation $k_{nn}(k)$ defined by the average degree of nodes adjacent to the nodes with degree k.

cording to increasing of the population with preference for aggregation.

III. EFFICIENCY FOR COMMUNICATION

We discuss the shortest path lengths and the minimum hops between any two nodes in the networks. Through simulations, each network model is numerically investigated in the averaging of 100 random realizations at size $N=1000$ generated from the initial configuration of a square graph positioned at $(\pm 1, \pm 1)$ on a plane, adding the center at (0,0) and the four diagonal links. We have similarly obtained the following results for the other initial configurations of triangle and hexagon.

A. Degree and correlation

Degree distribution is one of the important statistical characteristics related to SF properties. Using a rate-equation approach to our model, similar to that in the RA model [17], we derive an approximate form of a power law with exponential

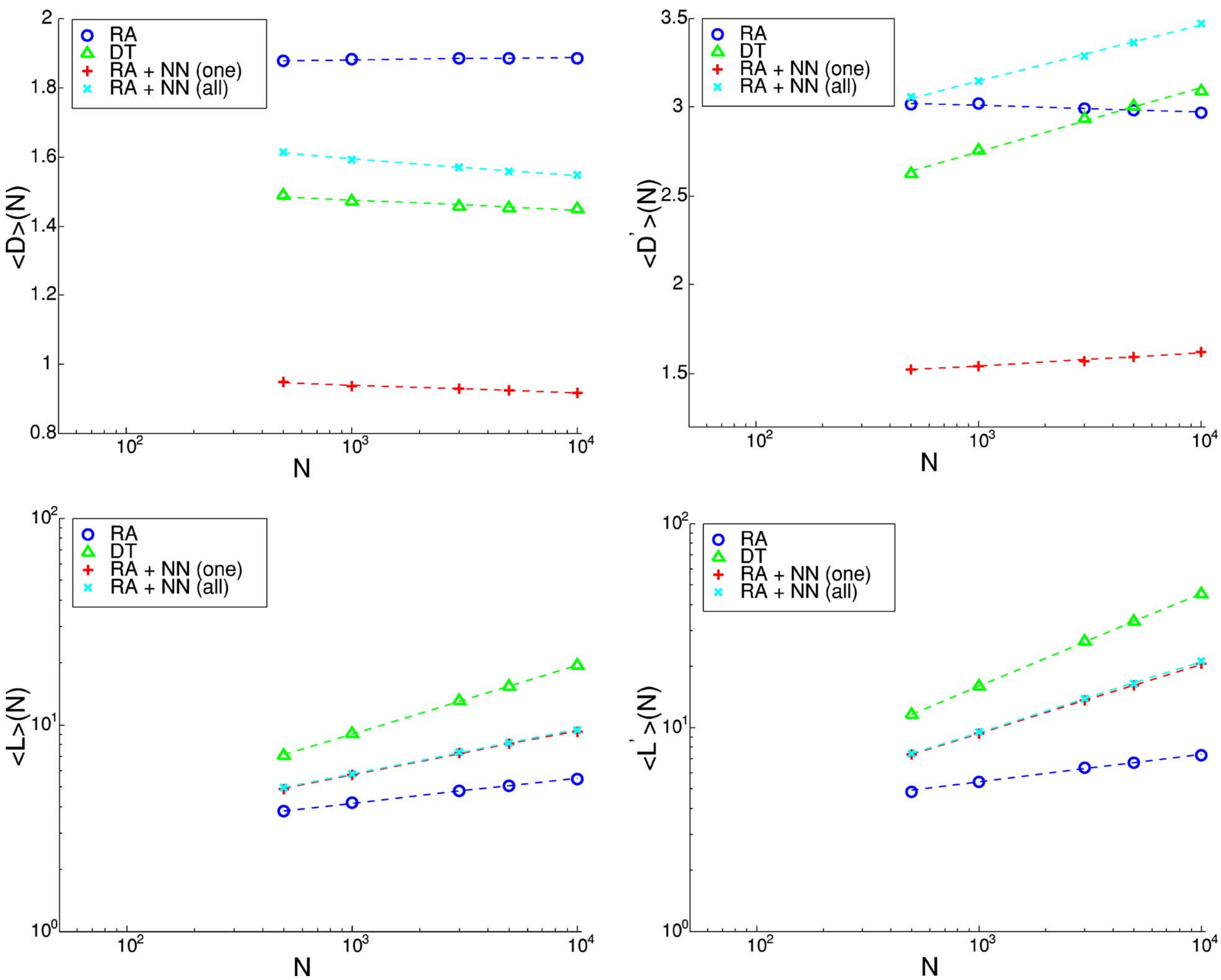

FIG. 5. (Color online) The average (a) distance $\langle D\rangle \sim (\ln N)^{\beta_d}$ and (b) number of hops $\langle L\rangle \sim N^{\alpha_l}$ on the optimal paths between any two nodes in each network over 100 realizations at size $N=500, 1,00, 3,000, 5,000, 10,000$. (c), (d) $\langle D'\rangle \sim (\ln N)^{\beta_{d'}}$ and $\langle L'\rangle \sim N^{\alpha_{l'}}$ on the paths of minimum hops and the shortest path, respectively. The dashed lines correspond to the numerical estimations whose exponents are shown in Table I as a function of size N.

cutoff $P(k) \sim k^{-\gamma} \exp(-ak)$ in the Appendix, and estimate the exponent γ and the parameter a which corresponds to the average rate of multiple diagonal flips. The occurrence of cutoffs also observed in real networks [34] is rather natural from the constraint on addition of new links to a node. Figure 3(a) shows the agreement of our approximation with the observed degree distribution. The