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Proof-Theoretic Study of substractural Logics having Modal Operation

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1 Introduction

Study of substructural logic started the beginning of 90's. Then a study of proof theory and a semantic study of an algebraic method have been performed. Late years in particular, a study of the moadal logic based on the logic that weakened the classic logic is performed flourishingly. The one was the modal logic in the intuitionistic logic, and, from 1960's to 1990's, it has been studied by R.Bull, Fischer-Servi, H.Ono and F.Wolter. For a proof-theoretic study of the aspect logic, there is the thing which We used natural deductionby tableau method by Fitting [2], Goré , Prawitz [6] and A.K.Simpson for other than a method to use a sequent calculation for.
On the other hand, as for the intuitionistic logic, it was captured the provable as one modal by Goré. And Goré pointed out that embedding was possible in the amodal logic in the intuitionism logic.From this, We can give semantics by Kripki which is similar to the modal logic for an intuitionistic logic. Ono (2005) [5] generally expanded translation of Gödel and introduced the substructural logic corresponding to $S4$. This is the logic (modal substructural logic) that expanded system $S4$ of the modal logic on basic system **FL** naturally of the substructural logic.In this study, we suggested a system of a sequent calculation of Gentzen style for the modal substructural logic introduced by Ono. And, about the system which We suggested, We confirmed that cut removal Craig's interpolation theorem holds and was aimed at syntax proving decision possibility. Furthermore, a result proved about the modal substructural logic semantically by Amano was given and compared it with the result. In addition, in this study, We intended for only a propositional Logic.

2 Intuitionistic Logic LJ

At first it seems that We have to explain a cut elimination theorem and a flow of proof of Craig's interpolation theorem. Therefore We gave proof about basic system **LJ** of intuitionism. Therefore We gave proof about intuitionistic logic **LJ** .

Definition 1 A sequent of **LJ** is expression of form of $A_1, \cdots, A_n \Rightarrow B$.

Therefore, We can think that the expression of **LJ** confined a number of Boolean expression to appear in the right side by a definition of an expression of**LK** to one at most. We have to introduce a mix rule so that a contraction rule is considered with this system. In addition, a definition about rank becomes complicated, too. As a result, next can say.

Theorem 1 Cut elimination theorem, Craig's interpolation and holds on intuitionistic logic **LJ** . and **LJ** is decidable.

3 Substructural Logic FL

Substructural logic is formalized as a system obtained from Gentzen's sequent calculus **LJ** by removing all of structural rule, i.e. weakening rule, contraction rule and exchange rule. Do not include one structural rule in system **FL** either, but add a structural rule to system **FL** voluntarily; can define a system of eight kinds of part structure logic in all by adding it. System FL does not have to introduce a mix rule so that a contraction rule is not considered. In addition, it is easily defined about rank. As a result, next can say.

Theorem 2 Cut elimination theorem, Craig's interpolation theorem and holds on Substructural logic **FL** and **FL** is decidable.

About decidability — Actually, a proof figure that does not include a cut rule is the sequent that it is easy for with a little number fomula, an upper sequent(s) to depend than a lower sequent. Thus, the number of sequents which can apper in a cut-free proof of given sequtnt is finite, and hence the number of possible sequents is also finite. Thus, we have the decidability of basic substructural logics witheout the contraction rule.

4 Modal Substructural Logic $\mathbf{S4_{FL}}$

In this chapter, We confirmed that cut elimination theorem and Craig's interpolation theorem consisted about each system which We suggested as the modal substructural logic. Furthermore, We proved it about decision decidability. We show the system which We suggested next.

- $K_{FL}, K_{FL_e}, K_{FL_w}, K_{FL_{ec}}, K_{FL_{ew}}andK_{FL_{ecw}}$
- $KT_{FL}, KT_{FL_e}, KT_{FL_w}, KT_{FL_{ec}}, KT_{FL_{ew}}andKT_{FL_{ecw}}$

- $S4_{FL}, S4_{FL_e}, S4_{FL_w}, S4_{FL_{ec}}, S4_{FL_{ew}}$ and $S4_{FL_{ecw}}$

A cut elimination theorem these each systems and proof of Craig's interpolation theorem of are about the same with **LJ** and proof of **FL** . Thus, We should add a case of a reasoning rule about functional symbol $\Box$ to proof of **LJ** or **FL** newly. Therefore, in this chapter, We proved it only about $\Box$. Furthermore, when a cut elimination theorem held good, We showed decision decidability and compared it with findings by Amano[1]

5 Conclusion

Each system of the modal substructural logic that We suggested was able to confirm that cut elimination theorem and Craig's interpolation theorem made ends meet.Furthermore, it is each system of the modal substructural logic $S4_{FL}, S4_{FL_e}, S4_{FL_w}, S4_{FL_{ew}}$ and $S4_{FL_{ecw}}$ is decidable. In addition, by comparison with decision possibility shown by Amano [1] semantically, the same result was provided. Other systems $K_{FL}, K_{FL_e}, K_{FL_w}, K_{FL_{ew}}, K_{FL_{ecw}}$ and $KT_{FL}, KT_{FL_e}, KT_{FL_w}, KT_{FL_{ew}}, KT_{FL_{ecw}}$ is decidable.

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