cumulative rate a is numerically estimated by the average convergent value as shown in Fig. 3(b). When we have no nearest node in the local circle or can only decrease the minimum angle of the triangles by any exchange of links, the diagonal flip is skipped; therefore the rate is smaller than 1. Note that the time step t is equivalent to the network size N, because a new node is added at each time step.

Figure 4(a) shows that the degree distributions in RA, RA+NN (one/all) , and DT models follow power-law, power-law with exponential cutoff, and lognormal distributions, respectively. In other words, DT models are not SF, while the other models are. Figure 4(b) shows different degree-degree correlations; RA models have a negative slope of $k_{nn}(k)$, a disassortative correlation, RA+NN (one/all) have weaker ones, while DT models have a positive slope of $k_{nn}(k)$, an assortative correlation. Although our models resemble RA models in the shape of the correlation, there exists the structural difference that the strong connections of stubs in RA models are relaxed in our models (see the star-like stubs at the four corners and the center in Fig. 2).

B. Path lengths and hops

We investigate, as a function of the size N, the average distance of path length $\langle D\rangle$ on the shortest paths, the distance $\langle D'\rangle$ on the paths of minimum hops, the average number of hops $\langle L\rangle$ on these paths, and the number of hops $\langle L'\rangle$ on the shortest paths between any two nodes in geographical networks. The distance is defined by the sum of link lengths on the path, and the average means a statistical ensemble over the optimal paths in the above two criteria (with respect to distance and hop) for each size N. Figures 5(a)–5(d) show that RA+NN (one) has the shortest distance and the intermediate number of hops in these models. With the nearly double hops compared to those in the RA+NN (one/all) , DT model

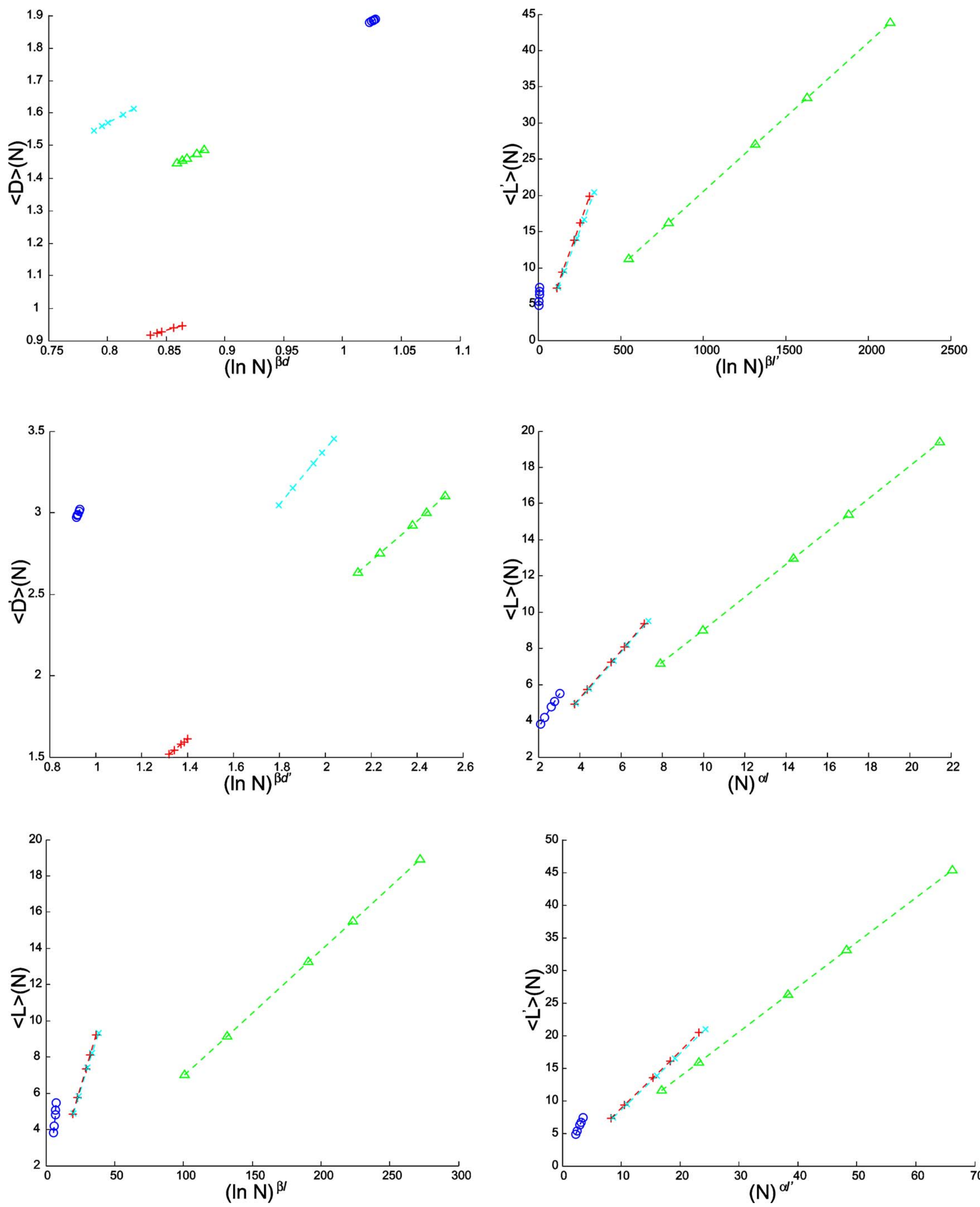

FIG. 6. (Color online) Scaling laws of the average path lengths and hops for different network size. The dashed lines show the estimations and the symbols correspond to RA, DT, and RA+NN (one/all) as in Fig. 5.

are no longer optimal using this criterion of minimum hops. Note that the shortest path and the path of minimum hops may be distinct, these measures are related to the link cost or delay and the load for transfer of a message. It is better to shorten both the distance and the number of hops; however their constraints are generally conflicted (see Fig. 5).

Figures 6(a)–6(d) show the dependencies of average shortest distances and minimum hops on the size, $\langle D\rangle \sim (\ln N)^{\beta_d}$, $\langle D'\rangle \sim (\ln N)^{\beta_{d'}}$, $\langle L\rangle \sim (\ln N)^{\beta_l}$, and $\langle L'\rangle \sim (\ln N)^{\beta_{l'}}$, as rescaled plots from Figs. 5(a)–5(d). We also investigate the fitting of other polynomial forms, $\langle L\rangle \sim N^{\alpha_l}$ and $\langle L'\rangle \sim N^{\alpha_{l'}}$ in Figs. 6(e) and 6(f). Each exponent is numerically estimated as shown in Table I by the mean-square-error (MSE) method. The values of β_d and $\beta_{d'}$ differ from $\gamma - 1 \approx 2$ numerically, suggesting that at the strong-disorder limit [35] only the longest link is dominant in the shortest paths, although the values of β_l and $\beta_{l'}$ are relatively close to 2. Since each network has various link lengths [36], the strength of disorder may affect these differences. In addition, the values of α_l and $\alpha_{l'}$ are close to $1/3$ predicted at the limit [35,37] for the Erdös-Rényi (ER) model as the classical random network and the Watts-Strogatz (WS) model as a SW

TABLE I. Estimated values of the exponents in the forms $\langle D\rangle \sim (\ln N)^{\beta_d}$, $\langle D'\rangle \sim (\ln N)^{\beta_{d'}}$, $\langle L\rangle \sim (\ln N)^{\beta_l}$, $\langle L'\rangle \sim (\ln N)^{\beta_{l'}}$, $\langle L\rangle \sim N^{\alpha_l}$, $\langle L'\rangle \sim N^{\alpha_{l'}}$ by the MSE method for each network.

Network model	β_d	β'_d	α_l	α'_l	β_l	β'_l
RA	0.012	−0.039	0.121	0.136	0.920	1.036
DT	−0.068	0.416	0.332	0.455	2.525	3.452
RA+NN (one)	−0.080	0.151	0.213	0.341	1.622	2.587
RA+NN (all)	−0.106	0.320	0.216	0.346	1.641	2.628

network [3]. The nearest α_l is in the DT, probably because the lognormal degree distribution resembles the unimodal shapes in ER and WS models rather than a power law. In all cases of Figs. 6(a)–6(f), we obtain straight lines suggesting as the existence of the above scaling relations. The short lines are due to the limitation of network size in our computational power, since the size needs to be enlarged on the more than the order of 10^6 for the extension of lines in the logarithmic form with small exponents.

IV. TOLERANCE TO FAILURE OR ATTACK

The fault tolerance and attack vulnerability are known as typical SF properties [5]. We compare the tolerance in the giant component (GC) of geographical and nongeographical rewired networks with the same degree distribution [38] by ignoring the planarity and the link length, when a small fraction f of nodes is removed. We should remark that all networks have the same average degree $\langle k\rangle = 2(3N-7)/N = 5.986$ and the minimum degree $k_{min}=3$ at the initial state before removal of nodes.

Figure 7(a) shows the relative size S/N for the fraction of random failures in RA, DT, and our models, where S denotes the average size of the GC over the 100 realizations. The robustness in DT models is at the same level with the others in spite of being a non-SF network. Figure 7(b) shows the results in randomly rewired networks, whose high tolerance is similar to that in the BA model [5] without geographical structure. Figures 7(c) and 7(d) show the average sizes of isolated clusters except for the GC. At the peak, the GC breaks off and is divided into small clusters. The values of the critical fraction f_c are summarized in Table II. By the geographical effect, it becomes weaker in the order of RA, RA+NN (one/all), and DT models, in which the degree distribution varies from a pure power law to a strong cutoff. These results are not contradictory to the theoretical prediction of a power-law degree distribution with exponential cutoff [39], since the average degree $\langle k\rangle$ is not constant, but gets smaller as the cutoff gets stronger; therefore, the connectivity

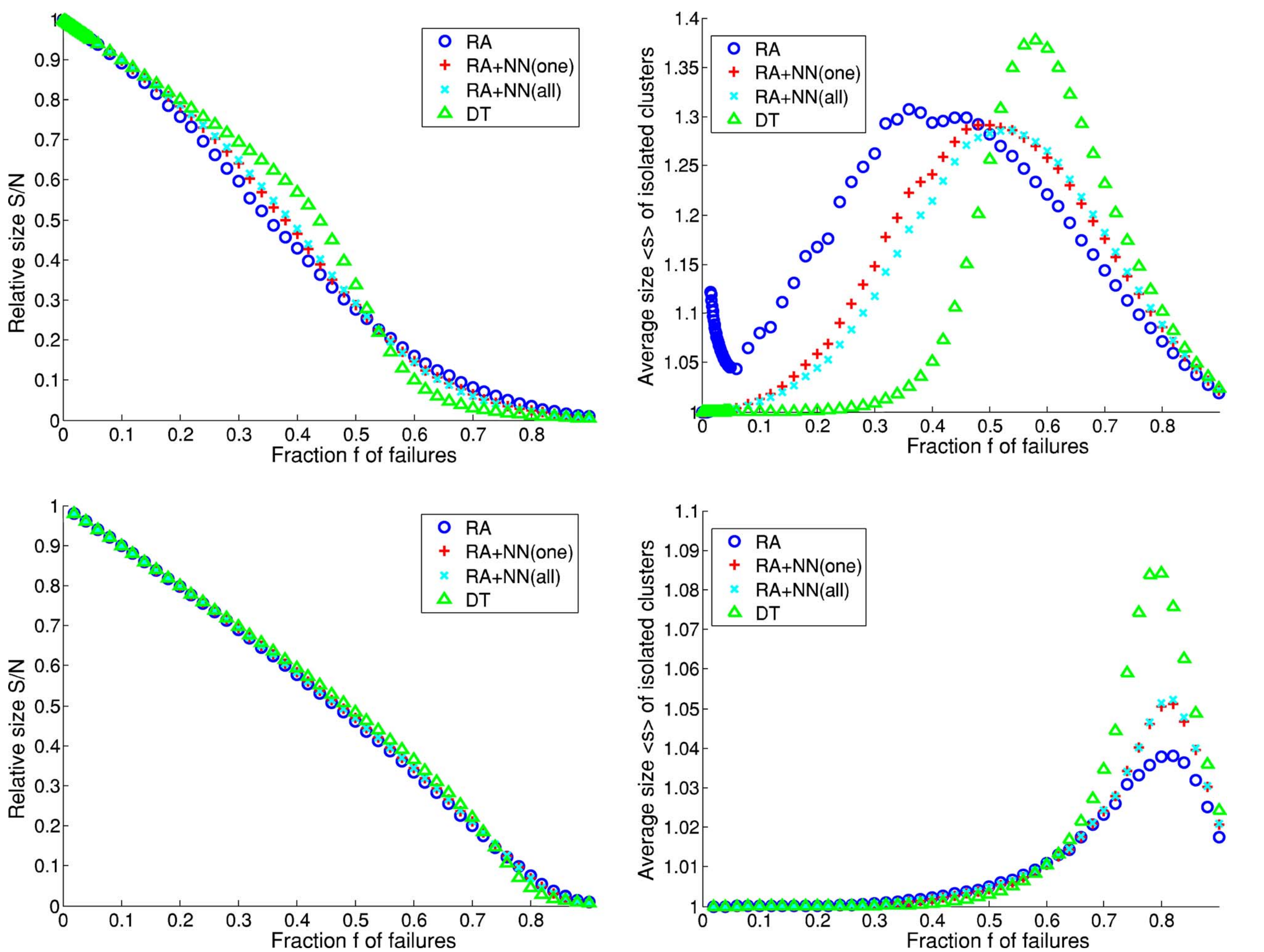

FIG. 7. (Color online) Relative sizes S/N of the GC against random failures in (a) the geographical and (b) the rewired networks. (c), (d) The corresponding average size $\langle s\rangle$ of isolated clusters other than the GC.

TABLE II. The critical fraction f_c of removed nodes in the geographical and the nongeographical rewired networks (denoted by geo. and rew., respectively). At the critical point, the average size $\langle s \rangle$ of isolated clusters shows a peak, and the GC disappears. See Figs. 7(c), 7(d), 8(c), and 8(d).

Removed-nodes network model	Random failure		Hub attack	
	geo.	rew.	geo.	rew.
RA	0.36	0.8	0.005	0.25
DT	0.58	0.78	0.3	0.58
RA+NN (one)	0.5	0.8	0.03	0.48
RA+NN (all)	0.52	0.8	0.04	0.52

is weaker; however, the corresponding strength of cutoff is in the inverse order of our results.

On the other hand, against attacks on hubs selected in decreasing order of degree, Figs. 8(a)–8(d) show the improvements in RA+NN [one (all)] from the extremely vulnerable RA model. Under a fixed $\langle k \rangle$, it is consistent with the previous result [17] that RA networks are less robust than a SW network with a unimodal degree distribution as in DT networks. By the geographical effect, each network also becomes more vulnerable than the rewired version. Thus, even with the same degree distribution, the robustness can be drastically changed by geographical constraints such as the strong connections of stubs that cover wide areas. Note that the weakly inhomogeneous DTs are different from a homogeneous random network, which has the same behavior against failures and attacks at a fraction of removed nodes [5].

V. CONCLUSION

We study the communication efficiency and the robustness of geographical network RA [17,23] and DT [28,29] models based on planar triangulation for sensor or *ad hoc* systems. In such networks, efficient online routing algorithms [27] can be applied, and the delivery of a message is guaranteed by using only local information. Moreover, to reduce the long-range links in RA models, we propose a modified model whose degree distribution is approximately derived as a power law with exponential cutoff.

We numerically investigate the shortest paths and the minimum hops between two nodes in these models, and sug-

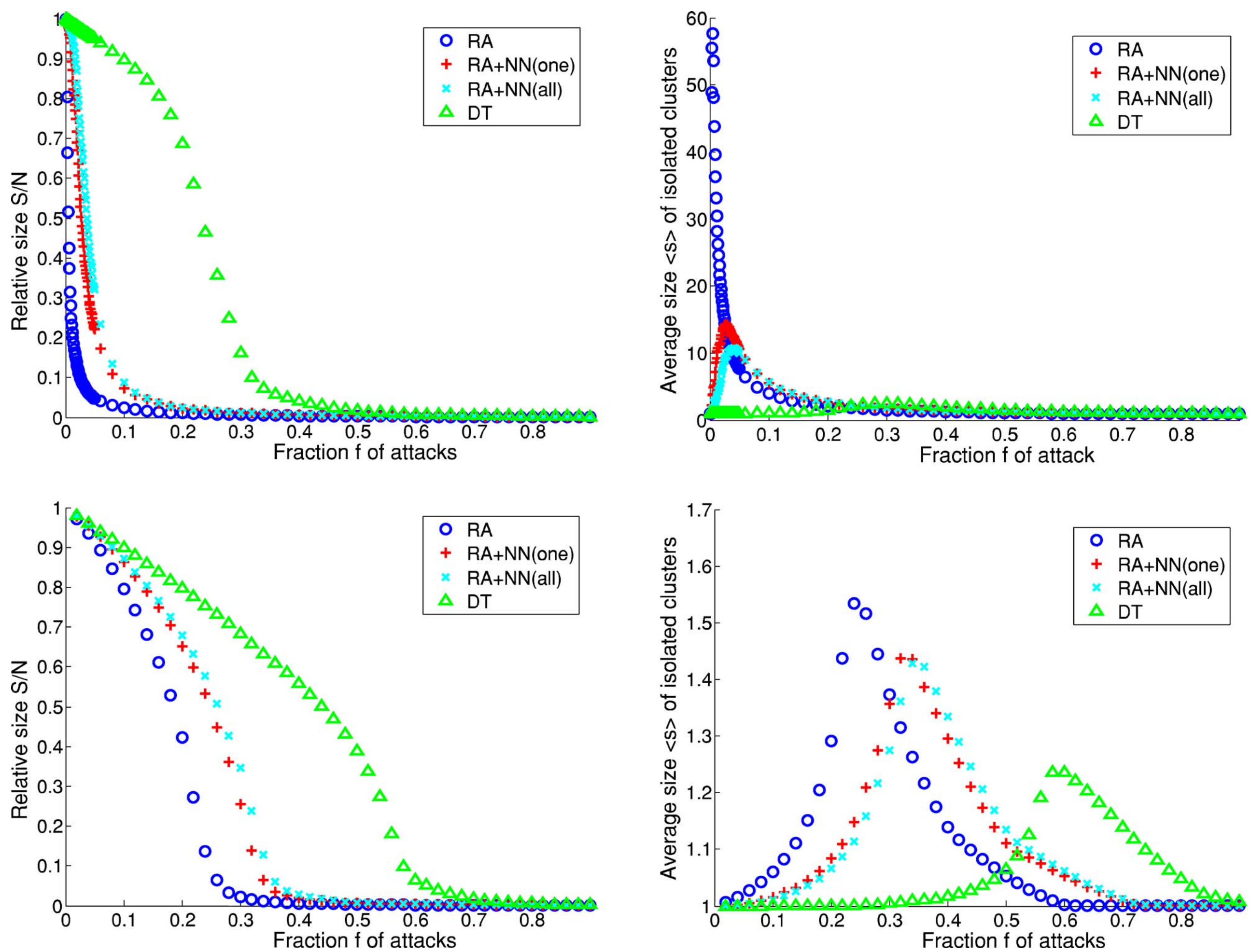

FIG. 8. (Color online) Relative sizes S/N of the GC against attack on hubs in (a) the geographical and (b) the rewired networks. (c), (d) The corresponding average size $\langle s \rangle$ of isolated clusters other than the GC.

gest the universal scaling laws of the average shortest path length $\langle D\rangle \sim (\ln N)^{\beta_d}$ and of the minimum hop $\langle L\rangle \sim N^{\alpha_l}$, similar to the case at the strong-disorder limit [35]. However, it is unclear what topological and spatial characteristics depend on the different values of exponents. A possible finite-size effect is also a further issue in comparing various network generation methods. From the simulation results, we conclude that RA networks have a path connected by a few hops but the path lengths tend to be long, including some long-range links, while DT networks have a zigzag path connected by many hops but each link is short. In contrast to the superior geometric properties [28], DT networks are no longer optimal in this criterion of the minimum hops. Our model is totally balanced: the shortest path length is the best, with an intermediate number of hops; the good result may be related to tradeoff optimization [40]. In addition, we find that the tolerance to failures and attacks is weakened by the geographical effect. In particular, RA models with a pure power-law degree distribution are extremely vulnerable. The geographical effect is consistent with the theoretical prediction for a general class of networks and numerical simulations for lattice-embedded SF networks [10]. The improvement of robustness by random rewirings [41] is probably based on a common mechanism: the majority of small-order cycles (connected with small hops) crucially influence the percolation threshold [10].

On the other hand, although DT models are the most robust among those investigated, it requires global configuration procedures that are unsuitable for *ad hoc* communication. Thus, there is a tradeoff between the localization and the robustness. However, the existence of robust non-SF networks is worth noting. We will further investigate the above effect in wider classes related to a family of SF networks, and also discuss a rapid recovery method by complementary rewiring of the damaged parts.

APPENDIX

We approximately derive an exponential decay in the tail of the degree distribution for the proposed network model. When some links are removed from a node by multiple diagonal flips as shown in Fig. 1, the dynamical equation of the number of nodes $n(k,N)$ with degree k at the size N is given by

$$n(k+1,N+1) = \frac{k}{N_\triangle} n(k,N) + \left(1 - \frac{k+1}{N_\triangle}\right) n(k+1,N) - a\frac{k}{N_\triangle} n(k+1,N),$$

where $N_\triangle$ and a denote the number of triangles and the average rate of multiple diagonal flips, respectively. The first and second terms in the right-hand side correspond to the preferential attachment through random selection of a triangle, and the third term is the statistical rewiring effect by multiple diagonal flips. Note that there is no other reason for decreasing the degrees. We neglect the other effects such as additional links to nodes with low degrees, because we focus on the tail of the degree distribution.

By using $P(k)=n(k,N)/N$, we have

$$\frac{N_\triangle + N}{N} P(k+1) + k(P(k+1) - P(k)) + akP(k+1) = 0.$$

From the continuous approximation $dp/dk \approx P(k+1) - P(k)$ and $\gamma \overset{\text{def}}{=} (N_\triangle + N)/N$, it is rewritten as

$$k\frac{dp}{dk} = -(\gamma + ak)p.$$

Thus, we obtain the solution $p(k) \sim k^{-\gamma} \exp(-ak)$ for large N. Furthermore, the finite-size effect may be estimated more precisely in the mean-field approximation by using generating function approaches [42], and the random walk algorithm [43] to generate a network is useful for discussing the statistical fluctuation in the degree and the scaling law.

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